Hi, my name is Yanming Chen. I am going to talk about Fourier transform in cs. Some of my sources are based on the paper I wrote in the physics department of UCSB before I transferred to UCLA.

In many disciplines of science and engineering, Fourier transform has long been an important analytical technique in the analysis of many systems. The essence of the Fourier transforms of a periodic function defined from −∞ to ∞ is to decompose the waveform into a sum of sinusoids in different frequencies. This property allows us to analyze the frequency components of signals. So, with this tool, we can solve a wide range of problems. The process of decomposing the signal can be applied to audio signal processing, image processing and many other areas. It is widely used in audio signal processing such as decoding and filtering sound clips. In image processing, it allows us to filter an image or to reduce the noise of an image.

In short, FT transforms a function from one domain to another. Domain here means the variable of that function. A variable can be in time domain, meaning that it is a function of time. It can also be in frequency domain, a function of frequency.

If we look at the math, Fourier transform comes from Fourier series. And Fourier series is a way to rewrite some periodic function as the sum of some sine functions. Fourier transform is continuous and integrating from neg inf to inf. Mathematically, it is defined like this. As we can see, Eq. 8 integrate with respect to t and 9 integrate with respect to omega. The transform or inverse transform can convert the variables between time variable t and frequency variable ω.

The discrete Fourier transform approximates the continuous Fourier transform. The fast Fourier Transform (FFT) is simply an algorithm. It is a particular method of performing a series of computations) that can compute the DFT much more rapidly than other available algorithms. The FFT improves the running time from O(n2) to O(n log n)

FFT is useful in audio. Imagine we are listening to a concert. It would not be difficult for us to distinguish the different pitch and timbre of some primary instruments. We can tell the difference between a cello and a violin be- cause their pitch ranges are different. The pitch is the frequency. This means that our brain is applying Fourier transform to the sound waves we received through our ears and decomposing the waves to frequency components. So, we can either follow the melody of the violin or the cello like changing the focus of a camera lens, even though many other instruments are playing simultaneously.

We also want the machines to have a similar ability. The sound input is digital signals. They are recorded through regular discrete samples. These samples are taken at a specified rate. The Figure is showing how an actual input is converted to the frequency domain using computer programs. I used the FFT function in SciPy package of python to complete this. The opening theme of star wars is a blend of different instruments playing in different pitches. FFT can distinguish them to be the figure on the right. The x-axis is the frequency or the pitch. So, this is how Fourier transform is used in audio.

In addition to the application on sound processing, Fourier transform is also powerful in image processing. Digital images are represented by a matrix of pixels where each pixel is a sample of an original image, storing the color information of the image on that position. The value of each pixel is a single sample representing some amount of information of that position. To obtain some specific features of the image (i.e., the shape of a figure in that image), we can use convolution to get all the information of the pixels. Convolution is another math definition. To better understand the physical meaning of convolution, let’s consider the following daily-life example: On Instagram, we can add filters to the pictures we post. Some filters are not simply adding a layer of colors to the original image. Instead, it has an effect on each pixel of the image. After filtering, we can get a very detailed picture. Using Fourier transform to do convolution allows us to characterize convolution operations in terms of changes to different frequencies. For example, it can remove noise by reducing high-frequency content.

Again, I used SciPy package of python to demonstrate the idea. The original picture is the physics building of UC Santa Barbara. After converting it to grey scale and applying a FFT to it, we get this spectrum. Then, applying Gaussian blur to it we get a vague image because Gaussian blur filter out the high frequency noise of the image.

And finally, the sparse Fourier transform is the latest algorithm of FT in recent years. It was first developed for or computing DFT on signals with a sparse frequency domain. The algorithm improves the asymptotic runtime compared to the prior methods based on pruning. With the new algorithm, streams of data can be processed 10 to 100 times faster than was possible with the FFT. This is a great advantage when dealing with big data. The speedup can occur because the information we care about most has a great deal of structure: music is not random noise; the frequencies of music is clear and sparse. Typical applications are [GPS](https://en.wikipedia.org/wiki/GPS) synchronization and spectrum sensing.

Basically, this is the end of my presentation. Thank you for watching.