Kerfing with Generalized 2D Meander-Patterns

Conversion of Planar Rigid Panels into Locally-Flexible Panels with Stiffness Control

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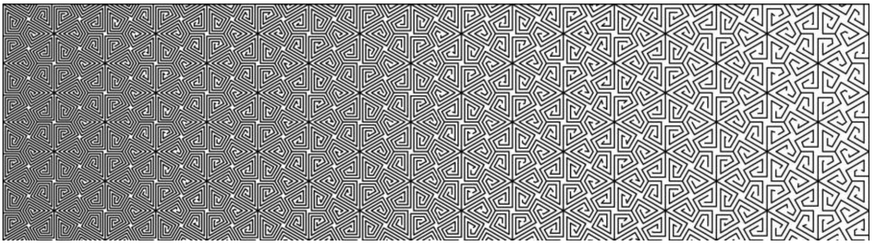
Abstract. In this paper, we present a kerfing (relief-cutting) method to turn rigid planar surfaces into flexible ones. Our kerfing method is based on a generalization of the 2D meander-pattern recently invented by Dujam Ivanišević. We have developed algorithms to obtain a large subset of all possible 2D meander-patterns with a simple remeshing process. Our algorithm can be applied to any polygonal mesh to produce 2D meander-patterns. The algorithm, when applied to regular (4,4) tiling pattern, in which every face is 4-sided and every vertex is 4-valence, provides the original 2D meander-pattern of Ivanišević. Moreover, since these meander-patterns are obtained by a remeshing algorithm, by changing parameters, we can control local properties of the pattern with intensity of images to obtain desired stiffness in any given region (See Fig.1). This approach provides a simple interface to construct desired patterns.

Keywords: Kerfing, Flexible Panels, Relief Cuts

1 Introduction

Relief cutting is a technique for turning rigid planar panels into flexible ones. In architectural practice, the process of relief cutting is called kerfing. There exist many relief cutting patterns that are recently invented by designers. Among them 2D meander patterns invented by Dujam Ivanišević [18] seems to be one of the most promising patterns to obtain very flexible panels that can provide double-curvature.

In this paper, we present an approach for kerfing rigid materials using 2D meander-patterns to convert planar rigid panels into locally-flexible ones with stiffness control. Fig. 1 and Fig. 2 show two examples of rigid panels with varying local flexibility. In our approach, the local properties of generalized meander patterns are manipulated by any 2D function that is given as an image. We chose to use images to simplify the interface for describing functions. One can simply draw an image to manipulate local properties of 2D meander-pattern to control local stiffness. Any B&W image can be used as shown in Fig. 3.



1. *A pattern whose density decreases based on the distance from line.*

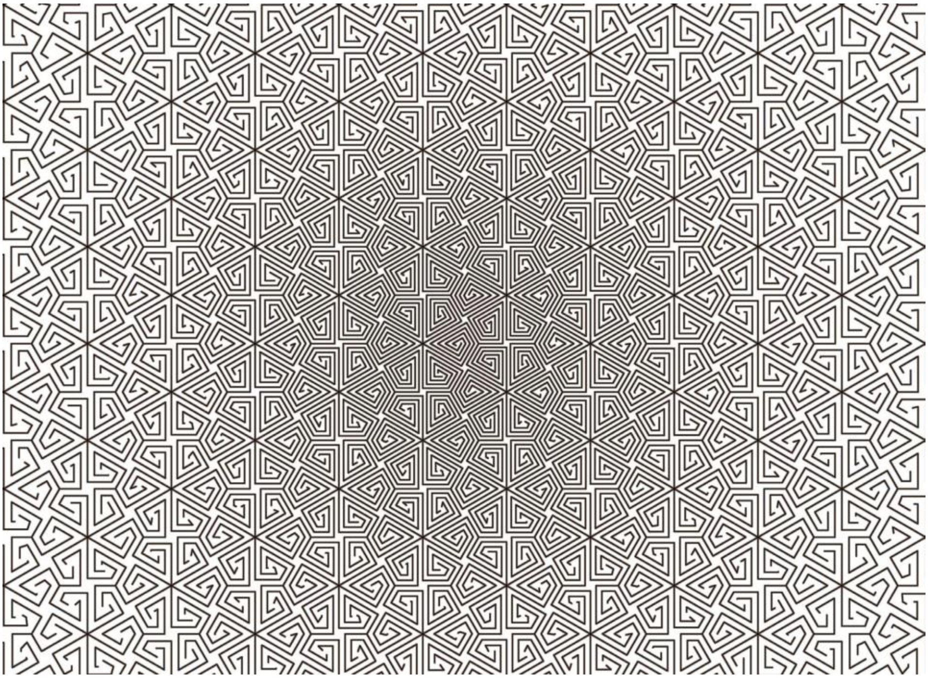
2. *When the pattern in Fig.1a is used for kerfing a planar wood panel, it turns a rigid wood panel into a gradually flexible rectangular panel which is rigid in one side and flexible in another.*

Fig. 1. An example of locally-flexible panels with varying stiffness obtained by kerfing using generalized 2D meander patterns

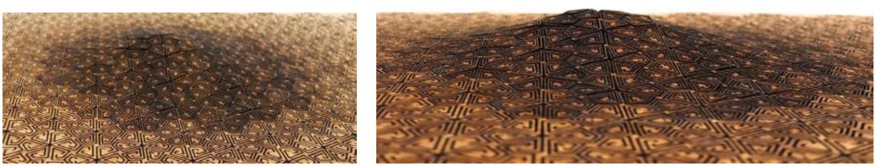
1.1 Motivation

Designing curved geometries is becoming easier as a result of the advancements in computer aided geometric design. Although we can design wide variety of shapes virtually, it is still hard to physically construct those shapes. One of the main challenges comes from the limitation of construction materials that are often produced in standard shape, size and rigidity in an industrialized mass production process. In the construction process, these mass-produced materials should be converted into flexible shapes with desired properties that meet designers' objectives. There is, therefore, a need for the development of methods to convert rigid materials into materials with desired flexibility properties.

The deformation of planar rigid panels results in strong forces in some regions. The panels cannot tolerate these forces in those regions and start to break starting from those regions. If we remove materials in those highly stressed regions of the original planar surface, it is expected to change the behavior of the material. The terms kerfing or relief-cutting is introduced to explain this phenomena of obtaining flexibility by cutting or, in other words, by removing materials. Many designers, in fact, have experimented with a wide variety of cut patterns and demonstrated that some specific cut patterns, such as spirals, generate significant amount of flexibility.

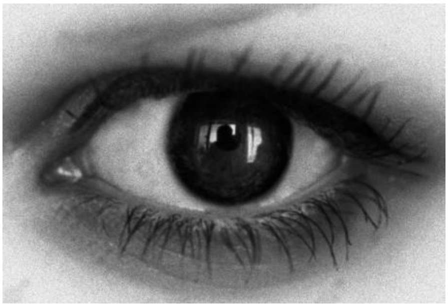


(a) A pattern whose density decreases based on the distance from a center point.

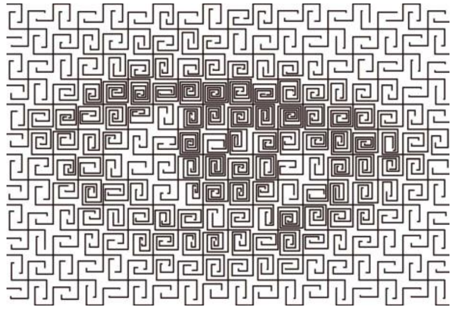


(b) When the pattern in Fig. 2a is used for kerfing a planar wood panel (also called relief cut), it makes the center of the wood panel flexible by allowing to turn it into a conical dome structure.

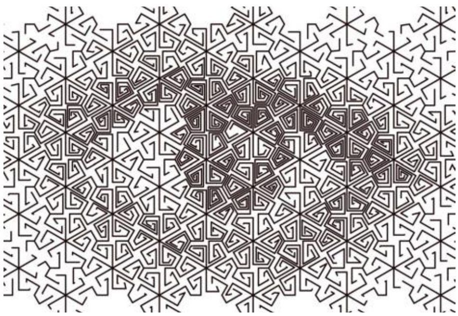
Fig. 2. Another example of locally-flexible panels with varying stiffness obtained by kerfing using generalized 2D meander patterns. In this case, we obtain double-curvature around a center point by changing the density of the pattern based on the distance from a center point.



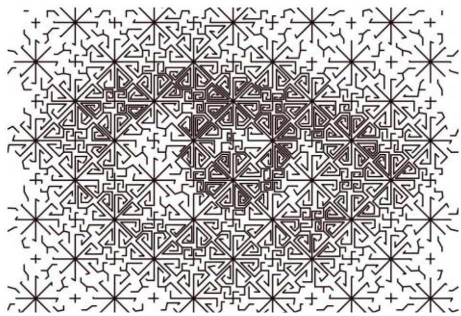
(a) Control Image.



(b) Resulting Pattern by applying our algorithm to a rectangular grid pattern.



(c) Resulting Pattern by applying our algorithm to a hexagonal grid pattern.



d) Resulting Pattern by applying our algorithm to a 4.8.4 semi-regular tiling pattern.

Fig. 3. An example that demonstrate the efficiency of our method to manipulate local flexibility using a control image. In this case, although the frequency of tiling patterns cannot be above Nyquist frequency [26] and it is hard avoid aliasing in resulting patterns [32], we are able to lessen the aliasing effects using a version of jittering technique introduced by Cook et al. [8]. Despite we use very low resolution patterns eye is still visible and no additional aliasing pattern is visible.

(a) An example of 1D symmetric periodic meander patterns from Etruscan amphora from 470 B.C. [30].

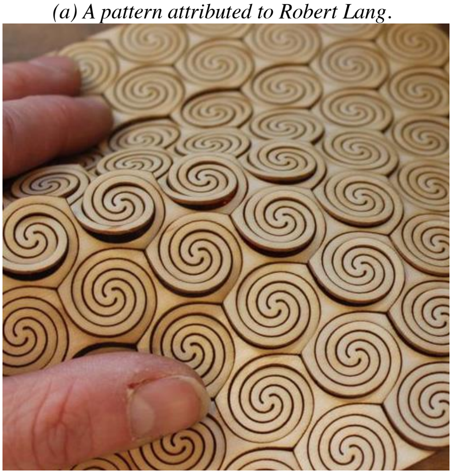
(b) 2D symmetric periodic meander Pattern designed by Dujam Ivanišević.

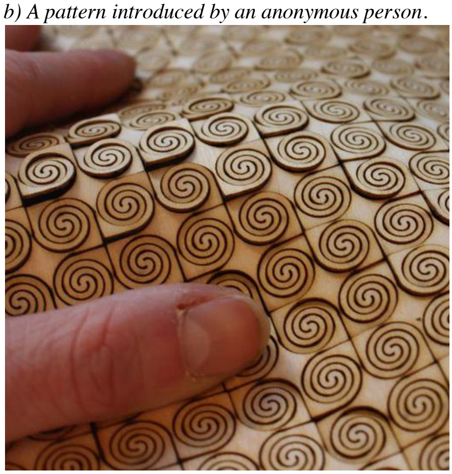
(c)We created these models using our generalized algorithm to demonstrate that our algorithm can produce the original 2D-meander-pattern and its flexibility.

Fig. 4. 1D and 2D meander patters. Super flexible plywood obtained by relief cutting by 2D meander pattern invented by Dujam Ivanišević in 2014 [18].

One of the problems with existing patterns the architects experiment with is that the cut patterns are regular structures. Since the flexibility of the surface is largely dictated by the cut lines, using predefined cut pattern do not necessarily produce a geometry that can morph to a desired shape and provide an anisotropic flexibility. To obtain such anisotropic behavior there is a need for the development of methods to obtain non-regular cut patters that can provide desired flexibility. In this work, therefore, we developed a method to obtain non-regular versions of one of the most popular cut patters: 2D meander-patterns (See Fig. 4).

(a) A pattern attributed to Robert Lang. *b) A pattern introduced by an anonymous person.*





c) Extended of the original pattern to circular patters and hexagonal grid by M. S. Raynolds [29].

Fig. 5. Other Symmetric Meander Patterns motivated by Dujam Ivaniˇsevi´c’s work.

Meander-patterns are repeated labyrinth-like motifs that are commonly used in Hellenistic art to decorate borders or artworks [20]. Edwin Reissman recently classified a few of meander patterns in Hellenistic art [30] [1]. The Fig. 4 shows one of the most common meander-patterns that consists of S-shaped patterns that are interlocked together. Examples known from Hellenistic period are almost always one-dimensional repeated patterns as shown in this figure. In 2014, Dujam Ivanišević discovered a new 2D wallpaper meander pattern to be used as relief cuts to turn rigid materials into flexible panels [18].

[1] The name “meander” comes from the twisting and turning paths of the two “Menderes” Rivers in Asia Minor.

This particular pattern was so successful for kerfing that designers and architects use it to obtain flexible panels from wood and shared their experiences in blogs and youtube such as [34] after Ivanišević shared his discovery in 2015 in instructables website [19] (See Fig. 4c). In 2016, a few new 2D meander-patterns are presented in architectural blogs [19, 29] (See Fig. 5). However, the resulting kerfing from these patterns do not become as flexible as the original pattern as visually demonstrated by Ivanišević [19].

One problem with these 2D meandering patterns is that they are wallpaper patterns that can only be generated using basic symmetry operations such as rotation and translation [31, 3]. Using symmetry operations, we can only create exact copies of original pattern and it is hard to generalize these motifs in general tilings to study for finding more variety of patterns. Another problem with the wallpaper approach, it is hard to change local properties of the panels. In other words, we cannot make one regions more rigid and another region more flexible. All local regions always have exactly the same flexibility. There is, therefore, a need for a practical approach to provide most possible versions of these motifs with a local control.

1.2 Contributions

In this paper, we present an algorithmic approach to obtain a large subset of all possible 2D meander-patterns with a simple remeshing method. Our algorithm is conceptually based on subdivision methods in computer graphics that can allow to obtain any semi-regular mesh [6]. Similar to subdivision algorithms, our algorithm can be applied to any polygonal mesh such as regular or semi-regular polyhedra and allow to create consistent meander patterns on any 2-manifold surface. In planar surfaces starting from regular (4,4) grid pattern; it provides the original 2D meander-pattern invented by Dujam Ivanišević. Moreover, since we obtain the pattern with a remeshing algorithm, by changing parameters we can manipulate local motifs to control local stiffness of the surface. This method is also useful to create artworks drawn using the meandering motifs. As a conclusion our contributions can be listed as follow:

- We have developed a remeshing algorithm to generalize the 2D meandering pattern into general setting.
- By applying this algorithm to polygonal meshes, we can cover any 2-manifold surface with this meandering pattern.
- We have developed a method to manipulate local properties of the original meandering patter. Using this method, we can obtain different stiffness in every point. In addition, we can obtain artworks made from this meander-pattern.

2 Related Work

Auxatic materials that demonstrate elastic property with a negative Poisson’s ratio is one of the recent interest in materials science research [35, 17]. It has recently been observed that Auxatic behavior can be obtained by relief cuts [17]. In fact, most of the existing architectural relief-cutting methods to obtain flexible panels comes from obtaining auxetic behavivor by using rotating polygons obtained by the cuts [16, 22]. Dukta is another type of architectural relief-cutting process that is introduced in 2007 by Serge and Pablo Lunin for turning rigid materials into flexible [23]. Formal analysis of folding and twisting by ducta process is recently provided [27]. In this work, we are working on a third type of relief-cuting process that are produced by meander-patterns [18].

Meander-patterns can be classified as spiral forms, which are one of the most common shapes found in nature, mathematics, and art [1, 11, 9]. Spiral shapes can be seen in many natural objects including snail shells, seashells and rams' horns, cochlea in our inner ear, galaxies [33]. Spiral forms also exist in almost all cultures as artistic and mystical symbols. Celtic crosses have whorls. Spirals can be found in Greek votive art and Tribal tattoos [21]. This widespread usage of the spiral form suggests that humans innately find them aesthetically pleasing and interesting.

Spirals are also popular in mathematics. They are among the most studied curves since ancient Greek times. Although spirals are usually represented by parametric equations, there are a wide variety of methods that can be used to construct and represent spirals. There are also many named spirals such as the Archimedean spiral, the Fermat spiral, the Logarithmic spiral, Fibonacci spiral, Euler and Cornu spirals [11]. Spirals even appear when we zoom in on the Mandelbrot and Julia sets [24]. Spiral forms are also used by Charles Perry [28, 14, 15] in his mathematically inspired sculptures.

A particularly interesting use of spirals that is related to meander-patterns is to cover surfaces with spiral patterns with interlocking N-branched spiral trees. Such spiral tree structures are used to construct Daniel Erdely's Spidrons [12, 13] and Ergun Akleman's Twirling Sculptures [1]. To develop an algorithm for creating generalized meander-patterns that consists N-branched spiral trees, there exist two possible approaches:

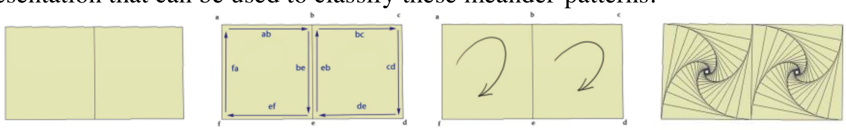
1. **Analog approach:** In this case, we can design differential equations from gradient fields that are produced from an implicit function. These differential equations can be used to create spiral vector fields [5], which can later to be used to construct N-branched spiral trees.
2. **Discrete approach:** In this case, we produce desired patterns by remeshing 2D polygonal meshes such as the ones that are used in creation of subdivision surfaces [6]. The remeshing algorithms of Spidrons [12, 13] or Twirling Sculptures [1] already produce spiral patterns that are similar to 2D meanderpatterns. We observe that those algorithms can be reformulate to obtain meander-patterns.

A particular set of implicit functions that can be used in the implementation of the analog approach is distance functions that are defined as distance to a set of points or lines. Such distance functions describe Voronoi structures as equi-distance from original points and lines. Resulting vector fields provide spirals emanating from original points and lines [5]. One advantage of this approach is that we can directly control the number of branches of the spiral trees. However, we observe that there exist two problems with this approach: (1) It is hard to control the shapes of the spirals and (2) It is hard to control distance between spiral arms. We, therefore, choose to use a discrete approach to produce N-branched spiral trees and initial algorithm for creating twirling sculptures provides a base for our algorithm (See Fig. 6). In the next section we present twirling algorithm and its shortcoming to obtain meander patterns.

3 Twirling Algorithm

The twirling sculpture algorithm starts with an initial 2-manifold polygonal mesh that is represented with the combinatorial structure of a graph and an associated ”rotation system” [10, 2]. Fig. 6a shows a portion of a 2-manifold mesh that consists of two quadrilaterals. In a 2-manifold mesh every edge can be represented by two half-edges pointing in opposite directions [25]. The boundary of every face in a 2-manifold mesh can always be described with a cyclically ordered sequence of half-edges, which can be considered as n vectors going around the face in consistent order (all clockwise or all counter-clockwise) (See Fig. 6b). This consistent order is the key to creating spiral branches using rotation order shown in Fig. 6c.

Our initial intuition was that a straightforward use of this twirling sculpture algorithm - with some minor adjustments - would be sufficient to obtain a generalized version 2D meander-patterns. However, this intuition turned out to be wrong. We obtained flexible sheets, however, our sheets did not seem as flexible as original meander-pattern. To identify differences between patterns, we first need a formal representation that can be used to classify these meander-patterns.



(a) *Initial Mesh* (b) *Half-Edges*. (c) *Rotation directions*. (d) *Interlocking Spirals*.

Fig. 6. The main algorithm for twirling sculptures creates interlocking spirals [1] (Images courtesy to Ergun Akleman).

4 Formalization of Meander Patterns

In 2D Meander-patterns invented by Dujam Ivanišević, each cut always consists of four spiral branches and each spiral branch of any 4-branched spiral tree is interlocked with another spiral branch of another 4-branched spiral tree. The pattern in Fig. 5b is different in the sense that each spiral tree consists of 6 branches and each spiral branch of any 6-branched spiral block is also interlocked with another spiral branch of another 6-branched spiral tree. The first meander-pattern shown in Fig. 5c is different from the other two in the sense that (1) each spiral tree is 3-branched and (2) each spiral branch of any 4-branched spiral tree is interlocked with two spiral branches of two other 3-branched spiral trees.

In conclusion, these meander-patterns can be uniquely classified by two numbers as [*N,K*]: where N denotes the number of spiral branches in a given spiral tree and K denotes the number of interlocking spirals. In this case, we use square bracket to differentiate from the regular mesh patterns. As a result, [4,2] refers the original meander pattern discovered by Dujam Ivanišević; [6,2] refers the meander pattern in Fig. 5b; and [3,3] refers the meander-pattern in Fig. 5c. On the other hand, the algorithm of twirling sculptures turns a n-valence vertex into an N-branched spiral tree and m-sided face gives m-number of interlocked spiral branches. Therefore, (1) a regular rectangular grid turns into a [4,4] meander-pattern; (2) a regular triangular grid (3,6), in which every face is regular and 3-sided, i.e. regula triangle and every vertex is 6-valence, turns into a [6,3] meander pattern; (3) a regular hexagonal grid, in which every face is regular and 6-sided, i.e, regular hexagon, and every vertex is 3-valence, turns into a [3,6] pattern. In other words, the straightforward application of twirling sculpture algorithm can obtain neither of [4,2] and [6,2]; nor [3,3].

5 Our 2D Meander-Pattern Algorithm

A careful examination of the differences between the patterns shows that [4,2], [6,2] and [3,3] are obtained by methodical “pruning” some of the branches that remove some of the trees. For instance, in [4,2] is obtained by removing every other 4-branched tree in a (4,4) grid sequence, [3,3] is obtained by removing every other 3-branched tree in a (6,3) sequence. [6,2] comes from a 6.3.6.3 semi-regular mesh [6] and obtained by removing 3-armed trees. Our initial FEA analysis also confirmed that the tree removal helps to make patterns more flexible.

Since spiral trees stems from vertices if the vertices of the initial mesh is 2-colorable, we can simply create every other tree. This corresponds to removing every other tree. In other words, if the mesh is vertex 2-colorable, we can always remove every other tree.

Although, a general mesh is not necessarily vertex 2-colorable, there exists remeshing algorithms that can turn any polygonal mesh into meshes whose vertices are 2-colorable. In this paper, we chose vertex insertion [4], which is the remeshing algorithm of popular Catmull-Clark subdivision scheme [7], to obtain vertex 2-colorable meshes.

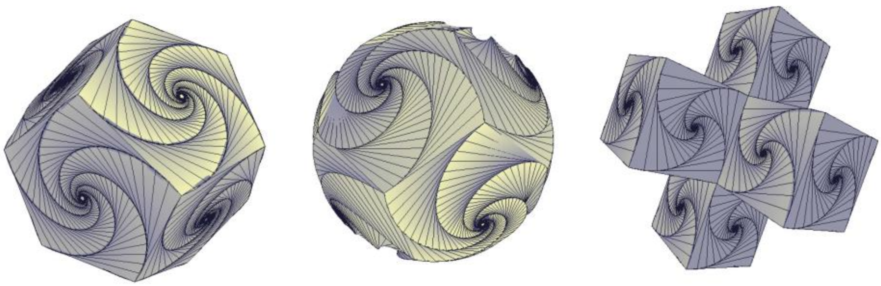


Fig.7. Interlocked Spirals on surfaces that is used to construct twirling sculptures (Images courtesy to Ergun Akleman).

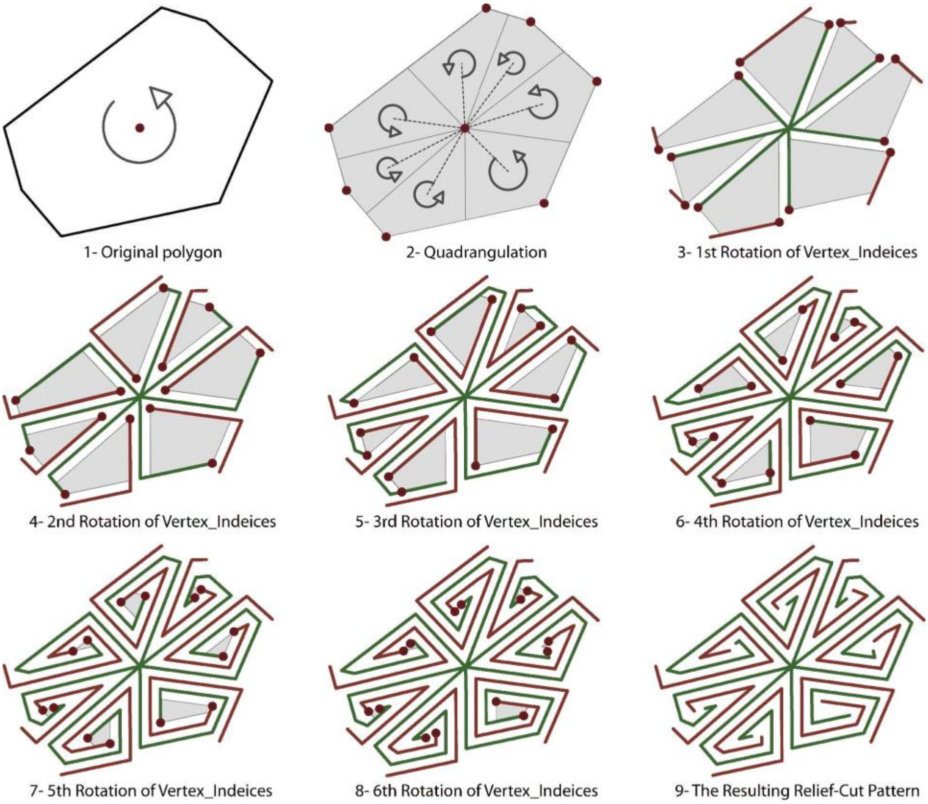


Fig. 8. The steps of basic algorithm.

5.1 Vertex Insertion Remeshing for 2D Meander-Patterns

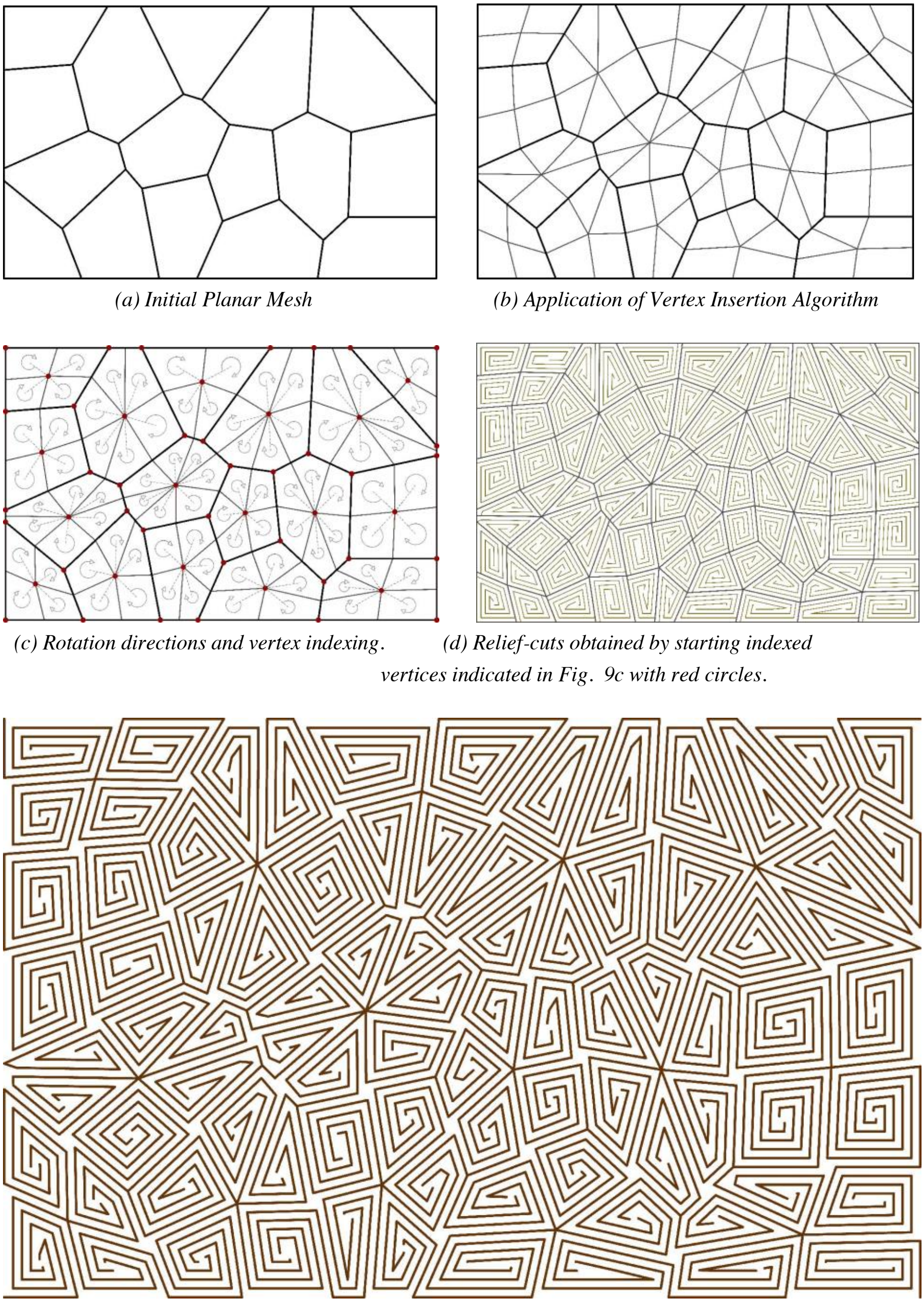
Vertex insertion is one of the simplest remeshing algorithms. It is obtained by first inserting a vertex in the center of each face and each edge. Then, each face-vertex (the vertices inserted to center of each each face) is connected with its edge-vertices (the vertices inserted to centers of boundary edges of the original face). This process turns each m-sided original face into m quadrilaterals (See Fig. 8.2). The vertices of the resulting mesh is always 2-colorable since we can always paint edge-vertices in one color and the rest (original vertices and face-vertices) in another color. The operation also creates a 2-manifold mesh, therefore, rotation orders are still consistent as shown Fig. 8.2.

We then apply a spiral creating algorithm starting from original and face-vertices. Since each spiral is created inside of a quadrilateral, this algorithm is relatively simple as shown in Fig. 8. Let the original vertex and face-vertex, i.e. two vertices that are selected as starting points of the two interlocked spiral-trees in every quadrilateral, are indexed to be labeled as selected as shown as red circles in Fig. 8.3. Then the two half-edges starting from these two selected-vertices. we draw two lines that are parallel to the two half-edges. Note that using this approach we can directly control exact distances between the lines.

An important property of our algorithm is that since we do not use edge-vertices to create spirals, the spiral associated with them are simply eliminated. This process turns any m-sided polygon into an mbranched spiral trees and each spiral branch of any spiral tree is interlocked with a spiral branch of another spiral tree. Therefore, the algorithm produces locally [m,2] structures where m can be any integer defined by number of sides of the initial faces of the initial mesh.

Fig. 10 shows two semi-regular meander pattern obtained by applying our algorithm to regular and semi-regular tilings. The meander-pattern shown in Fig. 10b turned out to be very flexible.

Based on this algorithm, the local control of flexibility is straightforward. For every step of line drawing, we obtain a integer value that comes from an underlying image. Based on this integer value, we simply control the distance between neighboring lines. To avoid aliasing we apply some randomization based on anti-aliasing approach introduced by Cook at al. [8]. As it can be seen in Fig. 3, the method allows to control local structure based on a given image. Two more examples are shown in Fig. 11.



(a) *Initial Planar Mesh*

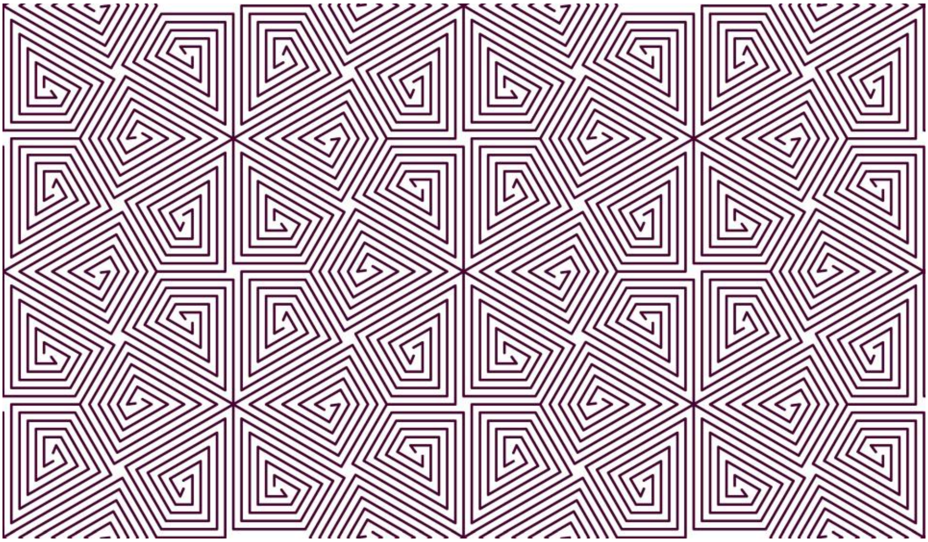
(b) *Application of Vertex Insertion Algorithm*

(c) *Rotation directions and vertex indexing.*

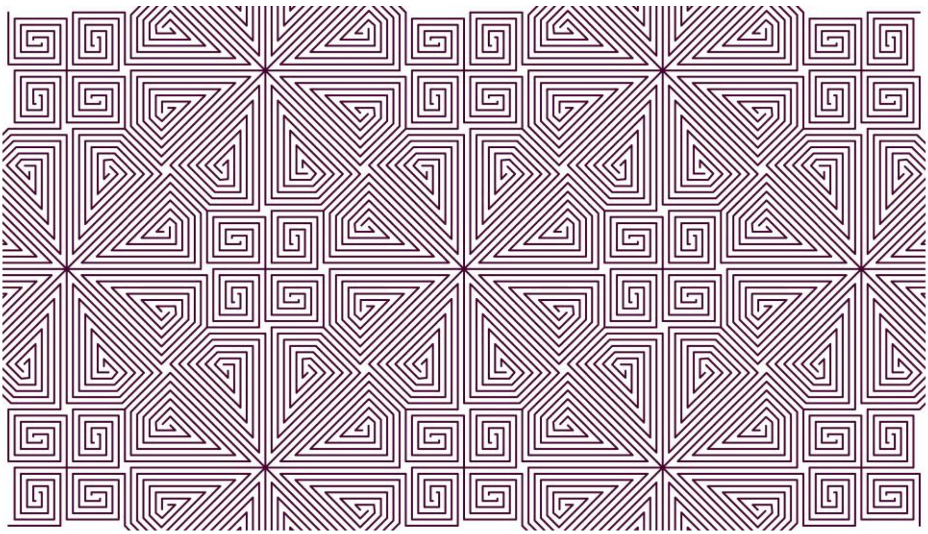
(d) *Relief-cuts obtained by starting indexed vertices indicated in Fig. 9c with red circles.*

(e) *Final Relief-Cuts obtained by removing original mesh and quads.*

Fig. 9. An example of overall algorithm working on a general mesh.

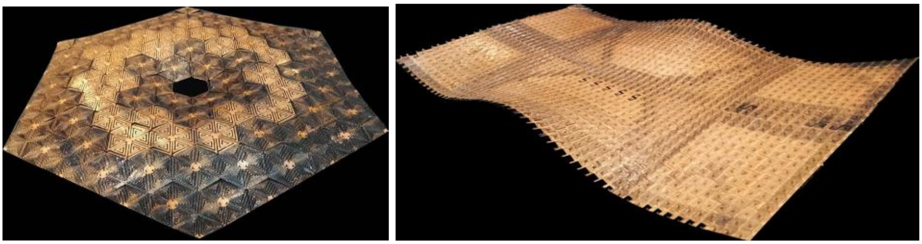


(a) 2D Meander-Patter obtained from Regular Hexagonal Tiling.



(b) 2D Meander-Patter obtained from Semi-regular 8.8.4 Tiling.

Fig. 10. Two examples of new 2D meander patterns obtained from regular and semi-regular tilings.



(a) A flexible shape with Negative and Positive curvature regions. *(b) Another flexible shape with Negative and Positive curvature regions.*

Fig. 11. Examples of panels obtained by our method.

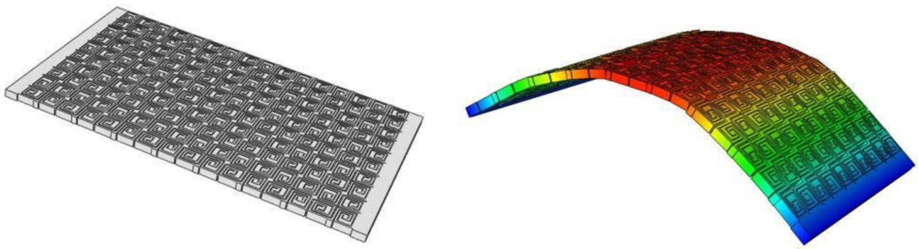
6 Finite Element Analysis

One way to optimally design the cuts pattern on the initial blank to achieve the desired shape of final product is to apply numerically analysis on how the material response to applied forces. Although by changing the cuts pattern parameters such as the pattern shape, density of cuts one may be able to deform the blank to desired shape but it is also important to keep the rigidity of wood. So cuts patterns may be chosen by trial and error but the easier and costly effective way is to use finite element analysis to predict the bank behavior under applied forces for different cuts parameters and then choose the optimal one for construction.

Fig. 12 shows Finite Element Analysis (FEA) result for modeling the flexible plywood under bending forces. As it can be seen from this example the deformation and displacement patterns of flexible plywood can be successfully captured by applying FEA and it can be possible to analyze the stress and strain patterns on the deformed plywood for optimally design the cuts parameters to achieve the pre-specified criteria. Using FEA analysis, we also observed that semiregular 8.8.4 tiling shown in Fig. 10b provides better flexibility than regular and hexagonal patters. For a complete analysis, more patterns need to be analyzed in a systematic way.

7 Conclusion and Future Work

In this paper, we have introduced a remeshing algorithm to obtain generalized 2D meander-patterns they are locally in the form of [n,2]. This method cannot create the [3,3] and [6,2] patterns shown in Fig. 5. We have recently observe that it is possible to obtain [6,2] pattern with a variation of this algorithm by replacing vertex insertion with another remeshing algorithm. We will report that in the near future. On the other hand, we are not sure if it is possible to generalize [3,3] pattern since we currently do not know any remeshing algorithm that can guarantee to produce 3-valent meshes with two-colorable vertices. [3,3] could possibly be a one of a kind pattern that can only be produced from hexagonal grid.



(a) Original 2D Meander-Pattern. *(b) Applying Forces*

Fig. 12. Flexible plywood under bending forces at two ends and displacement pattern on deformed plywood under bending.

The method is general and it can be applied to any polygon mesh. It will guarantee to produce meanderpatterns on any surface that is given by a polygonal mesh. Fig. 13 shows a few examples of covering arbitrary meshes with meander-patterns. We do not now an immediate architectural application of this; however, it can be helpful to produce flexible 3D shapes directly using 3D printer. Our local control algorithm works as desired avoiding antialising. There is now a need to develop methods to produce images based on desired flexibility levels.

(a) A cube covered with meander-pattern. *(b) A soccer-ball covered with meander-pattern.* *(c) Stanford Bunny covered with meander-pattern.*

Fig. 13. Examples of arbitrary shapes covered with meander-patterns.

Acknowledgements. This work was partially supported by the National Science Foundation under Grants NSF-EFRI-1240483, NSF-CMMI-1548243 and NSF-ECCS-1547075.

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