

# Iterative Deepening Search (IDS) and Iterative Deepening A\* (IDA\*)

CSX3009 Algorithm Design

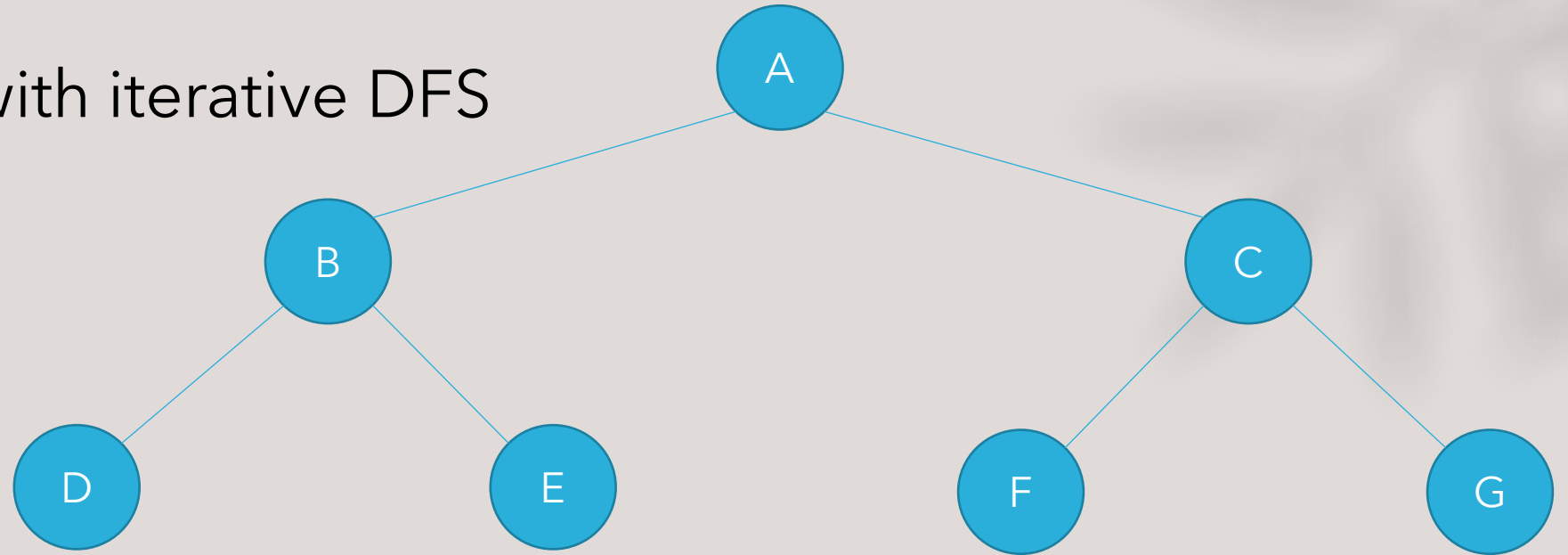
# DFS vs BFS

- The first solution that BFS discovers is at the minimum depth.
- Space complexity of BFS is exponential.
- Space complexity of DFS is  $O(\text{searching depth})$
- DFS may discover solution at deeper level than the minimum.

# IDS

Max depth = 2

Simulate BFS with iterative DFS



# A\* : BFS + Heuristics

- A heuristic lowerbound on the cost transforms BFS to UCS (Uniform Cost Search)
- The lowerbound on the **total** cost of a search:
  - cost from initial state to the current state
  - +
  - the lowerbound on the cost from current state to goal
- Like UCS, A\* searches out of the current lowest cost estimate.
- Difference from UCS:
  - UCS uses cost from initial state to current state.
  - A\* uses total cost estimate

# Admissible heuristic

- The key property of **A\* search** is its **admissibility**:
  - **an admissible heuristic**  $h(n)$  is one that **never overestimates** the cost to reach a **goal**,  $\{h(n) \leq h^*(n)\}$
- With an **admissible heuristic**, **A\* is cost-optimal**

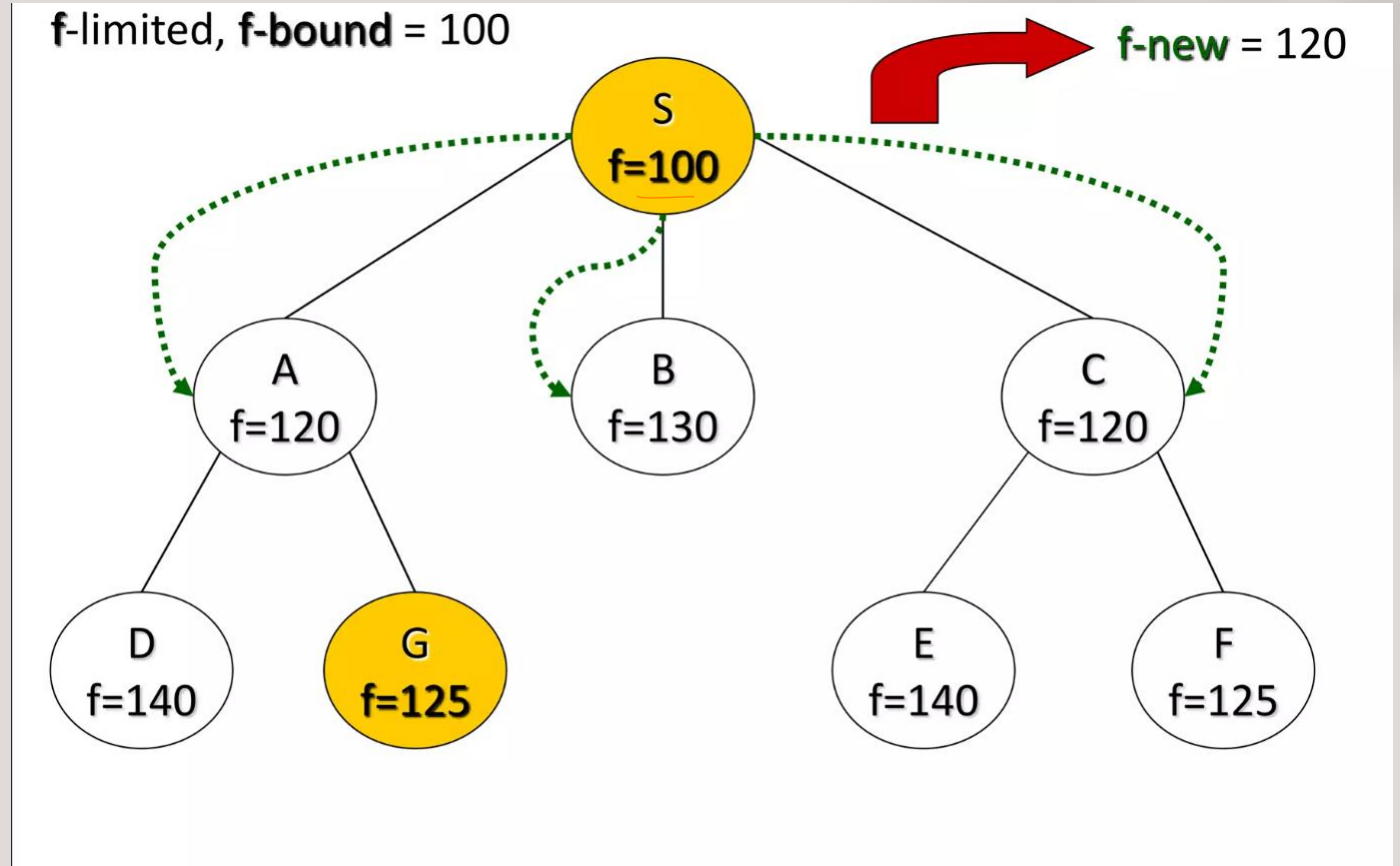
# Proof

The usual proof is by contradiction.

1. Assume that  $A^*$  with tree search and an admissible heuristic was not optimal.
2. Being non-optimal means that the first complete path from the start to the goal discovered by  $A^*$  (call this  $q$ ) will be longer than some other path  $p$ , which  $A^*$  explored up to some state  $s$ , but no further.
3. Since the heuristic is admissible, the estimated cost of reaching the goal from  $s$  must be smaller than the true cost.
4. By 3, and the fact that we know how much it costs to reach  $s$  along  $p$ , the estimated total cost of  $p$ , and thus the cost to expand  $s$  must be smaller than the true cost of  $p$ .
5. Since the true cost of  $p$  is smaller than the cost of  $q$  (by 2), the estimated cost to expand  $s$  must be smaller than the true cost of  $q$ .
6.  $A^*$  always picks the path with the most promising total cost to expand next, and the cost of expanding the goal state is given by the total path length required to reach it.
7. 5 and 6 form a contradiction, so our assumption in 1 must have been incorrect. Therefore  $A^*$  must be optimal.

# IDA\*

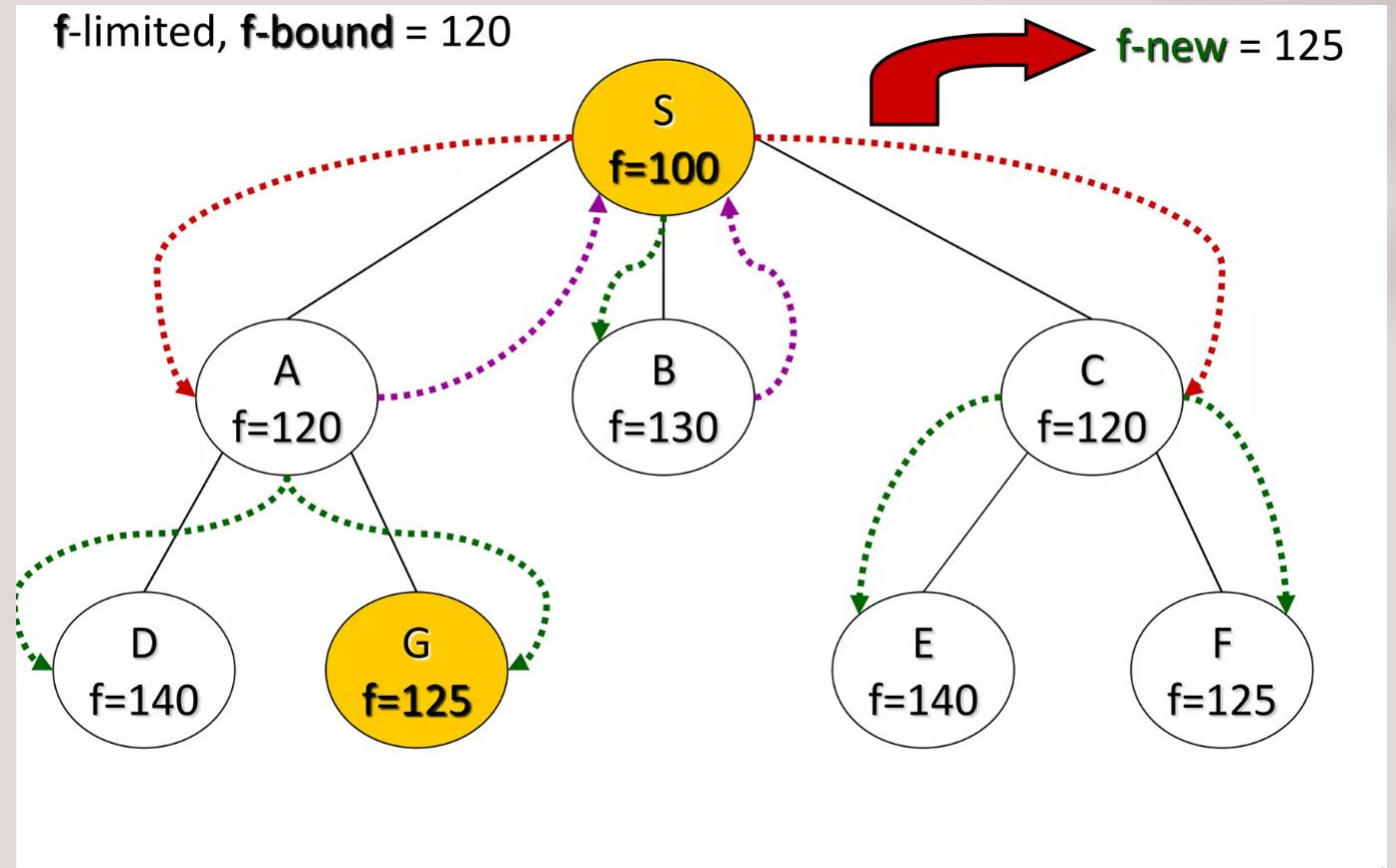
- Simulating A\* with DFS





# IDA\*

- Simulating A\* with DFS





# IDA\*

- Simulating A\* with DFS

