# MATH 3160

# Numerical Methods

## (Fall 2012)

**Instructor: Scott Zhang**

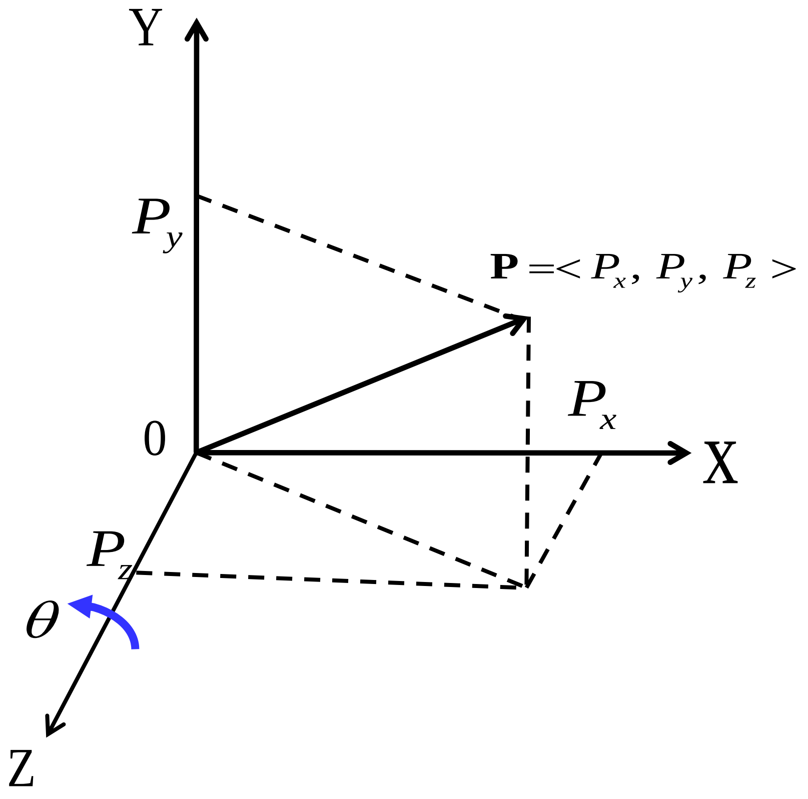
**School of Engineering & Information Technology**

# Rotation transform in 3 dimensions — about z-axis

$$\mathbf{P}' = < P_x', P_y', P_z' > = ?$$

$$\mathbf{P}' = \mathbf{R}_z(\theta)\mathbf{P}$$

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



## Rotation transform in 3 dimensions — about x-axis

$$\mathbf{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad \mathbf{P'} = \mathbf{R}_x(\theta)\mathbf{P}$$

## Rotation transform in 3 dimensions — about y-axis

$$\mathbf{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \qquad \mathbf{P'} = \mathbf{R}_y(\theta)\mathbf{P}$$

## ***Example***

Calculate the $3 \times 3$ rotation matrix that performs a rotation of 30 degrees about z-axis.

## Homogeneous coordinates

- A homogeneous coordinate is a 3D coordinate with an extra fourth coordinate w added to it.

- Written as $(x, y, z, w)$

- Example: For 3D coordinate (2,3,4) , the homogeneous coordinate is (2,3,4,1).

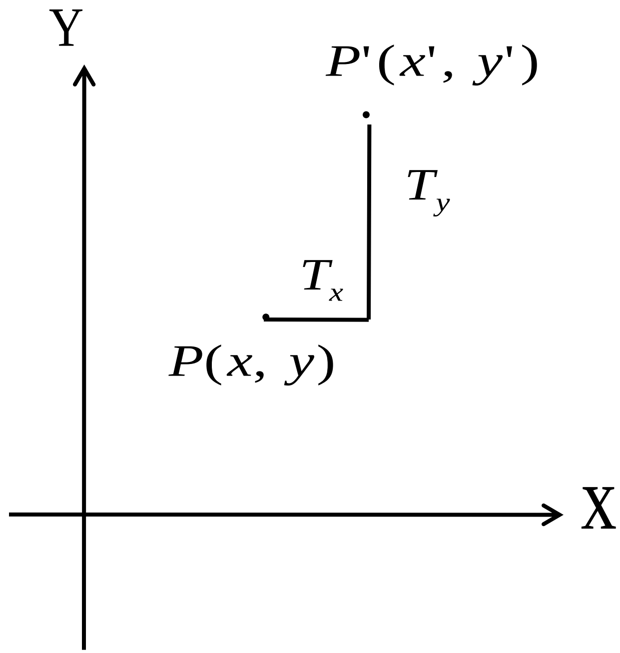
-How to convert a homogeneous coordinate back to a normal 3D coordinate

$$(x, y, z, w) \rightarrow (\frac{x}{w}, \frac{y}{w}, \frac{z}{w})$$

# **Translation** (in two dimensions)

$$x' = x + T_x$$

$$y' = y + T_y$$




Matrix addition:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{P} + \mathbf{T}$$

## **Translation** (in three dimensions)

$$P(x, y, z) \xrightarrow{(T_x, T_y, T_z)} P'(x', y', z')$$

Using matrix addition:

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

Using transform matrix:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$(x', y', z', w') \rightarrow \left(\frac{x'}{w'}, \frac{y'}{w'}, \frac{z'}{w'}\right) \rightarrow (x', y', z')$$

## ***Example***

A 3-D vector $< 2, -5, 3 >$ is translated 2 units along x-axis, 4 units along y-axis, and -6 units along z-axis. Determine the resulted vector.

# Combination of transforms using homogeneous coordinate

## Rotation about z-axis:

$$\mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad or\, \mathbf{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## Combination of first rotation (about z-axis) and then translation

$$\mathbf{P'} = \mathbf{MP} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & T_x \\ \sin\theta & \cos\theta & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \rightarrow (x', y', z')$$

**M** is the combination matrix

## Combination of rotation (about x-axis) and translation

$$\mathbf{P}' = \mathbf{MP} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & \cos\theta & -\sin\theta & T_y \\ 0 & \sin\theta & \cos\theta & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

## Combination of rotation (about y-axis) and translation

$$\mathbf{P}' = \mathbf{MP} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & T_x \\ 0 & 1 & 0 & T_y \\ -\sin\theta & 0 & \cos\theta & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

## Example

A 3-D vector $< 2, -1, 4 >$ rotates 60 degrees about z-axis and then is translated 2 units along x-axis, 4 units along y-axis, and 1 unit along z-axis. Determine the resulted vector.

$$\mathbf{P'} = \mathbf{MP} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & T_x \\ \sin\theta & \cos\theta & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 3.866 \\ 5.232 \\ 5 \\ 1 \end{bmatrix} \rightarrow (3.866,\ 5.232,\ 5)$$