Metodos matemáticos 2

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1. Función Gamma

Límite al infinito(Euler)

$$\begin{aligned}\Gamma(z) \equiv& \lim_{n\to\infty} \tfrac{1\cdot 2\cdot 3\cdots n}{z(z+1)(z+2)\cdots(z+n)} n^z, \quad z\in\mathbb{Z}^+ \ o \ z\in\mathbb{C}\\ \Gamma(z+1) =& \lim_{n\to\infty} \tfrac{1\cdot 2\cdot 3\cdots n}{(z+1)(z+2)\cdots(z+n+1)} n^{z+1}\\ =& \ z\cdot\Gamma(z) = \lim_{n\to\infty}\tfrac{nz}{z+n+1}\Gamma(z) = \Gamma(z+1) = z\cdot\Gamma(z)\end{aligned}$$

Aplicando lo anterior a z=1,2,3...n

$$\begin{aligned}\Gamma(1) =& \lim_{n\to\infty}\tfrac{1\cdot 2\cdot 3\cdots n}{(1)(2)\cdots(n+1)} n^z = \lim_{n\to\infty}\tfrac{n}{n+1} = 1\\ \Gamma(2) =& \ z\cdot\Gamma(z) = \Gamma(z+1) = \Gamma(1+1) = 1\\ \Gamma(3) =& \ 2\cdot\Gamma(1) = \Gamma(2+1) = 2\cdot 1\\ \Gamma(4) =& \ 3\cdot\Gamma(3) = \Gamma(3+1) = 3\cdot 2\cdot 1\\ &\vdots\\ \Gamma(n) =& \ (n-1)!\end{aligned}$$

Itegral Definida(Integral de Euler)

$$\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt, \quad Re(z) > 0$$

Ej.

Como aparecen en física

$$\Gamma(z) = 2\int_0^\infty e^{-t^2}t^{z+1}dt \quad o \quad \Gamma(z) = \int_0^1 \left[ln(\frac{1}{t})\right]^{z-1} dt$$

Si $z = \frac{1}{2} \Rightarrow \Gamma(\frac{1}{2}) = \sqrt{\pi}$ es integral error de Gauss

$$F(z,n) = \int_0^n \left(1-\frac{t}{n}\right)^n t^{z-1}dt, \quad \text{Re(z)} > 0 \text{ con n entero positivo} \ni$$

$$e^{-t} = \lim_{n\to\infty}\left(1-\frac{t}{n}\right)^n$$

$$\Rightarrow F(z,n) = \int_0^n \left(1-\frac{t}{n}\right)^n t^{z-1}dt = \int_0^1 (1-u)^n(un)^{z-1}ndu \ni u = \frac{t}{n}$$

$$\frac{F(z,n)}{n^z} = \int_0^1 (1-u)^n(u)^{z-1}du$$

Ahora por integración por partes usando $u = (1-u)^n$, $du = n(1-u)^{n-1}du$, $v = \frac{u^z}{z}$, $dv = u^{z-1}du$ tenemos

$$\begin{aligned}\tfrac{F(z,n)}{n^z} =& \ (1-u)^n \ \tfrac{u^z}{z}\Big|_0^1 - \textstyle\int_0^1 \tfrac{u^z}{z}n(1-u)^{n-1}du\\ F(z,n) =& \ n^z\cdot\tfrac{n(n-1)\cdots 1}{z(z+1)\cdots(z+n-1)} = \textstyle\int_0^1 u^{z+n-1}du\\ =& \ \tfrac{1\cdot 2\cdots n}{z(z+1)\cdots(z+n)}n^z \Rightarrow \lim_{n\to\infty}F(z,n) = \Gamma(z)\end{aligned}$$

Producto infinito(Weierstrass)

$$\frac{1}{\Gamma(z)} = ze^{\delta z}\prod_{n=1}^{\infty}\left(1+\frac{z}{n}\right)e^{-\frac{z}{n}}$$

$$\begin{gathered}\delta := \text{constante de Euler-Mascheron}\\ \delta := 0.5772156619\end{gathered}$$

Ecuación estadística de Maxwell-Boltzmann

$$e^{-E/KT} \qquad \begin{array}{l}\text{K es la constante de Boltzmann}\\ \text{T es la temperatura absoluta}\\ \text{E(energía): estado de energía ocupada}\end{array}$$

probabilidad de estar en estado de energía es $Y_{kt} = \beta$

$$\begin{array}{ll} P(E) = Ce^{-\beta E} & \text{Para un gas idel sin estructura}\\ n(E)dE & n(E)^{1/2}\\ 1 = C\int n(E)e^{-\beta E}dE & \text{E=energía cinética}\end{array}$$

$$\begin{aligned} 1 = &\quad c\int_0^{\infty} E^{1/2}e^{-\beta E}dE = \tfrac{C\Gamma(3/2)}{\beta^{3/2}}, \quad \beta E = T \Rightarrow dE = \tfrac{dt}{\beta}\\ 1 = &\quad C\int_0^{\infty} e^{-t}\left(\tfrac{t}{\beta}\right)^{1/2}\tfrac{1}{\beta}dt\\ &\quad = C\int_0^{\infty} e^{-t}t^{3/2-1}dt\cdot\tfrac{1}{\beta^{3/2}}\\ = &\quad \tfrac{C\Gamma(3/2)}{\beta^{3/2}}\\ = &\quad c\cdot\tfrac{\sqrt{\pi}}{2\cdot\beta^{3/2}}\end{aligned}$$

$$\therefore C = \frac{2\cdot\beta^{3/2}}{\sqrt{\pi}}$$

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Relaciones de funciones

$$\Gamma(z+1) = z\Gamma(z)$$

Fomula de reflexión

$$\Gamma(z)\Gamma(1-z) = \frac{\pi}{sen(z\pi)}$$

Fórmuláde duplicación de Legendre

$$\Gamma(1+z)\Gamma(z+\frac{1}{2}) = 2^{-2z}\sqrt{\pi}\Gamma(2z+1)$$

Notacíón doble factorial:

$$\begin{aligned} (2n+1)!! = &\quad 1\cdot 3\cdot 5\cdots(2n+1) = \tfrac{(2n+1)!}{2^n n} \\ (2n)!! = &\quad 2\cdot 4\cdot 6\cdots(2n) = 2^n n! \\ (-1)!! = &\quad 1 \end{aligned}$$

PROPIEDADES ANALíTICAS

$[\Gamma(z)]^{-1}$ tiene singularidades en $z = 0, -1, -2...$ y no tiene cuando $z = -1, 2, 3...$ y no tiene ceros en el plano complejo finito positivo.

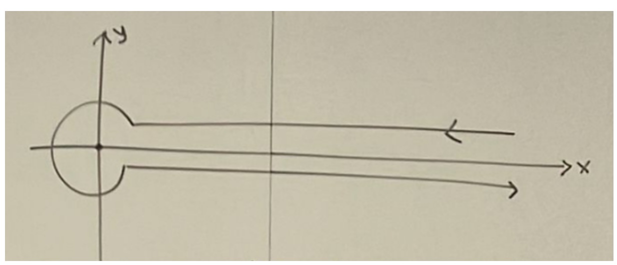
El residuo R_n cuando $z = -n$ donde n es un entero mayor o igual a cero.

$$\begin{aligned} R_n = &\quad \lim_{n\to 0}(\epsilon\Gamma(-n+\epsilon)) \\ = &\quad \lim_{n\to 0} \tfrac{\epsilon\Gamma(-n+1+\epsilon)}{-n+\epsilon} \\ = &\quad \lim_{n\to 0} \tfrac{\epsilon\Gamma(-n+2+\epsilon)}{(-n+\epsilon)(-n+1+\epsilon)} \\ = &\quad \lim_{n\to 0} \tfrac{\epsilon\Gamma(1+\epsilon)}{(-n+\epsilon)\cdots(\epsilon))} \\ = &\quad \tfrac{(-1)^n}{n!}, \text{el residuo alterna signos en } z=-n \end{aligned}$$

Integral de Schaefli

$$\int_C e^{-t}t^v dt = (e^{2\pi i v})\Gamma(v+1)$$

donde C es el contorno



Esta integral es útil cuando V no es entero.

Notación factorial:

$$\prod(z) = z! = \Gamma(z+1)$$

Función Digamma

$$\begin{aligned} \ln(\Gamma(z+1)) = &\quad z\cdot\Gamma(z) \\ = &\quad \lim_{n\to\infty} \tfrac{z}{z}\ln\left[\tfrac{n!\cdot n^z}{(z+1)(z_2)\cdots(z+n)}\right] \end{aligned}$$

$$\frac{d}{dz}\ln(\Gamma(z+1)) = \quad \frac{d}{dz}\lim_{n\to\infty}[\ln(n!) + z\cdot\ln(n) - \ln(z+1) - \ln(z+2) - ...\ln(z+n)]$$

$$\frac{d}{dz}\ln(\Gamma(z+1)) \equiv \underbrace{\varphi(z+1)}_{\substack{\text{Función}\\ \text{digamma}}} = \quad \lim_{n\to\infty}\left(\ln(n) - \frac{1}{z+1} - \frac{1}{z+2}\cdots - \frac{1}{z+n}\right)$$

$$\varphi(z+1) = \quad \frac{[\Gamma(z+1)]'}{\Gamma(z+1)}$$

Si sumamos y restamos un número armónico

$$H_n = \sum_{m=1}^{n} \frac{1}{m}$$

$$\varphi(z+1) = \lim_{n\to\infty} \left[\underbrace{(\ln(n) - H_n)}_{\substack{-\delta \text{ cuando} \\ n\to\infty}} - \sum \left(\frac{1}{z+m} - \frac{1}{m} \right) \right]$$

$$\Rightarrow \varphi(z+1) = -\delta + \sum_{m=1}^{\infty} \frac{z}{m(m+z)}, \quad \text{si z=0} \Rightarrow \varphi(1) = -\delta$$

para $n > 0$, n entero.

$$\begin{aligned} \varphi(n+1) = & \quad -\delta + \sum_{m=1}^{n} \tfrac{1}{m} \\ = & \quad -\delta + H_n \end{aligned}$$

Función Poligamma

Cuando se deriva muchas veces la función digamma

$$\begin{aligned} \varphi^{(m)}(z+1) \equiv & \quad \tfrac{d^{m+1}}{dz^{m+1}} \ln\Gamma(z+1) \\ \varphi^{(m)}(z+1) = & \quad (-1)^{m+1} m! \cdot \sum_{n=1}^{\infty} \tfrac{1}{(z+n)^{m+1}}, \quad m = 1, 2, 3... \end{aligned}$$

Si z=0 la función se conoce como zeta de Riemann

$$\zeta(m) = \sum_{n-1}^{\infty} \frac{1}{n^m}$$

$$\Rightarrow \varphi_{(1)}^{(m)} = (-1)^{m+1} \cdot m! \cdot \zeta(m+1), \quad m = 1, 2, 3...$$

Expansión de Maclaurin

$$\begin{aligned} \ln\Gamma(z+1) = & \quad \textstyle\sum_{n=1}^{\infty} \frac{z^n}{n!} \varphi_{(1)}^{(n-1)} \\ = & \quad -\delta \cdot z + \textstyle\sum_{n=2}^{\infty} (-1)^n \frac{z^n}{n} \zeta(n) \end{aligned}$$

converge en $|z| < 1$ para $z = x \Rightarrow$ se puede calcular $\Gamma(z+1)$ para números reales o complejos.

Suma de series

Transformar la serie por medio de fracciones parciales y expresar la serie infinita como sumas finitas de funciones gamma y poligamma.

Ej.

$$\begin{aligned} \varphi^{(m)}(z+2) = & \quad \varphi^{(m)}(z+1) + (-1)^m \tfrac{m!}{(z+1)^{m+1}} \\ \varphi^{(m)}(z+2) - \varphi^{(m)}(z+1) = & \quad \tfrac{(-1)^m m!}{(z+1)^{m+1}} \end{aligned}$$

$$\tfrac{d^{(m)}}{dz^{(m)}} [\varphi(z+2) - \varphi(z+1)]$$

$$\begin{aligned} \varphi(z+2) - \varphi(z+1) = & \quad \textstyle\sum_{m=1}^{\infty} \frac{z+1}{m(m+z+1)} - \sum_{m=1}^{\infty} \frac{z}{m(m+z)} \\ = & \quad \textstyle\sum_{m=1}^{\infty} \left[\frac{1}{m} - \frac{1}{m+z+1} - \frac{1}{m} + \frac{1}{m+z} \right] \\ = & \quad \tfrac{1}{z+1} \end{aligned}$$

$$\frac{d^m}{dz^m} [\varphi(z+2) - \varphi(z+1)] = \frac{d^m}{dz^m} \left[\frac{1}{z+1} \right] = \frac{(-1)^m m!}{(z+1)^{m+1}}$$

Demuestre que:

$$\frac{1}{2} \ln \left[\frac{\pi \cdot z}{sen(\pi \cdot z)} \right] = \sum_{n=1}^{\infty} \frac{\zeta(2n)}{2n} z^{2n}, \quad |z| < 1$$

- $\frac{1}{2}\ln\left[\underbrace{\Gamma(z)}_{z\Gamma(z)=\Gamma(z+1)} \quad \Gamma(z-1)\right] = \frac{1}{2}\ln\left[\frac{\pi}{sen(z\pi)}\right]$
- $\ln\Gamma(z+1) = \lim_{n\to\infty}\left[\ln(n!) + z\ln(n) - \ln(z+1) - ... - \ln(z+n)\right]$

$$\Gamma(z)\Gamma(1-z) \rightsquigarrow \Gamma(1+z)\Gamma(1-z)$$

$$\begin{aligned}
\ln\left[\Gamma(1+z)\Gamma(1-z)\right] = &\quad -\delta\cdot z + \sum_{n=2}^{\infty}(-1)^n\frac{z^n}{n}\zeta(n) \\
&\quad +(-\delta(-z)) + \sum_{n=2}^{\infty}(-1)^n\frac{(-z)^n}{n}\zeta(n) \\
= &\quad \sum_{n=2}^{\infty}\zeta(n)\underbrace{\left[\frac{z^n}{n} + \frac{(-z)^n}{n}\right]}_{z^{2n}} \\
= &\quad 2\sum_{n=1}^{\infty}\cancel{(-1)^{2n}}^{\,1}\zeta(2n)\frac{z^{2n}}{2n}
\end{aligned}$$

viernes 14 de julio
La Función Beta

$$\Gamma(p+q) \propto \Gamma(p)\Gamma(q)$$

$$n! = \Gamma(n+1) \& \int_0^\infty e^{-t} t^2 dt,$$ Variante de gamma para definir la funión beta

$$\Gamma(m)\Gamma(n) = \int_0^\infty e^{-u} u^m fu \int_0^\infty e^{-v} v^n dv$$

$$\begin{array}{cc} u = x^2 & v = y^2 \\ du = 2xdx & dv = 2ydy \end{array}$$

$$\begin{aligned} \Gamma(m)\Gamma(n) &= \int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2} x^{2m}(2x) y^{2m}(2y) dxdy \\ &= 4\int_0^\infty e^{-x^2} e^{-x^2} e^{-y^2} x^{2m+1} y^{2m+1} dxdy \end{aligned}$$

otro cambio de variables

$$x = rcos(\theta) \quad y = rsen(\theta)$$

Usando el jacobiano

$$\begin{vmatrix} \frac{\partial x}{\partial r} = cos\theta dr & \frac{\partial x}{\partial \theta} = -rsen\theta d\theta \\ \frac{\partial y}{\partial r} = sen\theta dr & \frac{\partial y}{\partial \theta} rcos\theta d\theta \end{vmatrix} = rdrd\theta$$

$$= \int_0^\infty \int_0^{\pi/2} \cancel{e^{-r^2}} \xrightarrow{} e^{-r^2 cos^2\theta} \quad r^{2m+2n+3} cos^{2m+1}\theta sen^{2n+1} drd\theta$$

$$\Gamma(m)\Gamma(n) = 2\Gamma(m+n+1) \int_0^{\pi/2} cos^{2m+1}\theta sen^{2n+1}\theta d\theta$$

$$\Gamma(z) = 2\int_0^\infty e^{-t^2} t^{2z-1} dt; \; Re(z>0)$$

$$\Gamma(q)\Gamma(p) = 4\int_0^\infty e^{-s^2} s^{2q-1} ds \int_0^\infty e^{-t^2} t^{2p-1} dt$$

$$\begin{array}{cc} s = rcos\theta & t = rsen\theta \\ r^2 = s^2 + t^2 & dsdt = rdrd\theta \end{array}$$

$$\begin{aligned} \Gamma(q)\Gamma(p) &= 4\int\int e^{-r^2 cos^2\theta} e^{-r^2 sen^2\theta} r^{2q-1} cos^{2q-1} r^{2p-1} sen^{2p-1} rdrd\theta \\ &= 2\cdot 2\int_0^\infty e^{-r^2} r^{2q+2p-1} cos^{2q-1} sen^{2p-1} drd\theta \end{aligned}$$

$$\Gamma(p)\Gamma(q) = 2\int cos^{2q-1} sen^{2p-1} d\theta$$

$$\frac{m!n!}{(m+n+1)!} \star = 2\int_0^{pi/2} cos^{2m+1}\theta sen^{2n+1}\theta d\theta$$

$$\left.\boxed{b(p,q) = \frac{\Gamma(q)\Gamma(p)}{\Gamma(q+p)}}\right\}$$ definición de función beta

$$b(p,q) = b(q,p)$$

Si se sustituye $t = cos^2\theta$ y $dt = -2cos\theta sen\theta d\theta$

$$\begin{aligned} \cancel{cos^2\theta} \xrightarrow{} t + sen^2\theta &= 1 \\ sen^2\theta &= 1 - t \end{aligned} \qquad \star = 2\int_0^1 cos^{2m}\theta \cancel{cos\theta} (1-t)^n \cancel{sen\theta} dt$$

$$\begin{aligned} sen^{2n+1}\theta &= sen^{2n}\theta sen\theta \\ &= (1-t)^n sen\theta \end{aligned} \qquad = \int_0^1 t^m (1-t)^n dt \Rightarrow$$ otra forma de escribir $B(m+1, n+1)$

lunes 17 de julio

Ej.

Una particula de masa m se mueve en un potencial simétrico esta descrito por $v(x) = A|x|^n$ y energía total como $\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + v(x) = E$

Resolviendo para $\frac{dx}{dt}$ e intgrando encontramos que el movimiento períodico es:

$$\zeta = \sqrt{2m}\int_0^{X_{max}} \frac{dx}{(E - Ax)^{1/2}}$$

donde X_{max} es el punto de innplexion clasico dado por $Ax^n_{max} = E$

Muestre que:

$$\tau = \frac{2}{n}\sqrt{\frac{e\pi m}{E}}\left(\frac{E}{A}\right)^{1/2}\left[\frac{\Gamma(1/n)}{\Gamma(\frac{1}{n}+\frac{1}{2})}\right]$$

$$\boxed{\begin{gathered} B(p+1, q+1) = B(1/n, 1/2) = \int_0^1 t^p(1-t^q dt) \\ \frac{1}{n} = p+1 \Rightarrow p = \frac{1}{n} - 1 \\ \frac{1}{2} = q+1 \Rightarrow q = \frac{1}{2} - 2 \\ \int_0^1 t^{\frac{1}{n}-1}(1-t)^{-1/2} \end{gathered}}$$

$$t = \frac{Ax^n}{E} \Rightarrow \left(\frac{Et}{A}\right)^{1/n}$$

$$dt = \frac{A}{E}n(x^{n-1})dx$$

$$dx = \underbrace{\frac{E}{A}}_{\frac{1}{n}\frac{x}{t}dt}\frac{1}{n}\frac{x}{x^n}dt$$

$$\begin{aligned} \tau = \quad & 2\frac{\sqrt{2m}}{\sqrt{E}\int_0^1 \frac{\frac{x}{nt}dt}{(1-t)^{1/2}}} \\ = \quad & \left(\frac{E}{A}\right)^{1/n}\frac{2}{n}\sqrt{\frac{2m}{E}}\int_0^1 \frac{\frac{\frac{(Et)^{1/n}}{A^{1/n}}}{nt}dt}{(1-t)^{1/2}} \\ = \quad & \frac{2}{n}\sqrt{\frac{2m}{E}}\left(\frac{E}{A}\right)^{1/n}\underbrace{\int_0^1 t^{\frac{1}{n}-1}(1-t)^{-1/2}dt}_{\frac{\Gamma(\frac{1}{n})\Gamma(\frac{1}{2}) \to \sqrt{\pi}}{\Gamma(\frac{1}{n}+\frac{1}{2})}} \end{aligned}$$

$$\tau = \quad \frac{2}{n}\sqrt{\frac{2\pi m}{E}}\left(\frac{E}{A}\right)^{1/n}\frac{\Gamma(\frac{1}{n})}{\Gamma(\frac{1}{n}+\frac{1}{2})}$$

2. SERIES DE STIRLING

$$\ln(n!) \rightsquigarrow \ln\Gamma(z)$$

para z no entero, $|z|$ muy grande, no Series de Maclaurin

$$\Gamma(z+1) = z\Gamma(z)$$

Se usa una expansión asintótica para $\ln(\Gamma(z))$ esta expansión es la que se conoce como Serie de Stirling o Formula de Stirling

Series asintóticas→ Series semi-convergentes→Series de Poincaré

Se consideran 2 tipos de integrales

$$\begin{aligned} I_1(x) = & \int_x^\infty e^{-u} f(u) du \\ I_2(x) = & \int_0^\infty e^{-u} f\left(\frac{u}{x}\right) du \end{aligned}$$

Qué podemos hacer con las series asintóticas:

1. Si multiplicamos 2 series asintóticas vamos a tener otra serie asintótica.
2. Se pueden integrar término a término u el resultado será, otra serie asintotica de la forma

$$\int_x^\infty f(x) dx$$

3. La diferenciación es válido únicamente bajo ciertas condiciones.

Método para generar una serie asintótica:

$$\left.\begin{array}{l} \textbf{Método de Steepest Descents} \end{array}\right\} \begin{array}{l} \text{a lo largo del camino} \\ \text{máximo decrecimiento} \end{array}$$

→ Saddle point → punto silla
→ Método de integrandos Holomorfos

Derivación de la fórmula de integración de Euler-Maclaurin

⟶ uno de los usos de los polinomios de Bernoulli ⟵

Números de Bernoulli ⟶ Polinomios de Bernoulli

$$B_n$$

$$\frac{t}{e^t - 1} = \sum_{n=0}^{\infty} \frac{B_n t^n}{n!}$$

Como $\frac{t}{e^t-1} = \sum_{n=0}^\infty \frac{B_n t^n}{n!}$ es una serie de Taylor se pueden identificar a B_n como una sucesión de derivadas de la función generadora valuadas en cero.

$$\Rightarrow B_n = \frac{d^n}{dt^n} \left(\frac{t}{e^t - 1} \right) \bigg|_{t=0}$$

$$\begin{aligned} B_0 = & \lim_{n\to 0} \frac{t}{e^t-1} = 1 \\ B_1 = & \lim_{n\to 0} \frac{d}{dt} \left[\frac{t}{e^t-1}\right]\bigg|_{t=0} = -\frac{1}{2} \\ B_1 = & \lim_{n\to 0} \left(\frac{1}{e^t-1} - \frac{te^t}{(e^t-1)^2}\right) \\ B_1 = & \lim_{n\to 0} \left(\frac{e^t-1-te^t}{(e^t-1)^2}\right) = \lim_{n\to 0} \frac{\cancel{e^t} - t\cancel{e^t} - \cancel{e^t}}{2(e^t-1)\cancel{e^t}} \\ = & -\frac{1}{2} \end{aligned}$$

La fórmula Euler-Maclaurin para evaluar integrales definidas en $(0, \infty)$

$$\begin{aligned} \int_0^\infty = & \frac{1}{2} f(0) + f(1) + f(2) + f(3...) \\ & + \frac{B_2}{2!} f'(0) + \frac{B_4}{4!} f^{(3)}(0) + \frac{B_6}{6!} f^{(5)}(0) \end{aligned}$$

Donde B_n son los números de Bernoulli

$$B_2 = \frac{1}{6},\ B_4 = -\frac{1}{30},\ B_6 = \frac{1}{42},\ B_8 = -\frac{1}{30}, ...$$

$$\int_0^\infty \frac{dx}{(z+x)^2} = \frac{1}{z}$$

$$\text{z no está en el eje real negativo}$$

$$f(1)+f(2)+f(3)+...=\sum_{n=1}^{\infty}\frac{1}{(z+n)^2}=$$

$$\varphi^{(m)}(z+1) = \frac{d^{m+1}}{dz^{m+1}}\ln\Gamma(z+1)$$

$$\underset{m=1,2,3...}{=} (-1)^{m+1}m!\sum_{n=2}^{\infty}\frac{1}{(z+n)^{m+1}}$$

m=1

$$(-1)^2\cdot 1!\sum_{n=1}^{\infty}\frac{1}{(z+n)^2}=\varphi^{(1)}(z+1)$$

$$f_{(0)}^{(2n+1)}=\left(\frac{d}{dx}\right)^{2n-1}\frac{1}{(z+x)^2}\bigg|_{x=0}=-\frac{(2n)!}{z^{2n+1}}$$

$$\int_0^{\infty}f(x)dx=\int_0^{\infty}\frac{dx}{(z+x)^2}= \frac{1}{z}=\frac{1}{2}\frac{1}{z^2}+ \underbrace{\varphi^{(1)}_{(z+1)}}_{\text{poligamma con m=1}} -\frac{B_2}{z^3}-\frac{B_4}{z^5}-...$$

$$\varphi^{(1)}_{(z+1)} = \frac{d}{dz}\underbrace{\varphi(z+1)}_{digamma}$$

$$\varphi^{(1)}_{(z+1)} = \frac{1}{z}-\frac{1}{2z^2}+\frac{B_2}{z^3}+\frac{B_4}{z^5}+...$$

$$= \frac{1}{z}-\frac{1}{2z^2}+\underbrace{\sum_{n=1}^{\infty}\frac{B_{2n}}{z^{2n+1}}}_{\text{Es una serie asintótica que es util si se utilizan pocos términos}}$$

Al integrar $\varphi^{(1)}_{(z+1)}$ obtenemos $\varphi_{(z+1)}$ [Digamma]

$$\varphi_{(z+1)} = C_1+\ln z+\frac{1}{2z}-\frac{B_2}{2z^2}-\frac{B_4}{4z^4}-...$$

$$= C_1+\ln z+\frac{1}{2z}-\sum_{n=1}^{\infty}\frac{B_{2n}}{\cdot z^{2n}}$$

viernes 21 de julio Fórmula de stirling

$$\begin{aligned}\varphi_{(z+1)} &= C_1 + \ln z + \tfrac{1}{2z} - \tfrac{B_2}{2z^2} - \tfrac{B_4}{4z^4} - ..., \quad B_{2n} \text{ Números de Bernoulli}\\ &= C_1 + \ln z + \tfrac{1}{2z} - \sum_{n=1}^{\infty} \tfrac{B_{2n}}{2nz^{2n}}\end{aligned}$$

$$\varphi_(z+1) \equiv \frac{d}{dz}\ln\Gamma(z+1)$$

$$\begin{aligned}\ln\Gamma(z+1) = \quad & C_2 + \left(z+\tfrac{1}{2}\right)\ln z + (c_1-1)z + \tfrac{B_2}{2z} + ... + \tfrac{B_{2n}}{2n(2n-1)z^{2n-1}} + ...\\ & \Rightarrow C_1 = 0 \ y \ c_2 = \tfrac{1}{2}\ln 2\pi\end{aligned}$$

$$\begin{aligned}\ln\Gamma(z+1) = \quad & \tfrac{1}{2}\ln 2\pi + \left(z+\tfrac{1}{2}\right)\ln z - z + \tfrac{1}{12z} + \tfrac{1}{360z^3} + \tfrac{1}{1260z^5} + ...\\ \lim_{n\to\infty} \tfrac{z!}{\sqrt{z}z^z e^{-z}} &= \sqrt{2\pi}\\ z! &\approx \sqrt{2\pi}\sqrt{z}z^z e^{-z}\\ \Gamma(z+1) &\approx \sqrt{2\pi}z^{z+\frac{1}{2}}e^{-z}\\ \ln\Gamma(z+1) &\approx \tfrac{1}{2}\ln 2\pi + \left(z+\tfrac{1}{2}\right)(\ln z) - z\\ \ln n! &\approx n\ln n - n\end{aligned}$$

Ej.

n es natural.

$$\lim_{n\to\infty} \sqrt[n]{n} = 1 \qquad\qquad \lim_{n\to\infty} \sqrt[n^3]{n} = 1$$

$$\lim_{n\to\infty} \sqrt[n^3]{n^7} = 1$$

$$\lim_{n\to\infty} \sqrt[p(n)]{q(n)} = 1$$

pero...

$$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$$

$$\begin{aligned}\Rightarrow \lim_{n\to\infty} \sqrt[n]{n!} = \quad & \lim_{n\to\infty} \sqrt[n]{\sqrt{2\pi}\sqrt{n}n^n e^{-n}}\\ = \quad & \lim_{n\to\infty} \cancelto{1}{\sqrt[2n]{2\pi n}} \cdot n \cdot \cancelto{\frac{1}{e}}{e^{-1}}\\ = \quad & \infty\end{aligned}$$

$$\lim_{n\to\infty} \frac{2n!}{4^n(n+1)!n!} = 0$$

$$\begin{aligned}(n+1)! = \quad & (n+1)n!(\sqrt{2\pi n}n^n e^{-n})\\ 2n! = \quad & \sqrt{2\pi}\sqrt{2n}(2n)^{2n}e^{-2n}\end{aligned}$$

$$\lim_{n\to\infty} \frac{\sqrt{2\pi}\sqrt{2n}\cancel{(2n)^{2n}}\cancel{(e)^{-2n}}}{\cancel{4^n}(n+1)(\sqrt{2\pi n}\cdot \cancelto{2\pi n}{n^n}\cancel{e^{-n}})^2}$$

$$\lim_{n\to\infty} \frac{\sqrt{\pi n}}{(n+1)(\pi n)} = 0$$

Ej.

Un conjunto de N partículas distiguibles es asignado a los estados $\psi_i \ni i = 1, 2, ..., M$. Si el número de partículas en los diversos estados son $n_1, n_2, n_3, ..., n_M$ (con $M \ll N$). El número de formas que se pueden hacer es:

$$w = \frac{N!}{n_1!n_2!...n_M!}$$

La entropía asociada es $S = k\ \ln w$ donde k es la constante de Boltzmann.
En el límite cuando $N \to \infty$, con $n_i = P_i N$, (P_i es la fracción del as partículas en le estado i). Encuentre S en función de N y P_i.

$$\text{Pista: } \sum_i P_i = 1$$

Sol.

$$\begin{aligned} \ln N! \approx&\quad N \ln N - N \ \ \text{usando la condición} \textstyle\sum_i n_i = N \\ S = k \ln W =&\quad l \left[\ln N! - \ln n_i!\right] \\ =&\quad k\left[(N \ln N - N) - \left(\textstyle\sum_{i=1}^{M} n_i + nn_i - n_i\right)\right] \end{aligned}$$

lunes 24 de julio

$$\begin{aligned} \zeta(z) &= \sum_{n=1}^{\infty} n^{-z} \quad \text{por definición} \\ \zeta(z) &= \int_0^{\infty} \frac{u^{z-1}}{e^u-1} du \end{aligned}$$

$$\begin{array}{ll} \zeta(1) = \infty & \zeta(2) = \frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \\ \zeta(3) = 1{,}202056 & \zeta(4) = \frac{\pi^4}{90} \\ \zeta(5) = 1{,}0369 & \zeta(6) = \frac{\pi^6}{945} \end{array}$$

$$Re(z) > 1$$

$$\begin{aligned} \zeta(-1) &= \frac{1}{1^{-1}} + \frac{1}{2^{-1}} + ... \\ &= 1+2+3+4+... \\ &= -\frac{1}{12} \end{aligned}$$

$$\zeta(-2n) = 0$$

Ceros "triviales"

$$\begin{array}{ll} z = 2+i & \\ n = \frac{1}{2} & n^z = \frac{1}{2}^{2+i} = \frac{1}{2}^2 \frac{1}{2}^i \end{array}$$

Otra forma de definir Zeta de Riemann

$$\zeta(s) = \prod_{\substack{\text{primos} \\ \text{p= 2}}}^{\infty} (1-p^{-s})^{-1}$$

franja crítica:

$$\begin{aligned} \zeta(z) &= \frac{1}{\Gamma(z)} \int_0^{\infty} \frac{t^{z-1}}{e^t-1} dt \frac{e^{-t}}{e^{-t}} = \frac{1}{\Gamma(z)} \int_0^{\infty} \frac{t^{z-1} . e^{-t}}{1-e^{-t}} \\ &= \int_0^{\infty} t^{z-1} e^{-t} \sum_{m=1}^{\infty} e^{-mt} dt \qquad \left(\frac{1}{1-x} = \sum_{m=0}^{\infty} x^m\right) \\ &= \frac{1}{\Gamma(z)} \int_0^{\infty} t^{z-1} \sum_{m=1}^{\infty} e^{-mt} dt \qquad = \frac{1}{\Gamma(z)} \int_0^{\infty} \sum_{m=1}^{\infty} \left(\frac{x}{m}\right)^{z-1} e^{-x} \frac{dx}{m} \\ &\qquad = \frac{1}{\Gamma(z)} \left(\sum_{m=1}^{\infty} \frac{1}{m^z}\right) \int_0^{\infty} x^{z-1} e^{-x} dx \end{aligned}$$

Ej.

$$\int_0^{\infty} \frac{x^n \cdot e^x}{(e^x-1)^2} dx = n!\zeta(n) \qquad \begin{array}{ll} u = x^n & v = -\frac{1}{e^x-1} \\ du = nx^{n-1}dx & dv = \frac{e^x}{(e^x-1)^2} dx \end{array}$$

$$\begin{aligned} &= -\cancel{\frac{x^n}{e^x-1}\Big|_0^{\infty}} \to 0 - \int_0^{\infty} -\frac{1}{e^x-1} nx^{n-1} dx \qquad \zeta(z) = \frac{1}{\Gamma(z)} \int_0^{\infty} \frac{t^{z-1}}{e^t-1} dt \\ &= \int_0^{\infty} \frac{x^{n-1}}{e^x-1} dx = n(n+1)!\zeta(z) \end{aligned}$$

Funciones Incompletas

Gamma incompleta

$$\delta(a,x) = \int_0^x e^{-t} t^{a-1} dt \qquad R(a) > 0$$

$$\Gamma(a,x) = \int_x^{\infty} e^{-t} t^{a-1} dt$$

$$\delta(a,x) + \Gamma(a,x) = Gamma(a) = \Gamma(a)$$

Cuando a es entero:

$$\delta(n,x) = (n-1)! \left(1 - e^{-x} \sum_{s=0}^{n-1} \frac{x^s}{s!}\right)$$

$$\Gamma(n,x) = (n-1)! e^{-x} \sum_{s=0}^{n-1} \frac{x^s}{s!}$$

Cuando a no es entero:

$$\begin{array}{lll} \delta(a,x) \rightarrow & \text{serie de potencias para x pequeno} & \Rightarrow \delta(a,x) = x^a \sum_{n=0}^{\infty}(-1)^n \frac{x^n}{n!(a+n)} \\ \Gamma(a,x) \rightarrow & \text{serie asintótica} & \Rightarrow \Gamma(a,x) \sim x^{a-1}e^{-x} \sum_{n=0}^{\infty} \frac{\Gamma(a)}{\Gamma(a-n)} \cdot \frac{1}{x^n} \end{array}$$

Cuando a=0

$$\Gamma(0,x) \sim \frac{e^x}{x}\sum_{n=0}^{\infty}(-1)^n\frac{n!}{x^n}$$

Beta Incompleta

$$B_x(p,q) = \int_0^x t^{p-1}(1-t)^{z-1}dt \quad 0 \leq x \leq 1, p > 0, q > 0$$

$$Si \quad x = 0 \Rightarrow B(p,q) \text{ función Beta}$$

Aparece en:

- Funciones hipergeométricas
- Probabilidades

Viernes 28 de julio

Lunes 31 de julio

Derivaci'on bajo el signo de la integral

Integral de Feynmann

$$\frac{d}{dt}\int f(x,t)dx = \int \frac{\partial(x,t)}{\partial t}dx$$

Derivaci'on de la funci'on Gamma

$$F(t) = \int_0^\infty e^{-tx}dx$$

$F(t) = \frac{1}{t}$ para $t > 0$

$$\begin{aligned} \frac{dF(t)}{dt} = F'(t) = & \int_0^\infty \frac{d}{dt}e^{-tx}dx = \frac{d}{dt}\left[\frac{1}{t}\right] \\ \Rightarrow & \int_0^\infty -xe^{-tx}dx = -\frac{1}{t^2} \\ \Rightarrow & \int_0^\infty x^n e^{-tx}dx = \frac{n!}{t^{n+1}} \end{aligned}$$

$n! = \int_0^\infty e^{-x}x^n dx$

$$n! = t^{n+1}\int_0^\infty x^n e^{-tx}dx = \int_0^\infty x^n e^{-x}dx$$

$$\begin{aligned} u &= tx \\ du &= tdx \end{aligned} \qquad \int_0^\infty \cancel{t^n} t \frac{u^n}{\cancel{t^n}} e^{-u}\frac{du}{\cancel{t}}$$

$$\int_0^1 \frac{x^2-1}{\ln(x)}$$

$$\begin{aligned} \frac{d}{dt}I(t) = & \int_0^1 \frac{\partial}{\partial t}\frac{x^t-1}{\ln(x)} \\ = & \int_0^1 \frac{1}{\ln(x)}\frac{\partial}{\partial t}[x^t-1]dx \\ = & \int_0^1 \frac{1}{\cancel{\ln(x)}}x^t\cancel{\ln(x)}dx \\ = & \int_0^1 x^t dx = \frac{1}{t+1}x^{t+1}\Big|_0^1 \\ = & \frac{1}{t+1} \\ \int \frac{d}{\cancel{dt}}I(t) = & \int \frac{1}{t+1}dt \\ I(t) = & \ln|t+1| + c \end{aligned}$$

si t=0

$$\begin{aligned} I(2) = & \ln|2+1| = \ln|3| \\ I(2) = & \int_0^2 I'(t)dt \\ = & \int_0^2 \frac{1}{t+1}dt = \ln|t+1|\Big|_0^2 \\ = & \ln|3| \end{aligned}$$

$$\int_{\mathbf{0}}^\infty \frac{\mathbf{e^{-x^2}sen(x^2)}}{\mathbf{x^2}}dx$$

$$\frac{d}{dx}sen(ax^2) = x^2cos(ax^2)$$

$$\begin{aligned}
\frac{d}{da}I(a) = &\quad \frac{\cancel{d}}{\cancel{da}}\int_0^\infty \frac{\partial}{\partial a}\frac{e^{-x^2}sen(x^2)}{x^2}dx \\
= &\quad \int_0^\infty \frac{e^{-x^2}}{\cancel{x^2}}\cancel{x^2}cos(ax^2)dx \\
I'(a) = &\quad \int_0^\infty e^{-x^2}cos(ax^2)dx \\
= &\quad Re\int_0^\infty e^{-x^2}e^{iax^2}dx \\
= &\quad Re\int_0^\infty e^{-x^2(1-ia)}dx \\
\int I'(a)da = &\quad \int \tfrac{1}{2}\sqrt{\pi}Re\left[\tfrac{1}{\sqrt{1-ia}}\right]da \\
I(a) = &\quad \frac{\sqrt{\pi}}{\cancel{2}}Re\left[\cancel{2}i\sqrt{1-ia}\right]+c
\end{aligned}$$

$$a=0 \Rightarrow sen(ax^2)=0 \Rightarrow c=0$$

a=1

$$I(1) = \sqrt{\pi}Re\left[i\sqrt{1-i}\right]$$

$1-i$ en forma polar: $re^{i\theta}$

$$\begin{aligned}
r = &\quad \sqrt{1^2+(-1)^2} = \sqrt{2} \\
\theta = &\quad tan^{-1}\left[\tfrac{-1}{1}\right] = -\tfrac{\pi}{4} \\
1-i = &\quad \sqrt{2}e^{-\frac{\pi}{4}i} \\
\sqrt{1-i} = &\quad \sqrt{\sqrt{2}}e^{-\frac{\pi}{8}i} \\
= &\quad \sqrt{\sqrt{2}}\left[cos\tfrac{\pi}{8} - isen\tfrac{\pi}{8}\right] \\
i\sqrt{1-i} = &\quad \sqrt{\sqrt{2}}\left[icos\tfrac{\pi}{8} - i^2sen\tfrac{\pi}{8}\right]
\end{aligned}$$

$$\int_0^\infty \left[\ln(\mathbf{x})\right]\left[\ln(\mathbf{tanh}(\mathbf{x}))\right]\mathbf{dx}$$

$$I(m) = \int_0^\infty x^m\ln(tanh(x))dx$$

a usar:

$$\frac{d}{dm}x^m = x^m\ln(x)$$

$$tanh(x) = \frac{e^x-e^{-x}}{e^x+e^{-x}} = \frac{1-e^{-2x}}{1+e^{-2x}}$$

$$\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$$

Viernes 04 de agosto Ej.

$$\int_0^\infty [\ln(x)]\,[\ln(tanh(x))]$$

$$\begin{aligned} I(m) = &\quad \int_0^\infty x^m \left[\ln(1-e^{-2x} - \ln(1+e^{-2x})\right] dx \\ = &\quad \int_0^\infty x^m \left[\sum_{n=1}^\infty \frac{(-1)^{n+1}(-e)^{-2nx}}{n}\right] dx - \int_0^\infty x^m \sum_{n=1}^\infty \frac{(-1)^{n+1}(e)^{-2nx}}{n} dx \end{aligned}$$

$$\boxed{\begin{aligned} tanh(x) =& \frac{e^x - e^x}{e^x + e^{-x}} \\ \frac{d}{dm} x^m =& x^m log(x) \\ tanh(x) =& \frac{1-e^{-2x}}{1+e^{-2x}} \\ \text{Serie de MacLaurin} & \\ \ln(1+x) =& \sum_{n=1}^\infty (-1)^{n-1}\frac{x^n}{n} \end{aligned}}$$

PEDIR COPIA

Funciones Bessel

$$x^2y'' + xy' + (x^2 - p^2)y = 0 \qquad \text{EC. Bessel de orden P.}$$

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = 0 \qquad \text{EC. Bessel de orden } \frac{1}{2}$$

M'etodo por series de Frobenius

$$y = \sum_{n=0}^\infty c_n x^{n+r}$$

$$xy'' + y' + xy = 0 \qquad \text{Ec. Bessel de orden 0}$$

$$y_2 = y_1 \ln(x) + x^{r_1+1} \sum_{n=0}^\infty b_n x^n$$

$$x\sum_{n=0}^\infty (n+r)(n+r-1)C_n x^{\overset{n+r-1}{n+r-2}} + \sum_{n=0}^\infty (n+r)C_n x^{n+r-1} + \sum_{n=0}^\infty C_n x^{\overset{n+r-1}{n+r}} = 0$$

$n=0$ $\qquad (r)(r-1)C_0x^{r-1} \qquad rC_0x^{r-1}$

$n=1$ $\qquad (1+r)(r)C_1x^r \qquad (1+r)C_1x^r$

$$\sum_{n=0}^\infty (n+r)(n+r-1)C_n x^{n+r-1} \qquad \sum_{n=0}^\infty (n+r)C_n x^{n+r-1}$$

lunes 07 d e agosto

$$\boxed{xy'' + y' + xy = 0}$$

$$y_1 = C_0 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

$$\begin{aligned} y_2 = {} & y_1 \ln(x) + x^{r_2+1} \sum_{n=1}^{\infty} b_n x^n \\ & \boxed{xy'' \ln(x)} + 2y_1' - \cancel{\tfrac{1}{x} y_1} + \sum_{n=1}^{\infty} n(n-1) b_n x^{n-1} + \\ & + \boxed{y_1' \ln(x)} + \cancel{\tfrac{1}{x} y_1} + \sum_{n=1}^{\infty} n b_n x^{n-1} + \\ & + \boxed{xy_1 \ln(x)} + \sum_{n=1}^{\infty} b_n x^{n+1} = 0 \end{aligned}$$

$$= 2 \sum_{n=1}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{2^{2n}(n!)^2} + \sum_{n=1}^{\infty} n(n-1) b_n x^{n-1} + \sum_{n=1}^{\infty} n b_n x^{n-1} + \sum_{n=1}^{\infty} b_n x^{n-1} = 0$$

$\underline{x^0}$: $\quad n = 1 \quad\quad 1(0)b_1 + 1b_1 = 0 \quad\quad \Rightarrow b_1 = 0$

$\underline{x^1}$: $\quad \dfrac{2(-1)(2)}{2^2(1!)^2} + 2(2-1)b_2 + 2b_2 = 0 \quad\quad \Rightarrow -1 + 2b_2 + 2b_2 = 0$

$$\Rightarrow b_2 = \frac{1}{4}$$

$\underline{x^2}$: Ya funciona para todas las series

$$2\sum_{n=2}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{2^{2n}(n!)^2} + \sum_{n=3}^{\infty} n(n-1) b_n x^{n-1} + \sum_{n=3}^{\infty} n b_j x^{n-1} + \sum_{n=1}^{\infty} b_n x^{n+1} = 0$$

$$2\sum_{n=2}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{2^{2n}(n!)^2} + \sum_{n=2}^{\infty} (n+1)(n) b_{n+1} x^n + \sum_{n=2}^{\infty} (n+1) b_{n+1} x^n + \sum_{n=2}^{\infty} b_{n-1} x^n = 0$$

$$2\sum_{n=2}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{2^{2n}(n!)^2} + \sum_{n=2}^{\infty} [n(n+1) b_{n+1} + (n+1) b_{n+1} + b_{n-1}] x^n = 0$$

$(n+1)^2 b_{n+1} \cancel{(n+1)} + b_{n-1}$

$$2\sum_{n=2}^{\infty} \frac{(-1)^n (2n) x^{2n-1}}{2^{2n}(n!)^2} + \sum_{n=2}^{\infty} \left[(n+1)^2 b_{n+1} + b_{n-1}\right] x^n = 0$$

$$\boxed{\text{n par} \to \text{n=2k}}$$

$(2k+1)^2 b_{2k+1} + b_{2k-1} \quad\quad$ para $k \geq 1$

$$b_{2k+1} = \frac{-b_{2k-1}}{(2k+1)^2}$$

<u>k=1</u>:

$$b_3 = \frac{-b_1}{3^2} = 0$$

<u>k=2</u>:

$$b_5 = \frac{-b_3}{5^2} = 0$$

$$\boxed{\text{n impar} \rightarrow \text{n=2k-1}}, k \geq 2$$

$$\frac{2(-1)^k 2k}{2^{2k}((k)!)^2} + (2k)^2 b_{2k} + b_{2k-2} = 0$$

$$(2k)^2 b_{2k} + b_{2k-2} = \frac{-(-1)^k 4k}{2^{2k}(k!)^2} \rightsquigarrow \left[(2k)^2 \frac{(-1)^{k+1} C_{2k}}{\underset{\star}{\cancel{2^{2k}(k!)^2}}} + \underbrace{\frac{(-1)^k C_{2k-2}}{\underset{\star\star}{2^{2k-2}((k-1)!)^2}}}_{\frac{2^2 k(k!)^2}{2^{2k-2}((k-1)!)^2} = 4k^2} = -\frac{(-1)^k 4K}{2^{2k}(k!)^2} \right] 2^2 k(k!)^2$$

suponemos:

$$\star b_{2k} = \frac{(-1)^{k+1} C_{2k}}{2^{2k}(k!)^2}$$

$$\underline{k=1} : b_2 = \frac{(-1)^2 C_2}{2^2(1!)^2} = \frac{C_2}{2^2} \Rightarrow C_2 = 1$$

$$\star\star b_{2k-2} = \frac{(-1)^k C_{2k-2}}{2^{2k-2}((k-1)!)^2}$$

entonces:

$$4k^2(-1)^k(-1)C_{2k} + (-1)^k C_{2k-2} 4k^2 = -(-1)^k 4k$$

$$c_{2k-2} - C_{2k} = \frac{-1}{k}$$

$$C_{2k} = C_{2k-2} + \frac{1}{k}$$

Buscando el patron:
<u>k=2</u>:

$$C_4 = C_2 + \frac{1}{2} = 1 + \frac{1}{2} = H_2$$

<u>k=3</u>:

$$C_6 = C_4 + \frac{1}{3} = \left[1 + \frac{1}{2}\right] + \frac{1}{3} = H_3$$

<u>k=4</u>:

$$C_8 = C_6 + \frac{1}{4} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = H_4$$

$$C_{2k} = H_k$$

$$\begin{aligned} b_{2k} =& \frac{(-1)^{k+1} C_{2k}}{2^{2k}(k!)^2} \\ =& \frac{(-1)^{k+1} H_k}{2^{2k}(k!)^2} \end{aligned}$$

$$\begin{aligned} y_2 =& J_0(x)\ln(x) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} H_n x^{2n-1}}{2^{2n}(n!)^2} \\ Y_o(x) =& \frac{2}{\pi}(\gamma - \ln(2))y_1 + \frac{2}{\pi} y_2 \end{aligned}$$

$$\gamma = \text{constante Euler-Mascheroni}$$

$$\gamma = \lim_{n \to \infty}(H_n - \ln(n)) = 0{,}577215$$

La soluci'on general

$$y = C_1 J_0(x) + C_2 Y_0(x)$$

$$x^2y'' + xy' + (x^2 - p^2)y = 0$$
$$\langle \text{EC. de Bessel de orden P} \rangle$$

$$y = \sum_{n=0}^{\infty} C_n x^{n+r}$$

$$(r-p)(r+p) = 0$$

3 casos:

1. $r_1 - r_2 \Rightarrow$ no es entero
2. $r_1 - r_2 \Rightarrow$ es entero
3. $r_1 = r_2 \rightsquigarrow$ p=0

Si $r_1 - r_2$ NO ES ENTERO

$$y_1 = C_0\Gamma(p+1)\sum_{n=0}^{\infty} \frac{(-1)^k}{k!\Gamma(p+k+1)}\left(\frac{x}{2}\right)^{2k+p}$$

$$J_p(x) = \sum_{n=0}^{\infty} \frac{(-1)^k}{k!\Gamma(p+k+1)}\left(\frac{x}{2}\right)^{2k+p}$$

$$= C_1 J_p(x + C_2 J_{-p}(x))]$$