Metodos matemáticos 2

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1. Función Gamma

L'imite al infinito(Euler)

$$
\begin{aligned}
\Gamma(z) \equiv & \lim_{n\to\infty} \tfrac{1\cdot 2\cdot 3\cdots n}{z(z+1)(z+2)\cdots(z+n)} n^z, \quad z \in \mathbb{Z}^+ \ o \ z \in \mathbb{C} \\
\Gamma(z+1) = & \lim_{n\to\infty} \tfrac{1\cdot 2\cdot 3\cdots n}{(z+1)(z+2)\cdots(z+n+1)} n^{z+1} \\
= & \ z\cdot\Gamma(z) = \lim_{n\to\infty} \tfrac{nz}{z+n+1}\Gamma(z) = \Gamma(z+1) = z\cdot\Gamma(z)
\end{aligned}
$$

Aplicando lo anterior a z=1,2,3...n

$$
\begin{aligned}
\Gamma(1) = & \lim_{n\to\infty} \tfrac{1\cdot 2\cdot 3\cdots n}{(1)(2)\cdots(n+1)} n^z = \lim_{n\to\infty} \tfrac{n}{n+1} = 1 \\
\Gamma(2) = & \ z\cdot\Gamma(z) = \Gamma(z+1) = \Gamma(1+1) = 1 \\
\Gamma(3) = & \ 2\cdot\Gamma(1) = \Gamma(2+1) = 2\cdot 1 \\
\Gamma(4) = & \ 3\cdot\Gamma(3) = \Gamma(3+1) = 3\cdot 2\cdot 1 \\
& . \\
& . \\
& . \\
\Gamma(n) = & \ (n-1)!
\end{aligned}
$$

Itegral Definida(Integral de Euler)

$$\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt, \quad Re(z) > 0$$

Ej. como aparecen en f'isica

$$\Gamma(z) = 2\int_0^\infty e^{-t^2} t^{z+1} dt \quad o \quad \Gamma(z) = \int_0^1 \left[ln(\frac{1}{t}) \right]^{z-1} dt$$

Si $z = \frac{1}{2} \Rightarrow \Gamma(\frac{1}{2}) = \sqrt{\pi}$ es integral error de Gauss

$$F(z,n) = \int_0^n \left(1 - \frac{t}{n}\right)^n t^{z-1} dt, \quad Re(z) > 0 \ con \ n \ entero \ positivo \ni$$

$$e^{-t} = \lim_{n\to\infty} \left(1 - \frac{t}{n}\right)^n$$

$$\Rightarrow F(z,n) = \int_0^n \left(1 - \frac{t}{n}\right)^n t^{z-1} dt = \int_0^1 (1-u)^n (un)^{z-1} n du \ni u = \frac{t}{n}$$

$$\frac{F(z,n)}{n^z} = \int_0^1 (1-u)^n (u)^{z-1} du$$

Ahora por integraci'on por partes usando $u = (1-u)^n$, $du = n(1-u)^{n-1}du$, $v = \frac{u^z}{z}$, $dv = u^{z-1}du$ tenemos

$$\begin{aligned}
\frac{F(z,n)}{n^z} =& \quad (1-u)^n \left.\frac{u^z}{z}\right|_0^1 - \int_0^1 \frac{u^z}{z} n(1-u)^{n-1}du \\
F(z,n) =& \quad n^z \cdot \frac{n(n-1)\cdots 1}{z(z+1)\cdots(z+n-1)} = \int_0^1 u^{z+n-1}du \\
=& \quad \frac{1\cdot 2\cdots n}{z(z+1)\cdots(z+n)} n^z \Rightarrow \lim_{n\to\infty} F(z,n) = \Gamma(z)
\end{aligned}$$

Producto infinito(Weierstrass)

$$\frac{1}{\Gamma(z)} = ze^{\delta z} \prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) e^{-\frac{z}{n}}$$

$\delta :=$ constante de Euler-Mascheron

$\delta := 0.5772156619$

Ecuaci'on estad'istica de Maxwell-Boltzmann

$e^{-E/KT}$

K es la constante de Boltzmann

T es la temperatura absoluta

E(energ'ia): estado de energ'ia ocupada

probabilidad de estar en estado de energ'ia es $Y_{kt} = \beta$

$P(E) = Ce^{-\beta E}$	*Para un gas idel sin estructura*
$n(E)dE$	$n(E)^{1/2}$
$1 = C\int n(E)e^{-\beta E}dE$	*E=energ'ia cin'etica*

$$\begin{aligned}
1 =& \quad c\int_0^\infty E^{1/2}e^{-\beta E}dE = \frac{C\Gamma(3/2)}{\beta^{3/2}}, \quad \beta E = T \Rightarrow dE = \frac{dt}{\beta} \\
1 =& \quad C\int_0^\infty e^{-t}\left(\frac{t}{\beta}\right)^{1/2}\frac{1}{\beta}dt \\
& \quad = C\int_0^\infty e^{-t}t^{3/2-1}dt \cdot \frac{1}{\beta^{3/2}} \\
=& \quad \frac{C\Gamma(3/2)}{\beta^{3/2}} \\
=& \quad c \cdot \frac{\sqrt{\pi}}{2\cdot\beta^{3/2}}
\end{aligned}$$

$$\therefore C = \frac{2\cdot\beta^{3/2}}{\sqrt{\pi}}$$