WSKmaths — Useful mathematical definitions.

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This package ensures that a number of useful mathematics packages are loaded, provides a number of useful theorem-like environments, and defines a number of significant mathematics macros. The concluding section offers a few general remarks about writing in LaTeX.

1 Packages

The mathematics packages loaded by this package are

- `amsmath`,
- `amsfonts`,
- `amsthm`,
- `framed`,
- `caption`,
- `xspace`.

It is worth looking at the documentation for `amsmath` (ftp://ftp.ams.org/pub/tex/doc/amsmath/amsldoc.pdf) and `amsthm` (ftp://ftp.ams.org/pub/tex/doc/amscls/amsthdoc.pdf); depending on your writing style, some constructions in these might save much time and effort.

2 Theorem-like environments

The package defines theorem-like environments as follows, all numbered consecutively as a whole:

- in theorem-style:
  - `thm` (theorems),
  - `prop` (propositions),
  - `lem` (lemmas),
  - `cor` (corollaries),
  - `exercise` (exercises);
- in separate styles:
  - `defn` (definitions, in definition style),
  - `rem` (remarks, in remark style).
  - `qn` (questions, in remark style).

Proofs should be enclosed in a `\begin{proof}` ... `\end{proof}` environment.

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3 Mathematical macros

It is convenient to replace commonly used mathematical constructions by LaTeX macros. This makes it easier to maintain notational consistency, and also it is then easier to change notation if (for example) it becomes apparent that notation clashes. The package defines these macros:

`\CFTP` • The Coupling from the Past algorithm is abbreviated by *CFTP*: use `\CFTP` (text mode). This and the following two textual macros do not save much typing, but facilitate textual consistency and coherent typographical style.

`\Ito` • For Itô (as in the Itô differential), use `\Ito` (text mode).

`\MCMC` • Abbreviate Markov chain Monte Carlo by *MCMC*: use `\MCMC` (text mode).

`\Borel` • Denote the Borel σ-algebra by $\mathfrak{B}$: use `\Borel`.

`\Expect` • Write $\mathbb{E}[X]$ for the expectation of the random variable X: use `\Expect{X}`. The enclosing square brackets will change size to encompass the argument. (For multi-line expectations, you'll have to do this the hard way, using `\mathbb{E}\Big[...\Big]`, or variants.)

`\Indicator` • Write $\mathbb{I}[A]$ for the indicator of the event A: use `\Indicator{A}`. The enclosing square brackets will change size to encompass the argument. Note that Williams (1991)'s notation $\mathbb{E}[X\ ;\ A]$ (`\Expect{X\;;\;A}`) is more concise and ultimately more readable than the equivalent $\mathbb{E}[X\ \mathbb{I}[A]]$. For conditional expectation use `\Expect{X\;|\;Y}`.

`\Integers` • Denote the set of integers by $\mathbb{Z}$: write `\Integers`.

`\Hess` • Denote the Hessian (covariant second derivative) of a function f in directions U and V by $\mathrm{Hess}\, f(U,V)$: use `\Hess f(U,V)`.

`\Law` • Write $\mathcal{L}(X)$ for the law of the random variable X: use `\Law{X}`.

`\Leb` • Write $\mathrm{Leb}(A)$ for the Lebesgue measure of a set A: use `\Leb(A)`.

`\Numbers` • Denote the set of natural numbers by $\mathbb{N}$: use `\Numbers`.

`\Prob` • Write $\mathbb{P}[A]$ for the probability of the event A: use `\Prob{A}`. The enclosing square brackets will change size to encompass the argument. (For multi-line probabilities use `\mathbb{P}\Big[...\Big]` *etc.*) For conditional probability use `\Prob{A\;|\;Y}`.

`\Rationals` • Denote the set of rational numbers by $\mathbb{Q}$: use `\Rationals`.

`\Reals` • Denote the set of real numbers by $\mathbb{R}$: use `\Reals`.

`\Var` • Write $\mathrm{Var}[X]$ the variance of the random variable X: use `\Var{X}`. The enclosing square brackets will change size to encompass the argument.

`\Cov` • Write $\mathrm{Cov}[X,Y]$ the covariance of the random variables X and Y: use `\Cov{X,Y}`. The enclosing square brackets will change size to encompass the argument.

`\ball` • The metric ball of centre x and radius r is denoted by $\mathrm{ball}(x,r)$: use `\ball(x,r)`. In exposition one should always be clear about whether one means the open or the closed ball!

`\origin` • The origin of Euclidean space is denoted by $\mathbf{o}$: use `\origin`.

`\d` • Some publishers (eg Cambridge University Press) advise that the differential notation in the integral $\int f(x)\,\mathrm{d}\,x$, or derivative $\frac{\mathrm{d}}{\mathrm{d}\,x}f(x)$ should use a specially defined symbol `\d`, as in `\int f(x) \d{x}` or `\tfrac{\d}{\d{x}}f(x)`. Note that this overloads an original LaTeX definition which uses `\d` to generate a dot below a symbol. Note also the use of a thin space `\,` before `\text{d}` in the definition of `\d`. This can clarify complicated expressions.

\dist
- Denote the metric distance between two points x and y by $\mathrm{dist}(x, y)$: use `\dist(x,y)`. Note that the alternative notation `d(x,y)` could lead to a clash with notation for differentials.

\eps
- An arbitrarily small positive scalar can be denoted by ε: use `\eps`.

\grad
- Denote the gradient of a function f by $\mathrm{grad}\, f$: use `\grad f`.

\half
- Write the fraction of one half as $\frac{1}{2}$: use `\half`. Note that `\mathchoice` is used to obtain different behaviour in superscript $a^{\frac{1}{2}}$, double superscript $a^{a^{\frac{1}{2}}}$, or displayed formulae:

$$\text{kinetic energy} \quad = \quad \frac{1}{2} v^2 \,.$$

  If required, other fractions can be added to emulate this style.

\sgn
- The sign of x can be written as $\mathrm{sgn}(x)$: use `\sgn(x)`. Be clear about the value of $\mathrm{sgn}(0)$!

4 General remarks

These remarks are aimed at making the whole process of LaTeX composition easier. I strongly advise you to read this thoughtfully, and change your writing habits accordingly. It may seem quicker and easier to dive straight in and write your LaTeX paper without worrying about these details. If you do this, you will end up working *much* harder in the long run. Your choice!

First of all, it helps to reduce the number of pages used by a document. (Every time you have to turn a page, you are going to have to make a continued mental effort to relate the mathematics on the current page with what came before!) In your LaTeX preamble, use `\usepackage{a4wide}` to ensure pages are typeset without the absurdly wide margins which are the default for LaTeX. Some writers allege that `a4wide` is deprecated (http://latex-community.org/forum/viewtopic.php?f=47&t=18598): however there appears to be no succinct alternative.

It is helpful to remember that much time will be spent staring at LaTeX source, and therefore it is worth spacing and laying this source out to make it easy to read and to debug! For example, I lay out equation-type environments with plenty of line-breaks, particularly taking care to ensure the "left-hand side" and the "right-hand side" are on separate lines.[1] [2] Here is an example:

$$\frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} \,\mathrm{d}\, u \quad \leq \quad \frac{1}{x} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \,.$$

The corresponding LaTeX code is

```
\[
  \frac{1}{\sqrt{2\pi}} \int_x^\infty          e^{-u^2/2} \d{u} \quad\leq\quad
  \frac{1}{x}           \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \,.
\]
```

Notice that I also ensure that the major relational operator in an equation ("=", "≥", "≤", ...) is bracketed by `\quad` spaces. This helps the reader to see immediately what the equation is all about, both in the final PDF and also in the source. Notice also how punctuation in a formula is preceded by a thin space `\,`. (And notice that one *should* add punctuation to ensure that the formula reads as part of the sentence.) Mathematics is always difficult: there is no need to make it harder by ignoring these small but systematic points.

Note also the use of `\[...\]` instead of `$$...$$` (similarly `\(...\)` instead of `$...$`). which protects against some otherwise confusing typos,[3] Also make liberal use of line-breaks to clarify

[1] But, of course, making sure no blank lines occur, which would trigger LaTeX errors.

[2] On the subject of footnotes, *avoid* using them wherever they might be confused with a superscript!

[3] Because LaTeX then can report more precisely *where* you opened a LaTeX mathematics mode without closing the previous one. At the cost of an easily-learned extra keystroke, you can save whole hours of frustration tracking down LaTeX bugs.

the text. In fact it is a good rule, when typing LaTeX source, to follow each full-stop and each major punctuation mark by a line-break. This aids clarity when reading the LaTeX source (a task which one performs continually when writing and editing a document), makes it easier to check which text has changed between one version and the next when using a file difference utility such as `diff` or its visual equivalents `vdiff` or `kdiff3`, and also works better if you are using a LaTeX editing environment such as *Kile* http://kile.sourceforge.net/ or *TeXnicCentre* http://www.texniccenter.org/ or *TeXshop* http://pages.uoregon.edu/koch/texshop/, any of which will compile LaTeX and then jump to the appropriate line in the previewer. (And there are *many* other advantages of using such an editing environment!)

I noted above, `amsmath` has a variety of excellent mathematics display environments: get familiar with them! Two of the most useful are `multline` for multi-line single equations, and `align` for aligned groups of equations. (But don't forget the use of `\nonumber` to suppress equation numbers for all but perhaps one of the aligned equations.)

While writing LaTeX, stay aware of the possibility of defining commonly used quantities using LaTeX macros. This makes it possible to change notation globally. Suppose you speak a great deal about a Soboloev space W^2 and then decide you want to change to $L^{2,1}$, for example. This is easy if you have used `\Sobolev`, coupled with `\newcommand{\Sobolev}{W^2}` in your preamble.

It is useful to follow a consistent pattern when labelling equations, sections, *etc.* Consider

$$\text{Poisson}(\lambda) \quad = \quad \frac{\lambda^n e^{-\lambda}}{n!} \qquad \text{for } n = 0, 1, 2, \ldots . \tag{1}$$

eq:poisson

This equation (1) is written as

```
\begin{equation}\label{eq:poisson}
  \text{Poisson}(\lambda) \quad=\quad
    \frac{\lambda^n e^{-\lambda}}{n!}
    \qquad\text{ for }n=0,1,2,\ldots\,.
\end{equation}
```

and is conveniently referred to by use of `\eqref{eq:poisson}`. Note that `\eqref` automatically surrounds the reference with brackets!

Similarly, for example, use `\label{thm:main-result}` for an instance of the theorem-like environment `thm`, and `\label{sec:intro}` for an introductory section.

Figures are made easier by the `Figure` environment. Label figures by `\label{fig:homer}`, *etc*, taking care to label *within* the `caption` content!). Note that it is often convenient to store all graphics in a subdirectory `image`. The preamble command `\graphicspath{{./image/}}` allows one to dispense with `image/` when referring to an graphic in `includegraphics`.

Figure

```
\begin{Figure}
  \includegraphics[width=3in]{under-construction}
    \caption{Example of simple figure.
      \label{fig:homer}}
 \end{Figure}
```

When drafting a paper, consider using `\usepackage{showkeys}`. This `usepackage` invocation must occur *after* the invocations of `\usepackage{amsmath}` and `\usepackage{hyperref}`. As seen above, this will, for example, place `eq:poisson` in the margin by the relevant equation *et cetera* and will annotate references (1) appropriately.

eq:poisson

Finally, a similar consistent labelling is useful for BibTeX references; consider for example this reference Goossens, Mittelbach, and Samarin (1994), which is labelled using my preferred `Authors-year` style[4] as follows: `\citet{GoosensMittelbachSamarin-1994}`. Perhaps this seems tedious to you? But (a) good LaTeX editing environments can be set up to auto-complete

[4] Why preferred? because I can then immediately and unambiguously check I have cited the correct reference in the PDF.

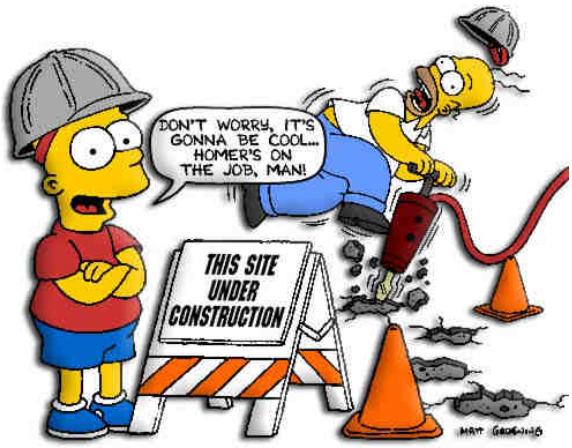

fig:homer

Figure 1: Example of simple figure.

`\citet{Goos...}`, (b) it is easy to copy-and-paste `GoosensMittelbachSamarin-1994` from the `.bib` file entry (see below), (c) you can then easily and consistently identify the cited reference from the source as well as the PDF output.

The `.bib` file contains this entry in the following form:

```
@book{GoosensMittelbachSamarin-1994,
  author = {Goossens, Michel and Mittelbach, Frank and Samarin, Alexander},
  booktitle = {Human Factors},
  pages = {528},
  title = {{The LaTeX Companion}},
  publisher = {Addison-Wesley},
  address = {reading, Massachusetts},
  year = {1994}
}
```

I prefer to use `\usepackage{natbib}` with a name-date system (the default for `natbib`). This offers considerable flexibility, for example using `\citep` for a reference in parentheses (Goossens et al., 1994). See the `natbib` quick reference for more details: http://merkel.zoneo.net/Latex/natbib.php. I have modified `\usepackage{showkeys}` to `\usepackage[notcite]{showkeys}`, so as to avoid annotating citations with keys, redundant if one constructs citations as `Authors-year`.

It is beyond the scope of this note, but be aware that one can save huge amounts of time by using a bibliography manager such as *Mendeley* http://www.mendeley.com/. My set-up allows collection of references from the web using *Zotero* https://www.zotero.org/ with a single click. These then transfer automatically to *Mendeley*. When dragged into a particular *Mendeley* group, they are stored in BibTeX form in various `.bib` files which can then be used directly by my various writing projects. Get this set up correctly, and you can then easily add references in final form *as you are drafting the LaTeX*. Why might you want to do this? (a) Completing the bibliography is a tedious chore at the end of a writing project, but easily done bit-by-bit when writing; (b) A steadily increasing bibliography is a great motivator when writing!

Saving time and effort in all these ways makes mathematical writing much easier.

References

bachSamarin-1994 Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, reading, Massachusetts, 1994.

Williams-1991 David Williams. *Probability with martingales*. Cambridge Mathematical Textbooks. Cambridge University Press, Cambridge, 1991. ISBN 0-521-40455-X; 0-521-40605-6.

5 Index

Numbers written in italic refer to the page where the corresponding entry is described; numbers underlined refer to the code line of the definition; numbers in roman refer to the code lines where the entry is used.