

Perfect Epidemics

Seminar at University College Dublin

W S Kendall S B Connor

Warwick, York

20 October 2025



Introduction

Homage to Dublin
(Book of Kells, 9th century)



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Work on perfect simulation ([CFTP](#)) for epidemics, now being written up.
WSK acknowledges the support of UK EPSRC grant EP/R022100.



Handout is on the web: use the QR-code or visit
wilfridskendall.github.io/talks/PerfectEpidemics.

Plan of talk

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Gregory: Is there any other point to which you would wish to draw my attention?

Holmes: To the curious incident of the dog in the night-time.

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- (e) Example with real data.

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- ➍ Simplest possible example: *random-walk-CFTP*
(can boost to use Ising model to do simple image reconstruction).

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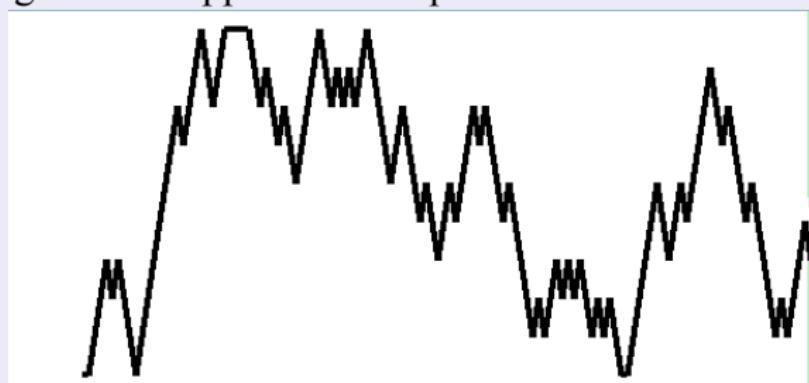
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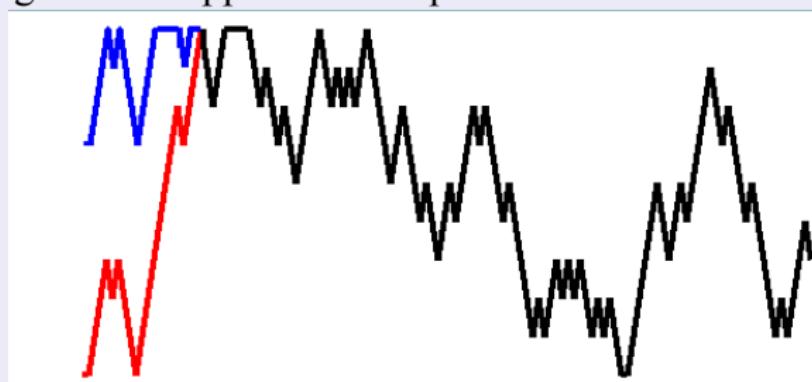
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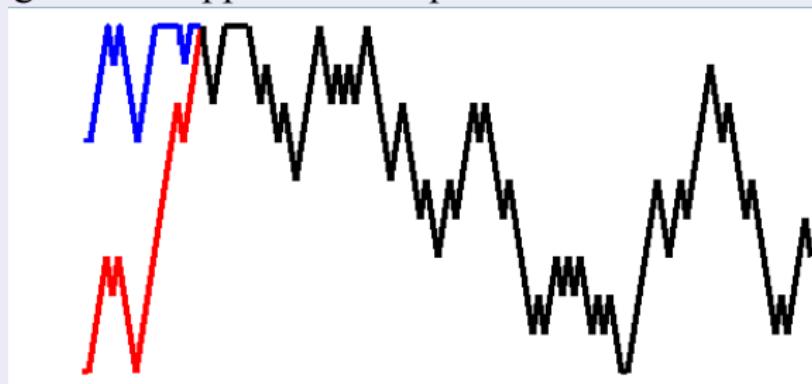
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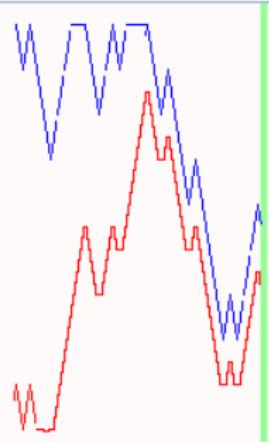
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- ④ Generally **not true** that location *at coupling* is a draw from equilibrium.

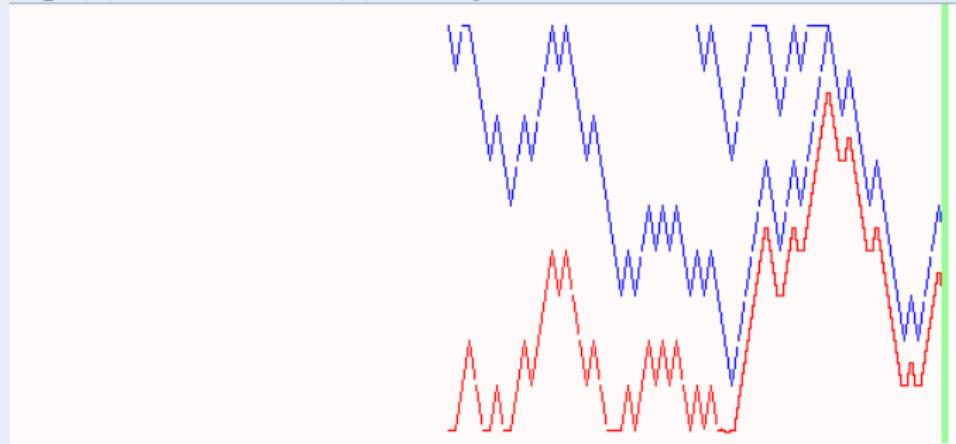
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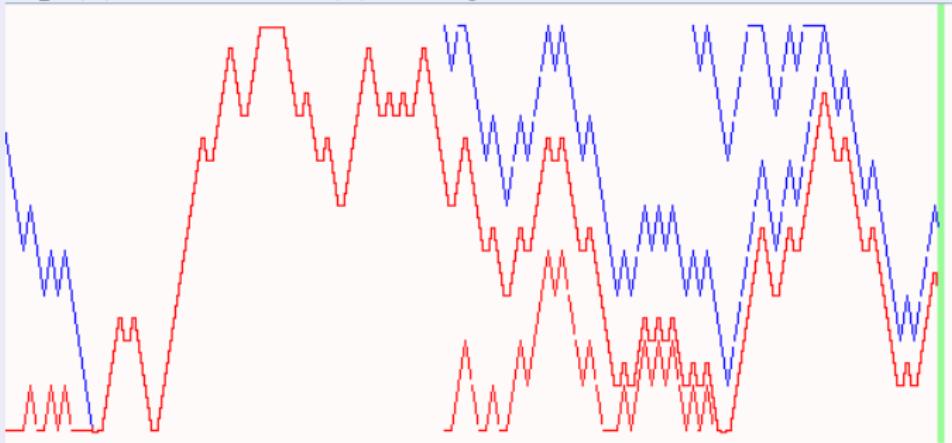


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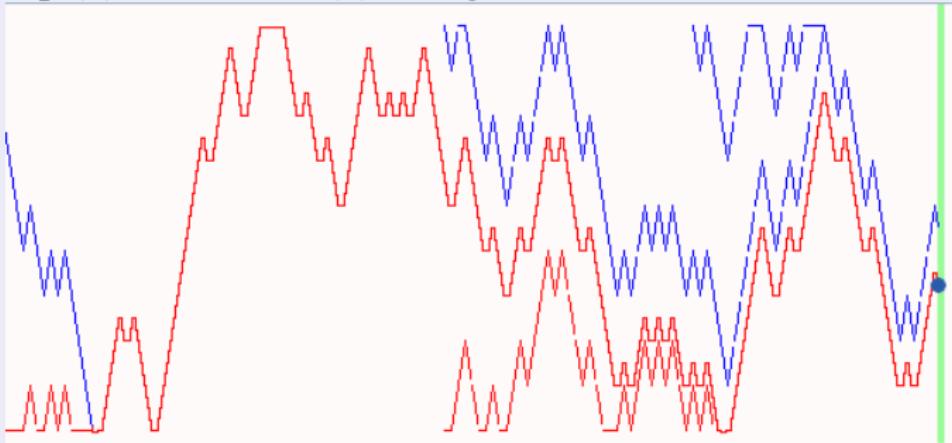
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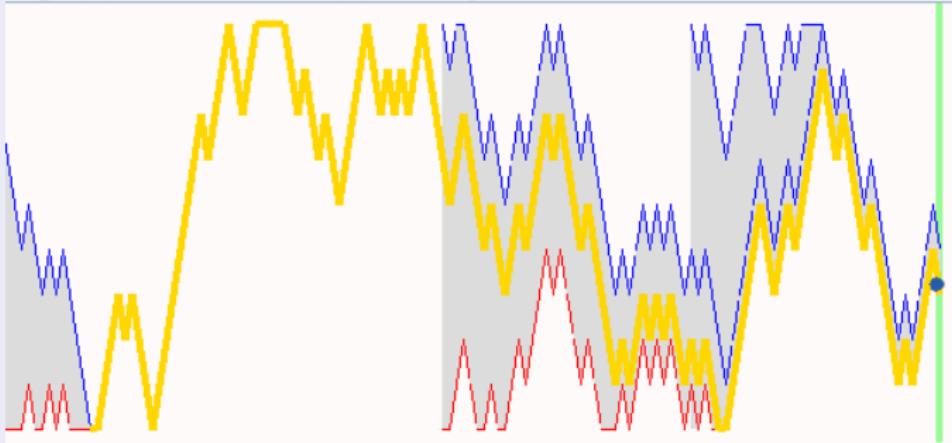
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- ⑤ The common value (golden thread) is an exact draw from equilibrium!

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- ③ Dominated CFTP delivers perfect simulation for stable point processes (WSK & Møller, 2000);
- ④ Detailed expositions: WSK (2005), Huber (2015).
(Want to implement CFTP in R? see WSK, 2015.)

2. Perfect Epidemics: a challenge problem for CFTP

S-I-R deterministic epidemic:

system of differential equations using susceptibles s , infectives i , removals r

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Both make an unrealistic assumption: homogeneous mixing.

In contrast, Fraser *et al* (2023) use a UK model with $N=10^6$ agents!

There are many important inferential questions (Cori & Kucharski, 2024).



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Wikipedia: “The British-registered *Diamond Princess* was the first cruise ship to have a major [COVID-19] outbreak on board, with the ship quarantined at Yokohama from 4 February 2020 for about a month. Of 3711 passengers and crew, around 700 people became infected and 9 people died.”

Evidently $\alpha s_0 / \beta \gg 1$ – as was sadly later confirmed, a sorrow for us all.



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- ➍ Markov chain Monte Carlo (MCMC) can be used (see next slide) but what about burn-in?
- ➎ Can we use **perfect simulation**?

An easier question: “Contact Tracing”

The simplest possible variant of contact tracing:

“When did the infections occur, supposing we only observe removals?”

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- ④ First step: evolve whole **S-I-R** trajectory in *algorithmic time* (alter potential infections and removals using immigration-death in discrete algorithmic time).
- ⑤ Result: *trajectory-valued chain*, unconditioned **S-I-R** as equilibrium.

From incidents to unconditioned epidemic trajectories (1/3)

Incidents defining an epidemic

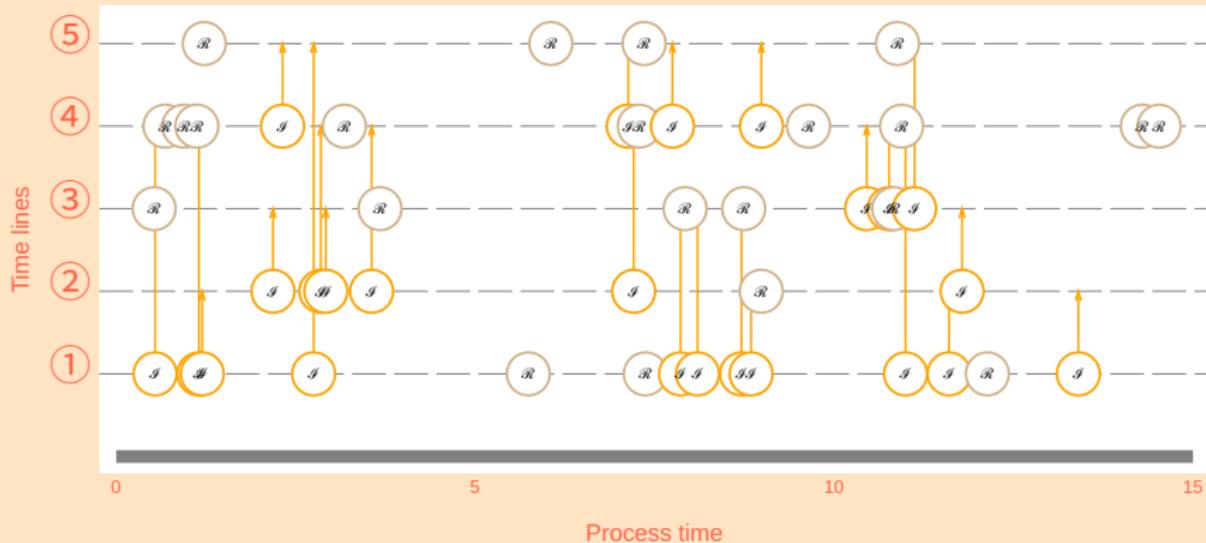


Figure 1: Light-orange circles denote potential infections (arrows point upwards to targets); light-brown circles denote potential removals.

From incidents to unconditioned epidemic trajectories (2/3)

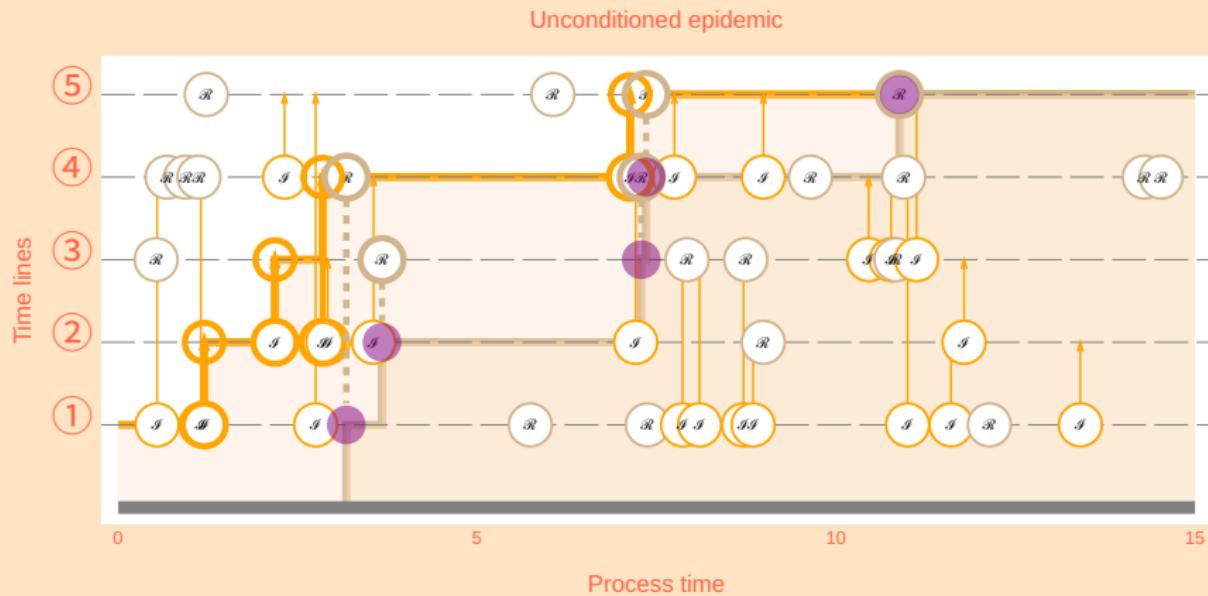


Figure 2: (a) *Infections* activate if on infected timeline and pointing to lowest uninfected timeline; (b) *Removals* activate if on infected timeline; remove lowest infected (purple disk).

From incidents to unconditioned epidemic trajectories (3/3)

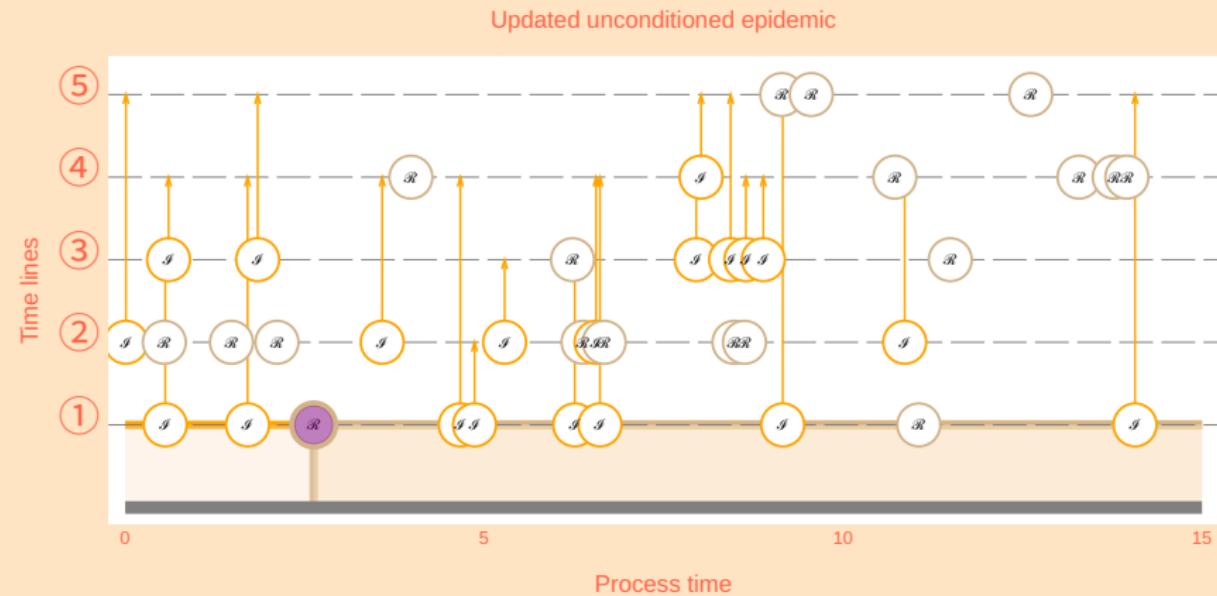


Figure 3: A step in algorithmic time for the unconditioned epidemic simply involves replacing all original incidents by an entirely new set of incidents.

Crucial technical point

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- Thus the original update is expressible as a (continuous) composition of updates, each of which satisfies detailed balance in equilibrium.
- The connection “restriction=conditioning” still holds.
- Crucially, re-sampling step 2 ensures composite evolution is irreducible over S ! (So equilibrium under conditioning is unique.)

Free evolution evolving in continuous algorithmic time

GIF MP4



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- Does this produce a *feasible* and suitably *monotonic* algorithm?
- **Housekeeping details** used to establish that monotonicity still works: *laziest feasible epidemic (LFE)* and *no-fly zone (NFZ)*.

Initial conditioned epidemic

The initial conditioned epidemic

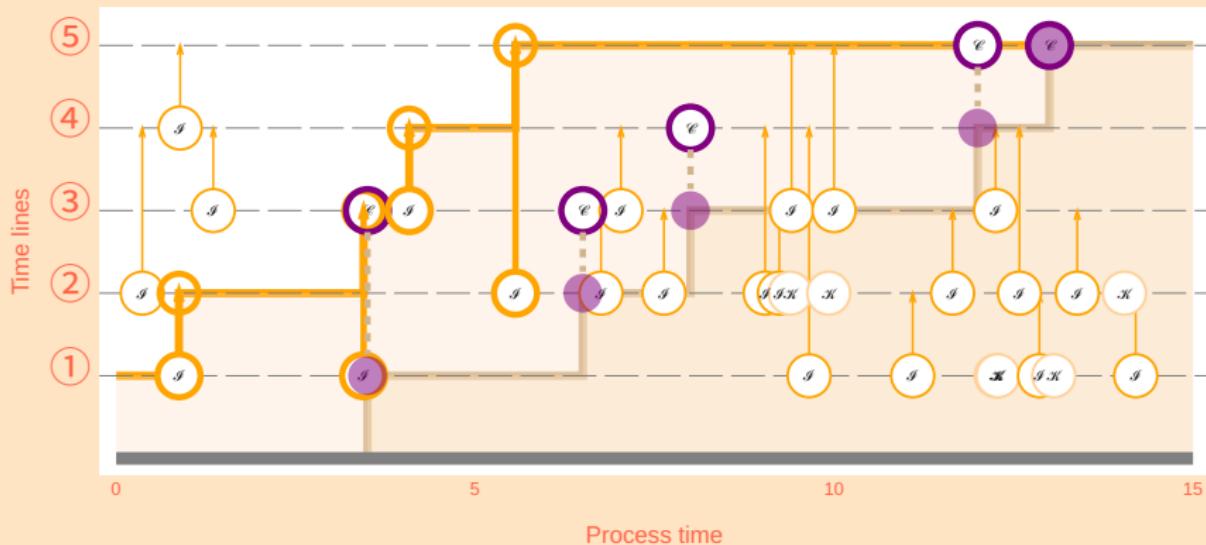


Figure 4: Initial conditioned epidemic, with conditioned removals indicated using purple circles (and purple disks when non-target timelines are infected).

Conditional epidemic update

Fully updated conditioned epidemic

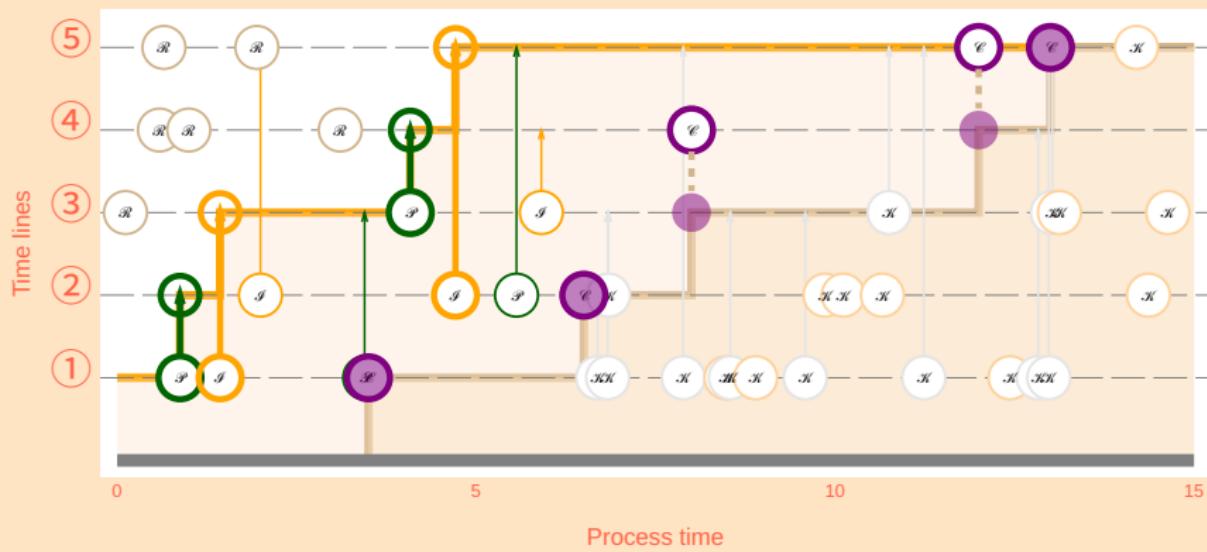


Figure 5: Epidemic updated under restriction: all conditioned removals remain activated, no new removals are activated. Green infections have been “perpetuated”.

Laziest feasible epidemic (LFE)

Fully updated conditioned epidemic with LFE

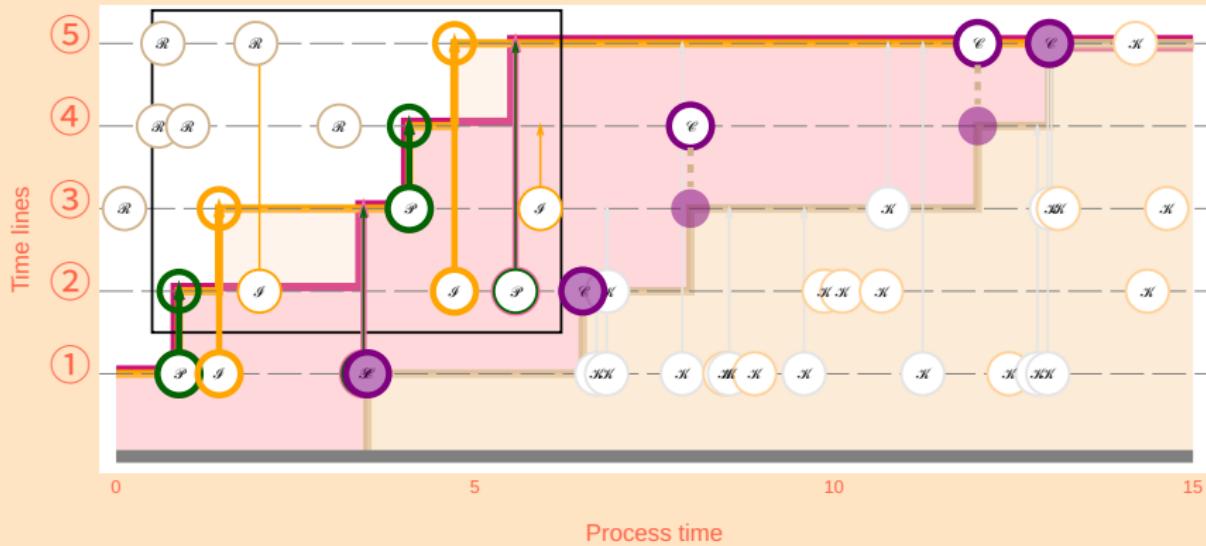
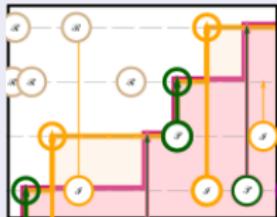


Figure 6: LFE computed recursively working right-to-left: slowest sequence of infections (and perpetuated infections) generating all conditioned removals. Can be used to identify perpetuated infections.

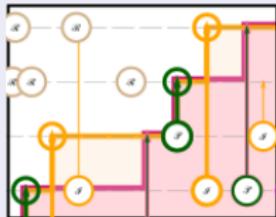
LFE: construction details



- ① Recursive definition of LFE: working over $[0, T)$,

$$\begin{aligned}s_N &= T \\ s_i &\leq \min \{s_{i+1}, \inf \{s : \text{there is a } \mathcal{C}_s^i\}\}.\end{aligned}$$

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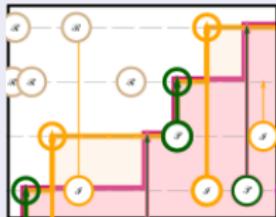
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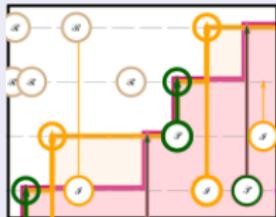
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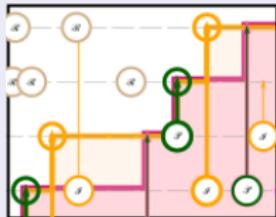
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- ③ Comparisons based on intrinsic definition show monotonic dependence of LFE on old epidemic history.

Fully updated conditioned epidemic with NFZ

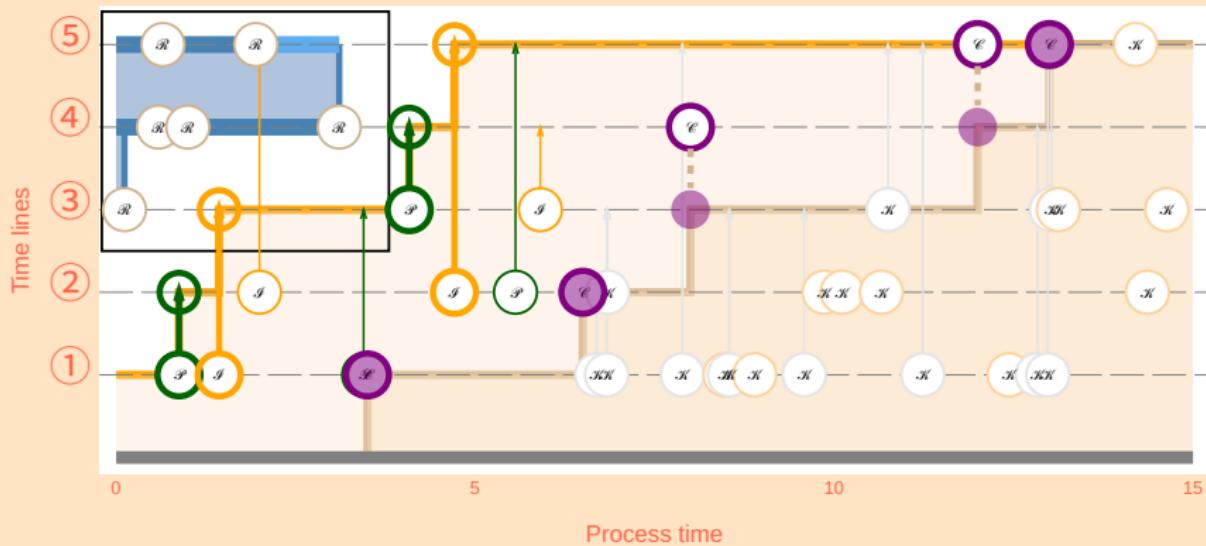
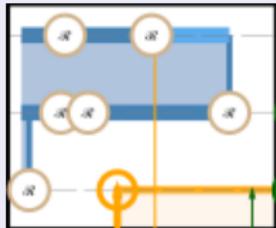


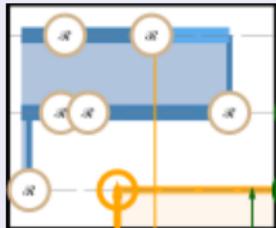
Figure 7: NFZ computed recursively working right-to-left: it traces a region of timelines such that unobserved removals are not activated if region not infected.

NFZ: construction details



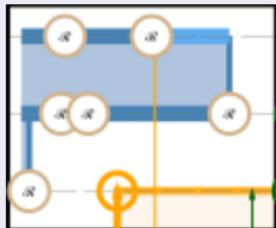
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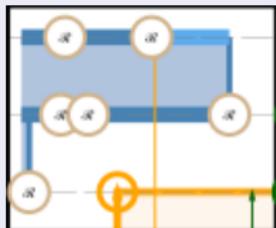
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- ➋ Can then show monotonic dependence of NFZ on old epidemic history.

GIF MP4

If a new $\mathcal{I}_t^{i < j}$ has i, j in infected zone then LFE is relevant;
if i in infected zone and j in susceptible zone then NFZ is relevant.

New epidemic is monotonic in LFE, NFZ!

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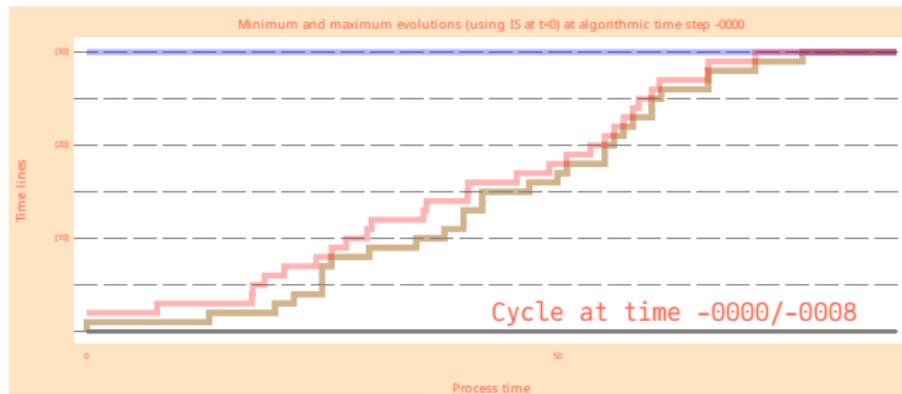
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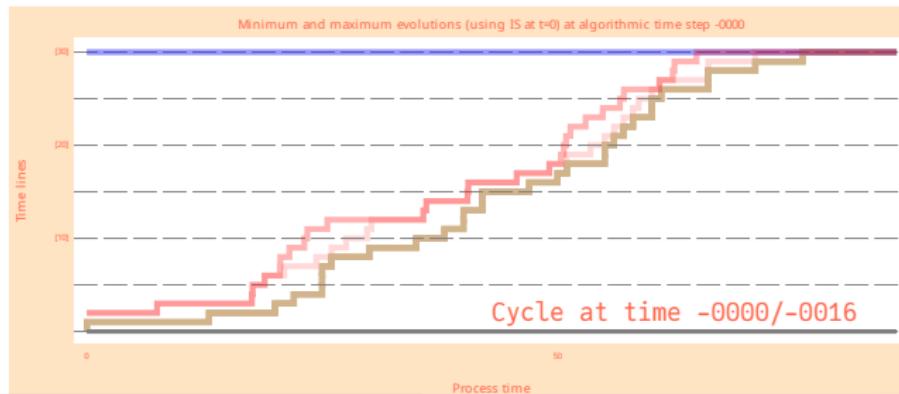
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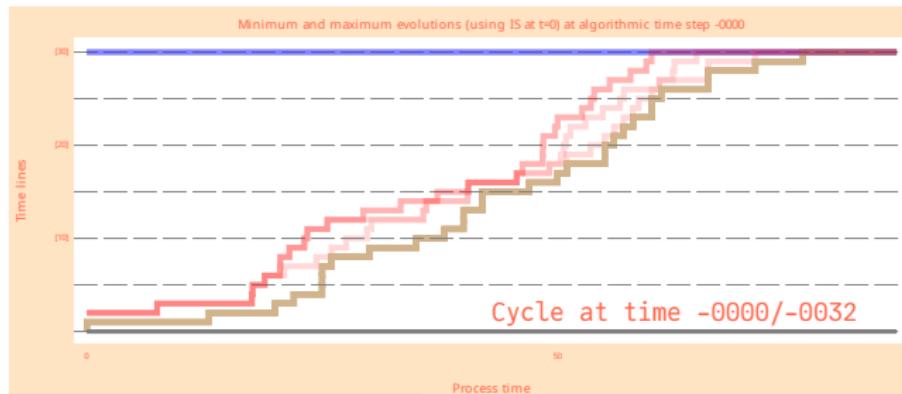
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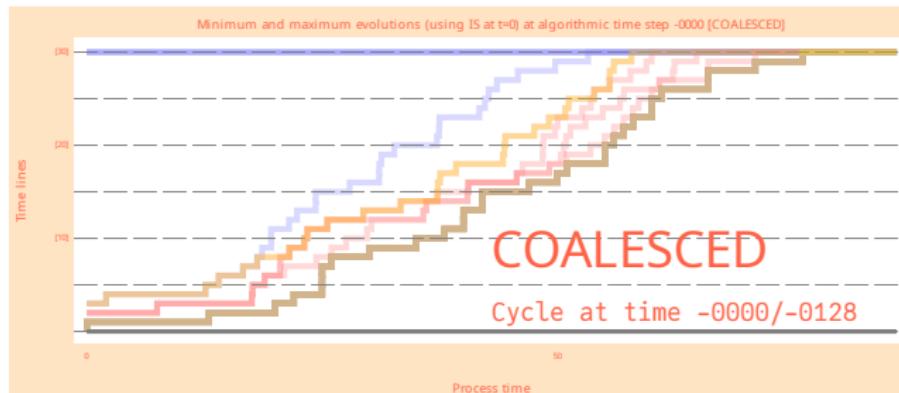
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- **Finally:** generalize to other suitable compartment models?

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- Thank you for your attention! **QUESTIONS?**



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Image information

<i>Image</i>	<i>Attribution</i>	
<i>Book of Kells</i>	Huber Gerhard	<i>CC BY 4.0</i>
Classic CFTP for a simple random walk	Result of code written by WSK	
<i>Diamond Princess</i>	Alpsdake	<i>CC BY-SA 4.0</i>
Epidemic CFTP images and animation	Result of code written by WSK	

Previous instances of this talk

<i>Date</i>	<i>Title</i>		<i>Location</i>
19/04/24	Perfect Epidemics	Short Research Talk	12mn Warwick
15/05/24	McMC+Perfect Simulation	Graduate Seminar, Aristotle Univ.	50mn Thessaloniki
17/01/25	Perfect Epidemics	Applied Probability Seminar	50mn Warwick
27/06/25	Perfect Epidemics	UK Research Network Stochastics	45mn Liverpool
20/10/25	Perfect Epidemics	Seminar	Dublin

Other technical information

Software used in computations

<i>Software</i>	<i>Version</i>	<i>Branch</i>	<i>Last commit</i>
quarto	1.6.39	—	
Running under julia	1.12.0	—	
EpidemicsCFTP	2.2.532	develop	Tue Jul 8 17:13:42 2025 +0100
EpidemicsUtilities	0.1.2.177	main	Fri Sep 26 15:35:26 2025 +0100
This quarto script	0.2.2.725	2025-10-09-Dublin-preparation	Tue Oct 14 18:01:39 2025 +0100

Project information

Version: 0.2.2.725 (2025-10-09-Dublin-preparation)

Author: Wilfrid Kendall <W.S.Kendall@warwick.ac.uk>

Date: Tue Oct 14 18:01:39 2025 +0100

Comment:

Near-final preparation for Dublin October 2025 talk. Detailed edits, and a few indicative notes towards monotonicity proofs.