

Perfect Epidemics

Applied Probability Seminar

Department of Statistics, University of Warwick

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Warwick, York

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Introduction

“Maybe the only significant difference between a really smart simulation and a human being was the noise they made when you punched them.”
(The Long Earth, Pratchett & Baxter, 2012)



Handout is on the web: use the QR-code or visit
wilfridskendall.github.io/talks/PerfectEpidemics.

This is initial work on using perfect simulation (CFTP) for epidemics.
WSK acknowledges the support of UK EPSRC grant EP/R022100.

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- Simplest possible example: *random-walk-CFTP*
(can boost to use Ising model to do simple image reconstruction).

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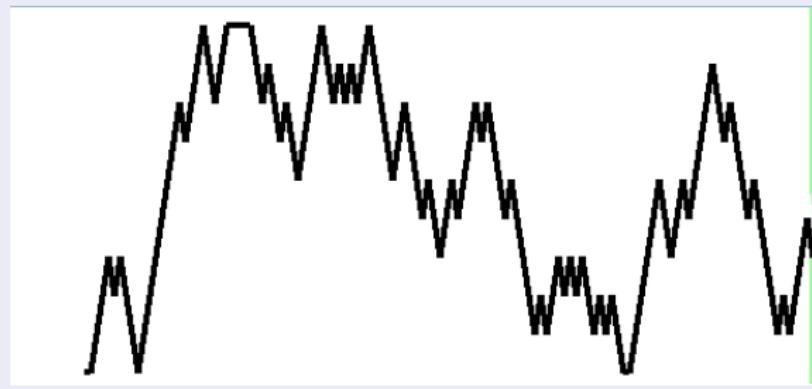
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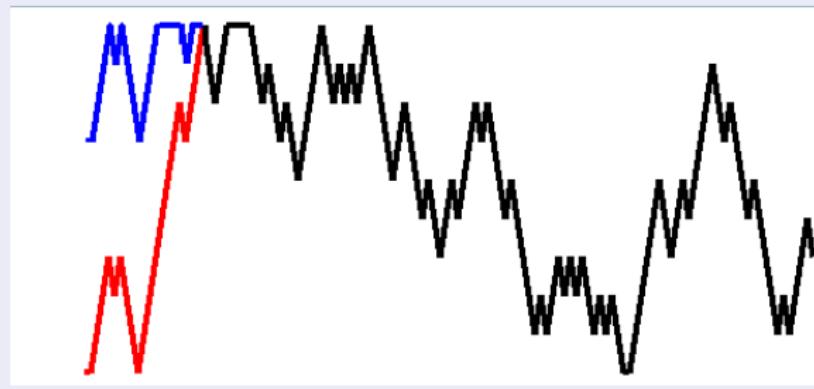
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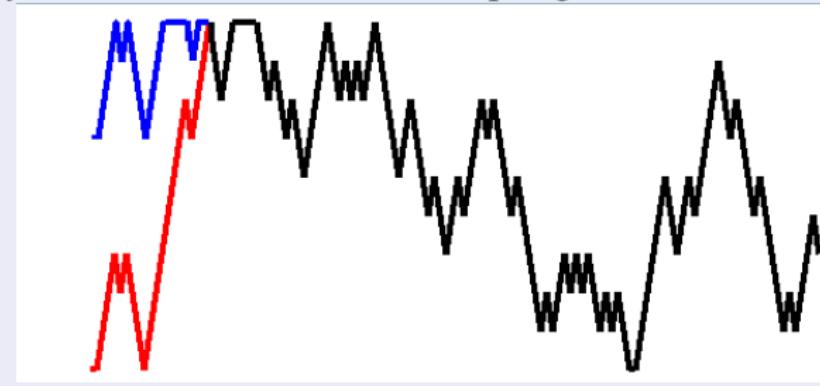
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- Generally **not true** that location *at coupling* is a draw from equilibrium.



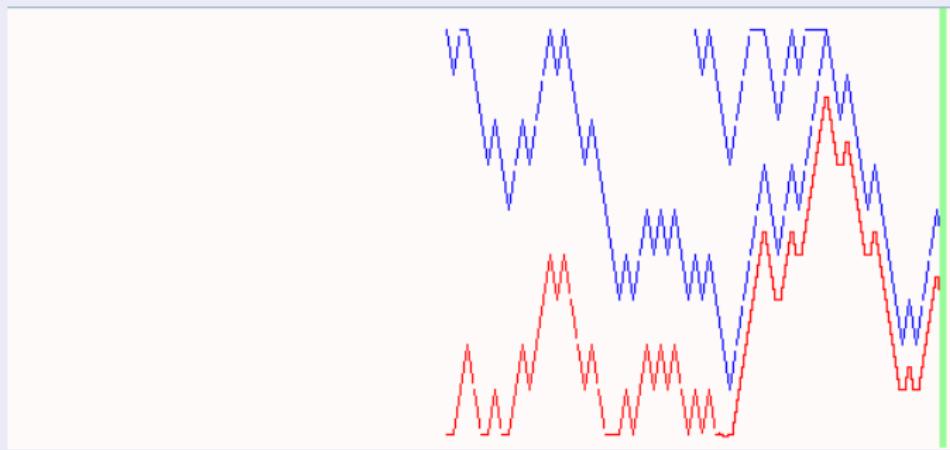
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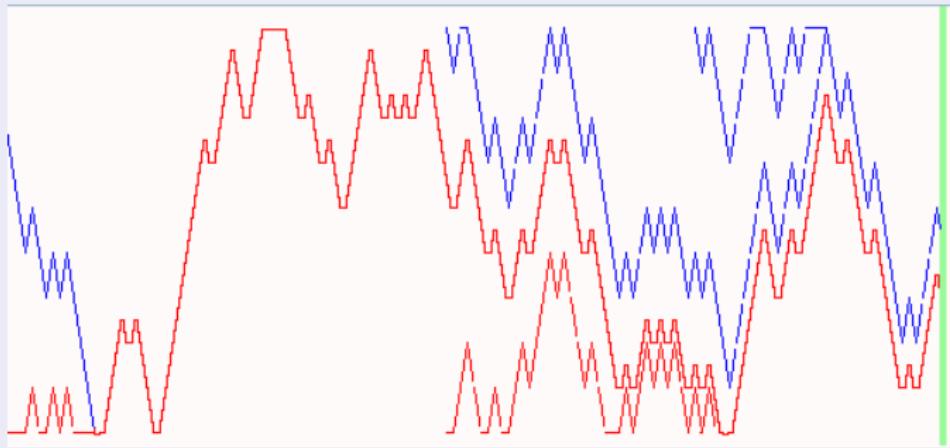
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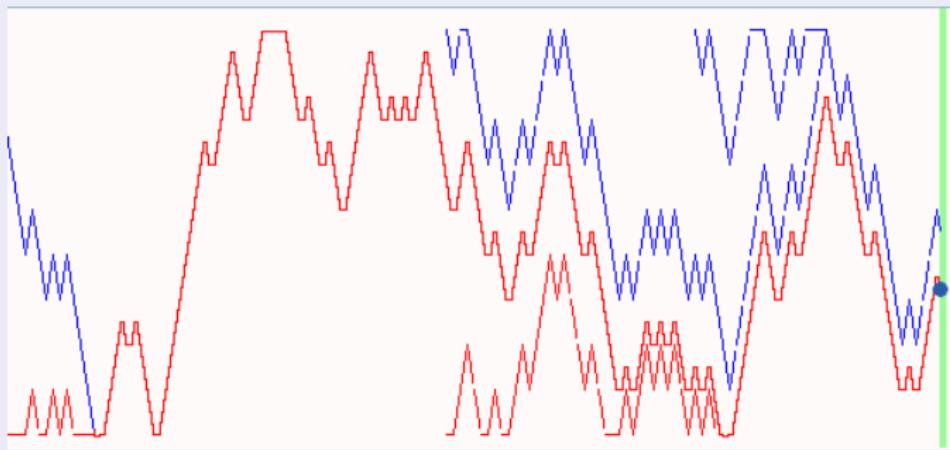
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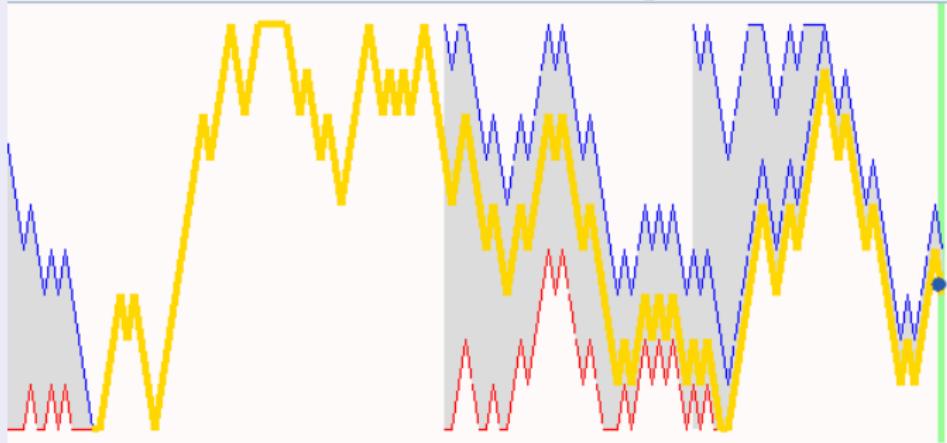
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- The common value is an exact draw from equilibrium!



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- Detailed expositions: **WSK (2005)**, **Huber (2015)**.
(Want to implement CFTP in R? see **WSK, 2015**.)

2. Perfect Epidemics: a challenge problem for CFTP

Many important inferential questions (Cori & Kucharski, 2024).

Simplest models (versus UK model with 10^6 agents!, Fraser & Others, 2023):

S-I-R deterministic epidemic: susceptibles s , infectives i , removals r
(constant total population $s + i + r = n$):

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Both models make an unrealistic assumption: homogeneous mixing.

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Wikipedia: “The British-registered *Diamond Princess* was the first cruise ship to have a major [COVID-19] outbreak on board, with the ship quarantined at Yokohama from 4 February 2020 for about a month. Of 3711 passengers and crew, around 700 people became infected and 9 people died.”

Evidently $\alpha s_0 / \beta \gg 1$ – as was sadly later confirmed, a sorrow for us all.



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- ⑤ Can we use **perfect simulation**?

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“When did the infections occur, supposing we only observe removals?”

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- ➎ Result: *trajectory-valued chain*, unconditioned S-I-R as equilibrium.

From incidents to unconditioned epidemic trajectories (1/3)

Incidents defining an epidemic

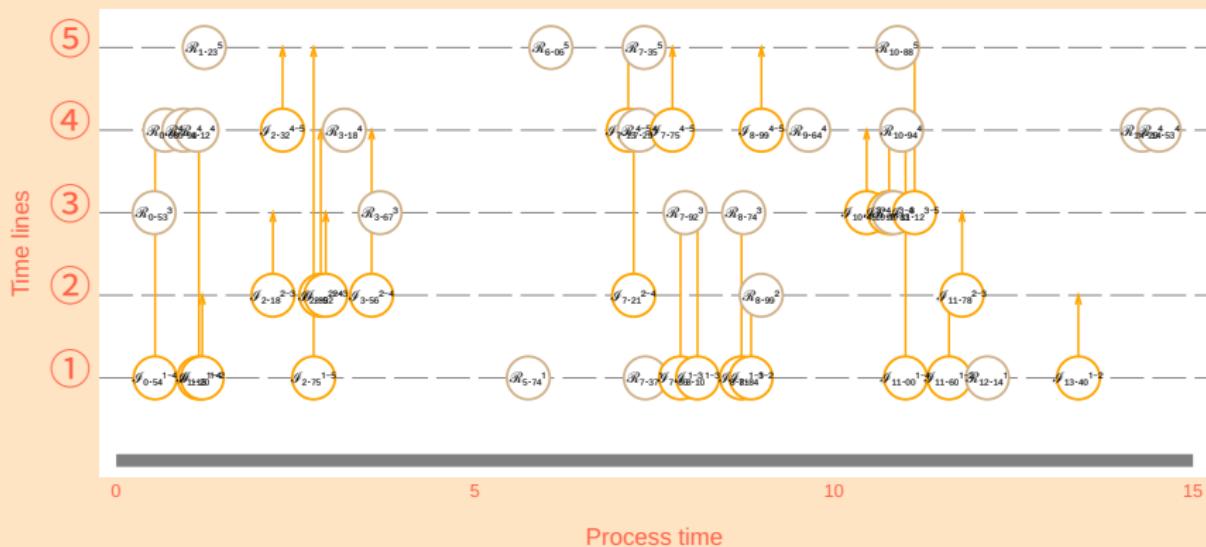


Figure 1: Light-orange circles denote potential infections (arrows point upwards to targets); light-brown circles denote potential removals.

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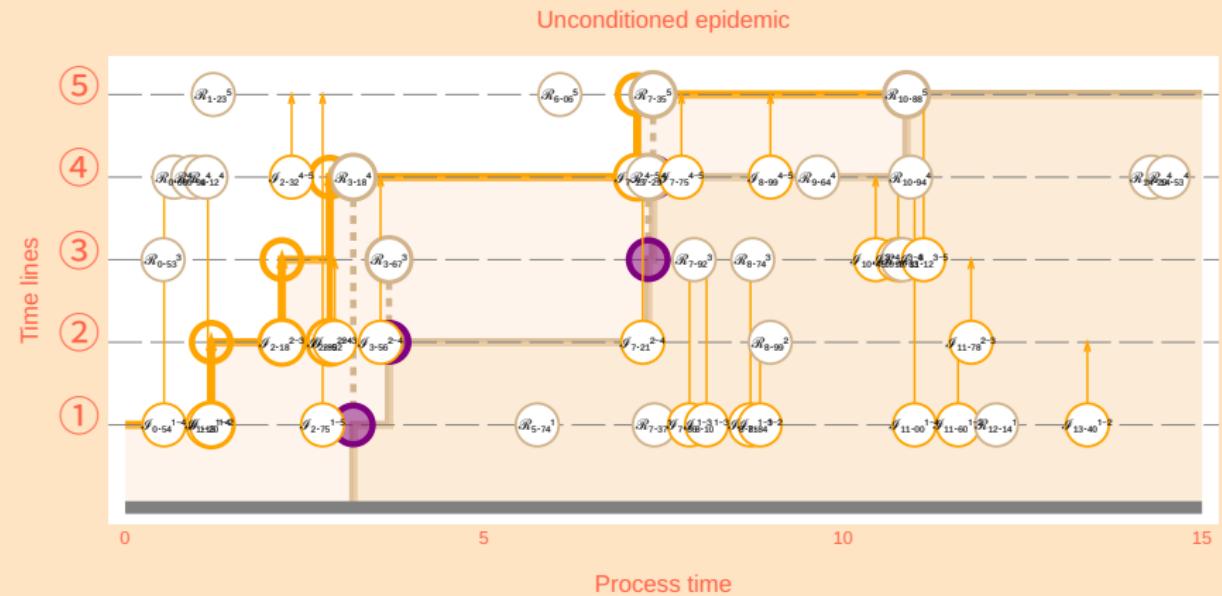


Figure 2: Activate (a) *infection* if target on lowest uninfected timeline; (b) *removal* if in infected region, then remove lowest infected (purple disk if different timeline).

From incidents to unconditioned epidemic trajectories (3/3)

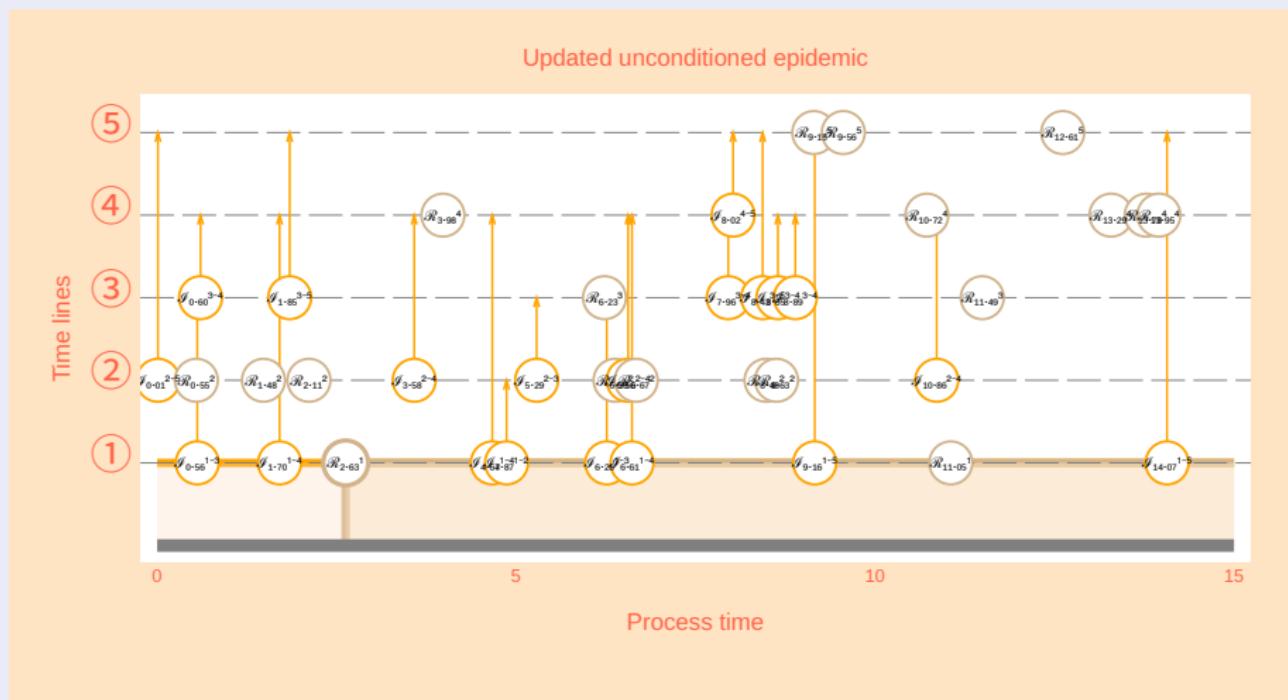


Figure 3: A step in algorithmic time for the unconditioned epidemic simply involves replacing the original incidents by a new set of incidents.

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- For later purposes it is convenient to stage the replacement as follows:
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- Thus the original update is expressible as a (continuous) composition of updates, each of which satisfies detailed balance in equilibrium.
- The connection “restriction=conditioning” is thereby preserved.
- Crucially, step 2 ensures composition action is irreducible over S !
(So equilibrium under conditioning is unique.)

Illustration of technical point (1/8)

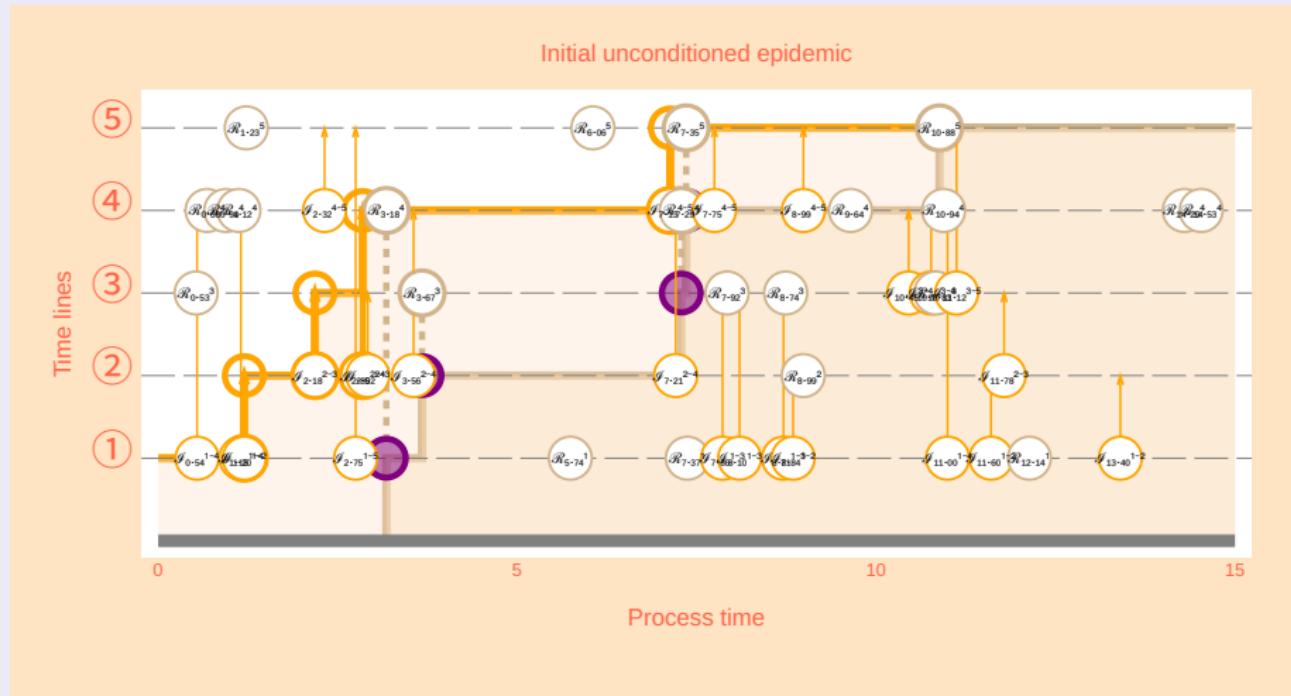


Figure 4: No change yet to removals or infections;

Illustration of technical point (2/8)

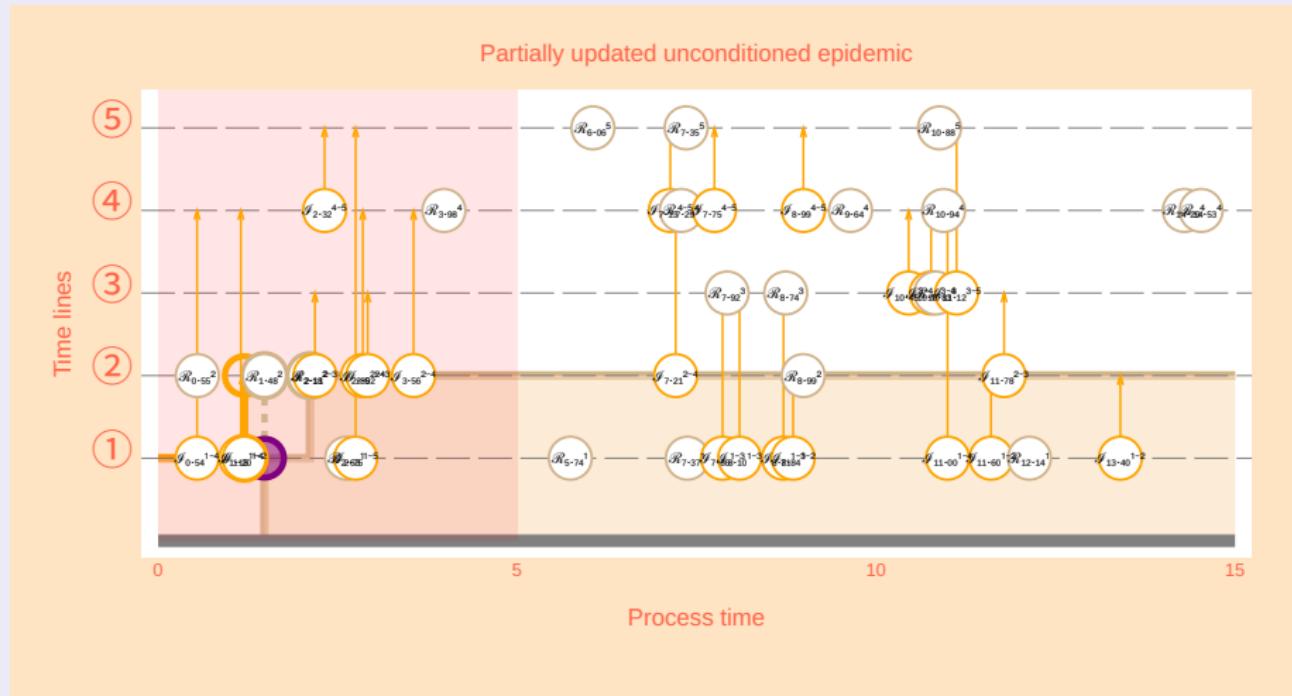


Figure 5: Replace first third of removals, infections unchanged;

Illustration of technical point (3/8)

Partially updated unconditioned epidemic

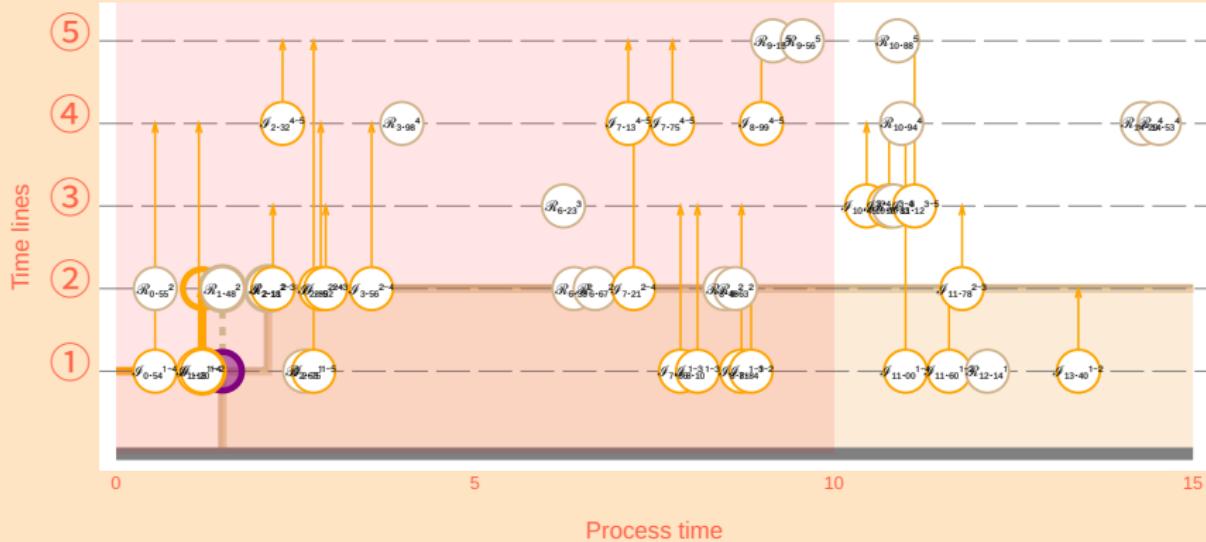


Figure 6: Replace first two-thirds of removals, infections unchanged;

Illustration of technical point (4/8)

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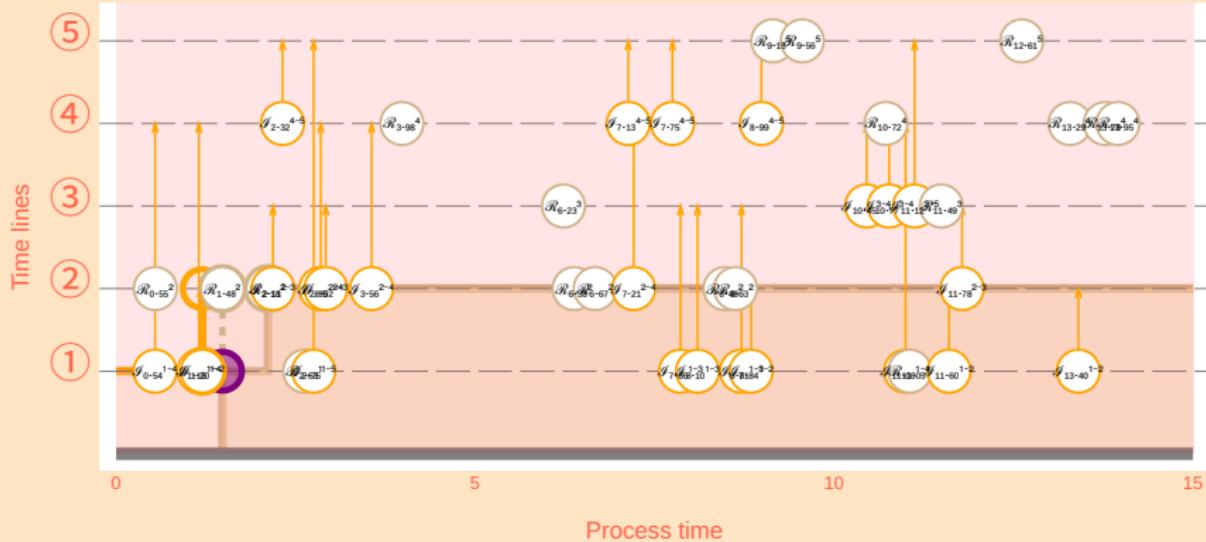


Figure 7: All removals resampled, infections as yet unchanged;

Illustration of technical point (5/8)

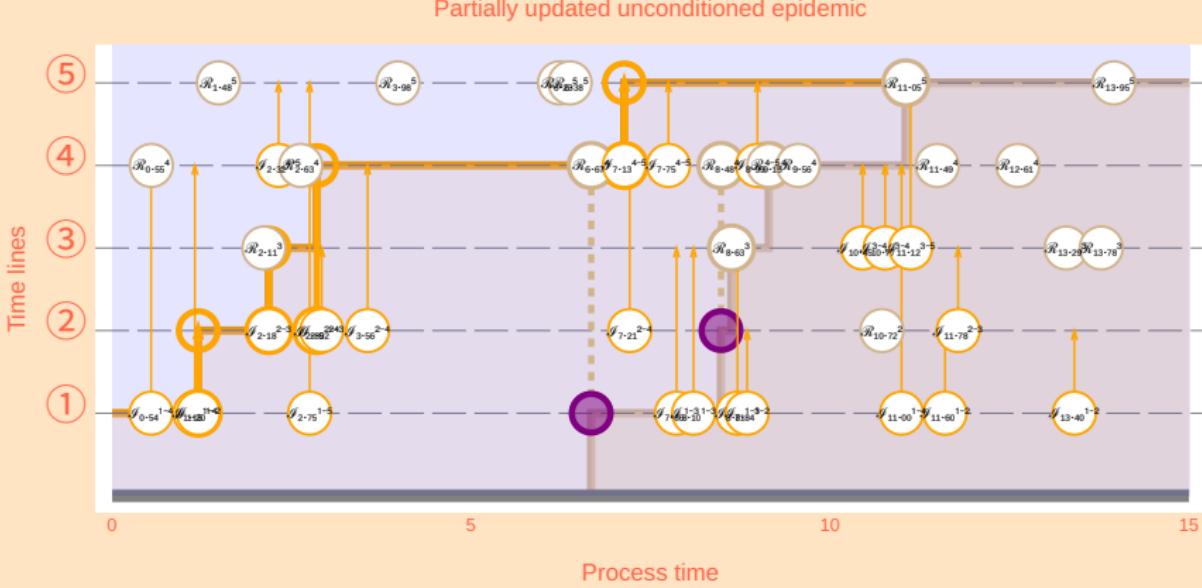


Figure 8: Re-sample all removal timelines, infections as yet unchanged;

Illustration of technical point (6/8)

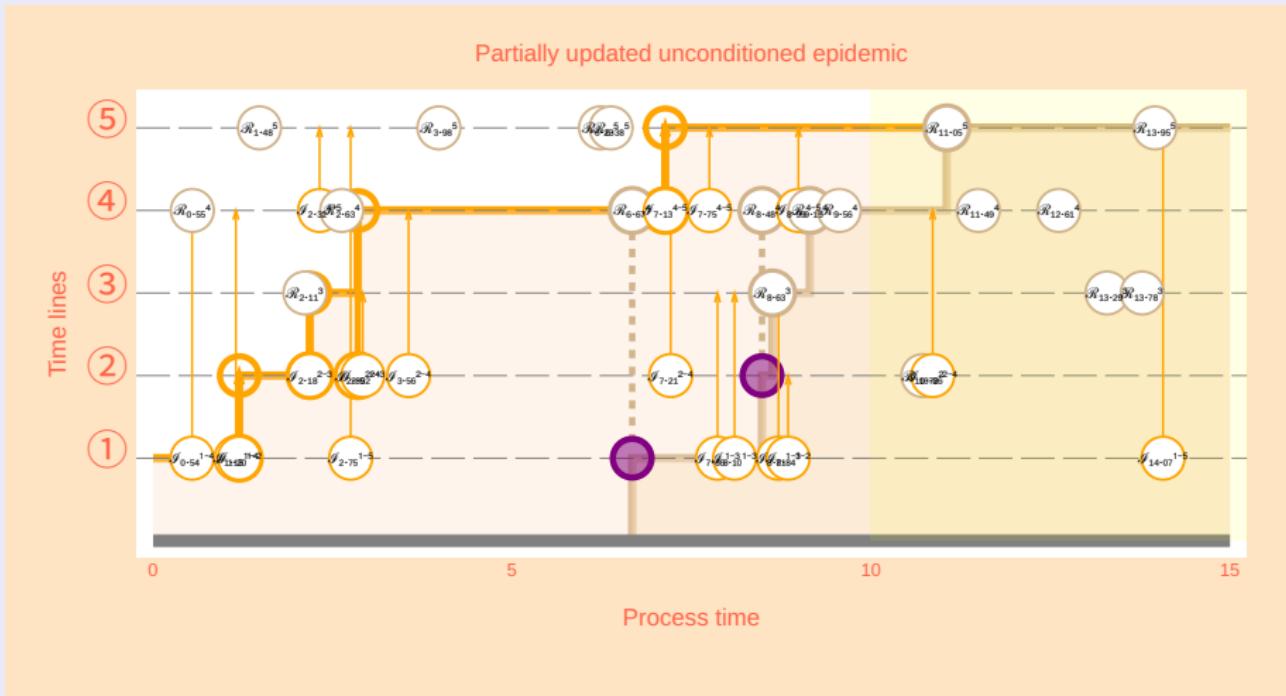


Figure 9: Re-sample last third of infections;

Illustration of technical point (7/8)

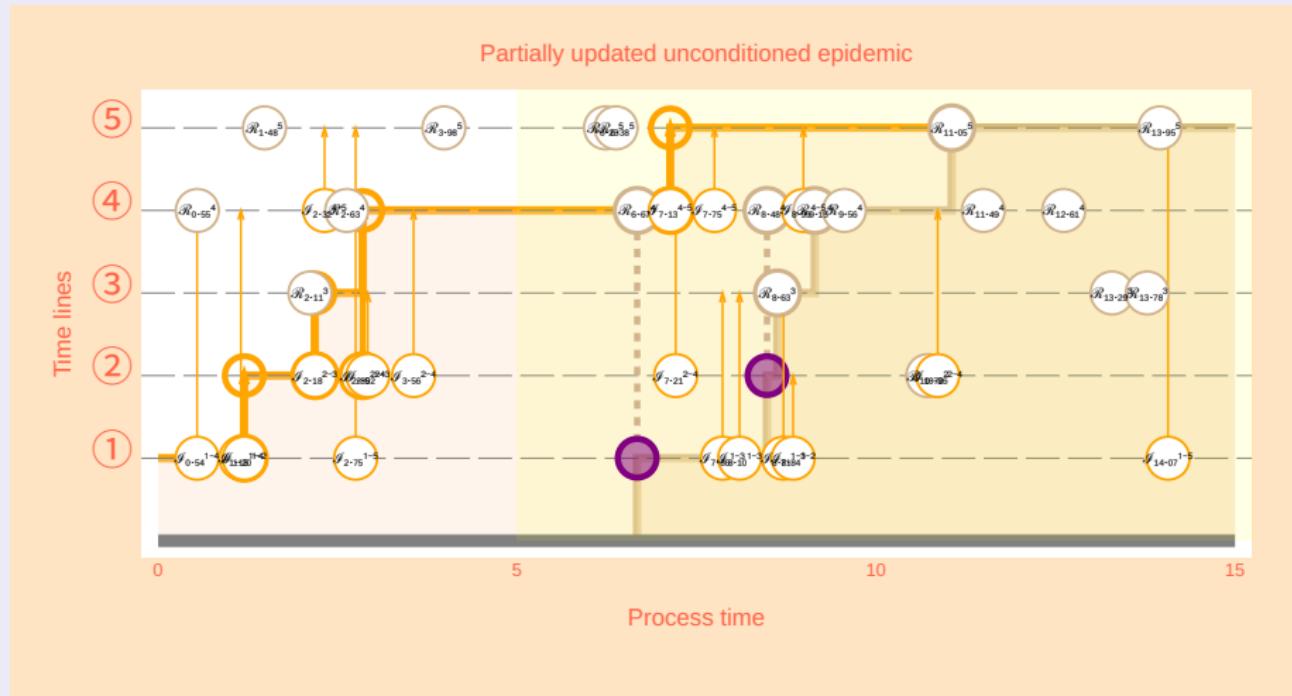


Figure 10: Re-sample last two-thirds of infections;

Illustration of technical point (8/8)

Fully updated unconditioned epidemic

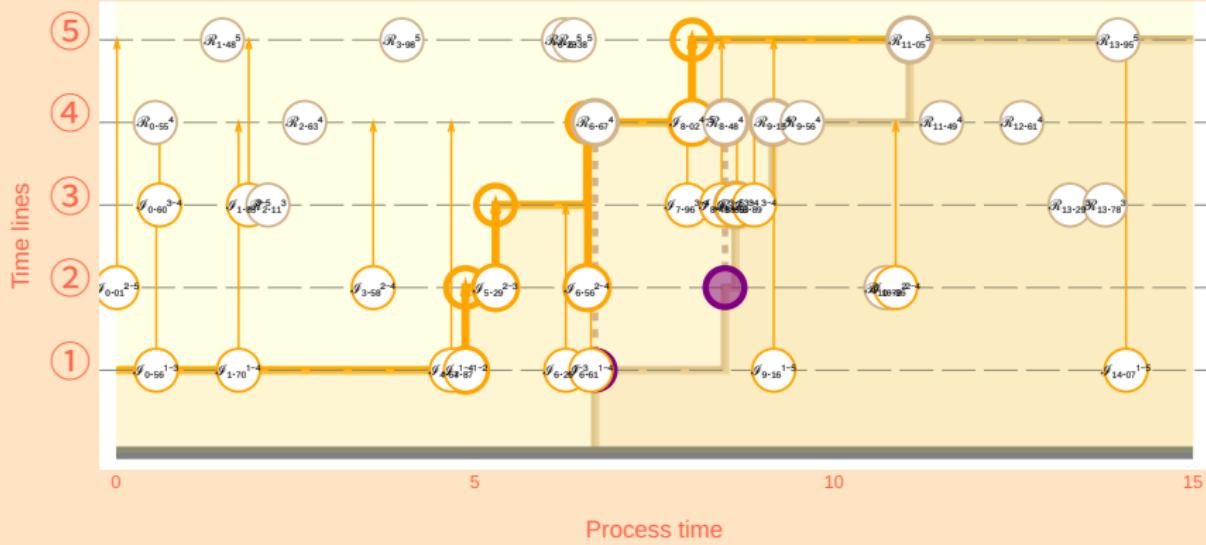


Figure 11: All infections now re-sampled.

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- Does this produce a *feasible* and suitably *monotonic* algorithm?
- **Housekeeping details** used to establish that monotonicity still works: *last feasible epidemic (LFE)* and *no-fly zone (NFZ)*.

Initial conditional epidemic

Initial conditioned epidemic

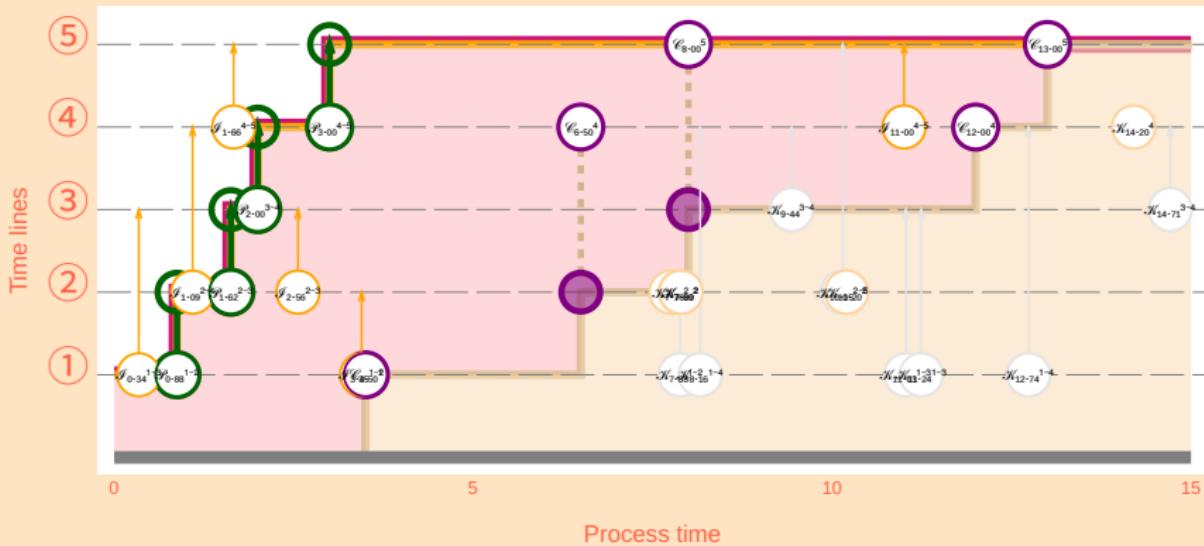


Figure 12: Initial epidemic with conditioned removals indicated using purple circles (and purple disks when different timelines are infected).

Conditional epidemic update

Fully updated conditioned epidemic

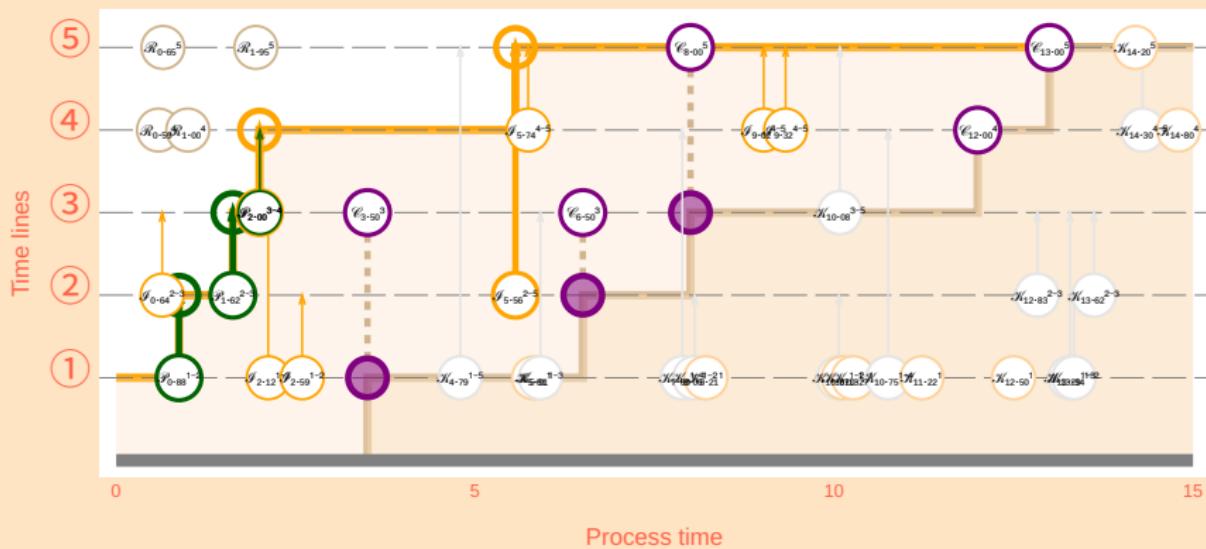


Figure 13: Epidemic updated under restriction: all conditioned removals remain activated, no new removals are activated. Green infections have been “perpetuated”.

Last feasible epidemic (LFE)

Fully updated conditioned epidemic with LFE

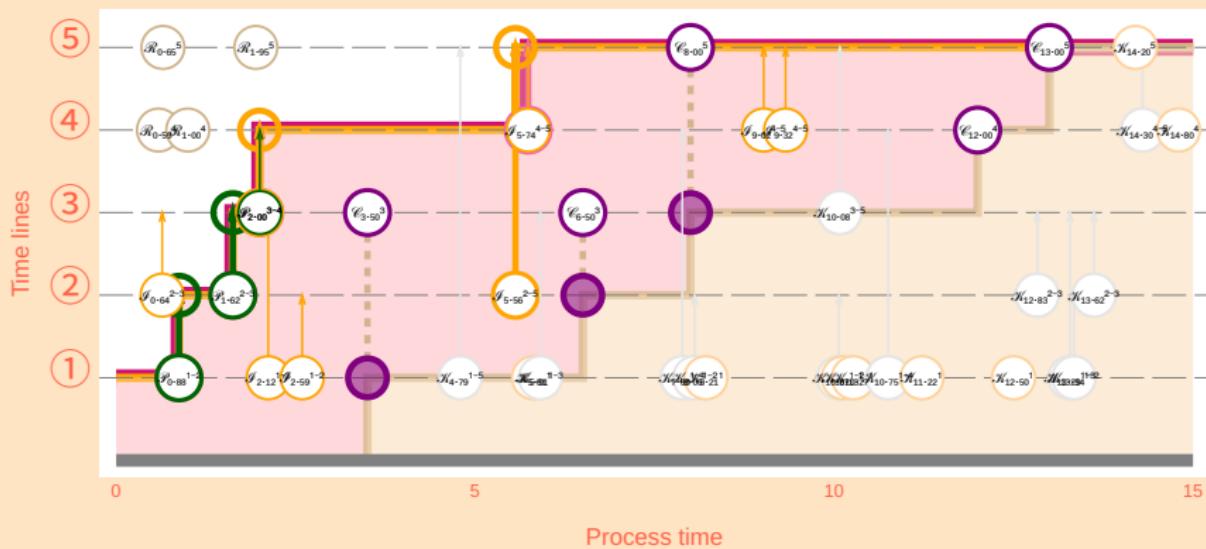


Figure 14: LFE computed recursively working right-to-left: the slowest sequence of infections deals with all infected timelines in order (includes perpetuated infections).

No-fly zone (NFZ)

Fully updated conditioned epidemic with NFZ

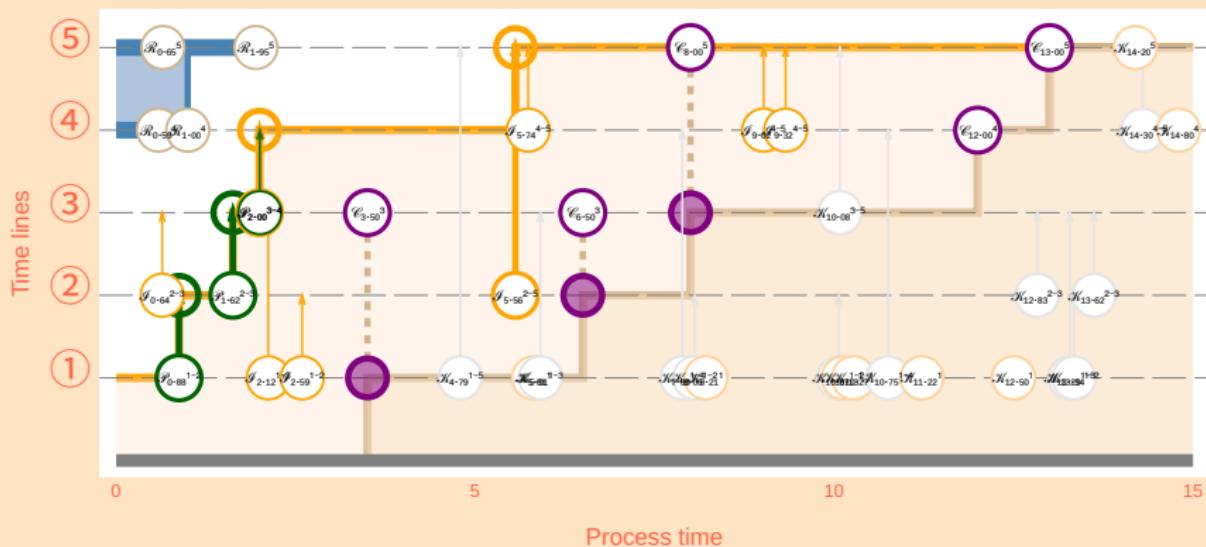


Figure 15: NFZ computed recursively working right-to-left: it traces the region of timelines that must not be infected if one is not to activate unobserved removals.

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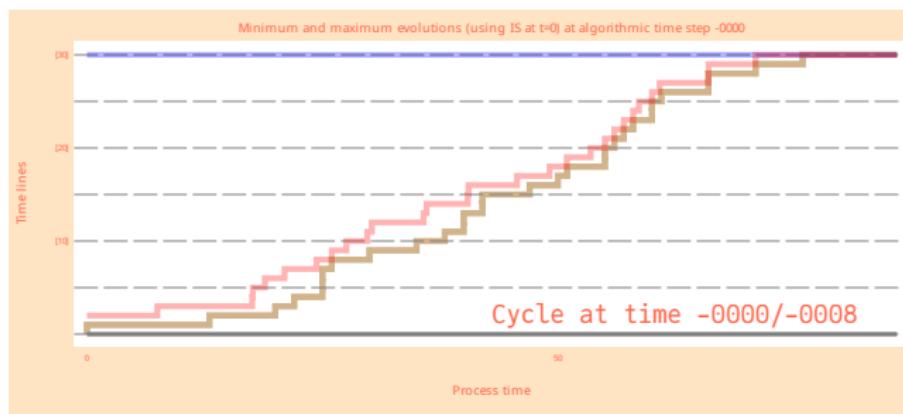
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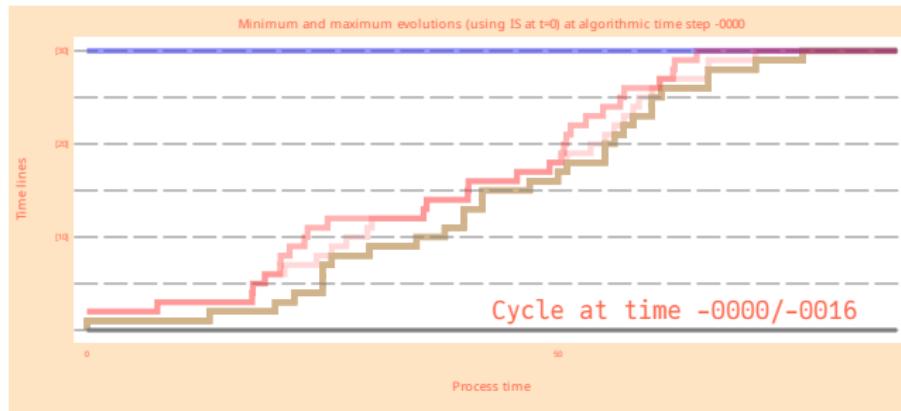
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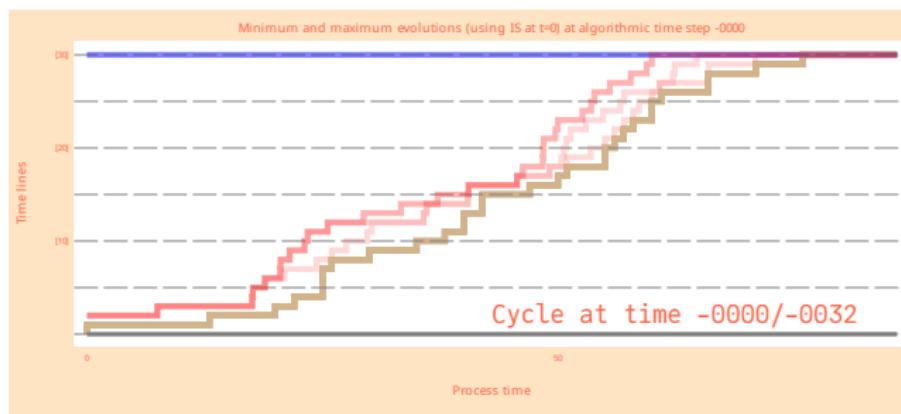
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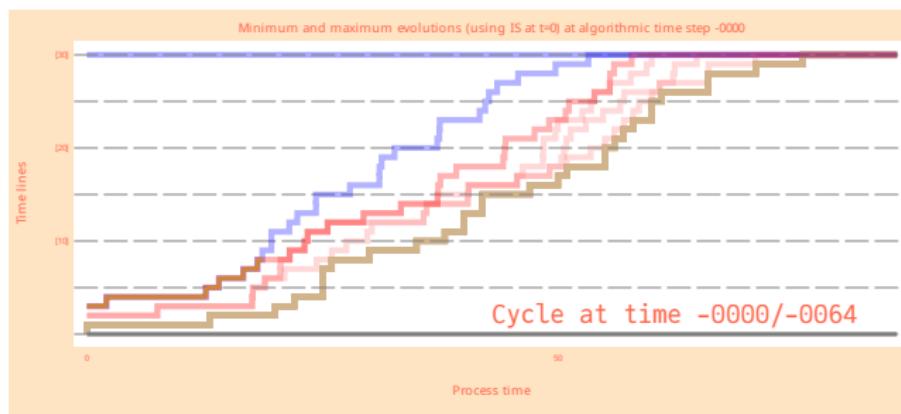
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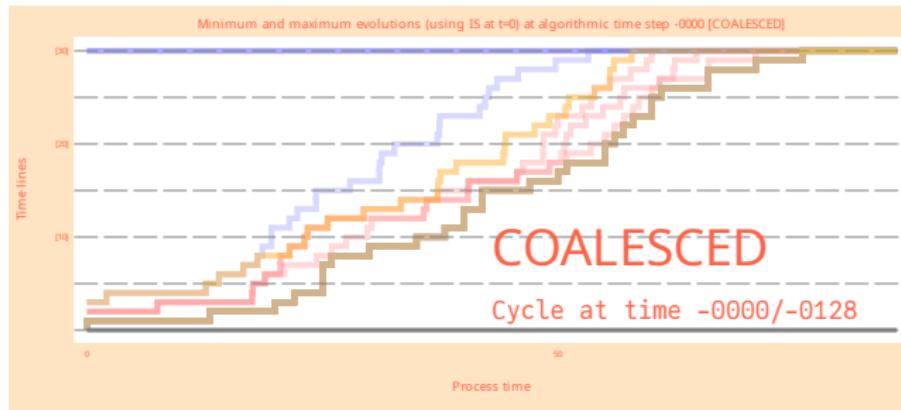
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- Finally: generalize to other suitable compartment models?

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- Thank you for your attention! **QUESTIONS?**



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Image information

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<i>Diamond Princess</i>	Alpsdake	<i>CC BY-SA 4.0</i>
Epidemic CFTP images and animation	Result of code written by WSK	

Previous instances of this talk

<i>Date</i>	<i>Title</i>	<i>Location</i>
19/04/24	Perfect Epidemics	Short Research Talk (12min)
15/05/24	McMC and Perfect Simulation	Graduate Seminar, Aristotle Univ. (50min)
17/01/25	Perfect Epidemics	Applied Probability Seminar (50min)

Appendix A: A “near-maximal” configuration

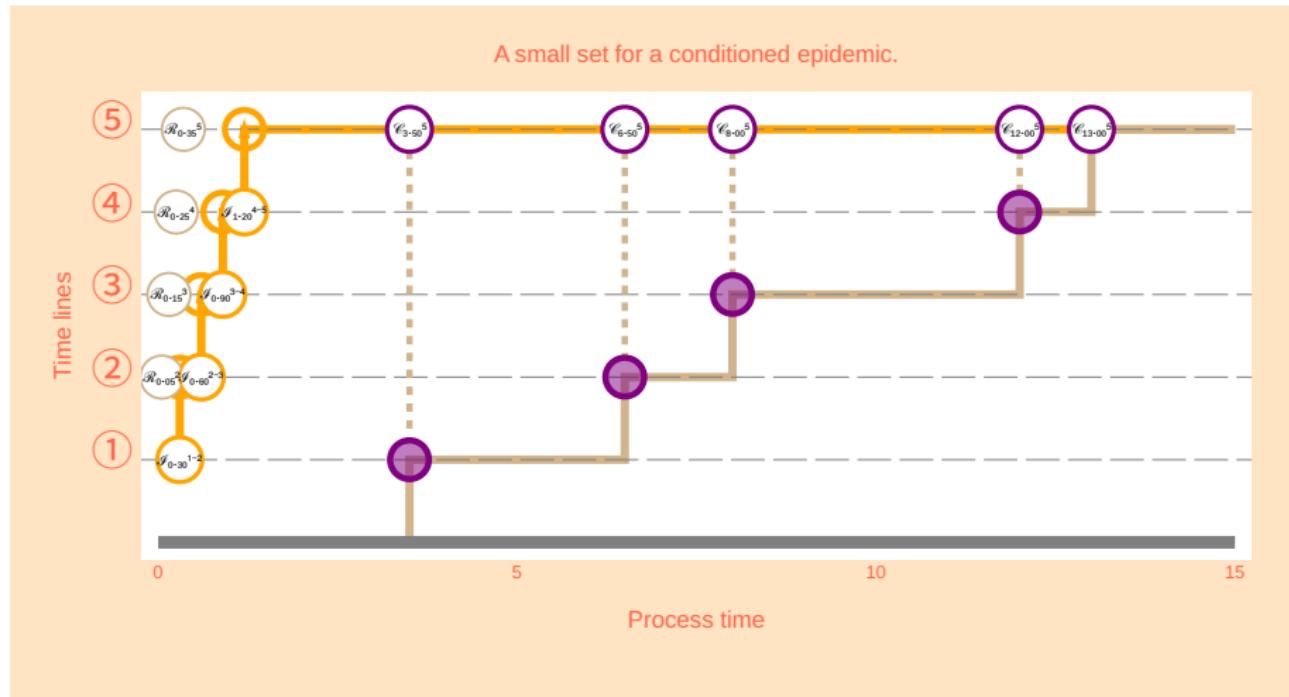


Figure 16: A conditional epidemic in which all activated infections occur before time 3.0, also before smallest observed removal time.

Appendix B: Updating a conditional epidemic INCOMPLETE

We now work through the update of the conditional epidemic in stages.

Initial conditional epidemic (1/8)

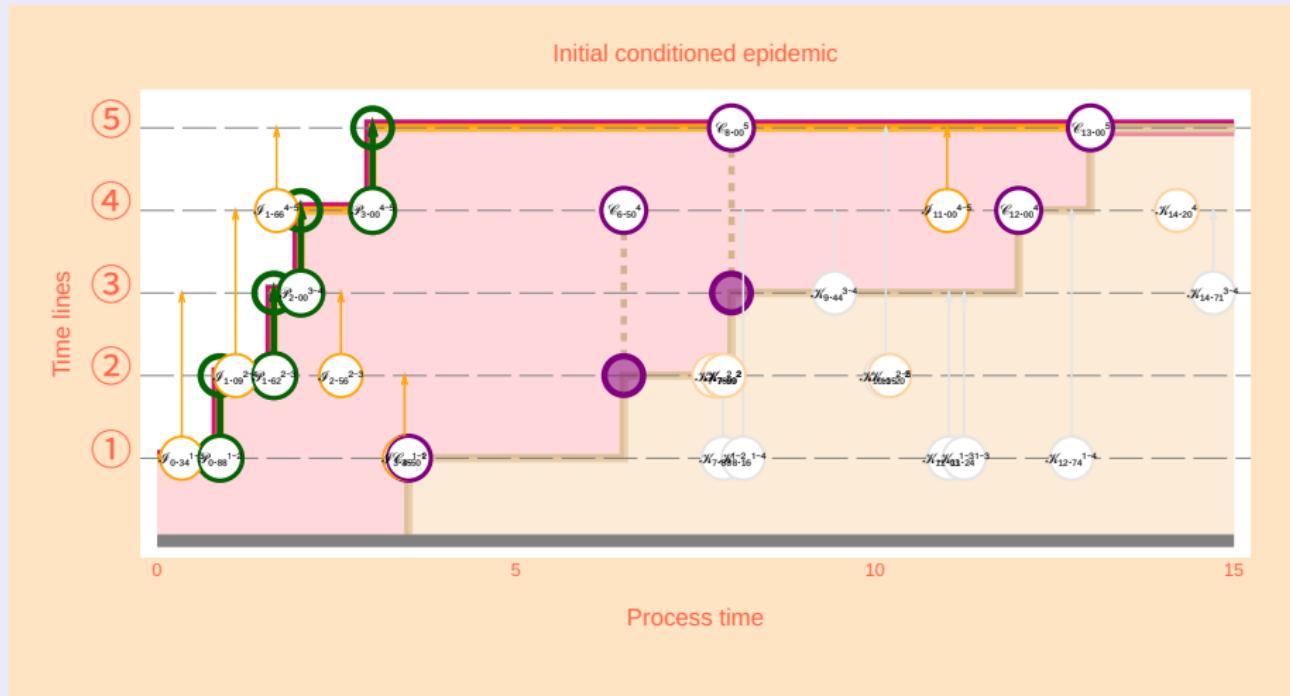


Figure 17: Initial epidemic with conditioned removals indicated by purple circles.

Conditional epidemic (2/8)

Partially updated conditioned epidemic

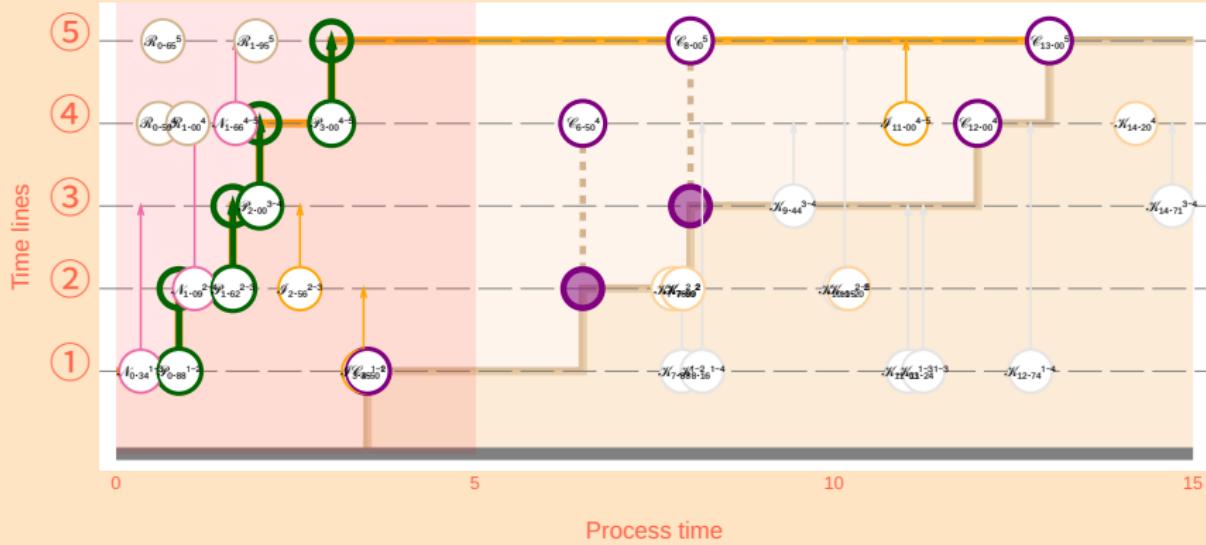


Figure 18: Replace first third of removals, infections unchanged;

Conditional epidemic (3/8)

Partially updated conditioned epidemic

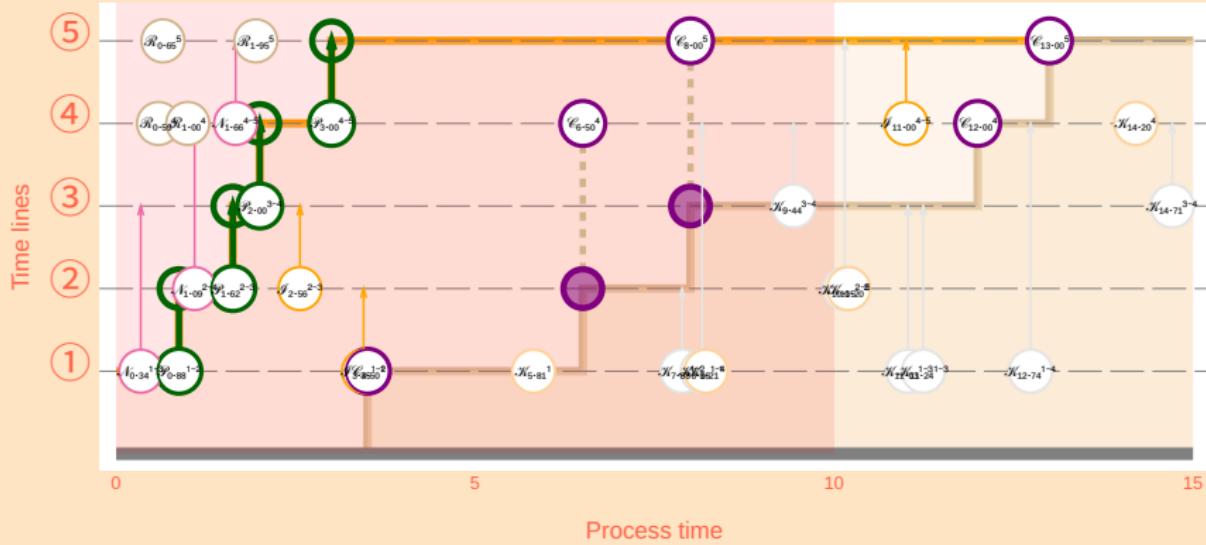


Figure 19: Replace second third of removals, infections unchanged;

Conditional epidemic (4/8)

Partially updated conditioned epidemic

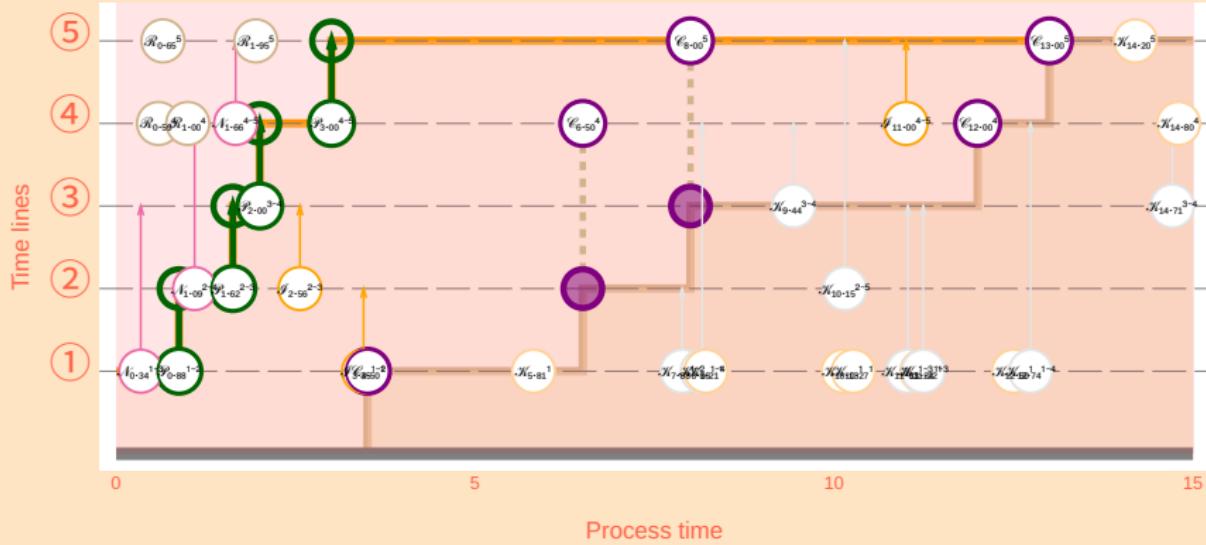


Figure 20: Replace remaining removals, infections unchanged;

Conditional epidemic (5/8)

Partially updated conditioned epidemic



Figure 21: Re-sample all removal timelines, infections as yet unchanged;

Eventual conditional epidemic after use of an innovation (6/8)

Still to be done: 1/3 of way through new infections, display current LFE and NFZ.

Eventual conditional epidemic after use of an innovation (7/8)

Still to be done: 2/3 of way through new infections, display current LFE and NFZ.

Eventual conditional epidemic after use of an innovation (8/8)

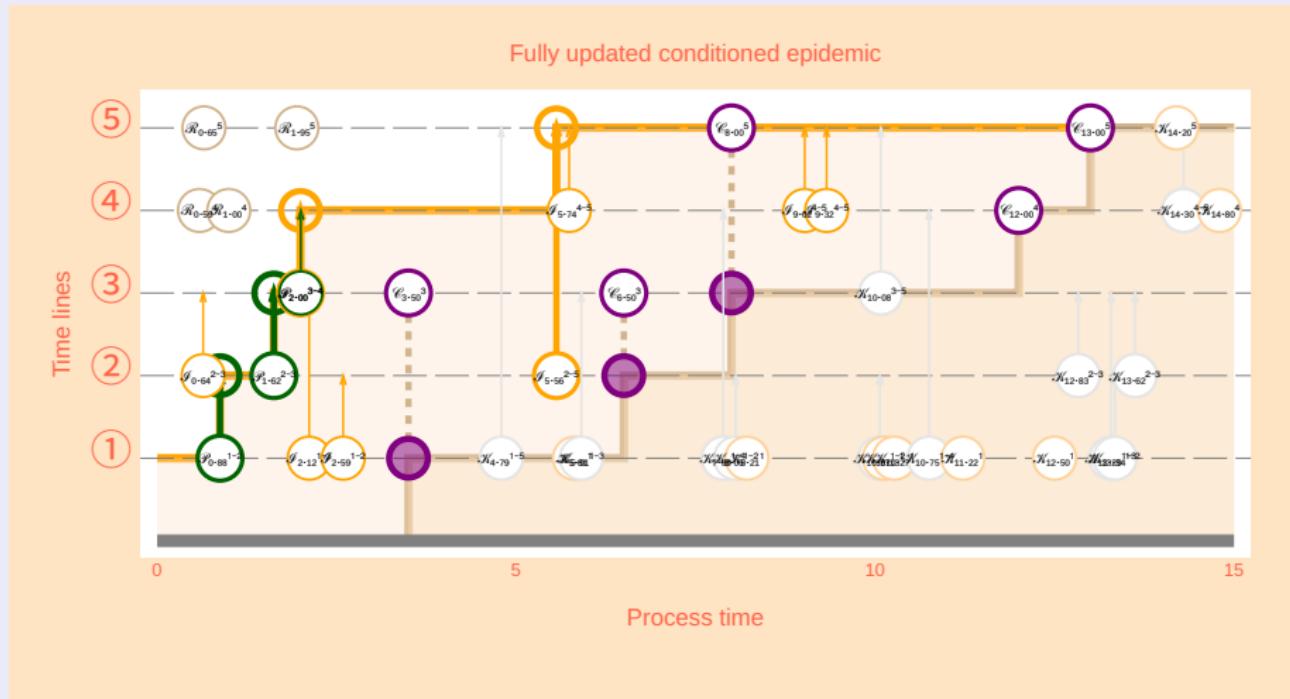


Figure 22: All infections now re-sampled. Green infections are “perpetuated”.

Appendix C: Naive approach to compartment models fails

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- This would apply, for example, in the case of the *Diamond Princess* if α , β depended on age and location of cabin on the ship.

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Recall that infections and removals *after* a conditional removal have been censored out. A valid configuration must satisfy the following, derived from the process dynamics:

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- So each timeline is divided into a *susceptible interval* (empty if it is initially infected), an *infected interval* (empty if it is never infected), and a *removed interval* (empty if it has no conditioned removal).

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As in S-I-R case, the conditioned epidemic is the unique equilibrium.

Requirements for monotonicity

For CFTP we need to know that, for coupled iterations (using the same pattern of innovations of new \mathcal{I} s and \mathcal{R} s), if two variants are started so that the infected region of one contains the other, then this persists through development of the algorithmic time.

It would suffice to prove two technical results:

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It would suffice to prove two technical results:

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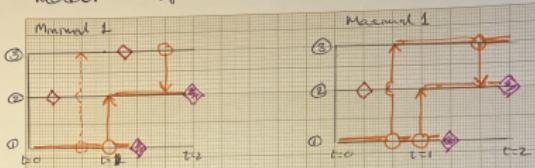
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Then CFTP would make sense, and it would only be necessary to show that accessibility of a set of near-maximal configurations guarantees eventual coalescence.

Counterexample to monotonicity

"Observe" generalization to compartmental model can fail to be monotonic!

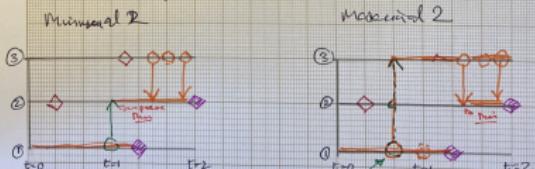
1/2/25



- ◆ Conditioned removal
 - ↑ Infection
 - Infectivity of virus

This is after deleting all old inactivated
removals and replacing by new inactivated
removals.

Now work from right - to left deleting all infections except where so doing would leave a candidate removal not infected. (and work)
At last we get to:

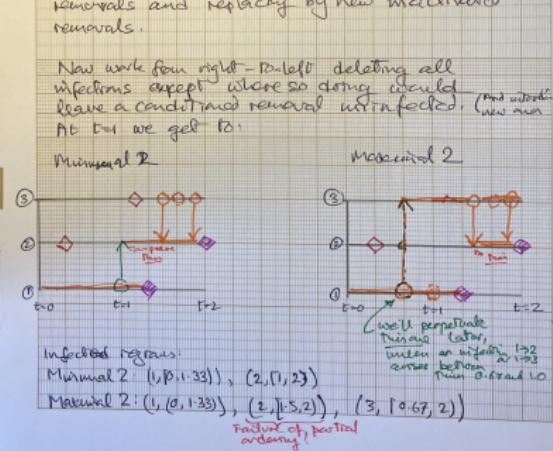


Infected neurons

Mirrored 2: (1, [0, 1, 33]), (2, [1, 23])

$$\text{Matrizen 2: } \begin{pmatrix} 1, (0, 1.33) \\ 2, (1.5, 2) \end{pmatrix}, \quad \begin{pmatrix} 3, (0.67, 2) \end{pmatrix}$$

Failure of partial
orderings?



Other technical information

Software used in computations

<i>Software</i>	<i>Version</i>	<i>Branch</i>	<i>Last commit</i>
quarto	1.6.39	—	
Running under julia	1.11.3	—	
EpidemicsCFTP	2.2.492	main	Thu Jan 23 20:50:07 2025
EpidemicsUtilities	0.1.2.156	main	Thu Jan 23 12:06:14 2025
This quarto script	2.2.619	Wilfrid-2025-01-30-compartment	Mon Feb 3 16:41:03 2025

Revision notes

These notes were produced from `PerfectEpidemics.qmd`:

Version:	2.2.620 [Wilfrid-2025-01-30-compartment]
Author:	Wilfrid Kendall <W.S.Kendall@warwick.ac.uk>
Date:	Mon Feb 3 17:54:28 2025 +0000
Summary:	Counterexample to naïve compartment model generalization. Stiffened up definition of evolution in algorithmic time. Added printing of <code>README.pdf</code> and <code>TODO.pdf</code> . Added <code>critic</code> filter using <code>_metadata.yml</code> .