

Perfect Epidemics

2nd Workshop, UK Research Network in Stochastics
University of Liverpool

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Warwick, York

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Introduction

“Once we came to accept the photographic image as reality, the way to its future simulation was open.”
[Lev Manovich]



Handout is on the web: use the QR-code or visit
wilfridskendall.github.io/talks/PerfectEpidemics.

Work on perfect simulation ([CFTP](#)) for epidemics, now being written up.
WSK acknowledges the support of UK EPSRC grant EP/R022100.

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- ④ Simplest possible example: *random-walk-CFTP*
(can boost to use Ising model to do simple image reconstruction).

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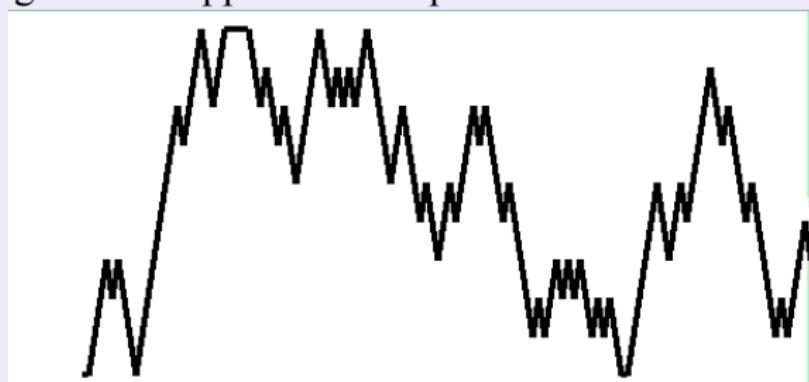
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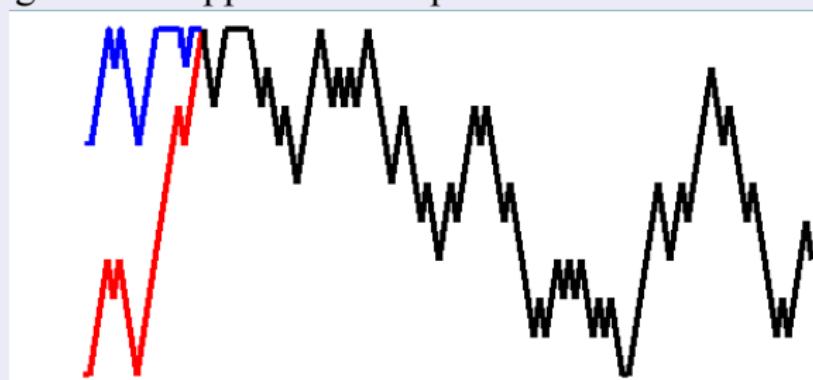
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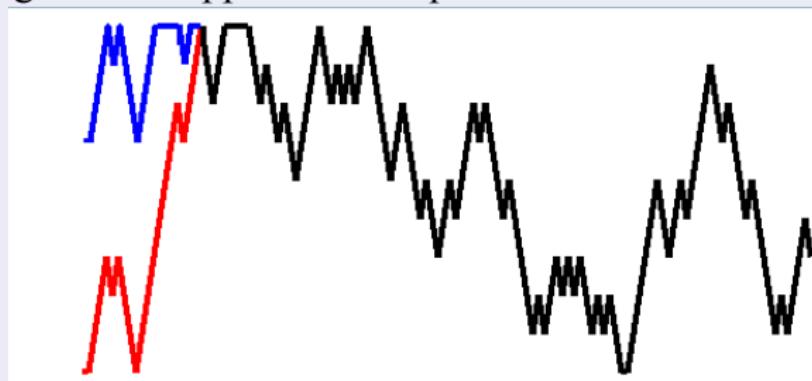
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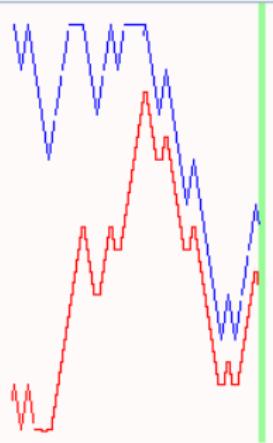
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- ④ Generally **not true** that location *at coupling* is a draw from equilibrium.

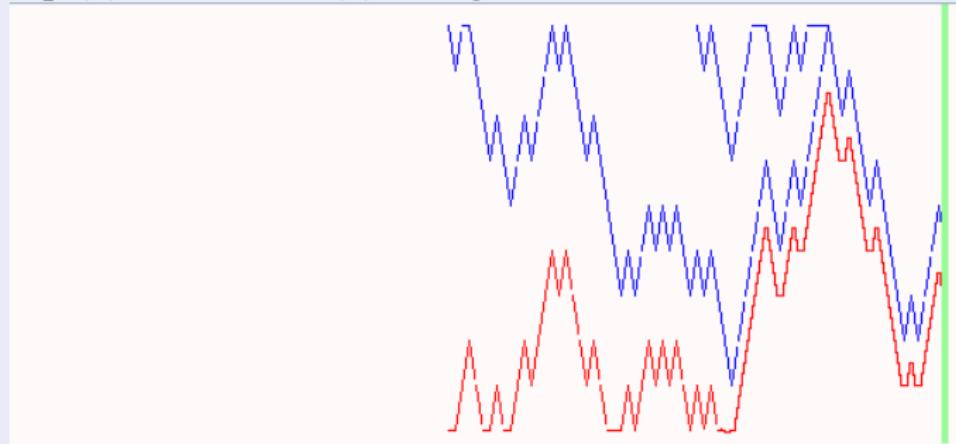
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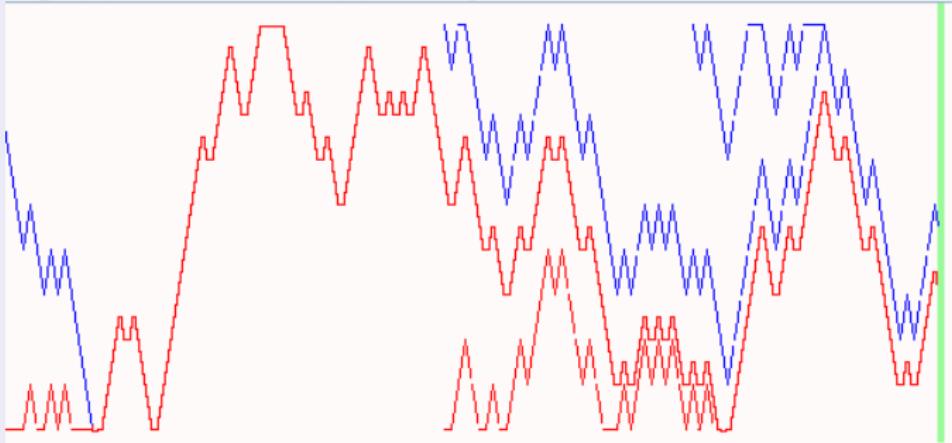


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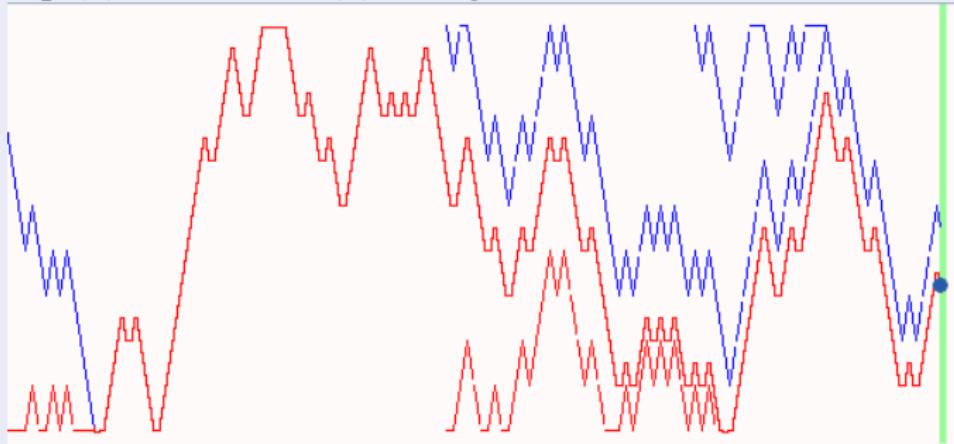
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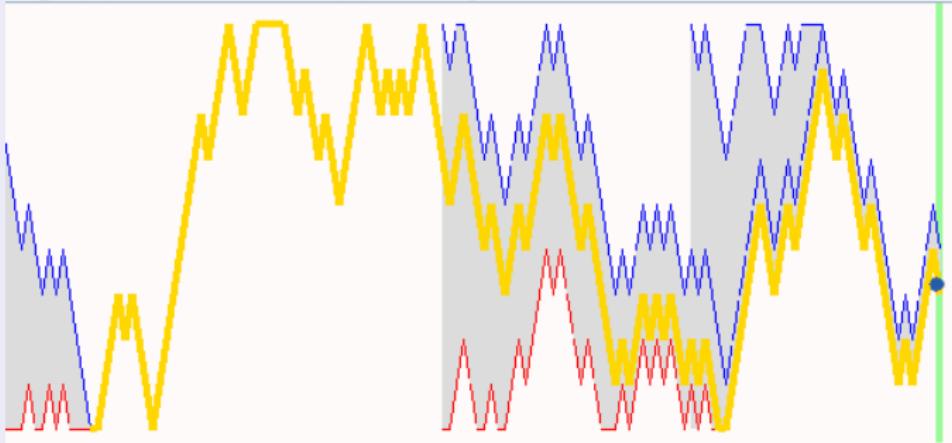
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- ④ When coupled, top and bottom yield a common value at time 0.
- ⑤ The common value (golden thread) is an exact draw from equilibrium!

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- ➍ Detailed expositions: WSK (2005), Huber (2015).
(Want to implement CFTP in R? see WSK, 2015.)

3. Perfect Epidemics: a challenge problem for CFTP

S-I-R deterministic epidemic:

based on susceptibles s , infectives i , removals r :

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Constant total population $s + i + r = n$.

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Both make an unrealistic assumption: homogeneous mixing.

In contrast, Fraser *et al.* (2023) use a UK model with 10^6 agents!

There are *many* important inferential questions (Cori & Kucharski, 2024).



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Wikipedia: “The British-registered *Diamond Princess* was the first cruise ship to have a major [COVID-19] outbreak on board, with the ship quarantined at Yokohama from 4 February 2020 for about a month. Of 3711 passengers and crew, around 700 people became infected and 9 people died.”

Evidently $\alpha s_0 / \beta \gg 1$ – as was sadly later confirmed, a sorrow for us all.



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- ➎ Can we use **perfect simulation**?

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- ⑤ Result: *trajectory-valued chain*, unconditioned S-I-R as equilibrium.

From incidents to unconditioned epidemic trajectories (1/3)

Incidents defining an epidemic

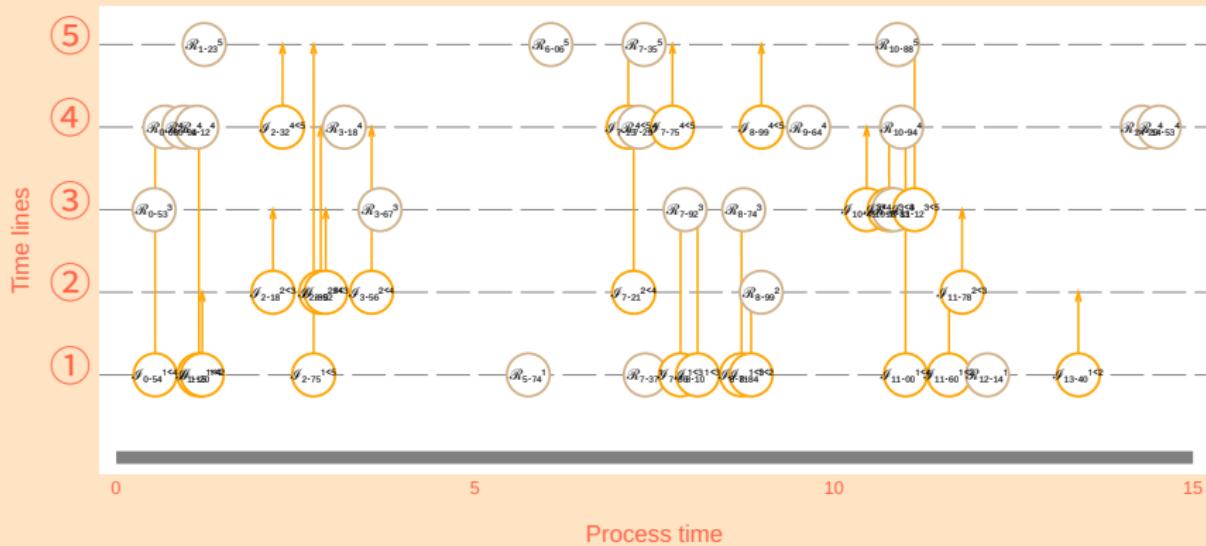


Figure 1: Light-orange circles denote potential infections (arrows point upwards to targets); light-brown circles denote potential removals.

From incidents to unconditioned epidemic trajectories (2/3)

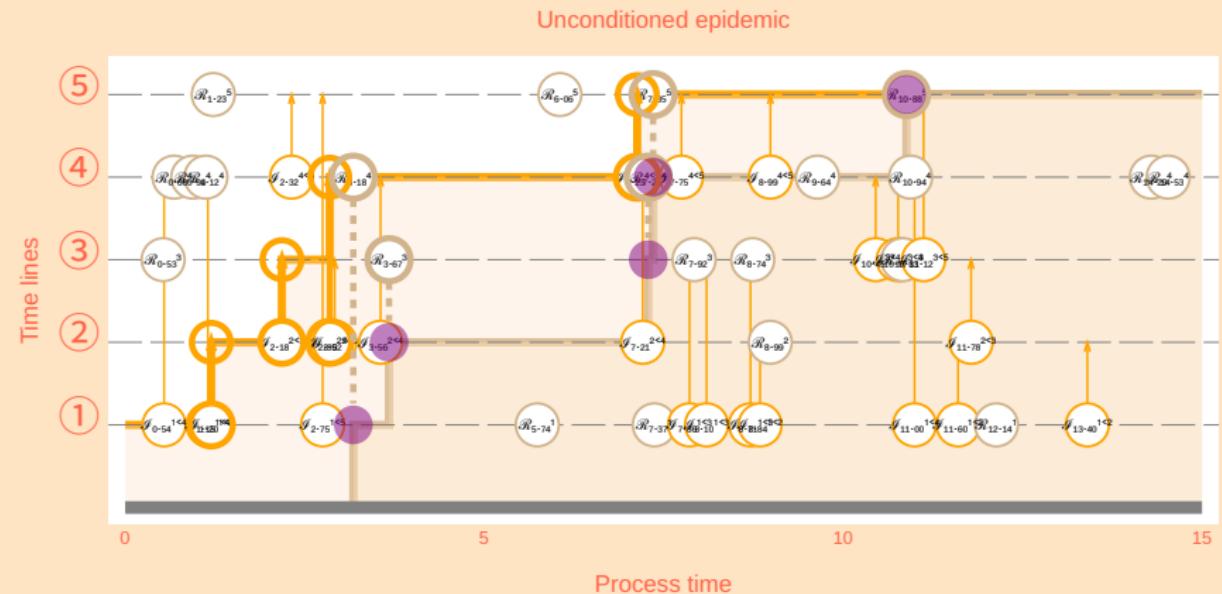


Figure 2: (a) *Infection* activates if target lies on the lowest uninfected timeline; and (b) *removal* activates if on infected timeline; remove lowest infected (purple disk).

From incidents to unconditioned epidemic trajectories (3/3)

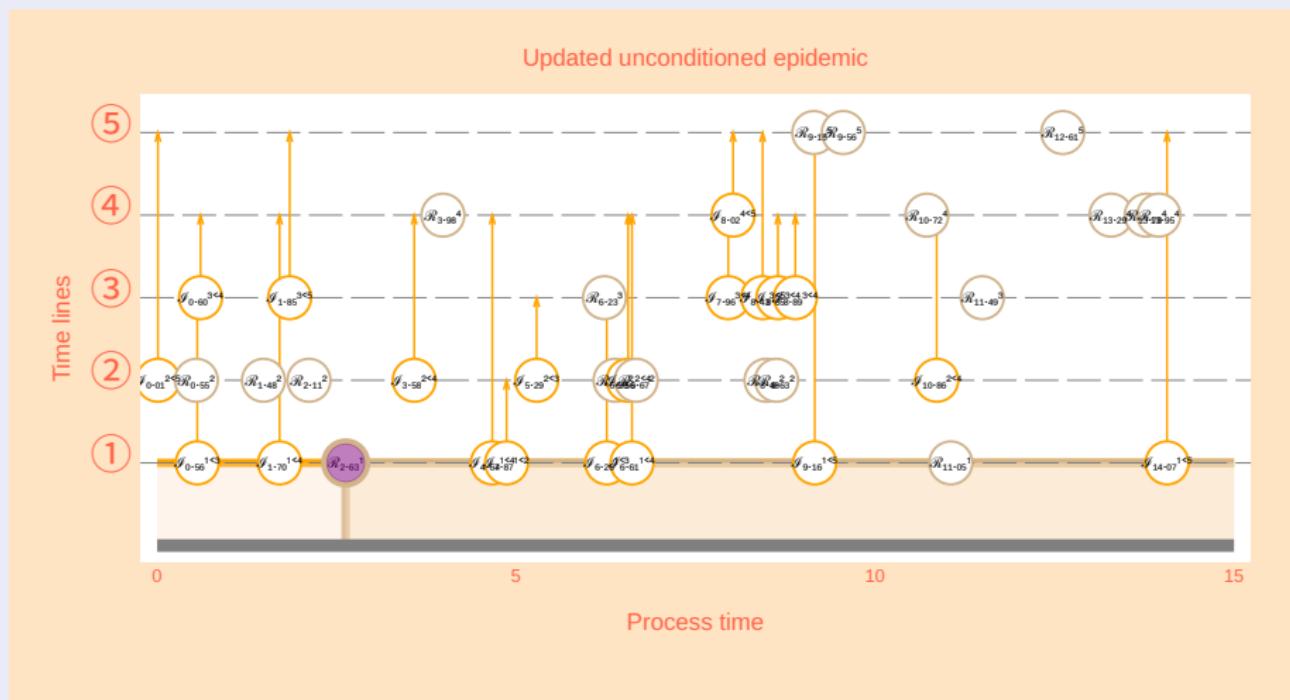


Figure 3: A step in algorithmic time for the unconditioned epidemic simply involves replacing all original incidents by an entirely new set of incidents.

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- Thus the original update is expressible as a (continuous) composition of updates, each of which satisfies detailed balance in equilibrium.
- The connection “restriction=conditioning” holds (**needs proof**).
- Crucially, re-sampling step 2 ensures composite evolution is irreducible over S ! (So equilibrium under conditioning is unique.)

Free evolution evolving in continuous algorithmic time

GIF MP4

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- Does this produce a *feasible* and suitably *monotonic* algorithm?
- **Housekeeping details** used to establish that monotonicity still works: *laziest feasible epidemic (LFE)* and *no-fly zone (NFZ)*.

Initial conditioned epidemic

The initial conditioned epidemic

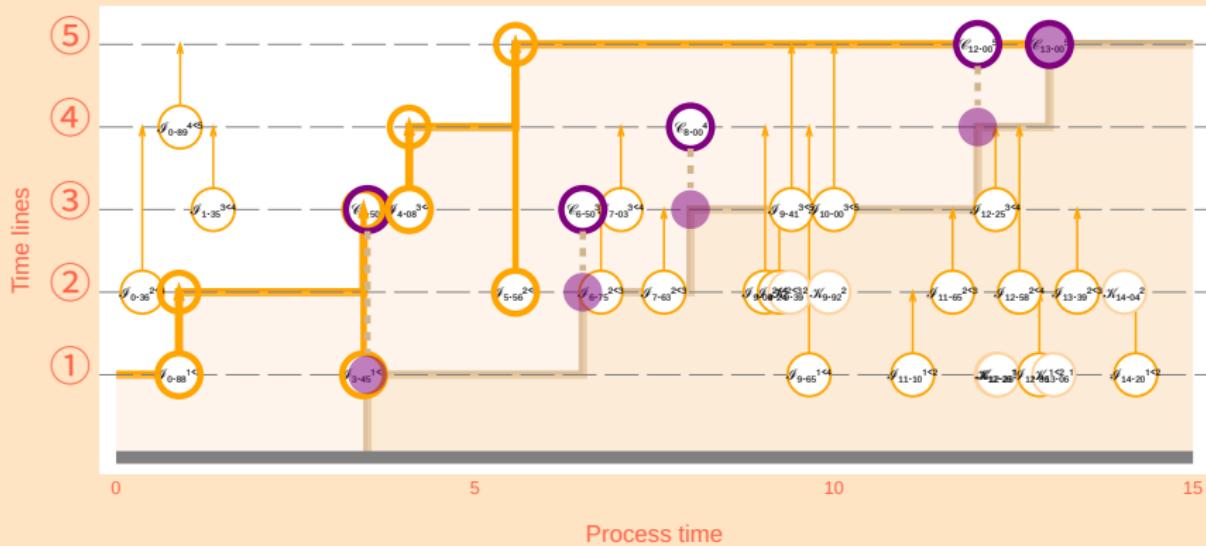


Figure 4: Initial conditioned epidemic, with conditioned removals indicated using purple circles (and purple disks when non-target timelines are infected).

Conditional epidemic update

Fully updated conditioned epidemic

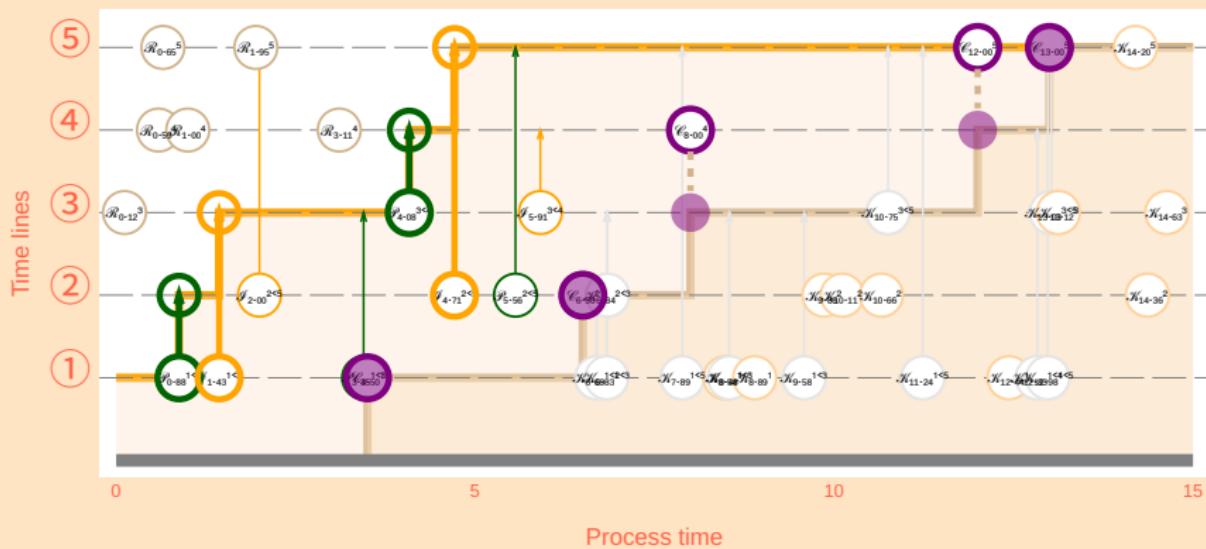


Figure 5: Epidemic updated under restriction: all conditioned removals remain activated, no new removals are activated. Green infections have been “perpetuated”.

Laziest feasible epidemic (LFE)

Fully updated conditioned epidemic with LFE

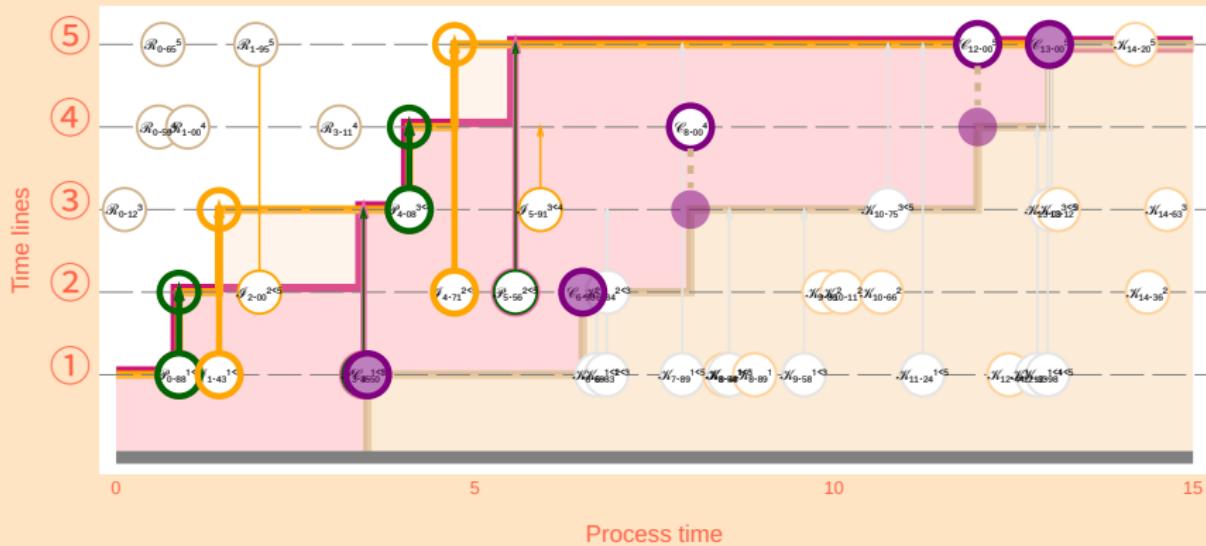


Figure 6: LFE computed recursively working right-to-left: the slowest sequence of infections deals with all infected timelines in order (includes perpetuated infections).

No-fly zone (NFZ)

Fully updated conditioned epidemic with NFZ

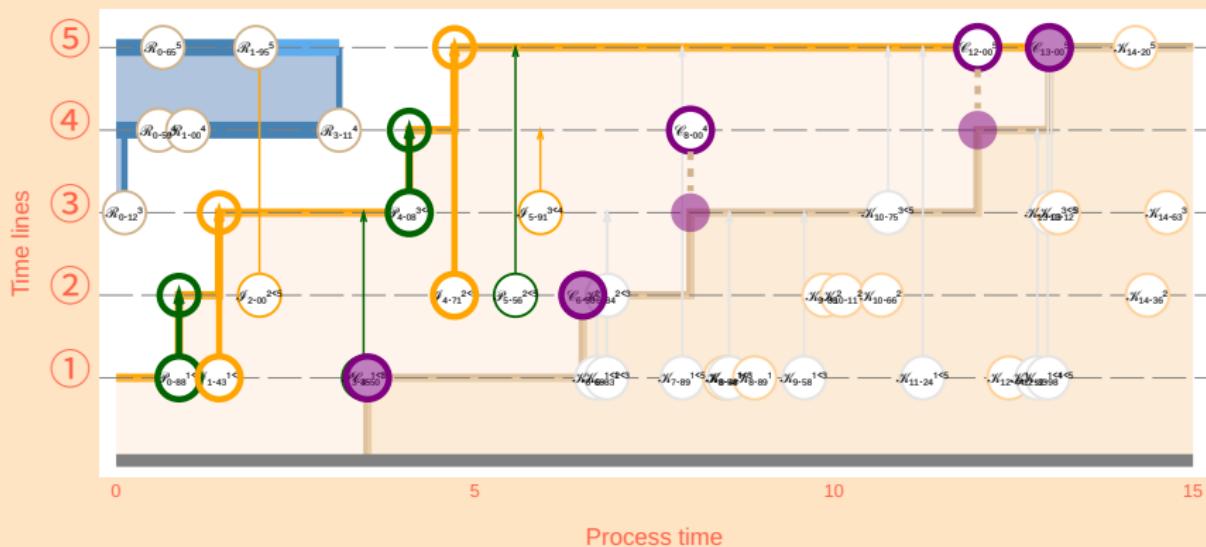


Figure 7: NFZ computed recursively working right-to-left: it traces the region of timelines that must not be infected if one is not to activate unobserved removals.

Conditioned evolution evolving in continuous algorithmic time

GIF MP4

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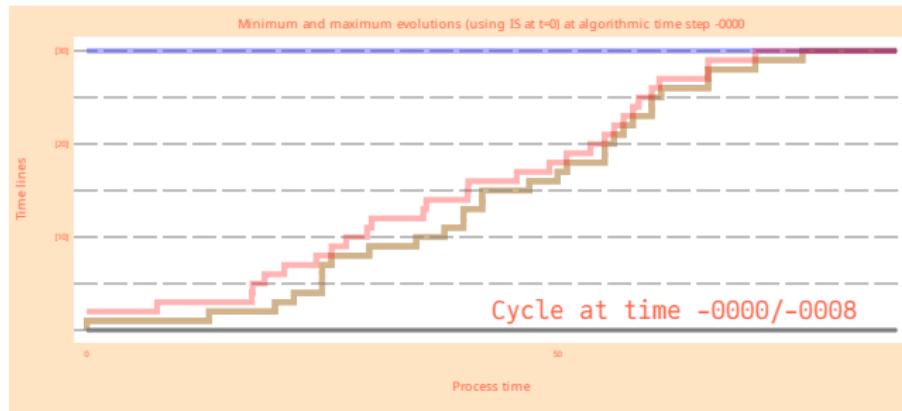
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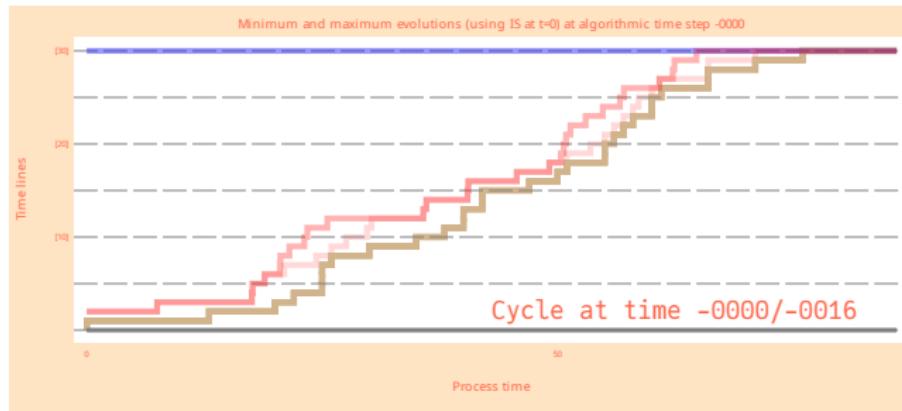
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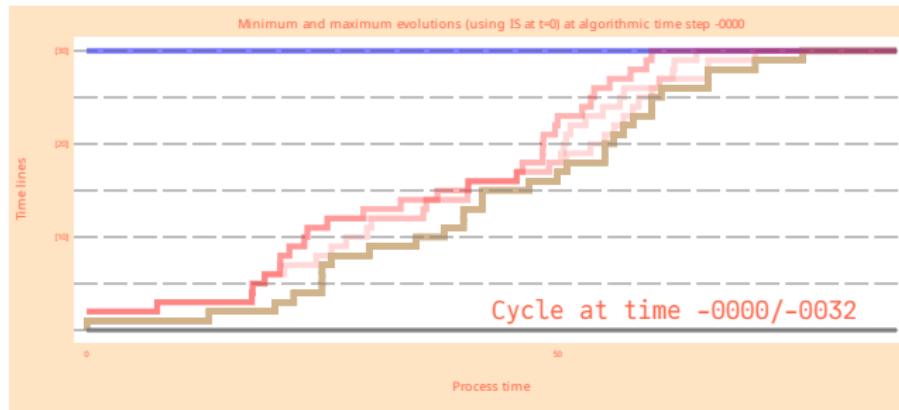
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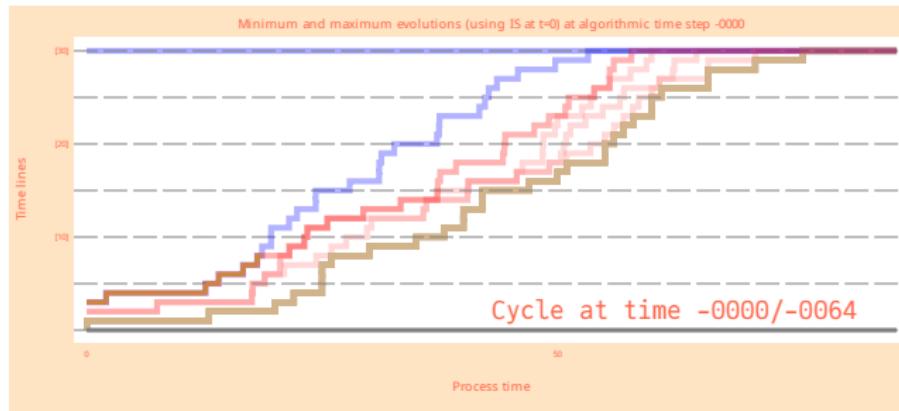
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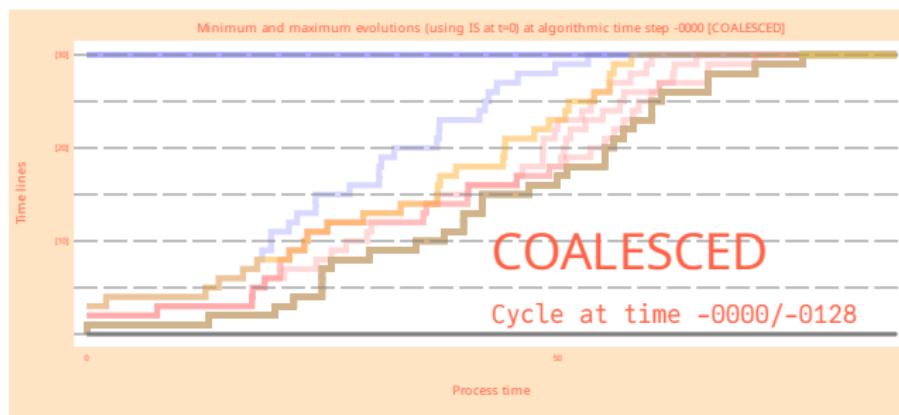
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- Finally: can we generalize to other suitable compartment models?

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- Thank you for your attention! **QUESTIONS?**



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Image information

<i>Image</i>	<i>Attribution</i>	
<i>Terry Pratchett</i> Classic CFTP for a simple random walk	Luigi Novi Result of code written by WSK	<i>CC BY 3.0</i>
<i>Diamond Princess</i> Epidemic CFTP images and animation	Alpsdake Result of code written by WSK	<i>CC BY-SA 4.0</i>

Previous instances of this talk

<i>Date</i>	<i>Title</i>		<i>Location</i>	
19/04/24	Perfect Epidemics	Short Research Talk	12mn	Warwick
15/05/24	McMC+Perfect Simulation	Graduate Seminar, Aristotle Univ.	50mn	Thessaloniki
17/01/25	Perfect Epidemics	Applied Probability Seminar	50mn	Warwick
27/06/25	Perfect Epidemics	UK Research Network Stochastics	45mn	Liverpool

Other technical information

Software used in computations

<i>Software</i>	<i>Version</i>	<i>Branch</i>	<i>Last commit</i>
quarto	1.6.39	—	
Running under julia	1.11.5	—	
EpidemicsCFTP	2.2.515	develop	Wed May 28 18:27:08 2025 +0100
EpidemicsUtilities	0.1.2.174	main	Tue Mar 4 16:32:10 2025
This quarto script	0.2.2.716	develop	Fri Jun 20 20:15:02 2025 +0100

Project information

Version: 0.2.2.716 (develop)

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Comment:

Sorted out movies, quote.