Perfect Epidemics

Seminar at University College Dublin

W S Kendall   S B Connor

Warwick, York

20 October 2025

Warwick Statistics

Introduction

Homage to Dublin
(Book of Kells, 9th century)

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Work on perfect simulation (CFTP) for epidemics, now being written up.
WSK acknowledges the support of UK EPSRC grant EP/R022100.

Handout is on the web: use the `QR-code` or visit `wilfridskendall.github.io/talks/PerfectEpidemics`.

Plan of talk

Gregory:	Is there any other point to which you would wish to draw my attention?
Holmes:	To the curious incident of the dog in the night-time.
Gregory:	The dog did nothing in the night-time.
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e. Example with real data.

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   - exploit monotonicity (*couple* maximal and minimal processes);
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4. Simplest possible example: *random-walk-CFTP* (can boost to use Ising model to do simple image reconstruction).

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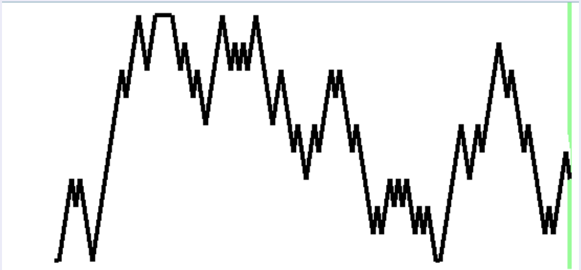
2. Conventional MCMC picks a starting point, then runs the simple random walk for long time till approximate equilibrium.



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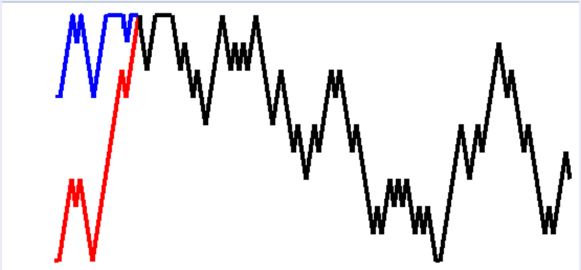

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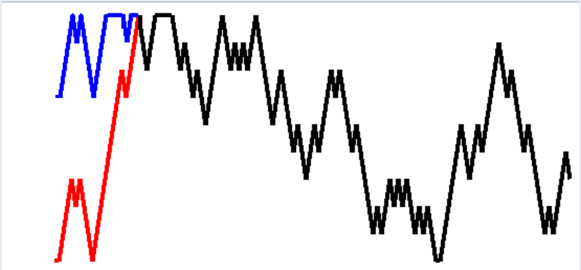

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4. Generally not true that location *at* coupling is a draw from equilibrium.

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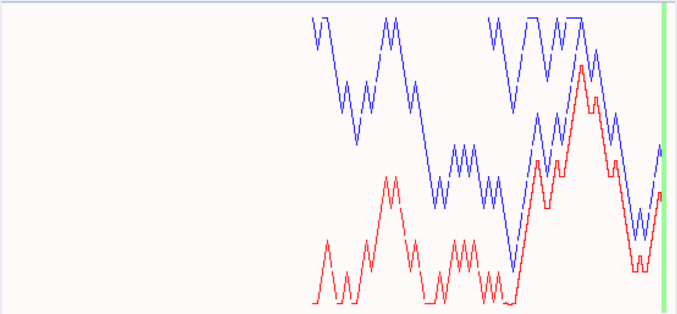

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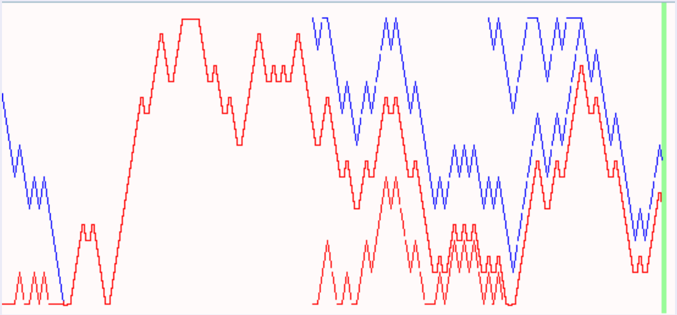

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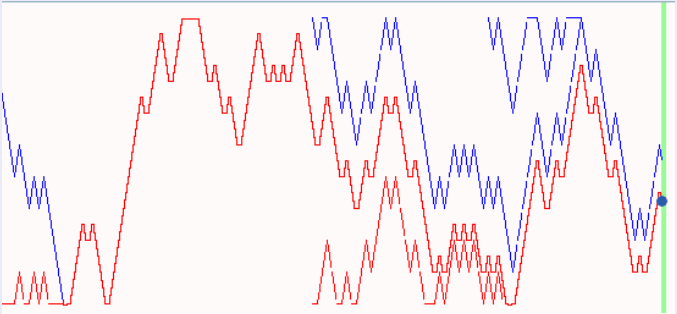

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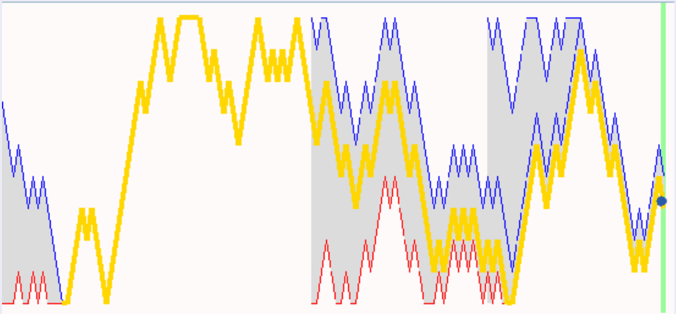

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4. When coupled, top and bottom yield a common value at time 0.
5. The common value (golden thread) is an exact draw from equilibrium!

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4. Detailed expositions: WSK (2005), Huber (2015).
   (Want to implement CFTP in R? see WSK, 2015.)

Warwick Statistics

2. Perfect Epidemics: a challenge problem for CFTP

S-I-R deterministic epidemic: differential equation system for (s, i, r)

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\begin{aligned}
&\textbf{Susceptible:} \qquad & s' &= -\alpha\, s\, i\,, \\
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Both models share an unrealistic assumption: homogeneous mixing.

In contrast, Fraser *et al* (2023) deploy a UK model with $N{=}10^6$ agents!

There are *many* important inferential questions (Cori & Kucharski, 2024).

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Wikipedia: "The British-registered *Diamond Princess* was the first cruise ship to have a major `[COVID-19]` outbreak on board, with the ship quarantined at Yokohama from 4 February 2020 for about a month. Of 3711 passengers and crew, around 700 people became infected and 9 people died."
Evidently $\alpha\ s_0/\beta \gg 1$ – as was sadly later confirmed, a sorrow for us all.

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5. Can we use perfect simulation?

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The simplest possible variant of contact tracing:

“When did the infections occur, supposing we only observe removals?”

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4. First step: evolve whole S-I-R trajectory in *algorithmic time* (alter potential infections and removals using immigration-death in discrete algorithmic time).
5. Result: *trajectory-valued chain*, unconditioned S-I-R as equilibrium.

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From incidents to unconditioned epidemic trajectories (1/3)

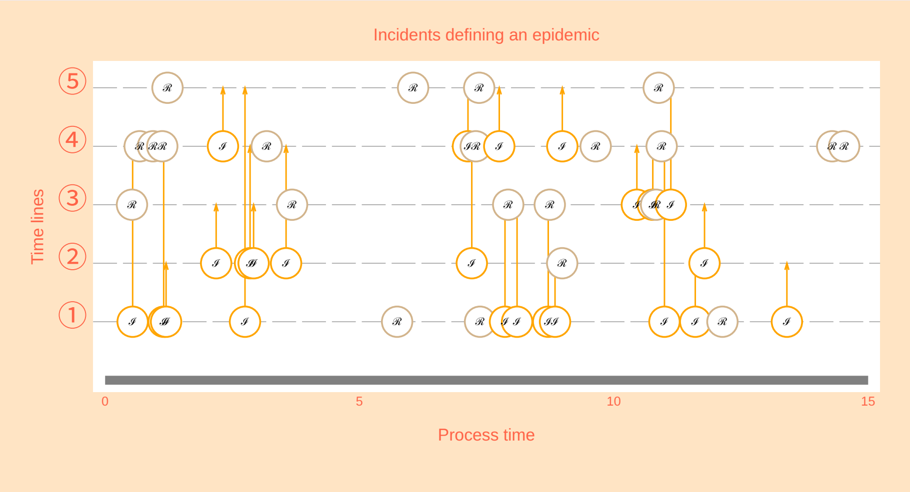

Figure 1: Light-orange circles denote potential infections (arrows point upwards to targets); light-brown circles denote potential removals.

From incidents to unconditioned epidemic trajectories (2/3)

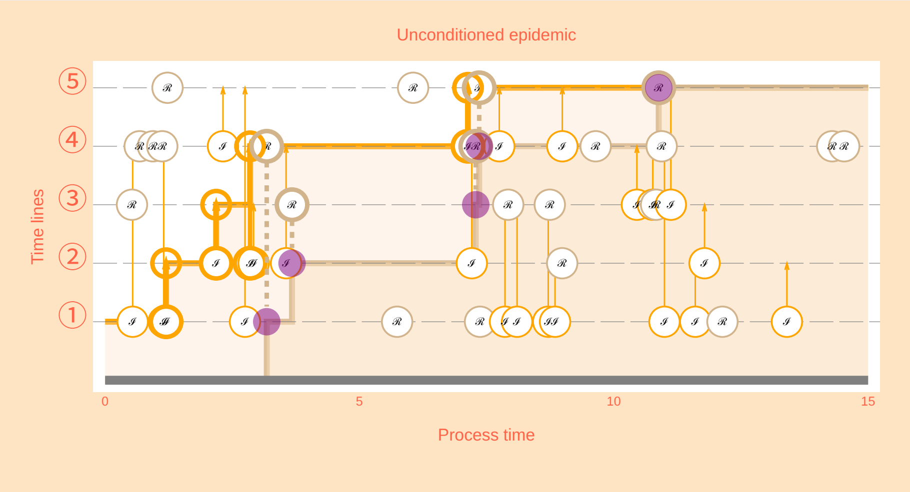

Figure 2: (a) *Infections* activate if on infected timeline and pointing to lowest uninfected timeline; (b) *Removals* activate if on infected timeline; remove lowest infected (purple disk).

From incidents to unconditioned epidemic trajectories (3/3)

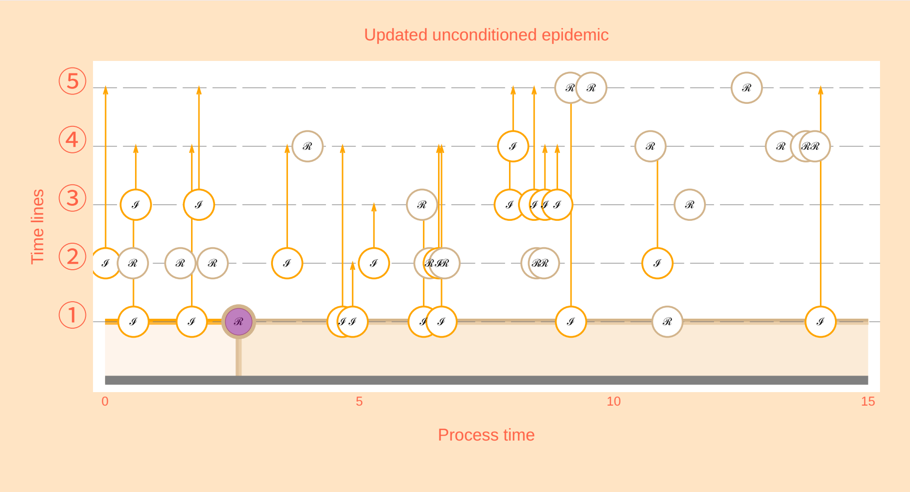

Figure 3: A step in algorithmic time for the unconditioned epidemic simply involves replacing all original incidents by an entirely new set of incidents.

Crucial technical point

- Updates in algorithmic time τ are then (algorithmic-)*time-reversible*: so restriction to a subset S of state-space (*activated* / *conditioned* removals must occur precisely at the specified set of times) implies a new equilibrium which is the old equilibrium conditioned to lie in S.

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- Crucially, re-sampling step 2 ensures composite evolution is irreducible over S! (So equilibrium under conditioning is unique.)

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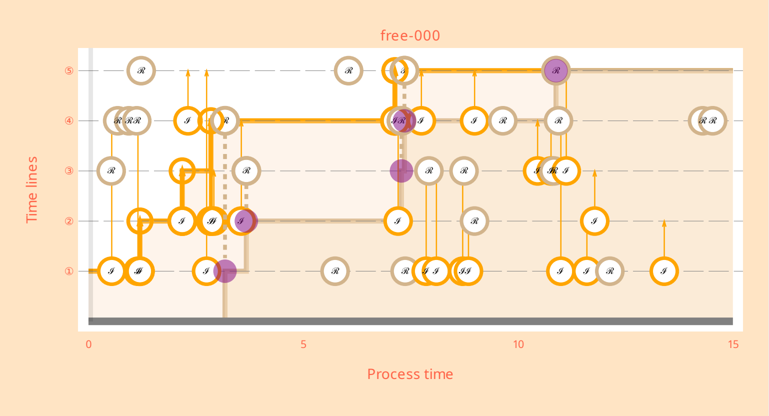
Free evolution evolving in continuous algorithmic time



GIF MP4

3. Conditioning on observed removals

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- Housekeeping details used to establish that monotonicity still works: *laziest feasible epidemic* (LFE) and *no-fly zone* (NFZ).

Initial conditioned epidemic

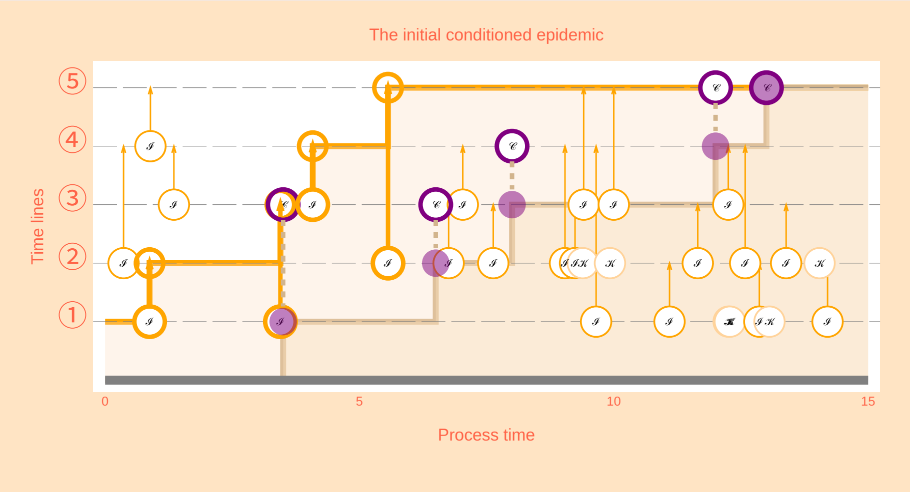


Figure 4: Initial conditioned epidemic, with conditioned removals indicated using purple circles (and purple disks when non-target timelines are infected).

Conditional epidemic update

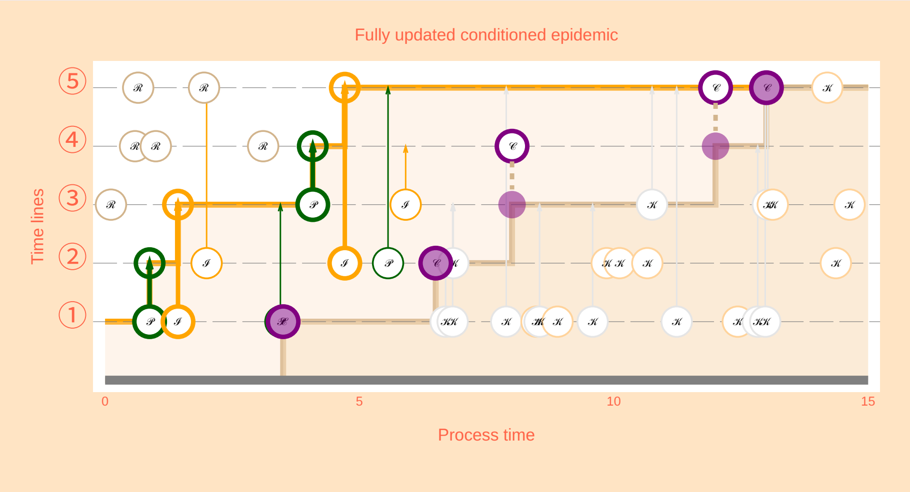


Figure 5: Epidemic updated under restriction: all conditioned removals remain activated, no new removals are activated. Green infections have been "perpetuated".

Laziest feasible epidemic (LFE)

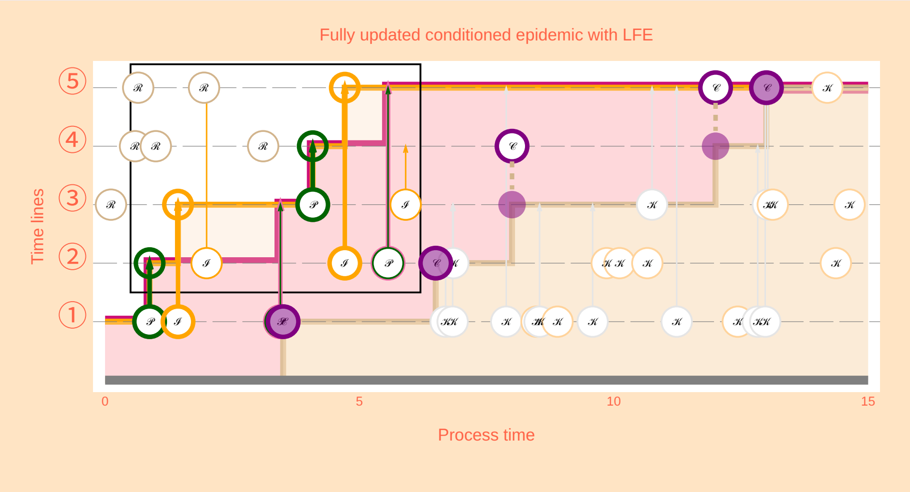
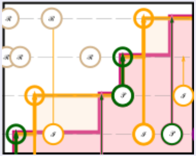


Figure 6: LFE computed recursively working right-to-left: slowest sequence of infections (and perpetuated infections) generating all conditioned removals. Can be used to identify perpetuated infections.

LFE: construction details



1. Recursive definition of LFE: working over $[0, T)$,

$$\begin{aligned} s_N &= T \\ s_i &\leq \min\left\{s_{i+1}\,, \inf\{s \;:\; \text{there is a } \mathcal{C}^i_s\}\right\}\;. \end{aligned}$$

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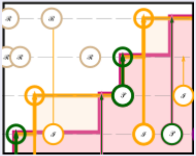

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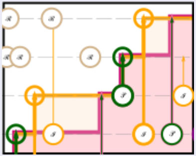

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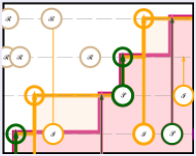

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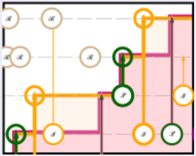

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3. Comparisons based on intrinsic definition show monotonic dependence of LFE on old epidemic history.

No-fly zone (NFZ)

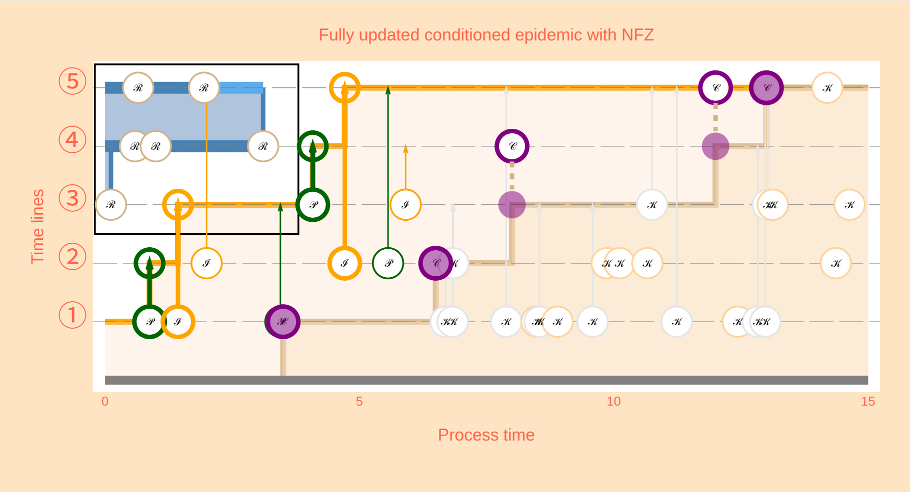
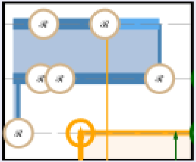


Figure 7: NFZ computed recursively working right-to-left: it traces a region of timelines such that unobserved removals are not activated if region not infected.

NFZ: construction details



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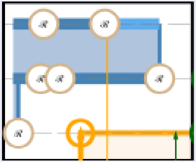

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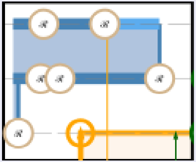

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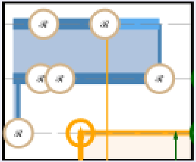

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2. Can then show monotonic dependence of NFZ on old epidemic history.

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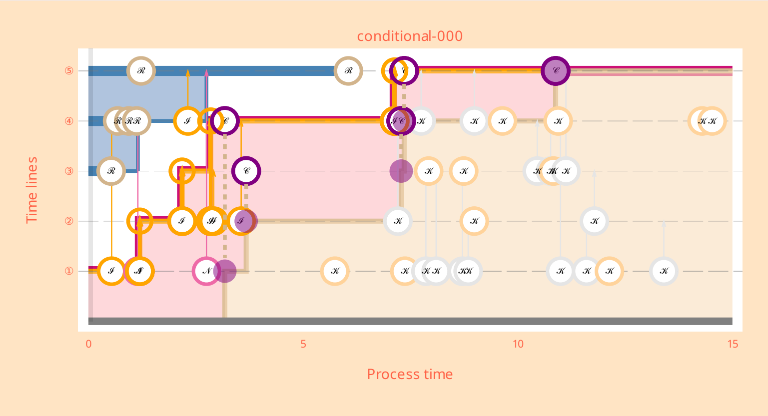
Conditioned evolution evolving in continuous algorithmic time



GIF MP4

If a new $\mathcal{J}_t^{i<j}$ has i, j in infected zone then LFE is relevant;
if i in infected zone and j in susceptible zone then NFZ is relevant.
New epidemic is monotonic in LFE, NFZ!

4. Example

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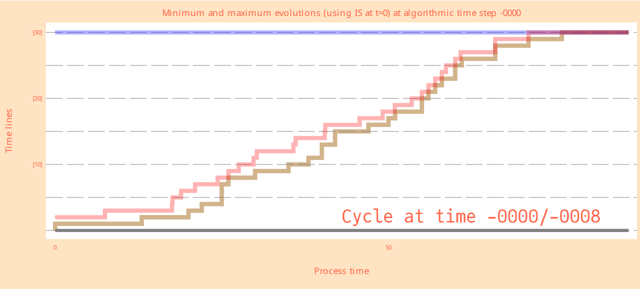

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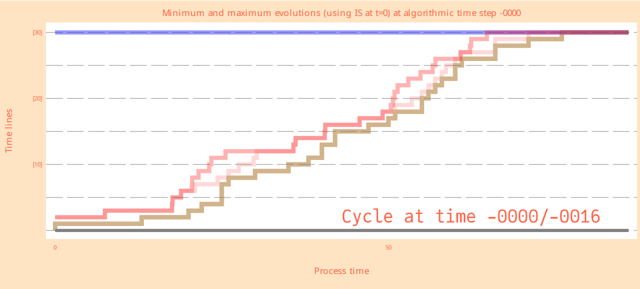

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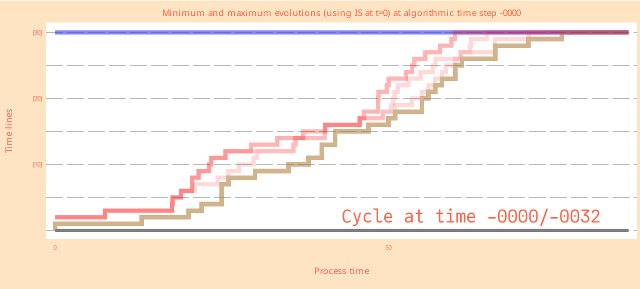

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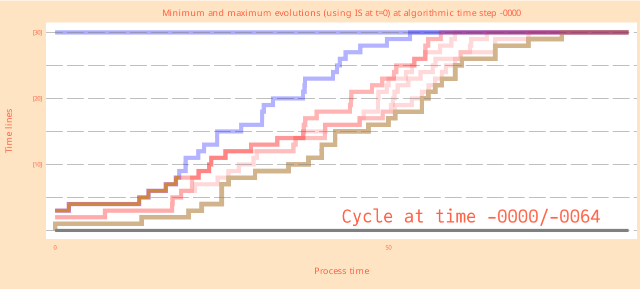

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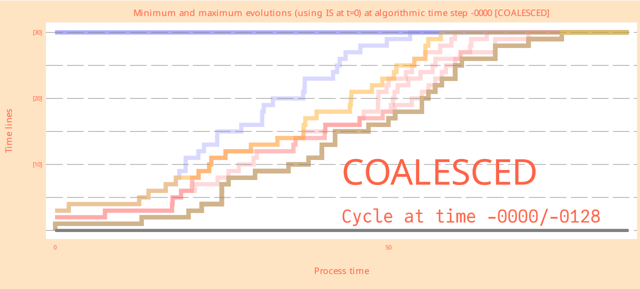

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Warwick Statistics

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- **Finally:** generalize to other suitable compartment models?

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- Thank you for your attention! **QUESTIONS?**

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Image information

Image	*Attribution*	
Book of Kells	Huber Gerhard	*CC BY 4.0*
Classic CFTP for a simple random walk	Result of code written by WSK	
Diamond Princess	Alpsdake	*CC BY-SA 4.0*
Epidemic CFTP images and animation	Result of code written by WSK	

Previous instances of this talk

Date	*Title*			*Location*
19/04/24	Perfect Epidemics	Short Research Talk	12mn	Warwick
15/05/24	McMC+Perfect Simulation	Graduate Seminar, Aristotle Univ.	50mn	Thessaloniki
17/01/25	Perfect Epidemics	Applied Probability Seminar	50mn	Warwick
27/06/25	Perfect Epidemics	UK Research Network Stochastics	45mn	Liverpool
20/10/25	Perfect Epidemics	Seminar		Dublin

Other technical information

Software used in computations

Software	*Version*	*Branch*	*Last commit*
`quarto`	`1.6.39`	—	
Running under `julia`	`1.12.0`	—	
`EpidemicsCFTP`	`2.2.532`	`develop`	`Tue Jul 8 17:13:42 2025 +0100`
`EpidemicsUtilities`	`0.1.2.177`	`main`	`Fri Sep 26 15:35:26 2025 +0100`
This `quarto` script	`0.2.2.725`	`2025-10-09-Dublin-preparation`	`Tue Oct 14 18:01:39 2025 +0100`

Project information

Version:	`0.2.2.727` (`2025-10-09-Dublin-preparation`)
Author:	Wilfrid Kendall <`W.S.Kendall@warwick.ac.uk`>
Date:	Wed Oct 15 19:53:35 2025 +0100

Comment:
Near-final preparation for Dublin talk 20 October 2025. Minor edits.