Perfect Epidemics

Applied Probability Seminar
Department of Statistics, University of Warwick

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Warwick, York

4 February 2025

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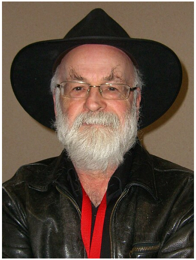
Introduction

"Maybe the only significant difference between a really smart simulation and a human being was the noise they made when you punched them."
(The Long Earth, Pratchett & Baxter, 2012)



Handout is on the web: use the `QR-code` or visit `wilfridskendall.github.io/talks/PerfectEpidemics`.

This is initial work on using perfect simulation (CFTP) for epidemics.
WSK acknowledges the support of UK EPSRC grant EP/R022100.

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- Key ideas of "*classic CFTP*":
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- Simplest possible example: *random-walk-CFTP*
  (can boost to use Ising model to do simple image reconstruction).

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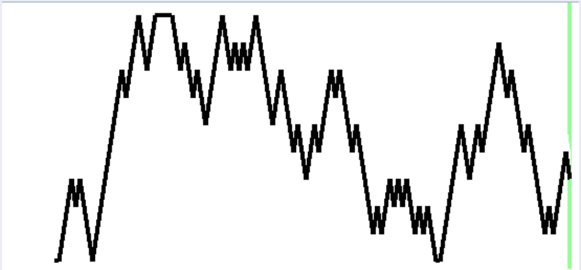

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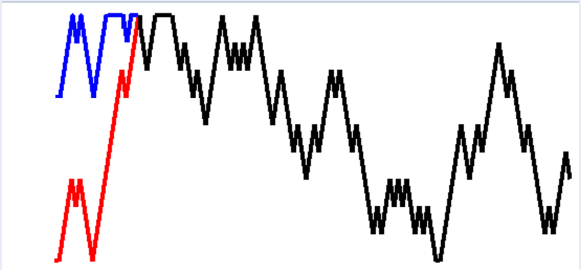

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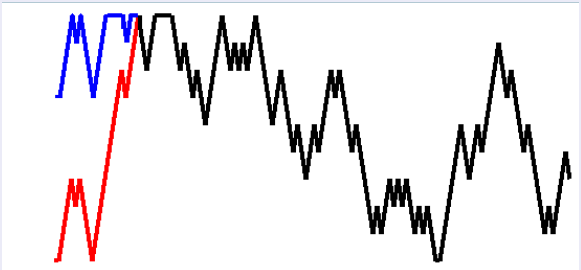

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- Generally not true that location *at* coupling is a draw from equilibrium.

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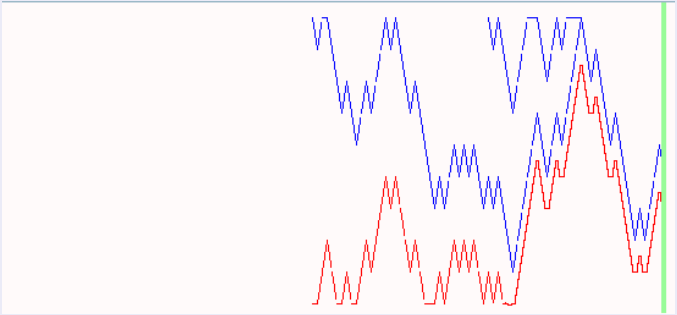

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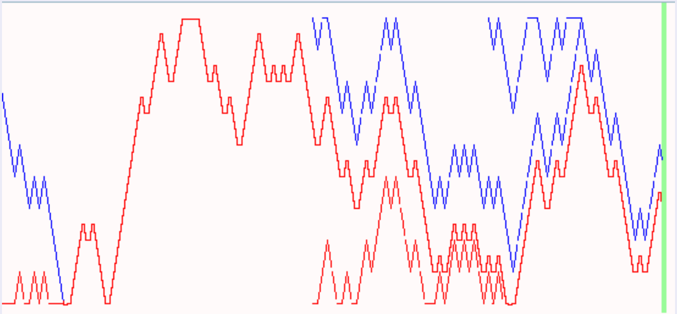

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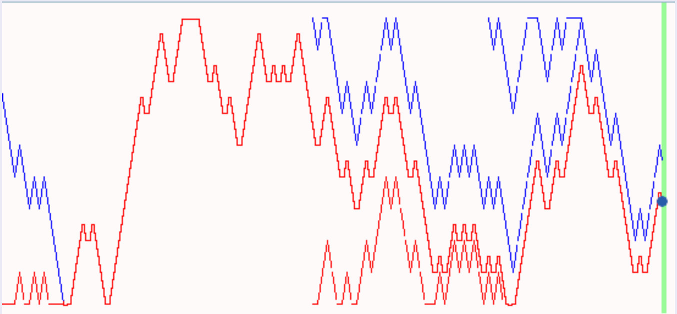

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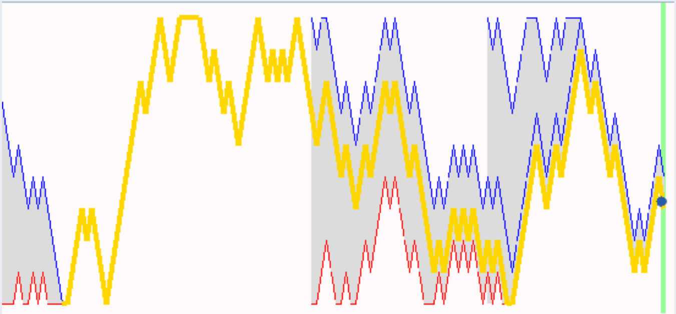

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- May need to iterate back-off doubling several times.
- When coupled, top and bottom yield a common value at time 0.
- The common value (golden thread) is an exact draw from equilibrium!

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  - Detailed expositions: WSK (2005), Huber (2015).
    (Want to implement CFTP in R? see WSK, 2015.)

2. Perfect Epidemics: a challenge problem for CFTP

Many important inferential questions (Cori & Kucharski, 2024).

Simplest models (versus UK model with 10^6 agents!, Fraser & Others, 2023):

S-I-R deterministic epidemic: susceptibles s, infectives i, removals r (constant total population $s + i + r = n$):

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S-I-R stochastic epidemic: a Markov chain (S, I, R) with transitions

Infection:	$S \to S-1\,,$	$I \to I+1$	at rate	$\alpha\, S\, I\,,$
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Both models make an unrealistic assumption: homogeneous mixing.

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Wikipedia: "The British-registered *Diamond Princess* was the first cruise ship to have a major `[COVID-19]` outbreak on board, with the ship quarantined at Yokohama from 4 February 2020 for about a month. Of 3711 passengers and crew, around 700 people became infected and 9 people died."
Evidently $\alpha\ s_0/\beta \gg 1$ – as was sadly later confirmed, a sorrow for us all.

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5. Can we use perfect simulation?

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5. Result: *trajectory-valued chain*, unconditioned S-I-R as equilibrium.

From incidents to unconditioned epidemic trajectories (1/3)

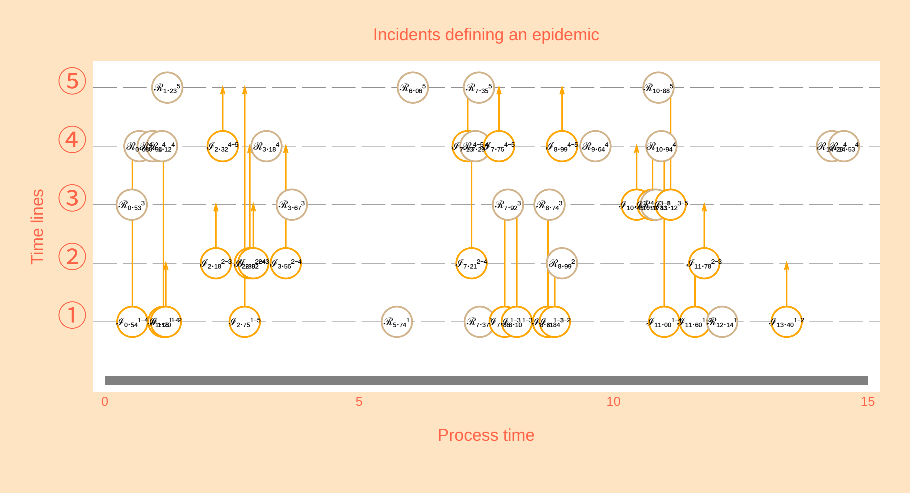

Figure 1: Light-orange circles denote potential infections (arrows point upwards to targets); light-brown circles denote potential removals.

From incidents to unconditioned epidemic trajectories (2/3)

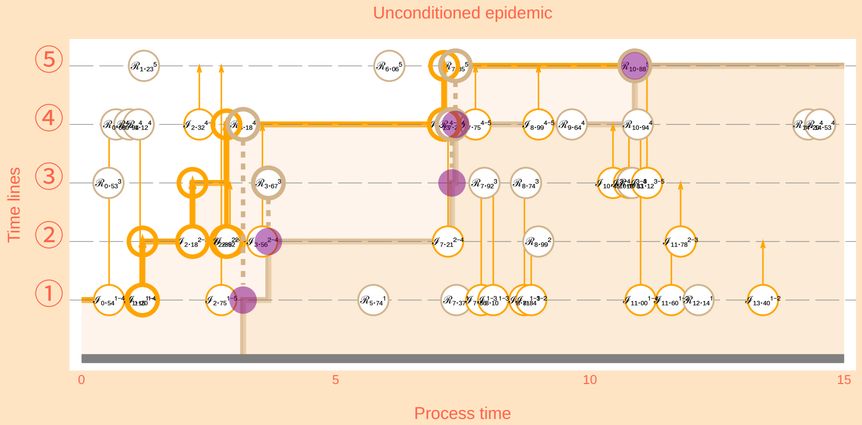

Figure 2: (a) *Infection* activates if target lies on the lowest uninfected timeline; and (b) *removal* activates if on infected timeline; remove lowest infected (purple disk).

From incidents to unconditioned epidemic trajectories (3/3)

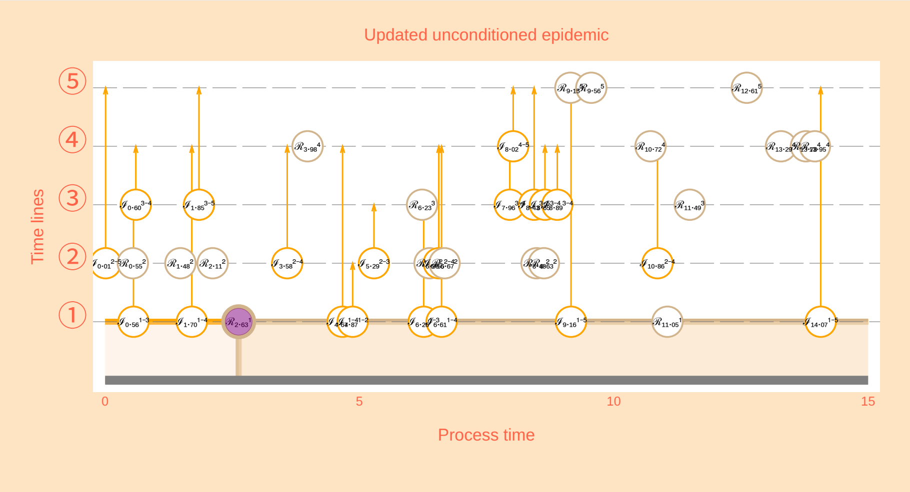

Figure 3: A step in algorithmic time for the unconditioned epidemic simply involves replacing all original incidents by an entirely new set of incidents.

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- Thus the original update is expressible as a (continuous) composition of updates, each of which satisfies detailed balance in equilibrium.
- The connection "restriction=conditioning" still holds.
- Crucially, step 2 ensures composition action is irreducible over S! (So equilibrium under conditioning is unique.)

Illustration of technical point (1/8)

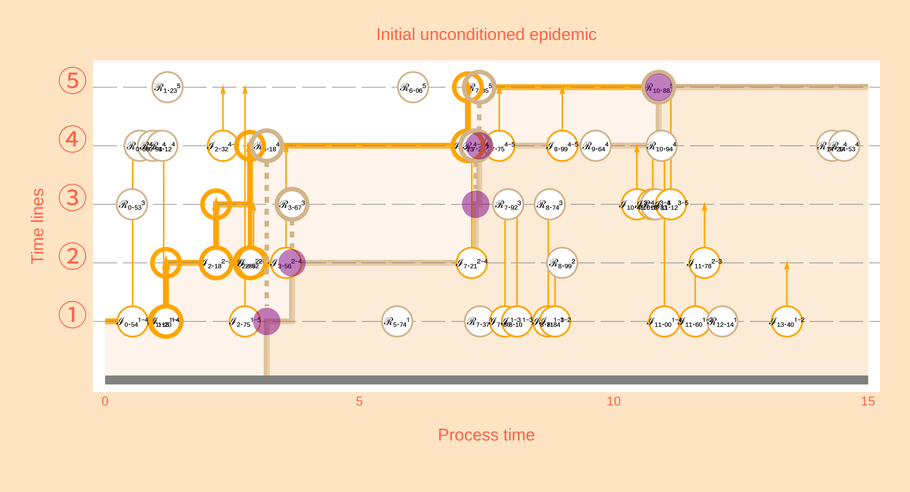

Figure 4: No change yet to removals or infections;

Illustration of technical point (2/8)

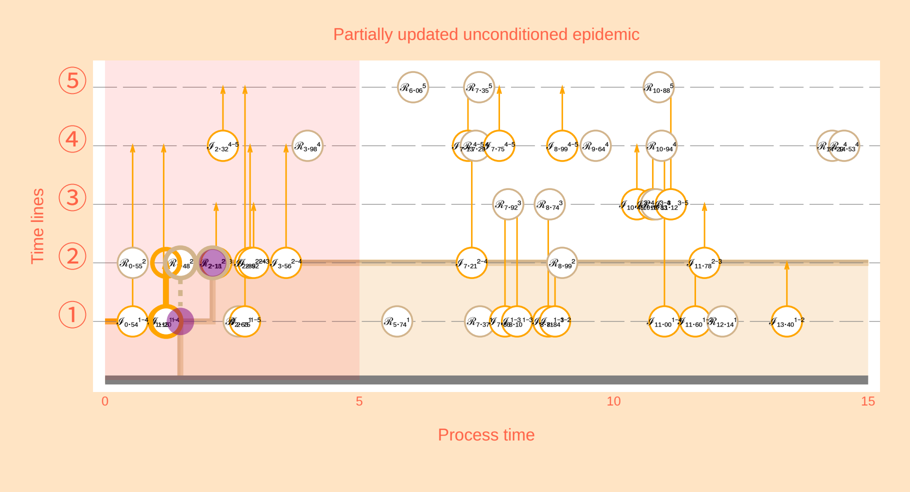

Figure 5: Replace first third of removals, infections unchanged;

Illustration of technical point (3/8)

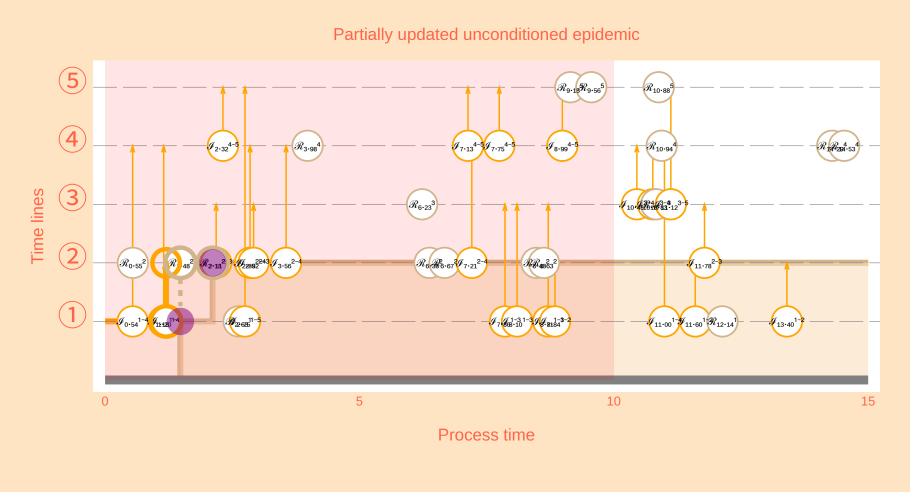

Figure 6: Replace first two-thirds of removals, infections unchanged;

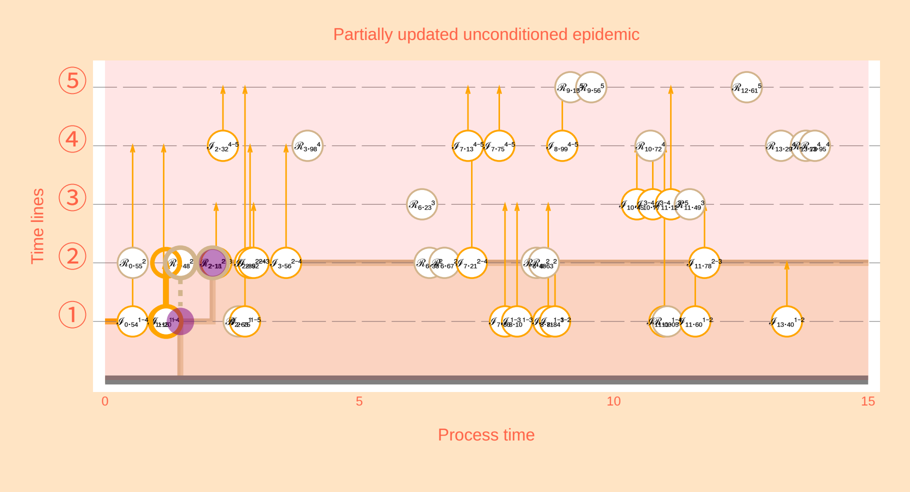

Figure 7: All removals resampled, infections as yet unchanged;

Illustration of technical point (5/8)

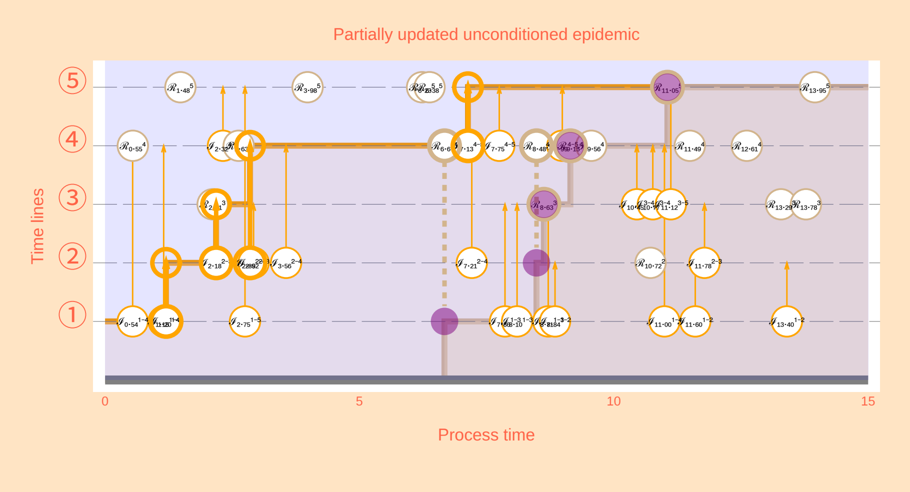

Figure 8: Re-sample all removal timelines, infections as yet unchanged;

Illustration of technical point (6/8)

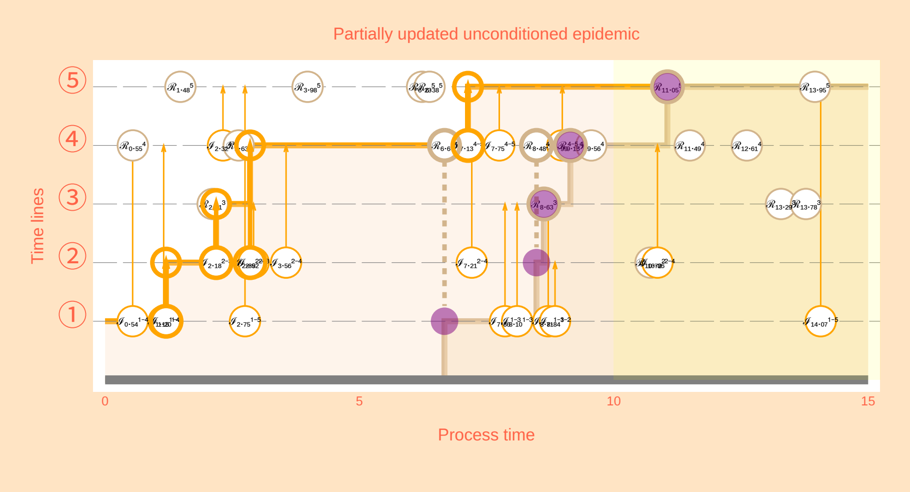

Figure 9: Re-sample last third of infections;

Illustration of technical point (7/8)

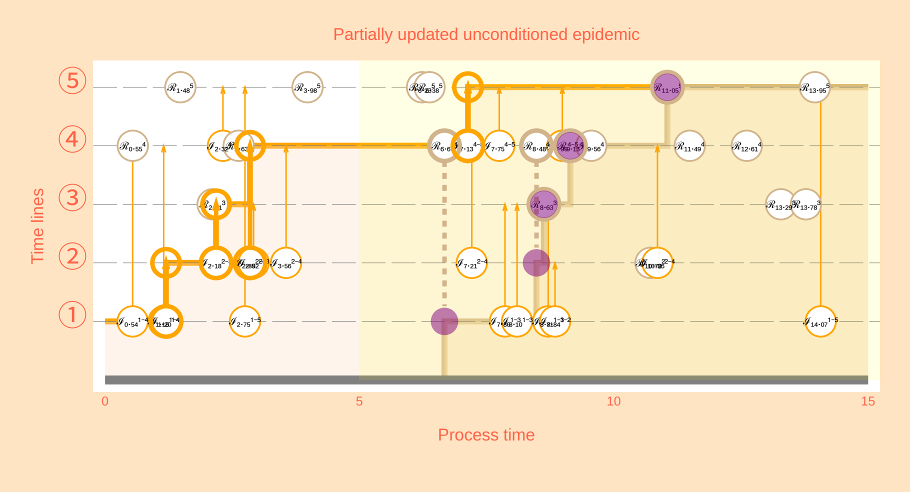

Figure 10: Re-sample last two-thirds of infections;

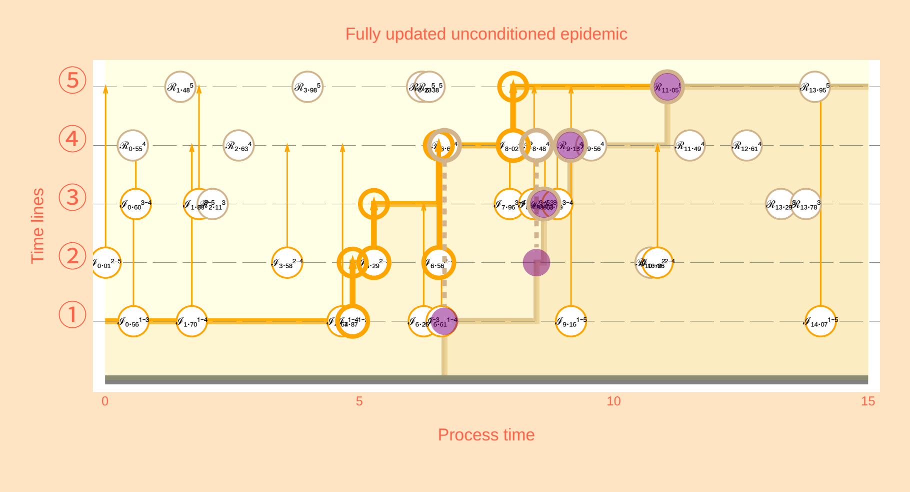

Figure 11: All infections now re-sampled.

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- Does this produce a *feasible* and suitably *monotonic* algorithm?
- Housekeeping details used to establish that monotonicity still works: *laziest feasible epidemic* (LFE) and *no-fly zone* (NFZ).

Initial conditioned epidemic

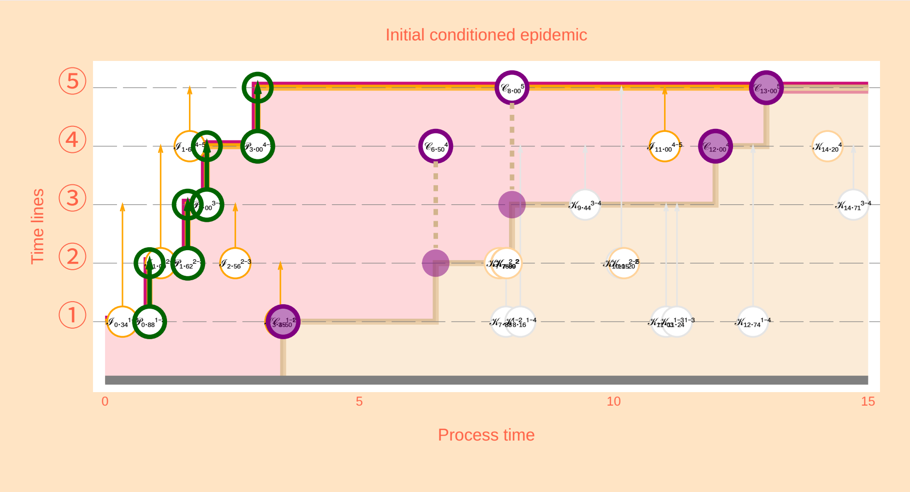

Figure 12: Initial epidemic with conditioned removals indicated using purple circles (and purple disks when different timelines are infected).

Conditional epidemic update

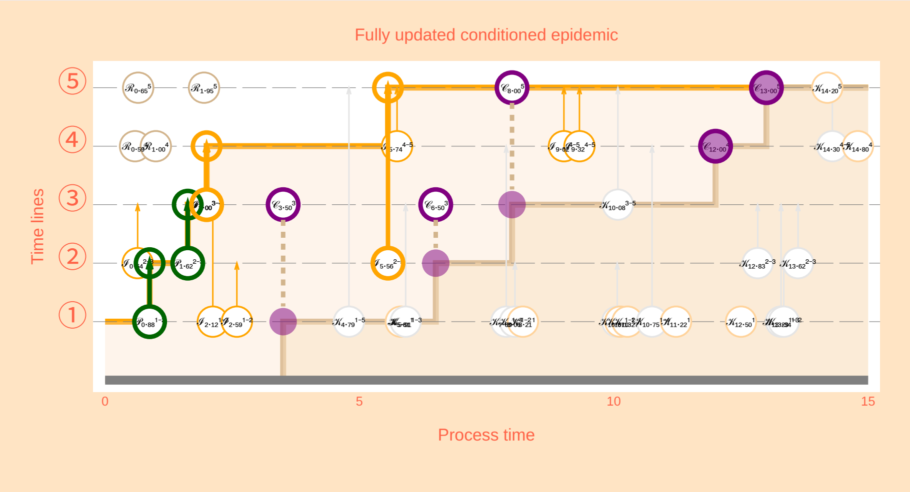

Figure 13: Epidemic updated under restriction: all conditioned removals remain activated, no new removals are activated. Green infections have been “perpetuated”.

Last feasible epidemic (LFE)

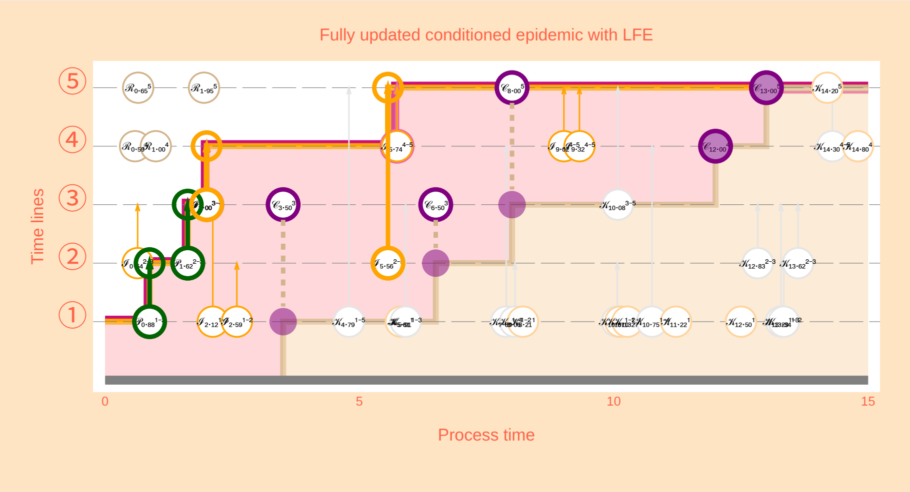

Figure 14: LFE computed recursively working right-to-left: the slowest sequence of infections deals with all infected timelines in order (includes perpetuated infections).

No-fly zone (NFZ)

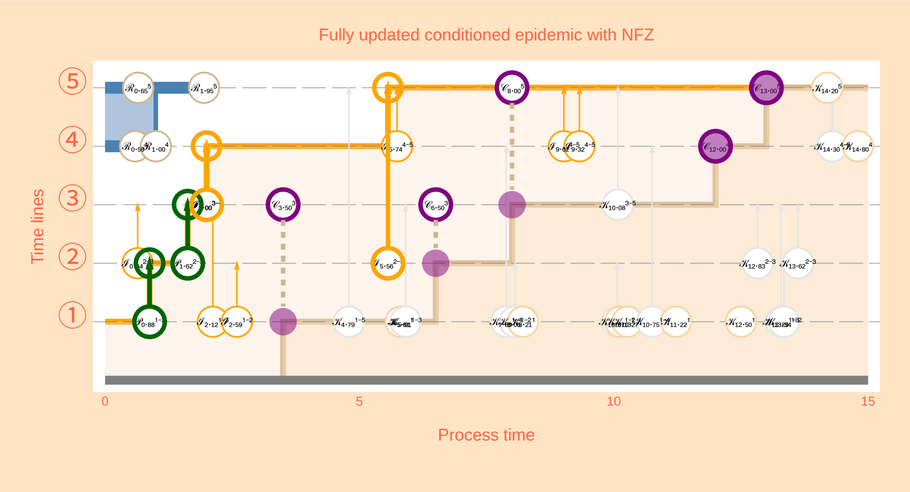

Figure 15: NFZ computed recursively working right-to-left: it traces the region of timelines that must not be infected if one is not to activate unobserved removals.

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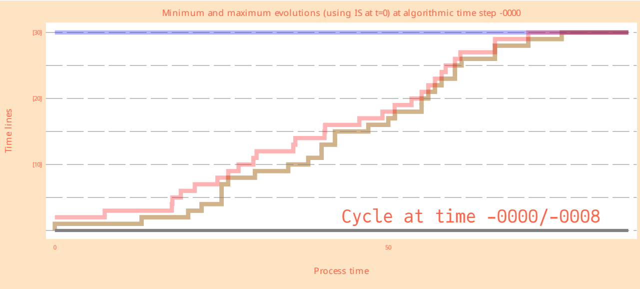

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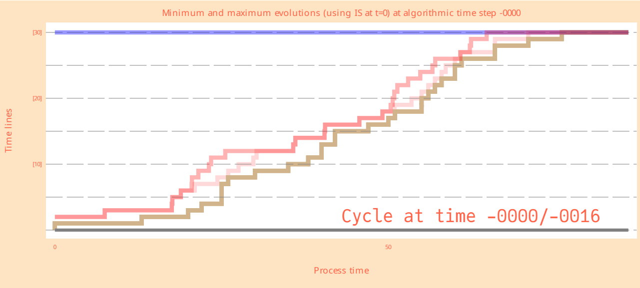

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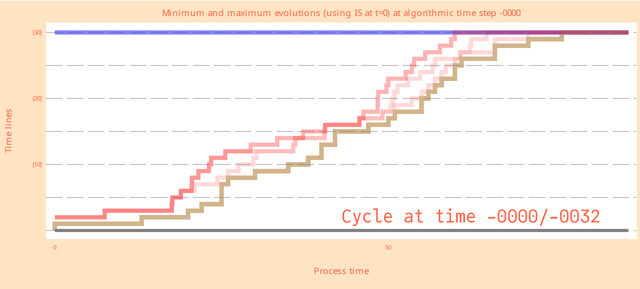

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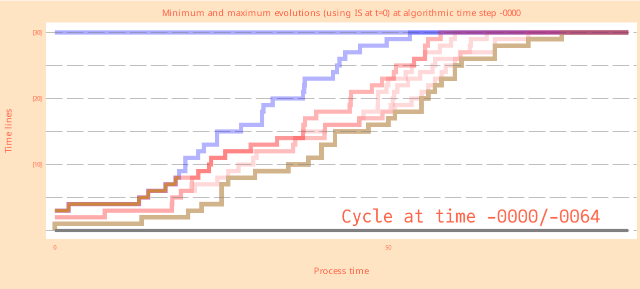

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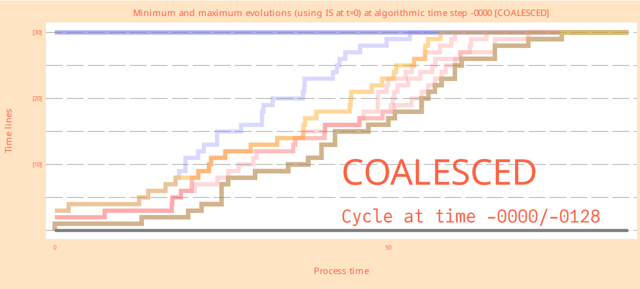

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- Finally: can we generalize to other suitable compartment models?

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- Thank you for your attention! **QUESTIONS?**

References I

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Image information

Image	*Attribution*	
Terry Pratchett	Luigi Novi	*CC BY 3.0*
Classic CFTP for a simple random walk	Result of code written by WSK	
Diamond Princess	Alpsdake	*CC BY-SA 4.0*
Epidemic CFTP images and animation	Result of code written by WSK	

Previous instances of this talk

Date	*Title*		*Location*
19/04/24	Perfect Epidemics	Short Research Talk (12min)	Warwick
15/05/24	McMC and Perfect Simulation	Graduate Seminar, Aristotle Univ. (50min)	Thessaloniki
17/01/25	Perfect Epidemics	Applied Probability Seminar (50min)	Warwick

Appendix A: A “near-maximal” configuration

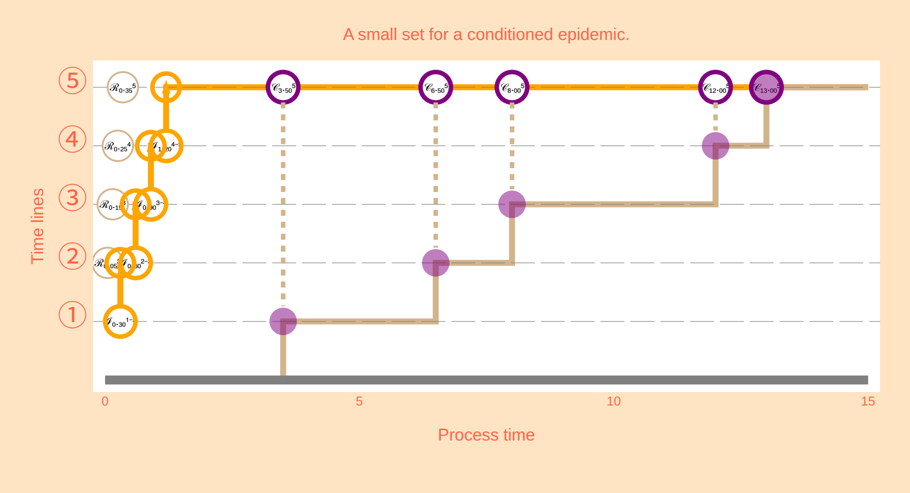

Figure 16: A conditioned epidemic in which all activated infections occur before time 3.0, also before smallest observed removal time.

Appendix B: Updating a conditioned epidemic INCOMPLETE

We now work through the update of the conditioned epidemic in stages.

Initial conditioned epidemic (1/8)

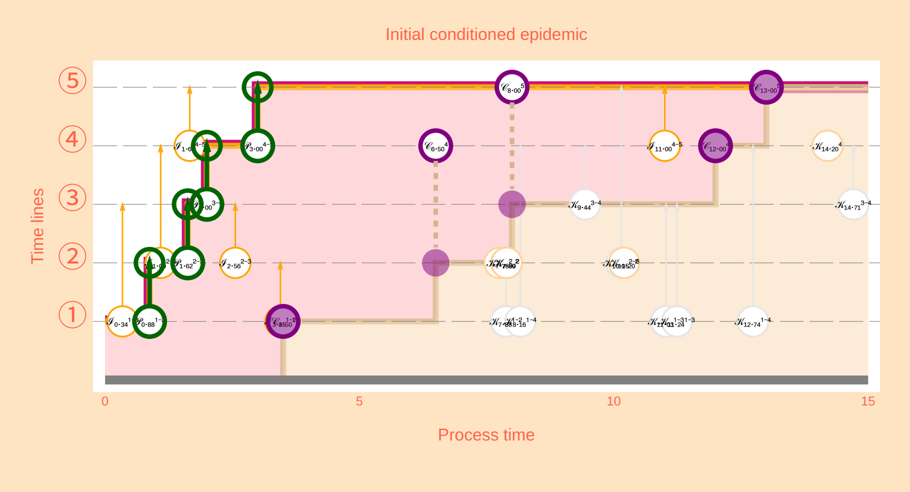

Figure 17: Initial epidemic with conditioned removals indicated by purple circles.

Conditional epidemic (2/8)

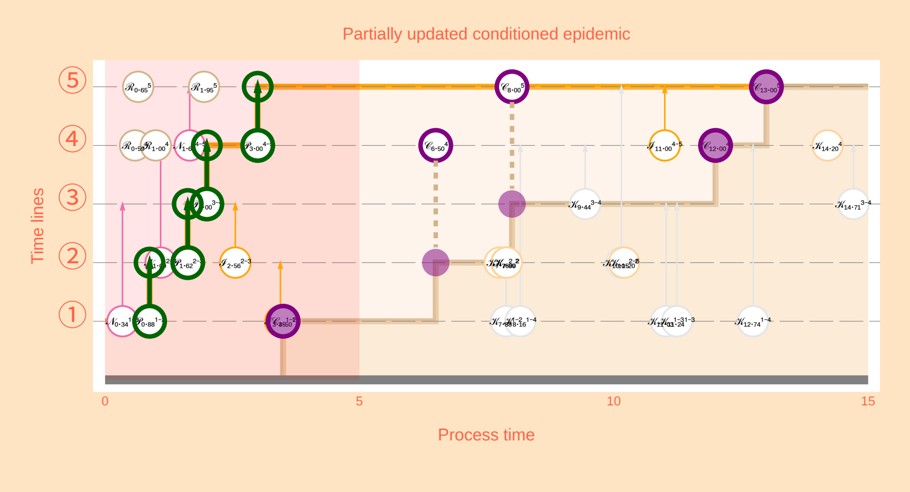

Figure 18: Replace first third of removals, infections unchanged;

Conditional epidemic (3/8)

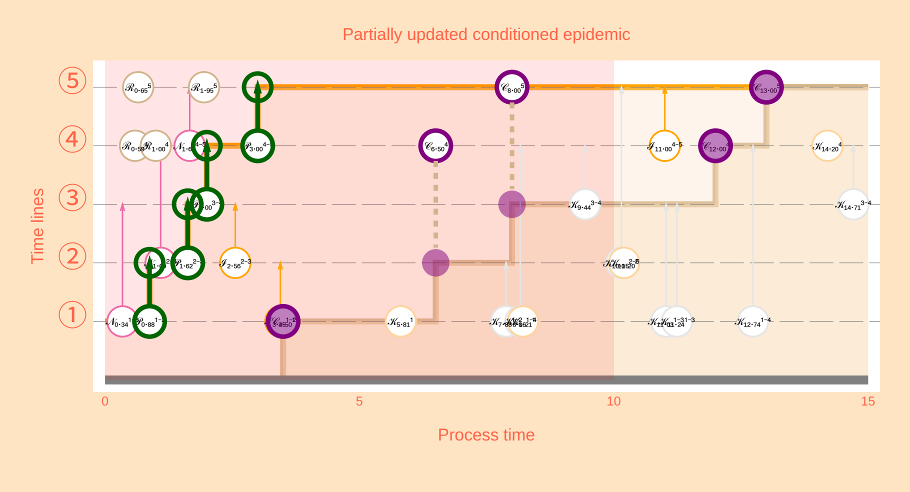

Figure 19: Replace second third of removals, infections unchanged;

Conditional epidemic (4/8)

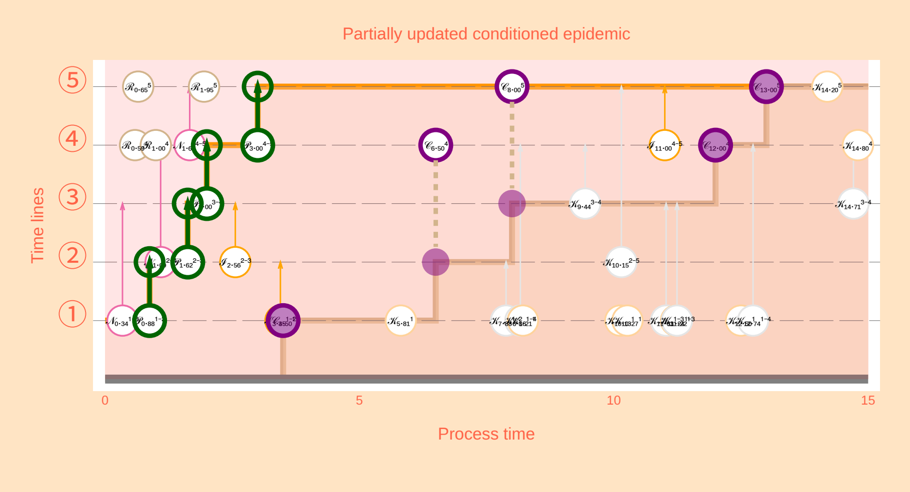

Figure 20: Replace remaining removals, infections unchanged;

Conditional epidemic (5/8)

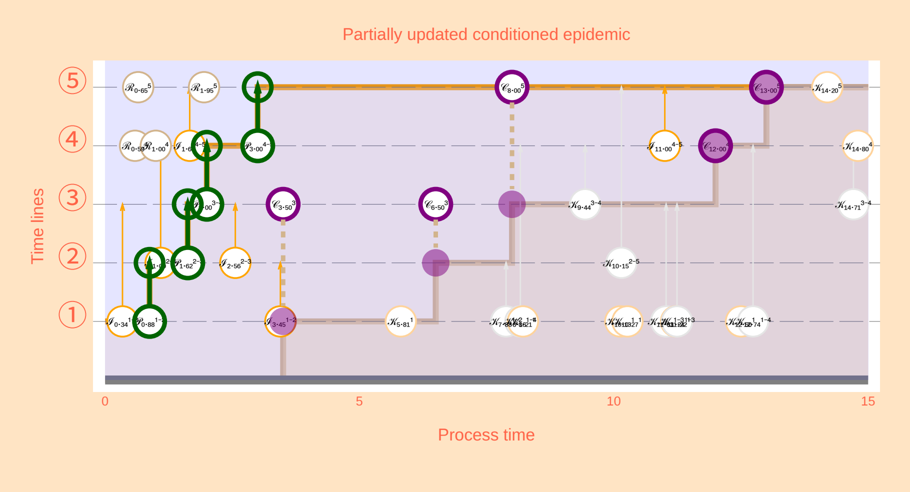

Figure 21: Re-sample all removal timelines, infections as yet unchanged;

Eventual conditioned epidemic after use of an innovation (6/8)

Still to be done: 1/3 of way through new infections, display current LFE and NFZ.

Eventual conditioned epidemic after use of an innovation (7/8)

Still to be done: 2/3 of way through new infections, display current LFE and NFZ.

Eventual conditioned epidemic after use of an innovation (8/8)

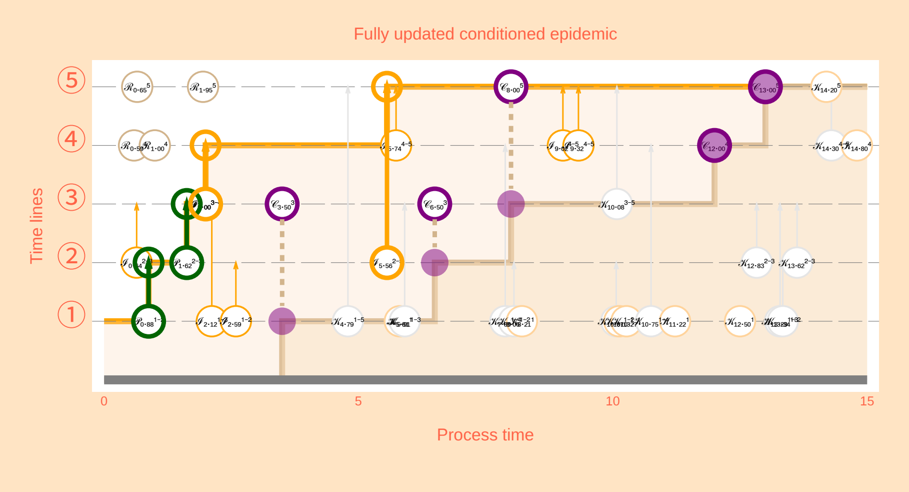

Figure 22: All infections now re-sampled. Green infections are “perpetuated”.

Appendix C: Naive approach to compartment models fails

- Consider a modification of this approach to the case of compartmentalized populations.

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Warwick Statistics

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- Suppose the conditioning on removals is specifically about named individuals j being removed at specified times r_j;
  suppose also there are no "occult" (unobserved) removals for any other individuals.

Warwick Statistics

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- Suppose the conditioning on removals is specifically about named individuals j being removed at specified times r_j;
suppose also there are no "occult" (unobserved) removals for any other individuals.
- This would apply, for example, in the case of the *Diamond Princess* if α, β depended on age and location of cabin on the ship.

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Recall that infections and removals *after* a conditioned removal have been censored out. A valid configuration must satisfy the following, derived from the process dynamics:

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So each timeline is divided into a *susceptible interval* (empty if it is initially infected), an *infected interval* (empty if it is never infected), and a *removed interval* (empty if it has no conditioned removal).

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Dynamics in algorithmic time

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4. Otherwise, at $\tilde{\mathcal{I}}_{a,b}(u)$, consider the latest update of NFZ.
   - If a infected at u and u is in b component of latest NFZ, discard $\tilde{\mathcal{I}}_{a,b}(u)$;
   - If a is not infected at u but u lies on b component of latest NFZ, update NFZ by adding/replacing interval of NFZ at a using $(a, [0, u))$;

Warwick Statistics

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    - If a infected at u and u is in b component of latest NFZ, discard $\tilde{\mathcal{I}}_{a,b}(u)$;
    - If a is not infected at u but u lies on b component of latest NFZ, update NFZ by adding/replacing interval of NFZ at a using $(a, [0, u))$;
    - Otherwise retain $\tilde{\mathcal{I}}_{a,b}(u)$ as $\mathcal{I}_{a,b}(u)$.

Warwick Statistics

Dynamics in algorithmic time

This closely corresponds to the evolution of the S-I-R epidemic above, but does not resample the mark i for each conditioned removal $\mathcal{C}_i$;

1. Remove all $\mathcal{R}$s, and re-sample inactivated removals on the susceptible intervals of each timeline. Recompute NFZ as a union of $(i, [0, t_i))$ using the latest time $t_i = t$ of re-sampled $\mathcal{R}_i(t)$.
2. List in *time-reverse order* original infections together with sampled *new* candidate $\tilde{\mathcal{I}}$s in complements of the removed intervals.
3. Work iteratively through this list. Would discarding original $\mathcal{I}_{i,j}(t)$ result in failure to infect a conditioned removal?. If so, retain $\mathcal{I}_{i,j}(t)$ as *perpetuated infection* $\mathcal{P}_{i,j}(t)$, otherwise discard.
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    - Otherwise retain $\tilde{\mathcal{I}}_{a,b}(u)$ as $\mathcal{I}_{a,b}(u)$.

As in S-I-R case, the conditioned epidemic is the unique equilibrium.

Requirements for monotonicity

For CFTP we need to know that, for coupled iterations (using the same pattern of innovations of new $\mathcal{I}$s and $\mathcal{R}$s), if two variants are started so that the infected region of one contains the other, then this persists through development of the algorithmic time.

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Warwick Statistics

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Warwick Statistics

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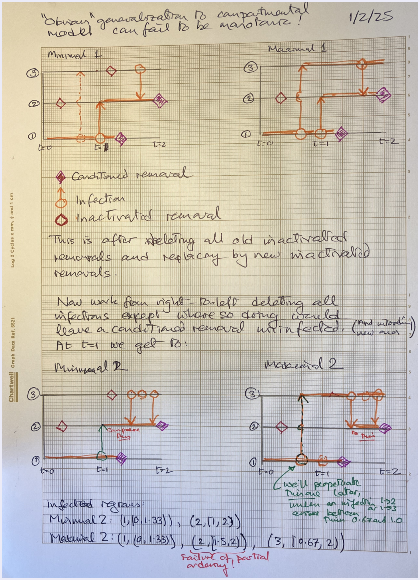
Then CFTP would make sense, and it would only be necessary to show that accessibility of a set of near-maximal configurations guarantees eventual coalescence.

Counterexample to monotonicity



Other technical information

Software used in computations

Software	*Version*	*Branch*	*Last commit*
`quarto`	`1.6.39`	—	
Running under `julia`	`1.11.3`	—	
`EpidemicsCFTP`	`2.2.492`	`main`	`Thu Jan 23 20:50:07 2025`
`EpidemicsUtilities`	`0.1.2.158`	`main`	`Tue Feb 4 17:55:46 2025`
This `quarto` script	`2.2.622`	`Wilfrid-2025-01-30-compartment`	`Tue Feb 4 17:00:20 2025`

Revision notes

These notes were produced from `PerfectEpidemics.qmd`:

Version:	`2.2.622` [`Wilfrid-2025-01-30-compartment`]
Author:	Wilfrid Kendall <`W.S.Kendall@warwick.ac.uk`>
Date:	Tue Feb 4 17:00:20 2025 +0000
Summary:	Cosmetic adjustments only,