Macroeconomics 2-Assignment 7

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Arellano & Ramanarayanan (2008)'s paper is entitled "Default and the Maturity Structure in Sovereign Bonds". Using a dynamic model of international borrowing and default, they document that in Argentina, Brazil, Mexico and Russia the maturity composition of new debt issuances, the levels of spreads, and the spread curve are all highly volatile.

Furthermore, they find the spread curve to be upward sloping and the average maturity to be longer term when spreads are low while the spread curve is inverted and the average maturity is shorter term when spreads are high. The spread curve captures the dynamics of the endogenous probability of default.

Finally, authors argue that while short term debt is advantageous to deliver the widest instant consumption, long term debt is useful in the sense that it can offset variations in short rates that are negatively associated to consumption.

Definition 1. *The recursive equilibrium for this economy consist of a set of policy functions for:*

- *Consumption $c(s)$, short term debt holdings $b^{'}(s)$, long term debt holdings $b_2^{'}(s)$, repayment sets $A(b_1^{'}, b_2^{'})$, and default sets $D(b, b_2)$, and*
- *the price for short term bonds $q^1(b^{'}, b_2^{'}, y)$ and long term bonds $q^2(b^{'}, b_2^{'}, y)$ such that:*

1. *Taking as given the bond price functions $q^1(b^{'}, b_2^{'}, y)$ and $q^2(b^{'}, b_2^{'}, y)$, the policy functions $b^{'}(s)$, $b_2^{'}(s)$ and $c(s)$, repayment sets $A(b_1^{'}, b_2^{'})$ and default sets $D(b, b_2)$ satisfy the representative domestic agent's optimization problem.*
2. *Bonds prices $q^1(b^{'}, b_2^{'}, y)$ and $q^2(b^{'}, b_2^{'}, y)$ reflect the domestic agent default probabilities such that lenders break even in expected value.*

Given initial states s, the value of the option to default is given by:

$$v^0(b, b_2, y) = max\left\{v^c(b, b_2, y), v^d(y)\right\}$$

$v^c(b, b_2, y)$, is the value associated with not defaulting and staying in the contract.
$v^d(y)$ is the value associated with default.

$$v^c(b, b_2, y), = \max_{b', b_2'} \left(u(c) + \beta \int_{y'} v^0(b', b_2', y) f(y', y) dy'\right) \tag{0.1}$$

$$v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^0(0, 0, y') - (1 - theta) v^d(y')\right] f(y', y) dy' \tag{0.2}$$

1 Prove that default decision is non-increasing in current bond holding

Given two current bond holding b and $\bar{b}$, such that $b \geq \bar{b}$. Let's prove that:
$v^c(b, b_2, y) \leq v^c(\bar{b}, b_2, y)$

We know that:

$$v^c(b, b_2, y) = \max_{b', b_2'} \left(u(c) + \beta \int_{y'} v^0(b', b_2', y) f(y', y) dy'\right)$$
$$b' = b_2 + \Delta b'$$

$c - q^1(b^{'}, b_2^{'}, y)\Delta b' - q^2(b^{'}, b_2^{'}, y) b_2^{'} = y - b$

According to the budget constraint, we can express c and $\bar{c}$ as:

- $c = q^1(b^{'}, b_2^{'}, y)\Delta b' + q^2(b^{'}, b_2^{'}, y) b_2^{'} + y - b$
- $\bar{c} = q^1(b^{'}, b_2^{'}, y)\Delta b' + q^2(b^{'}, b_2^{'}, y) b_2^{'} + y - \bar{b}$
- $u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}$
- $u(\bar{c}_t) = \frac{\bar{c}_t^{1-\sigma} - 1}{1-\sigma}$

Hence,

$$
\begin{aligned}
b &\geq \bar{b} \\
-b &\leq -\bar{b} \\
q^1(b^{'}, b_2^{'}, y)\Delta b' + q^2(b^{'}, b_2^{'}, y)b_2^{'} + y - b &\leq q^1(b^{'}, b_2^{'}, y)\Delta b' + q^2(b^{'}, b_2^{'}, y)b_2^{'} + y - \bar{b} \\
u(c) &\leq u(\bar{c}) \quad \textit{since utility is increasing in } c \\
u(c) + \beta \int_{y'} v^0(b', b_2', y) f(y', y) dy' &\leq u(\bar{c}) + \beta \int_{y'} v^0(\bar{b}', b_2', y) f(y', y) dy' \\
\max_{b', b_2'} \left(u(c) + \beta \int_{y'} v^0(b', b_2', y) f(y', y) dy' \right) &\leq \max_{\bar{b}', b_2'} \left(u(c) + \beta \int_{y'} v^0(\bar{b}', b_2', y) f(y', y) dy' \right) \\
v^c(b, b_2, y) &\leq v^c(\bar{b}, b_2, y)
\end{aligned}
$$

Hence, $v^c(b, b_2, y) \leq v^c(\bar{b}, b_2, y)$

So the default decision is non-increasing in current bond holding.

2 Prove that country will not choose to default if it holds positive assets

Let's consider two statements p and q. Proving that: $p \Rightarrow q$ is equivalent to proving that: $\neg q \Rightarrow \neg p$. I am going to use the later approach to prove this result.

Let's assume that a country default. Then the value of the option to default is,
$v^0(b, b_2, y) = max\left\{v^c(b, b_2, y), v^d(y)\right\} = v^d(y)$

$$
\begin{aligned}
v^d(y) = u(y^{def}) + \beta \int_{y'} \left[\theta v^0(0, 0, y') + (1-\theta) v^d(y')\right] f(y', y) dy' \\
v^d(y) = u(y^{def}) + \beta \int_{y'} \left[v^d(y')\right] f(y', y) dy'
\end{aligned}
$$

The above expressions of $v^d(y)$ reveal that under default, b' and b_2' will be lower or equal to 0. That's $b' = 0$ and $b_2' = 0$. Saying differently, under default, $(b', b_2') = (0, 0)$ is the optimal level of contract that maximizes the borrower's utility.

In this case, $q^1(0, 0, y) = 0$, $q^2(0, 0, y) = 0$, and the repayment sets $A(0, 0)$ is empty.

The budget constraint bolts down to: $c = y = y^{def}$