Requirements and Calculations of a Liquid Rocket Engine

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September 15, 2024

Contents

1 Nomenclature and Terms

$N2O$ - Nitrous Oxide
IPA - Isopropyl Alcohol
I_{sp} - Specific Impulse
O/F - Oxidizer/Fuel Ratio

2 System Requirements/Specifications

Requirement	Value	From?
Thrust	900 N	Rocket Weight
Specific Impulse	226 s	NASA CEA
Tank Pressure	5.127 MPa	Property (Vapor Pressure: 20 C)
O/F Mole Ratio	9	Ideal Reaction
O/F Mass Ratio	6.6	Calculated (O/F Mole Ratio)
Weight Flow Rate	3.6 N/s	Calculated (Thrust and Isp)
Mass Flow Rate	0.4059 kg/s	Calculated (Weight Flow Rate)
Ox Mass Flow Rate	0.3525 kg/s	Calculated (Mass Flow Rate)
Fuel Mass Flow Rate	0.0534 kg/s	Calculated (Mass Flow Rate)
Chamber Pressure	3 MPa	Assumed (Tank Pressure)
Chamber Temperature	2966 K	NASA CEA
Chamber Gas Density	3.3349 kg/m3	NASA CEA
Chamber Gamma	1.1459	NASA CEA
Chamber Mach Number	0.00	Assumption
Chamber SoS	1015 m/s	NASA CEA
Throat Pressure	1.7248 MPa	NASA CEA
Throat Temperature	2790 K	NASA CEA
Throat Gas Density	2.0577 kg/m3	NASA CEA
Throat Gamma	1.1471	NASA CEA
Throat Mach Number	1.00	Definition
Throat SoS	981 m/s	NASA CEA
Exit Pressure	0.1 MPa	Estimated (Altitude)
Exit Temperature	1907 K	NASA CEA
Exit Gas Density	0.17948 kg/m3	NASA CEA
Exit Gamma	1.2044	NASA CEA
Exit Mach Number	2.708	NASA CEA
Exit SoS	819 m/s	NASA CEA
Area Ratio	5.0683	NASA CEA
C*	1487 m/s	NASA CEA
Specific Gas Constant	32.4638 g/mol	NASA CEA
Throat Area	$65.591mm^2$	Calculated (Isentropic Equations)
Throat Diameter	9.1385 mm	Calculated
Exit Area	$332.4343mm^2$	Calculated (Area Ratio)
Exit Diameter	20.5734 mm	Calculated
Nozzle Exit Angle	15°	Determined (Hobby Rocket Engines)
Throat Entry Angle	30°	Determined (Youngblood)
Characteristic Length	1 m	Determined (Youngblood)
Chamber Diameter	30 mm	Determined
Chamber Length	84.36 mm	Calculated (Hobby Rocket Engines)
Chamber Wall Thickness	5 mm	Calculated & Determined

3 Derivations, Formulas & Calculation Process

3.1 Deriving Oxidizer/Fuel Ratio

Ideal Reaction: $C_3H_8O + 9N_2O \rightarrow 3CO_2 + 4H_2O + 9N_2$

In reality the reaction is more complicated. the heat of reaction causes many radicals and exotic species that can be calculated but are ignored for simplicity.

Mole O/F Ratio of 9 (9 Moles of N2O for 1 mole of C3H8O).
$M_{N2O} = 2 * 14 + 16 = 44$, $M_{C3H8O} = 3 * 12 + 8 * 1 + 16 = 60$

$O/F_{Mass} = \frac{9*44}{60} = 6.6$ (6.6 kg of N2O for 1 kg of C3H8O)

3.2 NASA CEA

Once these variables have been determined the NASA CEA (Chemical Equalibrium with Applications) or other equalibrium solver can be used to get many other critical parameters.

3.2.1 Procedure for running the NASA CEA

This is by no means the only way to do it. This is also intended as an early design exercise where other chamber pressures and O/F ratios should be analyzed. If you just want a single analysis point don't use the interval tabs and just use the number entry on the right side of the tabs for pressure and O/F ratio.

1. Look up NASA CEA on any browser. URL: https://cearun.grc.nasa.gov/
2. On the right hand side in the chemical equalibrium problem types check "Rocket". Then hit submit.
3. On the "Pressure" tab (should be autoselected) enter values for the chamber pressure.
   - This will be based on the tank pressure (5.127 MPa) and the losses in the plumbing to the chamber.
   - Since MPa, isn't a selectable option, multiply the MPa number by 10 to get the pressure in bar.
   - Assume 3 MPa as an approximate starting point. Since 3 MPa is our goal pressure, input 2 MPa (20 bar) as a Low Value and 5 MPa (50 bar) as a high value.
   - Make the interval smaller, maybe 0.5 MPa (5 bar) to see a range of pressure values.
   - Accept Input and Continue.
4. On the "Fuel(s)" tab check the "Use Periodic Table (mixtures)" since IPA isn't on the quick select list. Accept Fuel Selection and Continue.
5. On the "Periodic Table" check Hydrogen, Carbon and Oxygen (as these are the component elements in IPA). Accept Element Selections and Continue.

6. On the "Select your Fuel(s)" tab check C3H8O, 2propanol (This is the chemical formula and name for Isopropyl Alcohol). Accept Selected Reactant(s) and Continue. There should be a confirmation form, Fuel Mix form and Component Properties form. Accept inputs for all and continue to the next form.

7. On the "Select your Oxidizer(s)" tab check N2O. Accept Oxidizer Selection and Continue to the next form.

8. On the "Enter proportions of Oxidizer/Fuel" on the top selection area select Specify with Oxidizer/Fuel Wt. Ratio.

   - We calculated the stoichometric O/F weight ratio earlier as being 6.6.
   - If 6.6 is the target value enter a low value of 4 and a high value of 8 and use an interval of 0.2.

9. On the "Define Exit Conditions" tab define Chamber/Exit Pressure Ratios on the left hand side.

   - In rocket engines the most efficient engine is one that is perfectly expanded (meaning the pressure at the exit of the nozzle is at ambient pressures).
   - A rocket engine will fly in a multitude of different pressures as it travels upward (pressure decreases with altitude), but for now we will analyze with a exit pressure of 1 bar (or about 110m in altitude) as a baseline.
   - In order to get an exit pressure of 1 bar we take our chamber pressure range (2-5 MPa in steps of 0.5 MPa or 20-50 bar in steps of 5 bar) and divide it by the desired exit pressure.
   - So for this it's easy. The ratios are just the chamber pressure values. You will have to input each ratio manually (20, 25, 30 ... to 50) on the left column.
   - Accept Input and Continue to Next Form.

10. On the "Enter Your Final Choices Before Running CEA" the autogenerated selections should be good (Short, Mass-Fractions, Equilibrium). You can look at your entered ranges for chamber pressure, O/F ratio and chamber/exit pressure ratios before you run the CEA. Submit Input and Perform CEA Analysis.

11. The CEA should run in less than a couple of seconds. The file displayed in the embedded window is the output file. You can download the input and output files in the top right.

3.2.2 Procedure for analyzing the NASA CEA output

The output file of the NASA contains many valuable pieces of information that will be useful in designing a rocket engine later. A navigation tip: the CEA output file is sorted from smallest to largest O/F ratio first and within each O/F ratio it is sorted by chamber pressure.

1. Scroll through the output file to find the O/F Ratio of 6.6 and the Pin of 3 MPa (435.1 psi). For some reason the CEA output only displays the chamber pressures in psi no matter the input unit.

2. Once you have found the section where Pin = 435.1 psi and O/F = 6.6 find the exit condition where the P, bar equals 1.000 or Pinf/P = 30.
   - The chamber, throat and exit pressure column where P, BAR = 1.000 (or another exit pressure) are the only columns of interest per section.
3. Find and record the variables of interest. In our case this would be pressure (P), temperature (T), density (RHO), mach number, specific heat ratio (GAMMA), speed of sound (SON VEL), area ratio (Ae/At), c star, specific impulse (Isp) and the mass fractions. Here are the values:
   - Pressure at the throat: 17.248 Bar
   - Temperature in the combustion chamber: 2966 K, throat: 2790 K, exit: 1907 K
   - Density of the combustion gases in the combustion chamber: 3.3349 kg/m3, throat 2.0577 kg/m3, exit 0.17948 kg/m3
   - Mach Number at exit: 2.708
   - Specific Heat Ratio at the combustion chamber: 1.1459, throat: 1.1471, exit: 1.2044
   - Speed of Sound at the combustion chamber: 1015 m/s, throat: 981 m/s, exit: 819 m/s
   - Area Ratio: 5.0683
   - C star: 1487 m/s
   - Specific Impulse: 2218 m/s. In order to convert this to the familiar specific impulse with units of seconds divide by the gravitational constant g = 9.81. $I_{sp} = \frac{2218}{9.81} = 226s$
   - The mass fractions are listed for the main combustion gas products.
     - CO: Combustion Chamber - 0.05069, Throat - 0.03951, Exit - 0.00258
     - CO2: Combustion Chamber - 0.20943, Throat - 0.22700, Exit - 0.28503
     - H: Combustion Chamber - 0.00010, Throat - 0.00007, Exit - 0.00000
     - HO2: Combustion Chamber - 0.00003, Throat - 0.00002, Exit - 0.00000
     - H2: Combustion Chamber - 0.00083, Throat - 0.00065, Exit - 0.00006
     - H2O: Combustion Chamber - 0.14328, Throat - 0.14690, Exit - 0.15703
     - NO: Combustion Chamber - 0.01266, Throat - 0.00909, Exit - 0.00047
     - NO2: Combustion Chamber - 0.00002, Throat - 0.00001, Exit - 0.00000
     - N2: Combustion Chamber - 0.54681, Throat - 0.54849, Exit - 0.55252
     - O: Combustion Chamber - 0.00151, Throat - 0.00093, Exit - 0.00001
     - OH: Combustion Chamber - 0.01160, Throat - 0.00850, Exit - 0.00041
     - O2: Combustion Chamber - 0.02302, Throat - 0.01885, Exit - 0.00190

3.3 Deriving the Flow rates

Specific Impulse Equation: $I_{sp} = \frac{T}{\dot{m}g_0}$ → Weight Flow Version: $\dot{W} = \dot{m}g_0 = \frac{T}{I_{sp}}$
Solve: $\dot{W} = \frac{900N}{226s} = 3.982N/s$

Convert to mass flow: $\dot{m} = \frac{\dot{W}}{g_0} \rightarrow \dot{m} = \frac{3.9823}{9.81} = 0.4059 kg/s$

Separate into oxidizer and fuel mass flow rates:
Oxidizer - Nitrous Oxide: $\dot{m}_{N2O} = \dot{m} * \frac{6.6}{7.6} = 0.3525 kg/s$
Fuel - Isopropyl Alcohol: $\dot{m}_{IPA} = \dot{m} * \frac{1}{7.6} = 0.0534 kg/s$

4 Design Equations

4.1 Nozzle Design Equations

4.1.1 Solving for the specific gas constant

The Univeral Gas Constant $\bar{R}$ is 8.314 J/molK.
The specific gas constant is $R = \frac{\bar{R}}{M}$ where M is the molecular weight of the mixture of product gases. Need to solve for the molecular weight of all of the component gases and their moles in mass fraction.

- CO: $M = 12 + 16 = 28$ $x_{CO} = 28 \cdot 0.03951 = 1.1063$ moles
- CO2: $M = 12 + 16 + 16 = 44$ $x_{CO2} = 44 \cdot 0.2270 = 9.988$ moles
- H: $M = 1$ $x_H = 1 \cdot 0.00007 = 0.00007$ moles
- HO2: $M = 1 + 16 + 16 = 33$ $x_{HO2} = 33 \cdot 0.00002 = 0.00066$ moles
- H2: $M = 1 + 1 = 2$ $x_{H2} = 2 \cdot 0.00065 = 0.0013$ moles
- H2O: $M = 1 + 1 + 16 = 18$ $x_{H2O} = 18 \cdot 0.1469 = 2.6442$ moles
- NO: $M = 14 + 16 = 30$ $x_{NO} = 30 \cdot 0.00909 = 0.2727$ moles
- NO2: $M = 14 + 16 + 16 = 46$ $x_{NO2} = 46 \cdot 0.00001 = 0.00046$ moles
- N2: $M = 14 + 14 = 28$ $x_{N2} = 28 \cdot 0.54849 = 15.3577$ moles
- O: $M = 16$ $x_O = 16 \cdot 0.00093 = 0.01488$ moles
- OH: $M = 16 + 1 = 17$ $x_{OH} = 17 \cdot 0.0085 = 0.1445$ moles
- O2: $M = 16 + 16 = 32$ $x_{O2} = 32 \cdot 0.01885 = 0.6032$ moles
- Total: $x_{total} = 1.1063 + 9.988 + 0.00007 + 0.00066 + 0.0013 + 2.6442 + 0.2727 + 0.00046 + 15.3577 + 0.01488 + 0.1445 + 0.6032 = 30.1346$ moles

Molecular weight of mixture is $M = 28 \cdot \frac{1.1063}{30.1346} + 44 \cdot \frac{9.988}{30.1346} + 1 \cdot \frac{0.00007}{30.1346} + 33 \cdot \frac{0.00066}{30.1346} + 2 \cdot \frac{0.0013}{30.1346} + 18 \cdot \frac{2.6442}{30.1346} + 30 \cdot \frac{0.2727}{30.1346} + 46 \cdot \frac{0.00046}{30.1346} + 28 \cdot \frac{15.3577}{30.1346} + 16 \cdot \frac{0.01488}{30.1346} + 17 \cdot \frac{0.1445}{30.1346} + 32 \cdot \frac{0.6032}{30.1346} = 32.4645 g/mol$

Solve for specific gas constant $R = \frac{8.314}{32.4645} = 0.2561 J/gK = 256.1 J/kgK$

4.1.2 Throat Area

Mass Flow Rate at the throat: $\dot{m} = A_t \frac{P_o}{\sqrt{T_o}} \sqrt{\frac{\gamma}{R}} (\frac{\gamma+1}{2})^{\frac{\gamma+1}{2(1-\gamma)}}$
Area of Throat: $A_t = \frac{\dot{m}\sqrt{T_o}}{P_o} \sqrt{\frac{R}{\gamma}} (\frac{\gamma+1}{2})^{\frac{\gamma+1}{2(\gamma-1)}}$

Values used in the Equation:
Mass Flow Rate $\dot{m} = 0.4059 kg/s$
Stagnation Temperature $T_o = T_{chamber} = 2966K$
Stagnation Pressure $P_o = P_{chamber} = 3MPa$
Gamma (at the throat) $\gamma_{throat} = 1.1471$
Specific Gas Constant (at the throat) $R_{throat} = 256.1 J/kgK$

Plug in the numbers: $A_t = \frac{0.4059 \cdot \sqrt{2966}}{3 \cdot 10^6} \sqrt{\frac{256.1}{1.1471}} (\frac{1.1471+1}{2})^{\frac{1.1471+1}{2(1.1471-1)}} = 6.5591 \cdot 10^{-5} m^2$

Solve for Diameter $A_t = \pi \frac{{D_t}^2}{4} \to D_t = \sqrt{\frac{4A_t}{\pi}} = \sqrt{\frac{4 \cdot (6.5591 \cdot 10^{-5})}{\pi}} = 0.0091385m = 9.1385mm$

4.1.3 Exit Area

Use the throat area and determined A_e/A_t to get the exit area. $\frac{A_e}{A_t} = 5.0683$
$A_e = 5.0683 \cdot 6.5591 \cdot 10^{-5} = 3.3243 \cdot 10^{-4} m^2$
Diameter: $D_e = \sqrt{\frac{4A_e}{\pi}} = \sqrt{\frac{4 \cdot (3.3243 \cdot 10^{-4})}{\pi}} = 0.02057m = 20.5734mm$

4.1.4 Nozzle Angles

In normal commercial rocket engines the nozzle angle is optimized so that when the flow leaves it is as 1D as possible, to achieve max thrust. The profile that maximizes thrust for a certain throat area and length is called an "Rao optimum contour". This is is named after G.V.R Rao and is complicated to manufacture. Because of its complexity we will just use a 15° conical nozzle profile, common for hobby and small area ratio engines. It has 98.3% of the performance of an ideal nozzle, so close enough for me.

4.2 Combustion Chamber Equations

4.2.1 Chamber Length

For the combustion chamber Hobby Rocket Engines references a combustion chamber with a cross sectional area of at least 3 times the throat area. For this reason we choose a chamber diameter of 30 mm, much larger than the required size, subject to change based on the injector.

The chamber volume required for complete combustion is defined by the characteristic chamber length: $L^* = V_c/A_t \to L^* A_t = V_c$ where $V_c = A_c L_c +$ convergent volume
Youngblood references a characterisitc length of 1m being successful in nitrous, ethanol engines so that value will be used as a minimum.

For small combustion chambers the convergent volume is 1/10th the cylindrical portion so $V_c = 1.1 A_c L_c$
Combine Equations $L^* A_t = 1.1 A_c L_c \to L_c = \frac{L^* A_t}{1.1 A_c} = \frac{(1)(6.5591 \cdot 10^{-5})}{1.1(\pi \frac{0.03^2}{4})} = 0.08436m$ or $84.36mm$

4.2.2 Chamber Thickness

This system will definitely not be thin walled but it is a good starting point for minimums. The axial or longnitudinal stress in a thin walled pressure vessel is given by the equation: $\sigma_a = \frac{Pr}{2t}$ Where P is the pressure, r is radius and t is the thickness.

Rrearrange for thickness: $t = \frac{Pr}{2\sigma_a}$ In our case pressure is the chamber pressure (3 MPa), radius is 15 mm and sigma is material dependent. Since steel is cheap we will use that, heat transfer be damned (hopefully we don't run it for long enough). Steel rod stock on McMaster has a tensile yield stress of 60,000 psi or 413.685 MPa.
Plug this in: $t = \frac{3 \cdot 15}{2 \cdot 413.685} = 0.054389mm$ Remember this number is at ambient temperatures and the combustion chamber gas were calculated to be at 2966 K (Above the melting point of steel).

Recalculating with an operating temperature of 500°C or 773.15 K steel has about 40% of it's yield strength at 21C. Using this: $t = \frac{3 \cdot 15}{2 \cdot (0.4)(413.685) \cdot} = 0.13597mm$
Not really much of a worry, but let's use something thick enough to handle the heat loads as well, call the wall thickness 5mm.