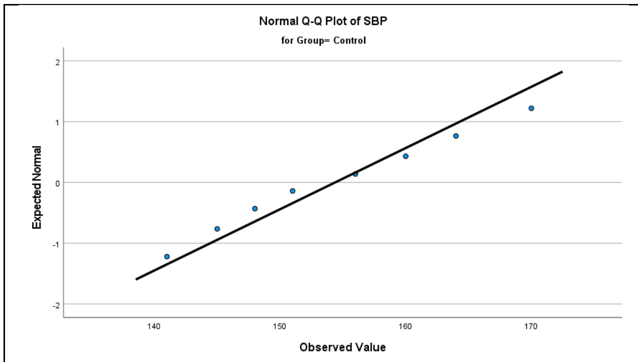
Practice Problem Topic 2-2: Inferences for Two Population Means

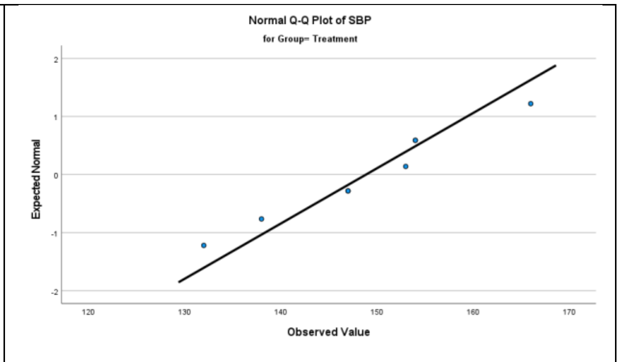
Does exercise (explanatory variable) have an effect on blood pressure (response variable)?

Comparing Two Population Means Using an Independent Sample Design
A sample of 16 people were randomly selected from a population that had high blood pressure (141-170 mm Hg), got very little exercise, and were 50-60 years of age. Then, from this sample, researchers randomly assigned 8 participants to a control group, who continued with their same lifestyle and 8 participants to a treatment group who performed moderate exercise, which consisted of 30 minutes per day of fast walking. After a period of 2 months, the systolic blood pressure (SBP) of participants in both groups was recorded, obtaining data as shown below. Note: this is a post-test design.

Participant	**Control Group (SBP in mm Hg)**		**Participant**	**Experimental Group (SBP in mm Hg)**
John	145		Aleisha	154
Xia	156		Coleen	166
Ikram	160		Bo	132
Halima	170		Martin	138
Monica	141		Fred	154
Nancy	164		Sandra	147
Henry	148		Jack	153
Natasha	151		Hashim	147



Control



Treatment

Tests of Normality

	Group	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SBP	Control	.133	8	.200*	.975	8	.937
	Treatment	.188	8	.200*	.957	8	.782

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Note: If the P-value of the Shapiro-Wilk Test is **greater than 0.05**, the distribution is not significantly different from a normal distribution, so the data are considered normal. If the P-value is below 0.05, the data significantly deviate from a normal distribution.

Group Statistics					
	Group	N	Mean	Std. Deviation	Std. Error Mean
SBP	Control	8	154.375	9.927	3.510
	Treatment	8	148.875	10.508	3.715

Independent Samples Test											
		Levene's Test for Equality of Variances		t-test for Equality of Means							
						Significance				99% Confidence Interval of the Difference	
		F	Sig.	t	df	One-Sided p	Two-Sided p	Mean Difference	Std. Error Difference	Lower	Upper
SBP	Equal variances assumed	.008	.930	1.076	14	.150	.300	5.500	5.111	-9.714	20.714
	Equal variances not assumed			1.076	13.955	.150	.300	5.500	5.111	-9.722	20.722

Although the SPPS output is shown, answer the questions below without using the numbers highlighted in yellow.

(a) At the 1% significance level, test whether there was a difference in mean SBP between the control and exercise group.

Step 1: Pooled-sample t-test is selected because of the following:

- The explanatory variable is categorical, that is, the control group versus experimental group (two-levels).
- The response variable is a continuous, quantitative variable, that is, SBP.
- **Purpose** of the study: To test for a difference between two population means (mean SBP in two groups)
- There is no pairing or link between participants in the control and experimental groups.
- **Assumptions:**
  1. Random selection from population and random allocation to groups
  2. Two independent samples
  3. Both populations are normally distributed. Shapiro Wilk test results in both P-values (0.937 and 0.782) being greater than 0.05, so the distributions are not significantly different from a normal distribution.
  4. The two standard deviations are nearly equal as indicated by Levene's test, which gives P = 0.930, which is greater than 0.05.

Step 2:
H_0: $\mu_1 = \mu_2$ or $\mu_1 - \mu_2 = 0$ (There is no difference in mean SBP between the control and exercise group.)
Ha: $\mu_1 \neq \mu_2$ or $\mu_1 - \mu_2 \neq 0$ (There is a difference in mean SBP between the control and exercise group.)
Parameter: $\mu_1 - \mu_2 = \mu_{Contol} - \mu_{Treatment}$

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Step 3:

Estimate of the difference between means = $\bar{y}_1 - \bar{y}_2 = 154.375 - 148.875 = 5.500 \text{ mm Hg}$

Estimate of the pooled population standard deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$
$$= \sqrt{\frac{(8-1)(9.927)^2 + (8-1)(10.508)^2}{8+8-2}} = 10.221629$$

Standard error of the estimate of the difference between means:

$$SE(\bar{y}_{Control} - \bar{y}_{Treatment}) = s_p\sqrt{(1/n_1) + (1/n_2)}$$
$$= 10.221629\sqrt{(1/8) + (1/8)} = 5.110815$$

$$t = \frac{\bar{y}_1 - \bar{y}_2}{s_p\sqrt{(1/n_1) + (1/n_2)}} = \frac{\bar{y}_1 - \bar{y}_2}{SE(\bar{y}_1 - \bar{y}_2)} = \frac{5.500}{5.110815}$$
$$= 1.0761$$

Step 4: $df = n_1 + n_2 - 2 = 8 + 8 - 2 = 14$

P-value: (0.15 > P > 0.10) x 2 = 0.30 > P > 0.20 [SSPS: P-value = 0.300]
There is weak evidence against H_0 because P-value is greater than 10% (Guidelines)
P > α (0.01), therefore do not reject H_0.

Step 5: At the 1% significance level, the data do not provide sufficient evidence to conclude that there is a difference in mean SBP between the control and exercise group.

Note: $P(t_{14} > 1.0763) \in (0.20, 0.30) \text{ OR } 0.20 < P < 0.30$

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(b) Determine a 99% confidence interval for the difference in mean SBP between the control and exercise group.
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Step 1: Critical value is:
For a 99% confidence interval, α = 1 – 0.99 = 0.01.
At $df = n_1 + n_2 - 2 = 8 + 8 - 2 = 14$, $t_{\alpha/2} = t_{0.01/2} = t_{0.005} = 2.977$

Step 2:

Parameter: $\mu_1 - \mu_2 = \mu_{Contol} - \mu_{Treatment}$

Estimate = $\bar{y}_1 - \bar{y}_2 = 154.375 - 148.875 = 5.500 \text{ mm Hg}$

Standard error of the estimate: $SE(\bar{y}_{Control} - \bar{y}_{Treatment}) = 5.110815$

Calculation of the confidence interval:

$$(\bar{y}_1 - \bar{y}_2) \pm t_{\alpha/2} \times SE(\bar{y}_1 - \bar{y}_2)$$

$$5.500 \pm 2.977 \times 5.110815$$

5.500 ± 15.2149

(-9.715, 20.715) mm Hg

Step 3: We can be 99% confident that the <u>difference</u> in mean SBP between the control and exercise groups is between -9.715 and 20.715 mm Hg.

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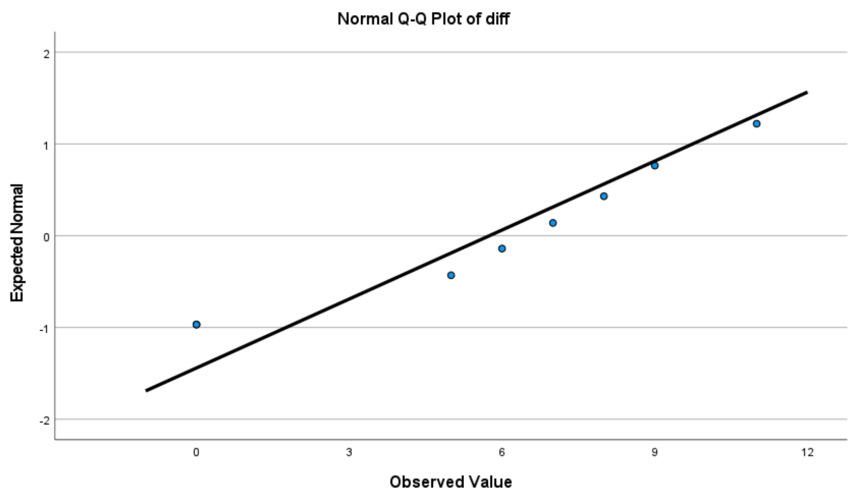
Comparing Two Populations Using a Paired Sample Design

A sample of 8 people were randomly selected from a population that had high blood pressure (141-170 mm Hg), got very little exercise, and were 50-60 years of age. The researchers recorded the initial systolic blood pressure (SBP) of each participant and then put them in a treatment program of moderate exercise, which consisted of 30 minutes per day of fast walking. After 2 months of this exercise treatment, the SBP of each participant was again recorded. Data are shown in the table below. At the 1% significance level, test whether there was a difference in the mean SBP before and after treatment with the exercise program.

Participant	Systolic blood pressure (mm Hg)	
	Before treatment	After treatment
Fred	144	137
Zainab	162	153
Ryan	143	132
Isabel	172	164
Hassan	160	154
Maryam	149	149
Cheng	153	153
George	150	145



Tests of Normality						
	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
diff	.175	8	.200*	.912	8	.369

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

Paired Samples Statistics					
		Mean	N	Std. Deviation	Std. Error Mean
Pair 1	Before	154.13	8	9.906	3.502
	After	148.38	8	10.197	3.605

Paired Samples Correlations					
				Significance	
		N	Correlation	One-Sided p	Two-Sided p
Pair 1	Before & After	8	.922	<.001	.001

Paired Samples Test										
		Paired Differences					t	df	Significance	
		Mean	Std. Deviation	Std. Error Mean	99% Confidence Interval of the Difference				One-Sided p	Two-Sided p
					Lower	Upper				
Pair 1	Before - After	5.750	3.991	1.411	.812	10.688	4.075	7	.00236	.00472

Although the SPPS output is shown, answer the questions below without using the numbers highlighted in yellow.

Step 1: Paired-sample t-test is selected because of the following:

- There explanatory variable is categorical, that is, "Before and After treatment" (two-levels).
- The response variable is a continuous, quantitative variable, that is, SBP.
- **Purpose** of the study: To test for a difference in mean SBP before and after treatment and the measurements taken before and after are paired, matched or linked on the same participants, as indicated by the arrows in the table of raw data.
- **Assumptions:**
  1. Random selection from population
  2. This is a paired sample because measures are paired on the same participants
  3. The paired differences are normally distributed. The Shapiro Wilk test results in P-value = 0.369, which is greater than 0.05, so the paired differences are not significantly different from normal distribution.

Step 2:

H_0: $\mu_1 = \mu_2$ or $\mu_d = 0$

(There was no difference in the mean SBP before and after treatment with the exercise program.)

Ha: $\mu_1 \neq \mu_2$ or $\mu_d \neq 0$

(There was a difference in the mean SBP before and after treatment with the exercise program.)

Parameter: $\mu_d = \mu_{Before} - \mu_{After}$

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Step 3:

Estimate of the mean difference: $\bar{d} = 5.750$

Standard error of the mean difference = $SE(\bar{d}) = \frac{s_d}{\sqrt{n}} = \frac{3.991}{\sqrt{8}} = 1.41103$

$$t = \frac{\bar{d}}{s_d / \sqrt{n}} = \frac{\bar{d}}{SE(\bar{d})} = \frac{5.750}{1.41103} = 4.075$$

Step 4: $df = n - 1 = 8 - 1 = 7$

P-value = (0.001 < P < 0.0025) x 2 = 0.002 < P < 0.005. [SSPS: P-value = 0.00472]

There is very strong evidence against H_0.

Since P-value < α (0.01), reject H_0.

Step 5: At the 1% significance level, there is sufficient evidence to conclude that there was a difference in the mean SBP before and after treatment with the exercise program.

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Compare the two-sample t-test for independent samples and the paired t-test

- The two-sample t-test for independent samples requires that the study units be randomly allocated to the two groups, whereas, in the paired design, all study units are subjected to the two conditions (before and after treatment).
- The variation from one participant to the other is much larger than the change from before to after, thus the paired design makes the test more powerful in rejecting H_0 because it eliminates the variation between study units
- The non-paired design did NOT give conclusive results.