Practice Problem 2: Inferences for One Population Mean

People having systolic blood pressure (SBP) above 140 mmHg are considered as being at risk of hypertension. The table below shows the SBP readings of a random sample of 14 teachers in a certain high school. Assume that the data are normally distributed.

Systolic Blood Pressure (mmHg)													
121	127	141	129	150	134	120	147	138	135	130	144	132	133

(a) Suppose you are provided with the sample statistics based on this random sample, that is, the sample mean is 134.3571 mmHg, and the sample standard deviation is 8.9838 mmHg. At the 5% significance level, test whether the mean SBP of teachers in this high school is different from the 140-mmHg risk level of hypertension.

Step 1: Purpose and assumptions

- SBP is a continuous, quantitative variable.
- Random sample
- Population is normally distributed
- Purpose: To test for a difference between one population mean (based on the sample mean for these high school students) and a hypothesized (mean or) value. In this case it is a value (not a mean), that is, SBP of 140 mmHg
- Select: One-sample t-test (parametric)

Step 2: State the null and alternative hypotheses.

H_0: μ = 140 (Mean SBP of teachers in this high school is not different from the 140-mmHg risk level of hypertension)
Ha: μ ≠ 140 (Mean SBP of teachers in this high school is different from the 140-mmHg risk level of hypertension)

Parameter = μ = mean SBP of teachers in this high school

Step 3: Obtain the Calculated or Observed Value of the test statistic
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Estimate of the population mean = $\bar{y} = 134.3571$ mmHg

Standard error of the estimate of the sample mean = $SE(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{8.9838}{\sqrt{14}} = 2.4010$

$$t = \frac{\bar{y} - \mu_0}{s \;/\sqrt{n}} = \frac{\bar{y} - \mu_0}{SE(\bar{y})}$$

$$t = \frac{134.3571 - 140}{2.4010} = \frac{-5.6429}{2.4010} = -2.350$$

Step 4: Decide to <u>reject H_0 or not reject H_0</u> and state the <u>strength of evidence</u> against H_0

df = n – 1 = 14 – 1 = 13
Since the t-table is one-tailed and we are doing a two-tailed test, we double the P-value.
P-value: $(0.01 < P < 0.02) \times 2 = 0.02 < P < 0.04$ [Note: SPSS gives an exact P-value of 0.035]
There is strong evidence against H_0.
P < α (0.05), therefore reject H_0.

Step 5: Interpretation (Conclusion in Words)
At the 5% significance level, the data provide sufficient evidence to conclude that the mean SBP of teachers in this high school is different from the 140-mmHg risk level of hypertension.
[Although this is not a necessary part of the conclusion, since the mean SBP of the teachers in less than 140 mmHg, this means they are not at risk.]

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SPSS Output for Hypothesis Test

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SBP	14	134.36	8.984	2.401

One-Sample Test

	Test Value = 140						
			Significance			95% Confidence Interval of the Difference	
	t	df	One-Sided p	Two-Sided p	Mean Difference	Lower	Upper
SBP	-2.350	13	.018	.035	-5.643	-10.83	-.46

(b) Based on the sample statistics given in part (a), determine a 95% confidence interval for the mean SBP of the teachers in this high school.

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Step 1: Critical value is:
For a 95% confidence interval, α = 1 – 0.95 = 0.05
At ***df = n – 1*** = 14 – 1 = 13: $t_{\alpha/2} = t_{0.05/2} = t_{0.025} = 2.160$

Step 2: Calculations
Parameter = μ = mean SBP of teachers in this high school
Estimate = $\bar{y} = 134.3571$ mmHg

Standard error of the estimate $SE(\bar{y}) = \frac{s}{\sqrt{n}} = \frac{8.9838}{\sqrt{14}} = 2.4010$

Calculation of the confidence interval:

$$\bar{y} \pm t_{\alpha/2} \times \frac{s}{\sqrt{n}}$$

$$134.3571 \pm 2.160 \times 2.4010$$

134.3571 ± 5.18616
(129.1709, 139.5433) mmHg

Step 3: Conclusion
It is estimated with 95% confidence that the mean SBP of the teachers in this high school is (129.17, 139.54) mmHg.
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SPSS Output for Confidence Interval: For two-sided 95% confidence interval

One-Sample Test							
	Test Value = 0						
			Significance			95% Confidence Interval of the Difference	
	t	df	One-Sided p	Two-Sided p	Mean Difference	Lower	Upper
SBP	55.958	13	<.001	<.001	134.357	129.17	139.54

(c) Suppose that, instead of being provided with the mean and standard deviation of this sample, you are provided with a 95% confidence interval for this same data set, which is (129.1700, 139.5442) [Note: any difference from the interval calculated in part (b) is just due to rounding.] Determine the sample mean and sample standard deviation.

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Sample mean: $\bar{y} = \dfrac{[\text{Lower endpoint + Upper endpoint}]}{2} = \dfrac{[129.1700 + 139.5442]}{2} = 134.3571$

Margin of error: $\text{ME} = \dfrac{[\text{Upper endpoint - Lower endpoint}]}{2} = \dfrac{[139.5442 - 129.1700]}{2} = 5.1871$

Use CV determined in part (b): $t_{\alpha/2} = t_{0.05/2} = t_{0.025} = 2.160$

$$ME = t_{\alpha/2} \times \frac{s}{\sqrt{n}}$$

$$5.1871 = 2.160 \times \frac{s}{\sqrt{14}}$$

$$\frac{5.1871 \times \sqrt{14}}{2.160} = s$$

Sample standard deviation = $s = 8.9853$

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(d) Suppose a confidence interval for the mean SBP of the teachers in this high school, calculated at a difference confidence level from the one given in part (c) is (127.1246, 141.5897). Determine the confidence level at which this interval was calculated. Use the sample standard deviation given in part (a) which is 8.9838 mmHg.

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Margin of Error (ME) $= \dfrac{[141.5897 - 127.1246]}{2} = 7.23255$

For a one mean interval:

$$ME = t_{\alpha/2} \times \frac{s}{\sqrt{n}} \text{ Or } t_{\alpha/2} \times \frac{s}{\sqrt{n}} = ME$$

$$t_{\alpha/2} = ME \times \frac{\sqrt{n}}{s}$$

$$t_{\alpha/2} = 7.23255 \times \frac{\sqrt{14}}{8.9838} = 3.012$$

At df = n – 1 = 14 – 1 = 13, $t_{\alpha/2} = 3.012 = t_{0.005}$ (By examining the t-table)

$t_{\alpha/2} = t_{0.005}$, so $\alpha / 2 = 0.005 \Rightarrow \alpha = (0.005)2 = 0.01$

Thus, the confidence level $= 1 - 0.01 = 0.99 \Rightarrow 99\%$

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(e) Compare the results you got in parts (a) and (b) and compare these results with the confidence interval given in part (d). Explain your answer.

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The hypothesis test in part (a) resulted in concluding that there is evidence that the mean SPB of the teachers in this high school is different from the risk level of 140 mmHg. This agrees with the confidence interval calculated in part (b), since this interval, (129.17, 139.54), does NOT contain 140, confirming that there is a difference.

However, the confidence interval given in part (d), (127.1246, 141.5897) DOES contain 140, indicating that this population of teachers in the high school are within the risk level. The reason for this different conclusion is that the confidence interval in part (d) has a level of 99%, which gives a wider confident interval than the 95% confidence interval obtained in part (b).

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