US Forest Service Timber Auctions: Estimating Risk-Aversion and its Implications

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Abstract:

This paper analyzes timber auction data from the United States Forest Service (USFS) in the western half of the United States in an attempt to answer three questions. First, what is the risk-aversion parameter? Second, does this matter to the USFS, and if so why? Third, which type of auction should the USFS use for timber auctions? The two types of auctions in our data sets were first-price sealed bid auctions for two and three bidders, and ascending auctions with two and three bidders. We were able to use this data to determine bid and valuation distribution and densities, and from that evaluate the risk aversion parameter for auctions with two bidders and auctions with three bidders. Risk-aversion is important to us because it allows us to determine the auction type that generates the highest revenue. In theory, under risk-aversion first price auctions should generate more revenue because risk-aversion affects bidding strategy in first price sealed bid auctions. Essentially, risk-aversion leads to less risky and higher bids. On the other hand, risk aversion does not affect ascending auctions because in an ascending auction it is always a dominant strategy to bid your valuation. We determined our risk-aversion parameter to be 0.2462 in auctions with two bidders, and 0.3308 in auctions with three bidders.

Motivation:

Because the USFS is a government entity, USFS timber auctions result in revenue for the government. Today, the USFS in the Western Half of the United States only uses sealed-bid auctions. However, our data from 1979 takes timber in the same geographical area and uses either first-price sealed bid auctions or ascending auctions. We are interested in using the data from 1979 to determine which type of auction will result in the highest revenue, and if the USFS should potentially change their auction type in order to generate more revenue.

We attempted to estimate the risk-aversion parameter of the bidders in these auctions, determine if the risk-aversion parameter does indeed matter to the USFS, and if so what type of auction the USFS should use to maximize their revenue. The risk-aversion parameter is important because it will help us determine how aggressively bidders will bid in auctions. The higher the risk-aversion factor, the riskier the bidders will bid. Riskier bidding generates less revenue for the seller, in this case the USFS.

Over the past few decades, the number of uses of timber has remained relatively the same. Construction and paper products are some of the major uses of timber. Therefore, the data from 1979 is still relevant for us today and we can use it to compare first-price sealed bid auctions and ascending auctions. From this comparison, we will be able to determine which auction type generates the most revenue, and if the USFS should use ascending auctions rather than first-price sealed bid auctions.

Data Section:

1.1 Data Summary

We analyzed four data sets of United States Forest Service (USFS) timber auctions from the western half of the United States in 1979. Our four unique data sets are as follows: a first-price sealed bid auction with two bidders, a first-price sealed bid auction with three bidders, an ascending auction with two bidders, and an ascending auction with three bidders. Each data point represents an independent auction. The two bidder ascending auction data describes 241 auctions, the three bidder ascending auction data describes 231 auctions, the two bidder sealed bid auction data describes 107 auctions, and the three bidder sealed bid auction data describes 108 auctions. Below are the other important variables:

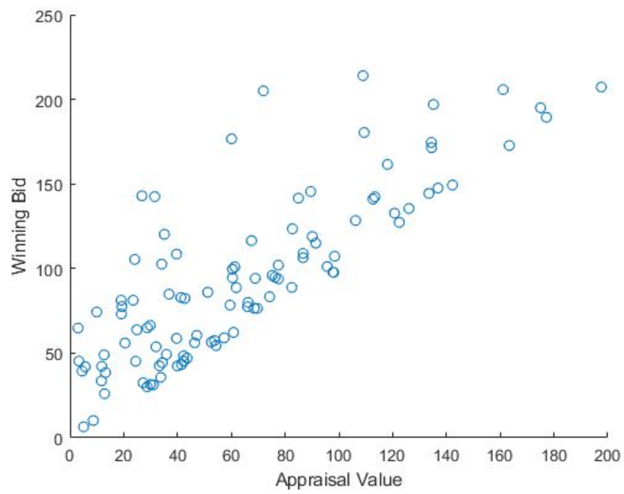
- Total volume of timber auctioned off (measured in MBF, 1000 board feet)
- The bids (measured in dollar amounts per unit of timber)
- Number of bidders
- Appraisal value (measured in dollar amounts per unit of timber)

Below are descriptions of the four data sets, arranged by column.

In each of the data sets, we dropped any auction with observations outside the first percentile and ninety-ninth percentile to eliminate outliers. This left us with 104 observations for two bidder first price sealed auctions, 234 observations for two bidder ascending auctions, 104 observations for three bidder first price sealed auctions, and 221 observations for three bidder ascending auctions.

- First price sealed bid auction with two bidders

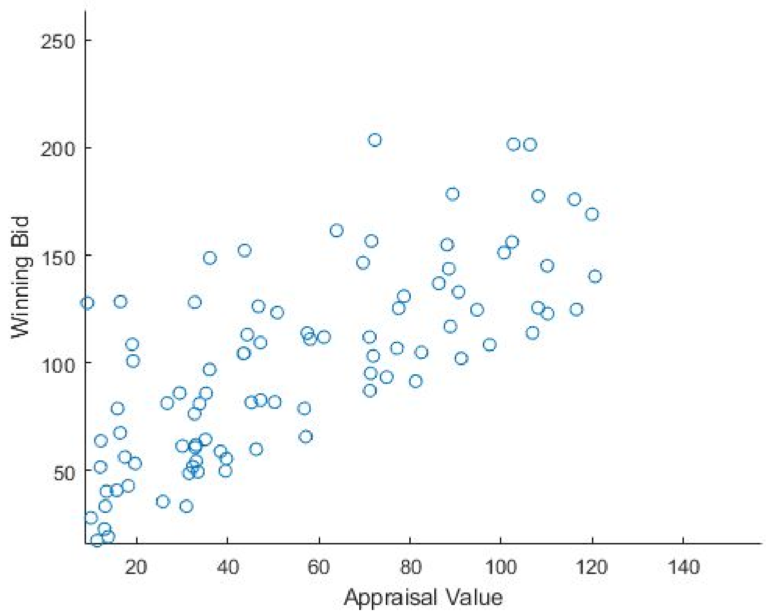
Volume of Timber	Winning Bid	Losing Bid	Appraisal Value	Number of Bidders
MBF of timber	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Appraisal Value (Dollars per unit of timber)	2



The scatter plot of appraisal values and winning bids clearly illustrates that winning bids and appraisal values are correlated. Specifically, appraisal values and winning bids are correlated with a coefficient of 0.8199. This shows that we must take appraisal values into account when we generate probability density functions and cumulative distribution functions of bids and values.

- First price sealed bid auction with three bidders

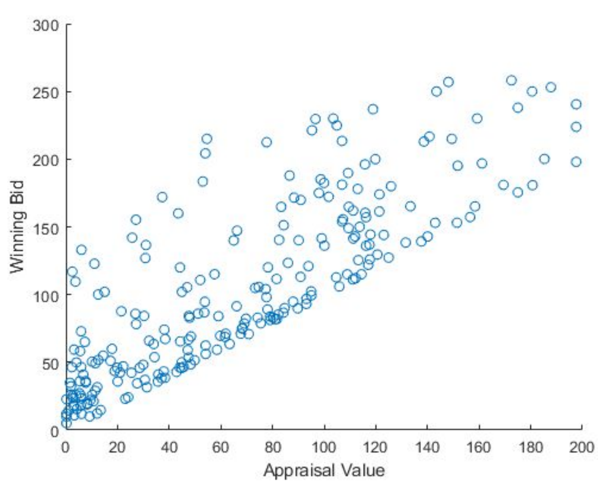
Volume of Timber	Winning Bid	Second-Highest Bid	Losing Bid	Appraisal Value	Number of Bidders
MBF of timber	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Appraisal Value (Dollars per unit of timber)	3



In the sealed first price auctions with three bidders, appraisal values and winning bids are correlated with a coefficient of 0.772. This relationship is clearly exhibited in the scatter plot of winning bids and appraisal values. This reinforces the need to condition our probability density functions and cumulative distribution function of bids and values to appraisal values.

- Ascending auction with two bidders

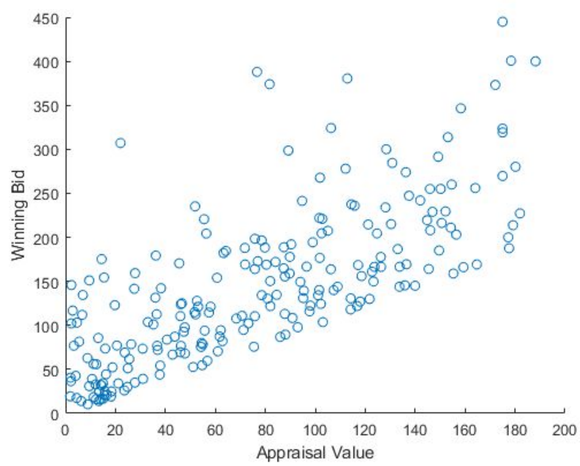
Volume of Timber	Winning Bid	Losing Bid	Appraisal Value	Number of Bidders
MBF of timber	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Appraisal Value (Dollars per unit of timber)	2



In the two bidder ascending auctions, the winning bids and appraisal values are correlated with a coefficient of 0.8215. Again, the relationship between appraisal values and winning bids is clearly visible.

- Ascending auction with three bidders

Volume of Timber	Winning Bid	Second-Highest Bid	Losing Bid	Appraisal Value	Number of Bidders
MBF of timber	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Bid (Dollars per unit of timber)	Appraisal Value (Dollars per unit of timber)	3



In the three bidder ascending auctions, the winning bids and appraisal values are correlated with a coefficient of 0.7431.

1.2 Auction Details:

For the sealed bid auctions, if at least two of the bidders submit the same bid, the seller draws out of a hat to determine the winner. The nature of ascending auctions does not permit their bidders to submit the same bid, therefore this problem is avoided in ascending auctions.

There is a large variation in the quantity of timber being auctioned off for all of the auctions, demonstrated by the large standard deviation for the quantity of timber and range for the quantity of timber. We expect the winning bids to increase in value as the number of bidders increase because of increased competition. For instance, we expect the winning bids to be higher in auctions with three bidders than auctions with two bidders.

The mean quantity of timber being auctioned off for sealed bid auctions is 1050 and the mean appraisal value is 59.6. The mean quantity of timber being auctioned off for ascending auctions is 3171 and the mean appraisal value is 72.3. From this, we can see that larger stands of higher quality timber tend to be auctioned by the USFS using ascending auctions rather than sealed bid auctions.

Model:

2.1 Model Setup

A single indivisible object is sold through a first-price sealed bid or ascending auction. There are $N \geq 2$ bidders. Bidder i's valuation for the object is $V_i \in [0, \omega]$, where $\omega < \infty$. Each bidders private valuation is identically and independently distributed (V_i i.i.d ~ F). This distribution is continuous and increasing on $[0, \omega]$, and is known by all bidders. Each bidder shares the same private value distribution.

For the ascending auction, it is a weakly dominant strategy to always bid your private valuation. Therefore, risk-aversion does not affect the equilibrium bidding strategy. This is the strategy equivalent to a second-price sealed bid auction. The winner is always the bidder with the highest valuation, but pays the winning bid of the second highest valuation. To visualize this, imagine two independent ascending auctions with three bidders. In auction one, the valuation distribution is (2,5,10), respectively. In auction two, the valuation distribution is (4, 5, 20), respectively. Because it is a dominant strategy to bid your valuation, the bids will be the same as the bidders valuations. Now, imagine the auction begins. Once the bids get to 5, two of the three bidders will drop out because 5 is the second-highest valuation. Therefore, even though the winner in the two auctions value the item differently (10 and 20), he/she pays 5 for the item. In observing this data, we know the winning bid in ascending auctions is the second highest valuation. We will use the distribution of winning bids/second-highest valuations in the ascending auction to estimate the distribution of valuations.

Bidding behavior becomes more complicated when considering a first-price sealed bid auction because risk-aversion is involved.

Below is the utility function that includes risk-aversion. X represents the difference between valuation and bids. α represents risk-aversion. When $\alpha = 1$, the equation shows risk-neutrality:

$$U(x) = X^{\alpha}\text{, where } 0 < \alpha \leq 1,\ U(0) = 0$$

For the first price sealed bid auction, the expected payoff is characterized as:

$$Expected\ Payoff = Pr(win)\ u(v - Price)\ + Pr(lose)\ u(0) = F(v)\ u(v - (v))$$

Because firms and businesses act rationally, we take the first order condition (F.O.C) of the expected payoff equation to maximize expected payoff. We can define the equilibrium bidding strategy with:

$$\boldsymbol{\beta}'(v) = (n-1)*u(v - \boldsymbol{\beta}(v))/\ u'(v - \boldsymbol{\beta}(v))\ *\ f(v)\ /\ F(v)$$

This F.O.C. characterizes the bidder distribution in a first-price sealed bid auction.

Estimation Strategy and Results

3.1 Estimation Strategy

There are n $\in (2, 3)$ bidders with valuations (V_i) ~ F on [0, w], w<∞. Each bidder's utility function is u(x)=x^{α} . Bidder i has valuation V_i and is under the assumption that all other bidders (n-1) are using the same bidding strategy β(·). Bidder i's expected utility is seen in the following equation:

$$u(V - b)[F(\beta^{-1}(b)]^{(n-`1)}$$

Because we can assume all bidders are rational and want to maximize their expected utility, we take the first order condition of the expected utility equation. To account for the correlation between bids and appraisal values, we held the appraisal value constant at the median appraisal value from the ascending auction data when generating distributions of bids. After taking the first order condition we get the following equation where V_i is the valuation, b_i is the bid, F ($V_i|x$) is the conditional cdf of valuations with respect to appraisal values, f(V_i |x) is the conditional pdf of valuations with respect to appraisal values, and α represents the risk aversion factor of the bidder.

$$\beta'(V) = (n - 1) \times \frac{f(V|x)}{F(V|x)} \times \frac{u(V-b)}{u'(V-b)}$$

We now plug in the utility function, $U(x) = X^{\alpha}$, and get the following:

$$\beta'(V) = (n - 1) \times \frac{f(V|x)}{F(V|x)} \times \frac{(V-b)}{\alpha(V-b)^{\alpha-1}}$$

Further simplification:

$$\beta'(V) = (n - 1) \times \frac{f(V|x)}{F(V|x)} \times \frac{(V-b)}{\alpha} \text{ (Equation 1)}$$

There are two unknowns in this equation, the cdf of valuations and α. Therefore, we need to solve for both of the unknowns. We will do this by first estimating F(v|x) and then solving the above equation for α.

At this stage, we will now solve for F(v). We will use $\widehat{G}(b|x,n)$ to represent the bid distribution and $\widehat{g}(b|x,n)$ to represent the density of bids. We use G to denote this because we are using the bid distribution rather than valuation distribution, and we are holding the number of bidders constant since the bid distributions are different for auctions with two and three bidders.

$\widehat{G}(b|x,n) =$ represents the estimated bid distribution given the number of bids.

$$\widehat{G}(b|x,n) = Pr(B < b|n) = Pr(\beta^{-1}(B) < \beta^{-1}(b)) =$$

$$= Pr(V < \beta^{-1}(b)) = F(\beta^{-1}(b))$$

$\widehat{G}(b|x,n)$ is the probability that a certain bid is less than a random bid drawn from distribution b given the number of bidders. We take the inverse function because we need to determine the valuation from the bid. After taking the inverse function, we have the probability that a certain valuation is less than a random valuation determined by taking the inverse function of bid distribution b. The final result is $\widehat{G}(b|x,n) = F(\beta^{-1}(b)$.

We differentiate this with respect to b in order to to get the estimated bid density.

$$\widehat{g}(b|x,n) = f(\beta^{-1}(b))/\beta'(\beta^{-1}(b))$$

Now, we have found $\widehat{G}(b|x,n)$ (the estimated bid distribution given the number of bids) and $\widehat{g}(b|n)$ (the estimated bid density given the number of bids) We will use these and substitute F(v) with $\widehat{G}(b|n)$ and f(v) with $\widehat{g}(b) \times \beta'(\beta^{-1}(b) = \widehat{g}(b) \times \beta'(v)$ in the equation representing how to maximize expected utility (Equation 1). After this substitution and simplification, we get the equation below solved for $\widehat{V}_{i,l}$, the estimated valuation for bidder i in auction l.

$$\widehat{V}_{i,l} = b_{i,l} + 1/(n-1) \times \widehat{G}(b|x,n)/\widehat{g}(b|x,n)) \times \alpha \text{ (Equation 2)}$$

There are still two unknowns in this equation, the distribution of valuations and α. In order to define the cdf of valuations, we will need to incorporate what type of auction was used. Specifically, we need to include whether the auction was an ascending auction or a sealed bid first price auction. By treating these two auctions seperately, we will be able to determine F(v|x). All bidders draw from the same distribution of valuations, regardless of the type of auction. This is true because the auction type was randomly selected. Therefore, we can use the data from the ascending auction and apply it to all bidder's valuations.

In an ascending auction it is always a weakly dominant strategy to bid your true valuation, and the winning bidder always pays the second-highest valuation. By taking the distribution of the winning bids in ascending auctions, we know we have taken the distribution of the second-highest valuations. The same can be said for taking the density of the winning bids, which is the density of the second-highest valuations. The second-highest valuations will be used to determine the distribution of the highest valuations We will call the cdf of second-highest values F˜(v|x) to distinguish it from F(v|x), the cdf of the valuations.

For every v $\in$ [0, ω] , the following equation holds true:

$$\tilde{F}(v|x) = (n)F(v)^{n-1} - (n-1)(F(v)^{n}$$

We can use the above equation to determine F(v) (the cdf of the highest valuation) for auctions with two and three bidders. We determined F(v) for both n=2 and n=3 and we will show the equations we used to solve for F(v) for n=2 and n=3 below. Assume we only took data from ascending auctions with n=2:

$$\tilde{F}(v|x) = 2 \times F(v|x) - F(v|x)^2$$

The above equation can then be used to solve for F(v) from F˜(v) for n=2.

Likewise, assume n=3 and our data comes from ascending auctions:

$$\tilde{F}(v|x) = 3 \times F(v|x)^2 - 2F(v|x)^3$$

As we did above, this equation can then be used to solve for F(v) from F˜(v).

We have estimated F(v|x). Now that we found F(v|x), one of the two unknowns, we can estimate α, the other unknown, for auctions with two and three bidders. We can utilize the fact that bidding strategy, with or without risk aversion, is strictly increasing.

Repeating equation 2:

$$\widehat{V}_{i,l} = b_{i,l} + 1/(n-1) \times \widehat{G}(b|x,n)/\widehat{g}(b|x,n)) \times \alpha$$

Solve this for α because α is the only unknown:

$$= (n-1) \times (\widehat{V}_{i,l} - b_{i,l}) \times \widehat{g}(b|x,n)/\widehat{G}(b|x,n)$$

Let λ ∈ [0, 1] denote the quintile. β(v (λ)) = b(λ). We are using the distribution we have of bids from first-price auctions, and finding the true valuation for the bidders using the distribution F(v) we recovered from the ascending auction data. From these distributions, given λ, we can recover a specific valuation by evaluating $F^{-1}(\lambda\,|x,n) = v$. Then, we can solve the equation for α. A bidder with median valuation (λ=0.5) will submit the median bid, bidder with λ=0.25 valuation (25th percentile) will submit the 25th percentile bids, etc..

$$\alpha = (n-1) \times (\widehat{V}(\lambda) - b(\lambda)) \times \frac{g(b(\lambda)|x,n)}{G(b(\lambda)|x,n)}$$

$G(b(\lambda)|x,n)$ is the probability that a certain bid is less than a randomly drawn bid in a distribution. Therefore, it is the same as a percentile. In this case, $G(b(\lambda)|x,n) = \lambda$. Then, we evaluation the conditional pdf of bids at the λth percentile bid. Finally, we are left with this equation with a single unknown, α.

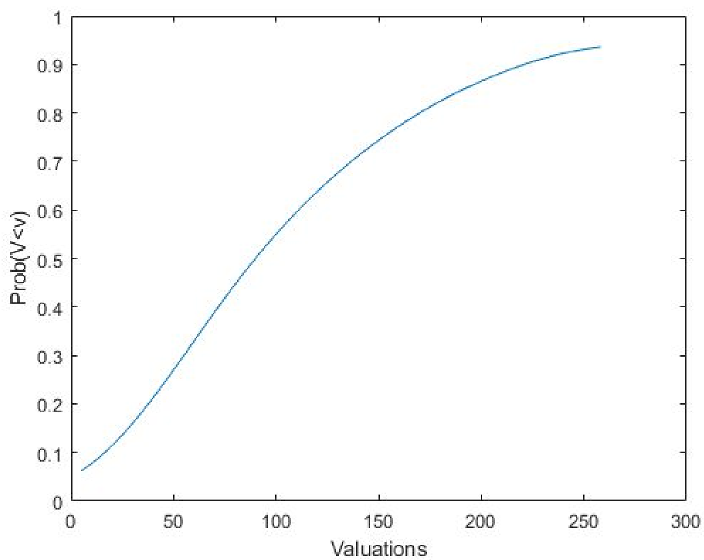
$$\alpha = (n-1) \times (\widehat{V}(\lambda) - b(\lambda)) \times \frac{g(b(\lambda)|x,n)}{\lambda}$$

Using this equation, pick any λ ∈ [0, 1] and solve for α. We did this using 100 different λ's, one for each percentile. After discarding any α outside of [0,1], we were left with 68 α's in the n=2 auction and 37 α's in the n=3 auctions.

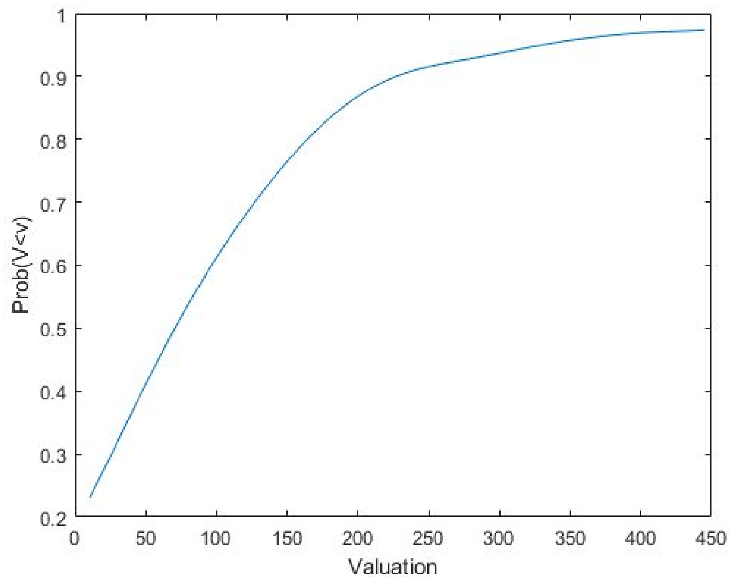
3.2 Estimation Results

We found F(v|x) for the n=2 and the n=3 cases.

N=2



N=3



We found the average α for the n=2 case and the n=3 case. For n=2, we averaged 68 α's while for n=3 we averaged 37 α's. We determined that for auctions with n=2 bidders α=0.2462, and auctions with n=3 bidders α=0.3308.

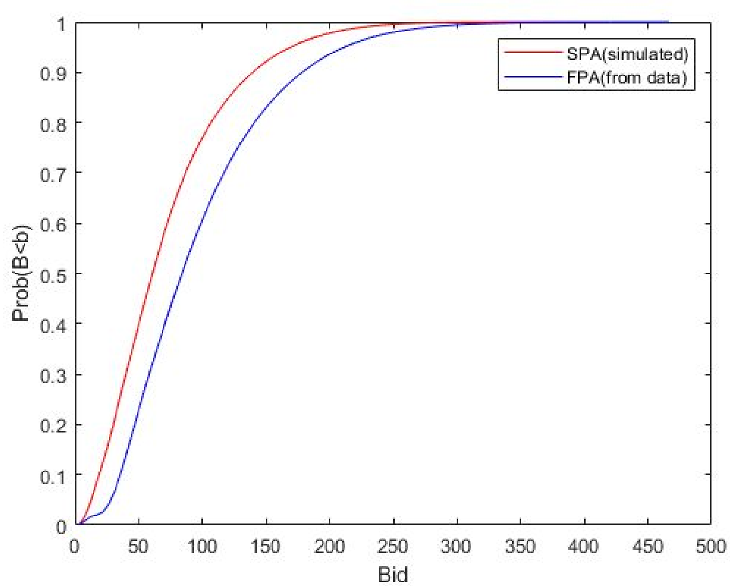
Counterfactual Analysis

We will now show why α matters to the USFS, how α affects revenue for the seller, and which auction format the USFS should prefer given our α values for auctions with n=2 bidders and n=3 bidders.

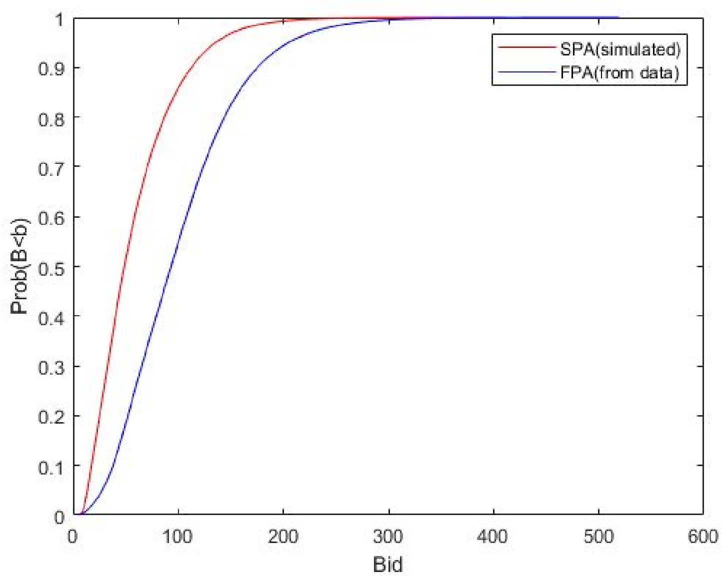
According to the revenue equivalence theorem, first-price auctions and ascending auctions should generate the same revenue given risk-neutral bidders. However, risk-aversion has an impact on the bidding strategy in first-price auctions while it has no effect on ascending auctions. For first-price auctions, risk-aversion leads to bidders bidding more aggressively because risk-averse bidders are more sensitive to losing. Aggressive bidding means higher bids, and therefore more revenue for the seller. Therefore, USFS should prefer the first price auction format over the ascending auction format given risk averse bidders.

To test this theory we simulated valuations for two and three bidder auctions from the conditional cumulative distribution function evaluated at the median appraisal value from the ascending auctions, F(v|x=median(X)). Then, we ran the valuations through a second price auction and found the prices paid. Then, we compare the prices paid in the simulated second price auction to the observed first price auctions, keeping in mind that the revenue from the second price auction should be equivalent to the revenue from a first price auction given risk neutrality. For the n=2 case, the simulated second price auctions generated an average price of \$52.21 per 1000 board feet of timber, while the observed first price auction generated an average revenue of \$81.00 per 1000 board feet of timber. For the n=3 case, the simulated second price auctions generated an average price of \$78.58 per 1000 board feet of timber, while the observed first price auction generated an average price of \$96.88 per 1000 board feet of timber. Therefore, given risk aversion the sealed first price auction generated \$28.79 per 1000 board feet of more in revenue compared to the second price auction. The cumulative distribution functions below directly compare the revenue across all percentiles and the result found at the average result holds.

N=2



<u>N=3</u>



The difference between the simulated second price auctions revenues and observed first price auctions revenues show that risk aversion should be important to the USFS, because the more risk averse bidders are the more revenue the USFS can expect from their first price auctions. Additionally, this analysis shows that the USFS should utilize first price auctions when their bidders are risk averse.

Furthermore, our results confirm the theoretically strong notion that revenue increases when the number of bidders increases. For simulated second price auctions, auctions with three bidders had on average $26.37 higher price per 1000 board feet than the auctions with two bidders. Similarly, for the observed first price auctions, auctions with three bidders had on average $15.88 higher price per 1000 board feet than auctions with two bidders.

Finally, we found that the difference between simulated second price auctions and observed first price auctions was smaller for auctions with three bidders in comparison to auctions with two bidders. The average difference in the two bidder case is $28.79, while the average difference in the three bidder case is $18.30.

Conclusion

In this paper we estimated the risk-aversion parameter in timber auctions held by the USFS, determined that this parameter is important for the USFS, and concluded that first-price sealed bid auctions generate more revenue than ascending auctions in this instance. Therefore, the USFS should continue to use the first-price sealed bid auction format in their timber auctions. More revenue is generated in first-price sealed bid auctions because of the presence of the risk-aversion factor. This factor is not present in ascending auctions, and risk-aversion does not affect the bidding strategy of the participants.

We began this paper with the data sets of timber auctions in 1979 attempting to estimate the risk-aversion parameter, determine if risk-aversion matters to the USFS, and then decide if a first-price sealed bid auction or ascending auction is best for the USFS. We accomplished this by creating an equation to maximize the bidder's expected utility. This equation had two unknowns--the cdf of valuations and the risk-aversion parameter α. We used data from ascending auctions to estimate the distribution of bidders and determine one of the unknowns. Finally, we solved the equation to maximize expected utility for the final unknown, α. For auctions with n=2 bidders α=0.2462, and auctions with n=3 bidders α=0.3308.

At this point, we determined that this does matter to the USFS because risk-aversion affects bidding behavior in first-price sealed bid auctions. For both auctions with n=2 and n=3 bidders, we determined that the first-price sealed bid format produces more revenue than the ascending auction format. Thus, the USFS should prefer to use first-price sealed bid auctions.

Appendix

Note: In our code appraisals are designated with “A”, highest bid is designated with “One”, second highest bid is designated with “Two”, and third lowest bid is designated with “Three.” Data from ascending auctions have the prefix “Asc”, while the data from sealed first price auctions have no prefix. Furthermore we do not include the code from our data entry below, because we manually created arrays for each variable (it would be a couple thousand lines).

Code for two bidder auctions:

```
%looking for outliers (outside of [.01,.99]) and removing observations with
%outliers
TFOne=isoutlier(One,'percentiles',[1,99]); % 55 99
TFTwo=isoutlier(Two,'percentiles',[1,99]);% 4 55
```

```
TFA=isoutlier(A,'percentiles',[1,99]);% 4 55

One([4,55,99],:) = [];
Two([4,55,99],:) = [];
A([4,55,99],:) = [];

TFAscOne=isoutlier(AscOne,'percentiles',[1,99]);% 1 2 240 241
TFAscTwo=isoutlier(AscTwo,'percentiles',[1,99]);% 35 131 240 241
TFAscA=isoutlier(AscA,'percentiles',[1,99]);% 231

AscOne([1,2,35,131,231,240,241],:) = [];
AscTwo([1,2,35,131,231,240,241],:) = [];
AscA([1,2,35,131,231,240,241],:) = [];

%histograms, scatter plots, and correlation coefficients after dropping outliers
scatter(A,One)
R=corrcoef(A,One)
histogram(One)
scatter(AscA,AscOne)
AscR=corrcoef(AscA,AscOne)
histogram(AscOne)
%All FPA bids
```

```
B=[One;Two];

%stacks appraisal so 2nd bids are aligned w/ appropriate appraisal

AA=[A;A];


%stacking all ascending bids

AscB=[AscOne;AscTwo];


%filling array of length(AscA) with the median appraisal from FPA

Q=zeros(234,1);

for N=1:length(AscA)

Q(N,1)=median(A) ;

end

%filling array of length(A) with the median appraisal from FPA

p=zeros(208,1);

for N=1:length(AA)

p(N,1)=median(A) ;

end


%joint pdf

g =mvksdensity([B,AA],[B,p],'function','pdf');


%marginal pdf wrt A
```

```
fa=ksdensity(AA,median(A),'function','pdf');

%conditional pdf: g(b|A)
ghat=g./fa;

%marginal distrubtion of A
Fa=ksdensity(AscA,median(A),'function','cdf');

%joint distribution of One & A
Fbar=mvksdensity([AscOne,AscA],[AscOne,Q],'Function','cdf');

%cdf of One|A
Ftilde=Fbar./Fa ;

% solve quadratic for Fhat (CDF of valuations)
Fhat=(2-sqrt(4-Ftilde.*4))./(2);
%plot estimated cdf of valuations (F(v|A=median,n=2))
plot(AscOne,Fhat)

% stack all bids
AscB=[AscOne;AscTwo];
%estimate alpha
```

```
alpha(Fhat,AscOne,B,ghat)

%Counter-factual

randomdraw(AscB,Fhat,1000,One)



function alpha(Fhat,AscOne,B,ghat)

a=zeros(99,1);

for P=1:99

[~,IA,~] = unique(Fhat);%for interp1 commands

pp=P*.01;

b=prctile(B,P); % finding bids(lamda)

v=interp1(Fhat(IA),AscOne(IA),pp); %evaluating CDF of valuations at lambda

gl=interp1(ghat,b);%evaluating pdf of bids at bids(lambda)

alp=(gl.*(v-b))./(pp); % solving for alphas

a(P,1)=alp; %creating an array of alphas

end

a= a(a>0 & a<1)% looking at alphas within (0,1)

nanmean(a)% average, ignoring NaNs

end

function randomdraw(B,Fhat,N,One)


% using Inverse transform Sampling
```

```
% https://en.wikipedia.org/wiki/Inverse_transform_sampling


u1= rand([N,1]); % draw N by 1 vector of uniform RVs

u2= rand([N,1]); % draw N by 1 vector of uniform RVs


[~,IA,~] = unique(Fhat); % interp1 needs unique values


%Simulate Valuations and sets NaNs to zero

ValuesOne=interp1(Fhat(IA),B(IA),u1,'linear') ;

ValuesOne(isnan(ValuesOne))=0;

ValuesTwo=interp1(Fhat(IA),B(IA),u2,'linear') ;

ValuesTwo(isnan(ValuesTwo))=0;

Values= [ValuesOne,ValuesTwo]; %Creating matrix of valuations

SecondPriceBids=Values; %in SPA valuations=bids

SecondHighestBid=min(SecondPriceBids,[],2); %price paid= 2nd highest bid


%finds observations less than zero

indices=find(SecondHighestBid<=0);

%removes above observations

SecondHighestBid(indices)=[];

%Average Price paid from SPA, and FPA given risk neutrality
```

```
AverageRevenueSPA=median(SecondHighestBid)

%Average Price paid from FPA given risk aversion

AverageRevenueFPA=median(One)

%compare cdf of bids from simulated SPA (which is equal to risk neutral

%FPA) with bids from FPA data where risk aversion is present


%plot showing cdf of simulated second highest valuation from SPA

% and cdf of winning bid in FPA data


[f1,SecondHighestBid]=ksdensity(SecondHighestBid,'support','positive','function','cdf');

hold on

[f2,One]= ksdensity(One,'support','positive','function','cdf');

hold off

plot(SecondHighestBid,f1,'r',One,f2,'b')

legend('SPA(simulated)','FPA(from data)')

end
```

Code for three bidder auctions:

```
%looking for outliers in FPA data (outside of [.01,.99]

TFOne=isoutlier(One,'percentiles',[1,99]); %94
```

```
TFTwo=isoutlier(Two,'percentiles',[1,99]);% 41, 96,

TFThree=isoutlier(Three,'percentiles',[1,99]);% 79 94

TFA=isoutlier(A,'percentiles',[1,99]); % 94

%Dropping Observations with outliers

One([41,79,94,96],:) = []

Two([41,79,94,96],:) = [];

Three([41,79,94,96],:) = [];

A([41,79,94,96],:) = [];


%looking for outliers in ascending data (outside of [.01,.99]

TFAscOne=isoutlier(AscOne,'percentiles',[1,99]);% 1 2 230 231

TFAscTwo=isoutlier(AscTwo,'percentiles',[1,99]);% 1 3 230 231

TFAscThree=isoutlier(AscThree,'percentiles',[1,99]); % 1 5 221 229

TFAscA=isoutlier(AscA,'percentiles',[1,99]) ;% 5 72 206 211

%dropping observations with outliers

AscOne([1,2,3,5,72,206,211,221,230,231],:) = []

AscTwo([1,2,3,5,72,206,211,221,230,231],:) = [];

AscThree([1,2,3,5,72,206,211,221,230,231],:) = [];

AscA([1,2,3,5,72,206,211,221,230,231],:) = [];


%histograms, scatter plots, and correlation coefficients after dropping outliers

scatter(A,One)
```

```
R=corrcoef(A,One)

histogram(One)

scatter(AscA,AscOne)

AscR=corrcoef(AscA,AscOne)

histogram(AscOne)


%Stacking All FPA bids into one array

B=[One;Two;Three];


%stacks appraisal so 2nd and 3rd bids are aligned w/ appropriate appraisal

AAA=[A;A;A];


%stacking all Ascending bids into one array

AscB=[AscOne;AscTwo;AscThree];


%creating array filled with the median Appraisal(from FPA).

% (We had trouble with repmat)

p=zeros(312,1);

for N=1:length(AAA)

p(N,1)=median(AAA) ;

end
```

```
%joint pdf

g =mvksdensity([B,AAA],[B,p],'function','pdf');


%marginal pdf wrt A

fa=ksdensity(AAA,median(AAA),'function','pdf');


%conditional pdf: g(b|A)

ghat=g./fa;


%creating array of length(AscA)filled with median appraisal (from FPA)

d=zeros(221,1);

for N=1:length(AscA)

d(N,1)=median(AAA) ;

end


%marginal distrubtion of One wrt A

Fa=ksdensity(AscA,median(AAA),'function','cdf');


%joint distribution of One & A

Fbar=mvksdensity([AscOne,AscA],[AscOne,d],'Function','cdf');


%cdf of One|A
```

```
Ftilde=Fbar./Fa ;


%Solving for cubic roots
Fhat=zeros(221,1);
for N=1:length(Fhat)
Ftilde(N);


T=[2;-3;0;Ftilde(N)]; %array of polynomial coefficients
Fhats=roots(T);%solves 222 cubics for three roots
Fhata=Fhats(2,1);%eliminates the nonsence roots
Fhat(N,1)=Fhata;%create array for Fhat
end
%Plot estimated cdf of valuations
plot(AscOne,Fhat)
% estimate alpha
alpha(Fhat,AscOne,B,ghat)
% counter-factual
randomdraw(AscB,Fhat,1000,One)


function alpha(Fhat,AscOne,B,ghat)
a=zeros(99,1);
for P=1:99
```

```
[~,IA,~] = unique(Fhat); %for interp1 commands

pp=P*.01;

b=prctile(B,P); % finding bids(lamda)

v=interp1(Fhat(IA),AscOne(IA),pp); %evaluating CDF of valuations at lambda

gl=interp1(ghat(IA),b);%evaluating pdf of bids at bids(lambda)

alp=(gl.*(v-b))./(pp); % solving for alphas

a(P,1)=alp; %creating an array of alphas

end

a= a(a>0 & a<1) % looking at alphas within (0,1)

nanmean(a) % average, ignoring NaNs

end


function randomdraw(AscB,Fhat,N,One)

% using Inverse transform Sampling

% https://en.wikipedia.org/wiki/Inverse_transform_sampling


u1= rand([N,1]); % draw N by 1 vector of uniform RVs

u2= rand([N,1]); % draw N by 1 vector of uniform RVs

u3= rand([N,1]); % draw N by 1 vector of uniform RVs

[~,IA,~] = unique(Fhat); % interp1 needs unique values

%Simulate Valuations and sets NaNs to zero

ValuesOne=interp1(Fhat(IA),AscB(IA),u1,'linear');
```

```
ValuesOne(isnan(ValuesOne))=0;

ValuesTwo=interp1(Fhat(IA),AscB(IA),u2,'linear') ;

ValuesTwo(isnan(ValuesTwo))=0;

ValuesThree=interp1(Fhat(IA),AscB(IA),u3,'linear') ;

ValuesThree(isnan(ValuesThree))=0;


%creating matrix of valuations & v=b in SPA

SecondPriceBids= [ValuesOne,ValuesTwo,ValuesThree];


%find second highest bid (max ignoring the max)

HighestBid= max(SecondPriceBids,[],2);

SecondPriceBids(bsxfun(@eq, SecondPriceBids, HighestBid)) = 0;

SecondHighestBid=max(SecondPriceBids,[],2);

%finds observations less than or equal to 0

indices=SecondHighestBid<=0;

%drop above observations

SecondHighestBid(indices)=[]

AverageRevenueSPA=median(SecondHighestBid)

AverageRevenueFPA=median(One)

%compare cdf of bids from simulated SPA (which is equal to risk neutral

%FPA) with bids from FPA data where risk aversion is present
```

```
%plot showing cdf of second highest bid of SPA and cdf of winning bid in FPA

[f1,SecondHighestBid]=ksdensity(SecondHighestBid,'support','positive','function','cdf');

hold on

[f2,One]= ksdensity(One,'support','positive','function','cdf');

hold off

plot(SecondHighestBid,f1,'r',One,f2,'b')

legend('SPA(simulated)','FPA(from data)')

end
```