ECM1416 CA1 (Formative assessment only) Jan 2020

[20%, marks] **Vector and matrix operations**

1) i) [2] **what is the value of this vector operation:**

$$\begin{bmatrix}1 & 1 & 2\\ 0 & 2 & 2\\ 1 & 0 & 3\end{bmatrix}\begin{bmatrix}0\\2\\1\end{bmatrix} = \text{ A)}\begin{bmatrix}0\\2\\1\end{bmatrix}, \text{B)}\begin{bmatrix}4\\6\\3\end{bmatrix}, \text{C)}\begin{bmatrix}1\\1\\1\end{bmatrix}, \text{D)}\begin{bmatrix}6\\3\\4\end{bmatrix}, \text{E)}\begin{bmatrix}4\\3\\6\end{bmatrix}$$

ii) [1] **What type of transformation is this?**:

A) rotation, B) translation, C) scaling, D) a combination or E) no transformation

2 i) [2] **what is the value of this vector operation?**:

$$\begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\1\end{bmatrix} = \text{ A)}\begin{bmatrix}0\\4\\2\end{bmatrix}, \text{B)}\begin{bmatrix}0 & 4 & 2\end{bmatrix}, \text{C)}\begin{bmatrix}4\\2\\2\end{bmatrix}, \text{D)}\begin{bmatrix}0\\2\\1\end{bmatrix}, E)\begin{bmatrix}0 & 2 & 4\end{bmatrix}$$

ii) [1] **What type of transformation is this?**:

A) rotation, B) translation, C) scaling, D) combination or E) no transformation

3 i) [2] **what is the value of this vector operation**:

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\2\\1\end{bmatrix} = \text{ A)}\begin{bmatrix}1\\1\\1\end{bmatrix}, \text{B)}\begin{bmatrix}0\\2\\2\end{bmatrix}, \text{C)}\begin{bmatrix}0\\2\\1\end{bmatrix}, \text{D)}\begin{bmatrix}1\\2\\0\end{bmatrix}, E)\ \begin{bmatrix}0 & 2 & 1\end{bmatrix}$$

ii) [1] **What type of transformation is this?**:

A) rotation, B) translation, C) scaling, D) combination, E) no transformation

4 With the following matrices

$$\boldsymbol{A} = \begin{bmatrix}1 & 1 & 2\\ 0 & 2 & 2\\ 1 & 0 & 3\end{bmatrix}; \boldsymbol{B} = \begin{bmatrix}1 & 1 & 2\\ 1 & 0 & 3\end{bmatrix}; \boldsymbol{C} = \begin{bmatrix}1\\0\\1\end{bmatrix}; \boldsymbol{D} = \begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}$$

i) [2] **what size matrix results from** $\boldsymbol{A}.\boldsymbol{B}^T$?

A) 3 x 3, B) 2 x 3, C) 3 x 2, D) 2 x 2 or E) none (i.e. it's not possible to evaluate)

What is

ii) [3] $(\boldsymbol{A}.\boldsymbol{C})^T =$ $A)\begin{bmatrix}1\\2\\1\end{bmatrix}, B)\begin{bmatrix}4 & 2 & 3\end{bmatrix}, C)\begin{bmatrix}3\\2\\4\end{bmatrix}, D)\begin{bmatrix}0\\2\\1\end{bmatrix}, E)\begin{bmatrix}3 & 2 & 4\end{bmatrix}$

iii) [1] $|\boldsymbol{D}| =$ A) 2 , B) 1 , C) 3 , D) 5 , E) 0

iv) [2] $|\boldsymbol{A}| =$ A) 4 , B) 0 , C) -1 , D) 8 , E) 6

v) [3] $\boldsymbol{D^{-1}} =$ $\text{A)}\begin{bmatrix}-2 & 3\\ -1 & 1\end{bmatrix}, \text{B)}\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}, \text{C)}\begin{bmatrix}1 & 2\\ 1 & 3\end{bmatrix}, \text{D)}\ \frac{1}{2}\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}, \text{E)}\begin{bmatrix}3 & 2\\ 1 & 1\end{bmatrix}$

5) [20%] **Solving equations with matrix algebra**

Solve the following problem by hand:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix};\ \boldsymbol{u_1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix};\ \boldsymbol{u_2} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

For

i) [10] $\boldsymbol{Ax_1} = \boldsymbol{u_1}$ find $\boldsymbol{x_1}$

ii) [10] $\boldsymbol{Ax_2} = \boldsymbol{u_2} - \boldsymbol{u_1}$ find $\boldsymbol{x_2}$

Show your working

6) [50%] **An analysis task: what is the correct £ value of these costs?** (use python)

You work for an audit office, checking quantity and costs for a client. This portfolio account consists of 20 suppliers who each produce and sell wholesale food items to the client. In this market, there are upto twenty differing items that the suppliers produce and sell. The accounts you have received give the total costs paid to supplier per year over 2017-2019, and the quantity of these products bought from the suppliers by the client over these three years. You also know that across this wholesale industry the suppliers have contractually agreed a fixed 3% per year price per item increase over this period to accommodate an industry increase in costs. Your client is not sure that the accounts are properly balanced and has tasked you to find out if this is the case, and if so where the issue lies.

Using the accounts data, provided: Costs C2017, C2018, C2019 (each row is a different supplier and they are the same by year) and Quantity 2017, Q2018, Q2019 (each 20 entries are the quantities of item 1, 2, … 20 for a given supplier). Read in the data and create the 20 x20 quantitiy matrix form this. The data text files. Answer the following questions

i) [10] **If prices of four of the twenty items in 2017 were £1.01,£2.00, £0.50 and £1.24, what should their prices be in 2018 and 2019?**

ii) [20] **Establish what the prices charged for each year have been. Can you explain and show what the price discrepancy is?**

iii) [20] **Establish what the actual costs each year should be and where has the error occurred?** Show your reasoning

7) [10%] **Inverse matrix problems, computational limits:**

Generate three inverse calculations for an arbitrary $n \times n$ matrix $\boldsymbol{A}$ with random entries, with $n = \{5, 10, 20, 50\}$. Use three inverse calculation methods: a) the recursive method, b) the Gauss-Jordan method and c) the python library command: numpy.linalg.inv().

i) [5] **What are the relative timing speeds of each as n varies?**

ii) [5] **Are their precision differences as n increases? Suggest a way to investigate and evaluate this, giving a table of results.**

Provide Answers as: Questions 1 to 4 answer in the ELE ECM1416 Multiple choice CA1 Question Test Book. Questions 5 to 7: provide written answers via BART (a report, include your python code file). **Set Fri 24th Jan 2020 (week 2), complete both entries by 5pm on 6th Feb (week 4).**