Econs514_A3_v2

May 5, 2023

[1]:
```
#####################################
#Econs 514 -- Assignment 3 - Rust (1987) Repl
#Due - 5/5/2023
#By -- Suhina Deol
#####################################

import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import minimize, fixed_point
from numba import njit, float64, float32, int64, prange
from numba.experimental import jitclass
from datetime import datetime
from tabulate import tabulate
```

[2]:
```
b_raw = pd.read_csv(r'C:\Users\suhin\iCloudDrive\Documents\Econs 514 (Metrics␣
↪IV)\Ass 3\bus1234.csv')
#count, id, group, year, month, replace, mile

b = pd.DataFrame(b_raw)
b.rename(columns = {'Unnamed: 0':'count'}, inplace = True)

b.describe()
```

[2]:

	count	id	group	year	month	\
count	8260.000000	8260.000000	8260.000000	8260.000000	8260.000000	
mean	4129.500000	99.454722	278350.605932	1981.080387	6.489831	
std	2384.600945	27.511206	265015.917102	2.617909	3.489873	
min	0.000000	59.000000	50.000000	1975.000000	1.000000	
25%	2064.750000	76.000000	203.000000	1979.000000	3.000000	
50%	4129.500000	94.000000	530875.000000	1981.000000	6.000000	
75%	6194.250000	122.000000	530875.000000	1983.000000	10.000000	
max	8259.000000	162.000000	530875.000000	1985.000000	12.000000	

	replace	miles
count	8260.000000	8260.000000
mean	0.007264	118235.158232

```
std       0.084924   85129.344989
min       0.000000       0.000000
25%       0.000000   46096.000000
50%       0.000000  101118.000000
75%       0.000000  179969.000000
max       1.000000  388254.000000
```

[3]:
```
b = pd.DataFrame(b_raw)
b = b[['id','group','year','month','replace','miles']]
b["day"] = 1

new_cols = ['id','group','year','month','day', 'replace', "miles" ]

b=b.reindex(columns=new_cols)

b =  b.astype({'year':'str'})
b =  b.astype({'month':'str'})
b =  b.astype({'day':'str'})

#b["date"] = b["year"].apply(str) + "/" + b["month"].apply(str) + "/" +␣
 ↪b["day"].apply(str)
b["date"] = b["year"] + b["month"]

#b["date"] = pd.to_datetime(b["date"])

print(type('date'))
```

```
<class 'str'>
```

[4]:
```
new_cols = ['date','id', "miles" ]
oldcols=b.reindex(columns=new_cols)
oldcols = pd.DataFrame(oldcols)

print(oldcols)
```

```
      date   id   miles
0     19815  144    2208
1     19816  144    5418
2     19817  144    8547
3     19818  144   11566
4     19819  144   15890
…     …   …      …
8255  19851   95  342004
8256  19852   95  343654
8257  19853   95  345631
8258  19854   95  347549
8259  19855   95  347549
```

[8260 rows x 3 columns]

[5]:
```
# The columns of the new data frame will be the values in col2 of the original
a = pd.DataFrame(oldcols)

# The columns of the new data frame will be the values in col2 of the original
newcols = list(set(oldcols['id']))
rows = list(set(oldcols['date']))

# Create the new data matrix
data = np.zeros((len(rows), len(newcols)))

# Iterate over each row and fill in the new matrix
for row in zip(a['date'], a['id'], a['miles']):
    rowindex = rows.index(row[0])
    colindex = newcols.index(row[1])
    data[rowindex][colindex] = row[2]

newf = pd.DataFrame(data)
newf.columns = newcols
newf.index = rows

print(newf)

#newf.describe()
```

```
             59        60        61        62        63        64        65   \
19813    58332.0  224753.0   43578.0   26243.0   77980.0  123849.0  273871.0
19786   118615.0  142429.0  164203.0  147696.0  144219.0   13879.0  170733.0
19818    70850.0  236619.0   47515.0   41155.0   88922.0  136702.0  283494.0
198211  101508.0  268085.0   80361.0   86664.0  120004.0  167959.0   34829.0
19808    41726.0  208989.0   26527.0  219181.0   61976.0  105815.0  255337.0
…            …         …         …         …         …         …         …
197511   10479.0    9064.0   12892.0   10924.0    7665.0   11997.0   12380.0
19776    84579.0   95928.0  104535.0   93889.0   92433.0   97651.0  107792.0
19772    74323.0   79929.0   89044.0   79567.0   74329.0   81364.0   90309.0
19783   110597.0  130014.0  147314.0  133632.0  131339.0      86.0  153719.0
19765    37631.0   37003.0   44432.0   41134.0   29658.0   41101.0   46165.0

             66        67        68   …  153  154  155  156  157  158  159  \
19813   261499.0  309716.0   80581.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19786   156911.0  170121.0  174133.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19818   276208.0  330953.0   97146.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
198211   33433.0  374383.0  148649.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19808   243977.0  287326.0   47150.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
…            …         …         …   …    …    …    …    …    …    …    …
197511   12693.0   13106.0   13075.0   …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```
19776   102865.0  105906.0  119254.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19772    88678.0   82572.0   94738.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19783   142490.0  153626.0  160841.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0
19765    47947.0   40007.0   45888.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0

        160  161  162
19813   0.0  0.0  0.0
19786   0.0  0.0  0.0
19818   0.0  0.0  0.0
198211  0.0  0.0  0.0
19808   0.0  0.0  0.0
…       …    …    …
197511  0.0  0.0  0.0
19776   0.0  0.0  0.0
19772   0.0  0.0  0.0
19783   0.0  0.0  0.0
19765   0.0  0.0  0.0

[117 rows x 104 columns]
```

[6]:
```
newf['index'] = newf.index
```

[7]:
```
#newf.sort_values(by=['index'])
newf['sort'] = newf['index'].str.extract('(\d+)', expand=False).astype(int)
newf.sort_values('sort',inplace=True, ascending=True)
newf = newf.drop('sort', axis=1)
newf = newf.drop('index', axis=1)

print(newf)
```

```
              59        60        61        62        63        64        65   \
19759     2353.0     129.0    3246.0     532.0    1667.0    1758.0    2486.0
19761    20326.0   18251.0   23993.0   21457.0   15095.0   21575.0   23211.0
19762    24898.0   22725.0   29378.0   26568.0   18735.0   26537.0   29290.0
19763    29349.0   27843.0   33604.0   29790.0   22240.0   31036.0   33696.0
19764    33304.0   32508.0   39581.0   35774.0   25886.0   36239.0   39938.0
…            …         …         …         …         …         …         …
198311  123511.0  292192.0  107623.0  111630.0  148163.0  191825.0   61291.0
198312  125254.0  293846.0  109722.0  113447.0  149325.0  193620.0   62965.0
198410  144557.0  312719.0  124500.0  135557.0  171245.0  212256.0   83251.0
198411  146465.0  315040.0  126422.0  137565.0  173234.0  214149.0   85887.0
198412  148396.0  316984.0  128500.0  139530.0  174745.0  215173.0   87897.0

             66        67        68   …       153       154       155       156   \
19759     2145.0    4091.0    3146.0   …       0.0       0.0       0.0       0.0
19761    24463.0   23419.0   23495.0   …       0.0       0.0       0.0       0.0
19762    30362.0   28431.0   29148.0   …       0.0       0.0       0.0       0.0
19763    35816.0   31299.0   34080.0   …       0.0       0.0       0.0       0.0
```

```
19764   41014.0  35441.0   40138.0  …      0.0      0.0      0.0      0.0
…         …      …        …  …      …      …      …      …
198311  59074.0  11948.0  173574.0  …  10589.0  27372.0  25864.0  22696.0
198312  60663.0  13214.0  175189.0  …  14055.0  32040.0  30610.0  28531.0
198410  82620.0  34173.0  195117.0  …  59885.0  84539.0  80984.0  78016.0
198411  84811.0  35875.0  197461.0  …  63437.0  89747.0  83764.0  81457.0
198412  86954.0  37088.0  199310.0  …  66721.0  95097.0  86367.0  86493.0

            157      158      159      160      161      162
19759       0.0      0.0      0.0      0.0      0.0      0.0
19761       0.0      0.0      0.0      0.0      0.0      0.0
19762       0.0      0.0      0.0      0.0      0.0      0.0
19763       0.0      0.0      0.0      0.0      0.0      0.0
19764       0.0      0.0      0.0      0.0      0.0      0.0
…         …      …       …       …       …       …
198311  23758.0  23631.0  23842.0  27282.0  23118.0  21170.0
198312  28834.0  29964.0  27637.0  31787.0  27032.0  25046.0
198410  76996.0  75425.0  71118.0  80916.0  73474.0  66774.0
198411  83086.0  80302.0  74699.0  86590.0  79438.0  70233.0
198412  87424.0  84302.0  78061.0  91785.0  82675.0  74987.0

[117 rows x 104 columns]
```

[8]:
```
# Calculate the states d = {0, 1, 2}
newf.replace(0, np.nan, inplace=True)
b_state = newf - newf.shift(1)

b_state[b_state <5000] = 0
b_state[b_state >9999] = 2
b_state[b_state>2] = 1

print(b_state)

b_state.describe()
```

```
        59   60   61   62   63   64   65   66   67   68   …  153  154  155  \
19759   NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  NaN  …  NaN  NaN  NaN
19761   2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  …  NaN  NaN  NaN
19762   0.0  0.0  1.0  1.0  0.0  0.0  1.0  1.0  1.0  1.0  …  NaN  NaN  NaN
19763   0.0  1.0  0.0  0.0  0.0  0.0  0.0  1.0  0.0  0.0  …  NaN  NaN  NaN
19764   0.0  0.0  1.0  1.0  0.0  1.0  1.0  1.0  0.0  1.0  …  NaN  NaN  NaN
…     …  …  …  …  …  …  …  …  …  …  …  …  …  …  …
198311  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  1.0  0.0
198312  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0
198410  2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  2.0  …  2.0  2.0  2.0
198411  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  1.0  0.0
198412  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  1.0  0.0
```

```
        156  157  158  159  160  161  162
19759   NaN  NaN  NaN  NaN  NaN  NaN  NaN
19761   NaN  NaN  NaN  NaN  NaN  NaN  NaN
19762   NaN  NaN  NaN  NaN  NaN  NaN  NaN
19763   NaN  NaN  NaN  NaN  NaN  NaN  NaN
19764   NaN  NaN  NaN  NaN  NaN  NaN  NaN
…       …    …    …    …    …    …    …
198311  0.0  0.0  0.0  0.0  0.0  0.0  0.0
198312  1.0  1.0  1.0  0.0  0.0  0.0  0.0
198410  2.0  2.0  2.0  2.0  2.0  2.0  2.0
198411  0.0  1.0  0.0  0.0  1.0  1.0  0.0
198412  1.0  0.0  0.0  0.0  1.0  0.0  0.0

[117 rows x 104 columns]
```

[8]:

```
              59          60          61          62          63          64   \
count  116.000000  116.000000  116.000000  116.000000  116.000000  116.000000
mean     0.293103    0.310345    0.353448    0.318966    0.310345    0.318966
std      0.659608    0.664815    0.675719    0.667235    0.651604    0.680142
min      0.000000    0.000000    0.000000    0.000000    0.000000    0.000000
25%      0.000000    0.000000    0.000000    0.000000    0.000000    0.000000
50%      0.000000    0.000000    0.000000    0.000000    0.000000    0.000000
75%      0.000000    0.000000    0.000000    0.000000    0.000000    0.000000
max      2.000000    2.000000    2.000000    2.000000    2.000000    2.000000

              65          66          67          68   …         153  \
count  116.000000  116.000000  116.000000  114.000000   …   23.000000
mean     0.396552    0.387931    0.491379    0.500000   …    0.347826
std      0.696317    0.694970    0.716218    0.731709   …    0.647281
min      0.000000    0.000000    0.000000    0.000000   …    0.000000
25%      0.000000    0.000000    0.000000    0.000000   …    0.000000
50%      0.000000    0.000000    0.000000    0.000000   …    0.000000
75%      1.000000    1.000000    1.000000    1.000000   …    0.500000
max      2.000000    2.000000    2.000000    2.000000   …    2.000000

             154         155         156         157         158         159  \
count  23.000000   23.000000   23.000000   23.000000   23.000000   23.00000
mean    0.695652    0.521739    0.565217    0.434783    0.478261    0.26087
std     0.702902    0.730477    0.727767    0.727767    0.730477    0.68870
min     0.000000    0.000000    0.000000    0.000000    0.000000    0.00000
25%     0.000000    0.000000    0.000000    0.000000    0.000000    0.00000
50%     1.000000    0.000000    0.000000    0.000000    0.000000    0.00000
75%     1.000000    1.000000    1.000000    1.000000    1.000000    0.00000
max     2.000000    2.000000    2.000000    2.000000    2.000000    2.00000

             160         161         162
count  23.000000   23.000000   23.000000
```

```
mean    0.608696   0.391304   0.304348
std     0.722315   0.722315   0.702902
min     0.000000   0.000000   0.000000
25%     0.000000   0.000000   0.000000
50%     0.000000   0.000000   0.000000
75%     1.000000   0.500000   0.000000
max     2.000000   2.000000   2.000000

[8 rows x 104 columns]
```

[9]:
```
N = b_state.notna().sum().sum()
p = (b_state == 0).sum().sum() / N
q = (b_state == 1).sum().sum() / N
(p, q)
```

[9]:
```
(0.7538994800693241, 0.11413716266402575)
```

[10]:
```
#Now you got that, go back and do b
```

[11]:
```
def discretize(miles, d=5000):
    '''Discretizes odometer data into buckets of length d'''
    return np.floor(miles / d)
```

[12]:
```
b['mileage']= b['miles'].apply(discretize)
```

[13]:
```
b.rename(columns = {'mileage':'state'}, inplace = True)
b.rename(columns = {'replace':'decision'}, inplace = True)

new_cols = ['id', 'date', 'state', 'decision']
b=b.reindex(columns=new_cols)

b.head()
```

[13]:

	id	date	state	decision
0	144	19815	0.0	0
1	144	19816	1.0	0
2	144	19817	1.0	0
3	144	19818	2.0	0
4	144	19819	3.0	0

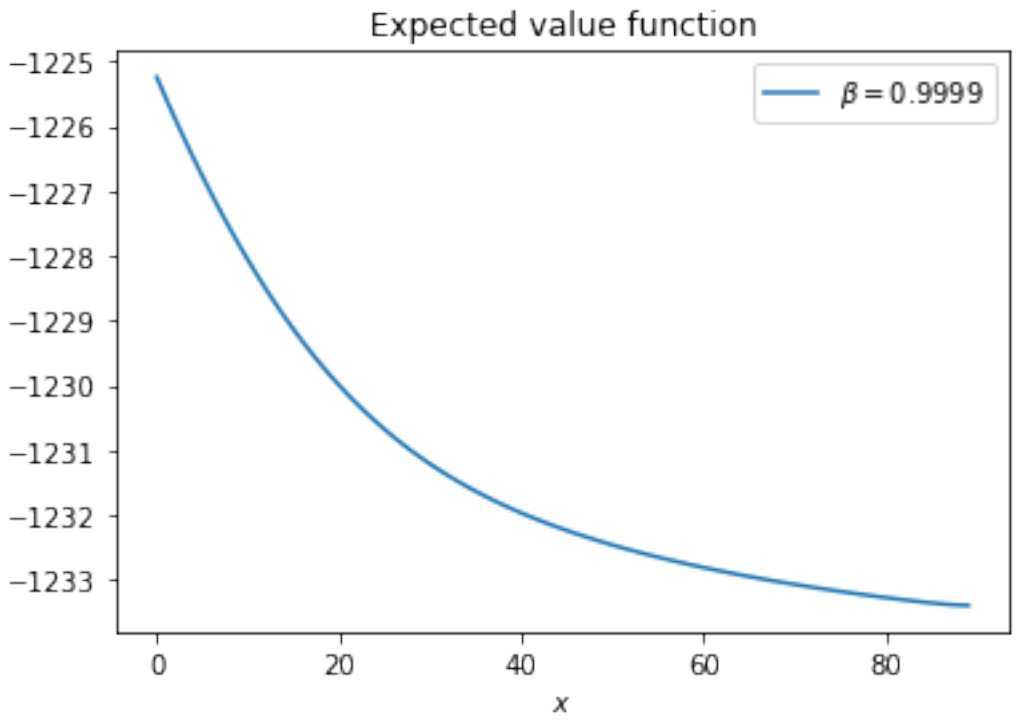
[15]:
```
spec = [(' ', float64),
        (' _1', float64),
        ('RC', float64),
        ('T', int64),
        ('p', float64),
        ('q', float64),
        ('scale', float64),
```

```
        ('P', float64[:, :]),
        ('x', float64[:])]

@jitclass(spec)
class Rust(object):

    def __init__(self,  =0.9999,  _1=3.6,
                 RC=10, T=90, p=0.3497,
                 q=0.6387, scale=0.001):
        self.  =
        self._1 =  _1
        self.RC = RC
        self.T = T
        self.p = p
        self.q = q
        self.scale = scale

        # Construct transition matrix
        P = np.zeros((T, T))
        np.fill_diagonal(P, p)
        P += np.diag(np.ones(T - 1) * q, 1)
        P += np.diag(np.ones(T - 2) * (1 - p - q), 2)
        P[:, -1] += 1 - P.sum(1)  # Adjust to sum to 1
        self.P = P

        # Statespace of x
        self.x = np.arange(T, dtype=np.float64)
```

[16]:
```
@njit
def c(x, rust):
    '''Linear cost function'''

    scale = rust.scale
     _1 = rust._1

    return scale *  _1 * x

@njit
def u(x, i, rust):
    '''Utility function'''

    if i == 0:
        return -c(x, rust)

    if i == 1:
        return -c(x, rust) - rust.RC
```

[17]:
```
@njit
def solve_EV(rust, tol=1e-3, maxiter=300000, print_flag=False):

    T = rust.T
      = rust.
    P = np.ascontiguousarray(rust.P)
    x = rust.x

    # Initial guess of value function
    EV = np.zeros(T)

    # Bellman operator
    def T(EV):
        wait = u(x, 0, rust) +   * EV
        replace = u(x[0], 1, rust) +   * EV[0]
        EV_new = np.exp(replace - EV) + np.exp(wait - EV)
        EV_new = P @ (np.log(EV_new) + EV)
        return EV_new

    # Find the fixed point of Bellman operator
    i = 0
    dist = 1e3
    for i in prange(maxiter):
        EV_new = T(EV)
        dist = np.linalg.norm(EV - EV_new)

        if dist < tol:
            if print_flag:
                print(f'Converged in {i} iterations')
            return EV
        else:
            i += 1
            EV = EV_new.copy()

    if print_flag:
        print('Reached max iterations')

    return EV
```

[18]:
```
r = Rust(p=p, q=q)
EV_star = solve_EV(r)
plt.plot(r.x, EV_star, label=rf'$\beta = {r. }$')

plt.title('Expected value function')
plt.xlabel('$x$')
plt.legend()
plt.show()
```



[19]:
```
b.head()
```

[19]:

	id	date	state	decision
0	144	19815	0.0	0
1	144	19816	1.0	0
2	144	19817	1.0	0
3	144	19818	2.0	0
4	144	19819	3.0	0

[20]:
```
@njit
def conditional_probability(rust):

      = rust.
    x = rust.x
    P = rust.P

    # Solve inner loop
    EV = solve_EV(rust)

    wait = u(x, 0, rust) +   * P @ EV
    replace = u(x[0], 1, rust) +   * EV[0]
    P_wait = 1 / (1 + np.exp(replace - wait))
    P_replace = 1 - P_wait
```

```
    return P_wait, P_replace
```

[21]:
```
def log_likelihood( ,              # Parameters to search over
                   b_data,         # Data consists of states and decisions
                   p,              # Absorbing state probability
                   q):             # Transition state 1 probability

    # Guess of parameters
    _1, RC =

    # Set up model with guess
    r = Rust( _1= _1, RC=RC, p=p, q=q)

    # Solve inner loop and find choice
    # probabilities conditional on x and i
    P_wait, P_replace = conditional_probability(r)

    # Log likelihood is sum of conditional probabilities given observed data
    logL = 0
    for decision, state in zip(b.decision, b.state):
        state = int(state)

        # Wait
        if decision == 0:
            logL += np.log(P_wait[state])

        # Replace
        if decision == 1:
            logL += np.log(P_replace[state])

    return -logL
```

[22]:
```
%%time
_star = minimize(log_likelihood, x0=(0.1, 10), args=(newf, p, q)).x
```

```
CPU times: total: 51 s
Wall time: 1min 13s
```

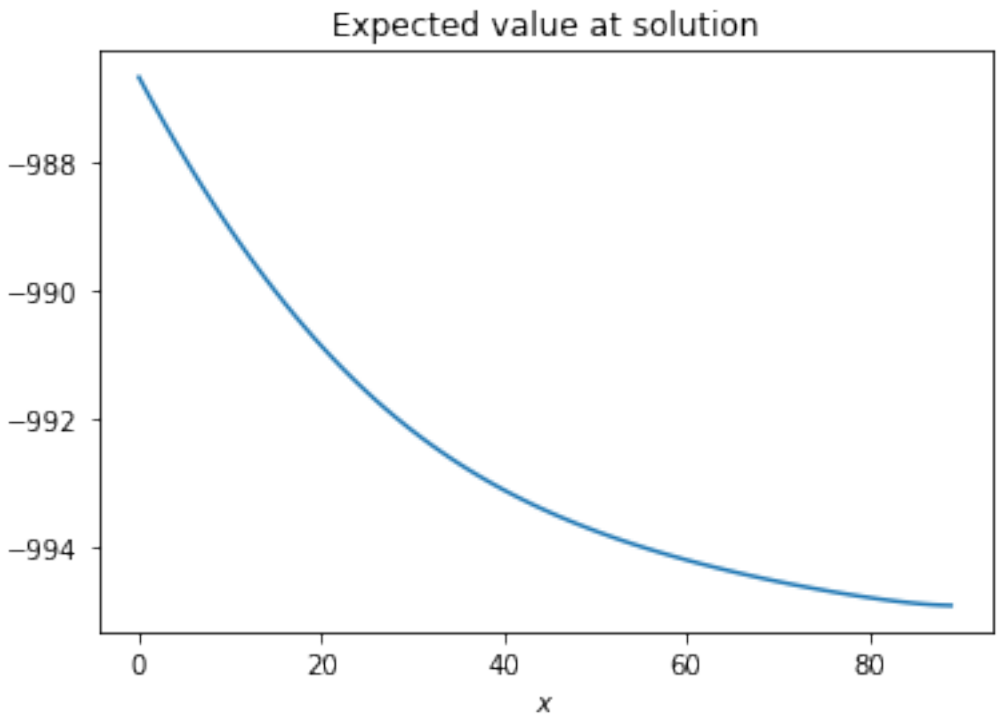
[23]:
```
_star

#Get following output, so handcode x1 and x2 into next result
# success: False
#: by natasha  ----- array([ 2.61808055, 10.03904057])
```

[23]:
```
array([ 2.13082394, 10.76908507])
```

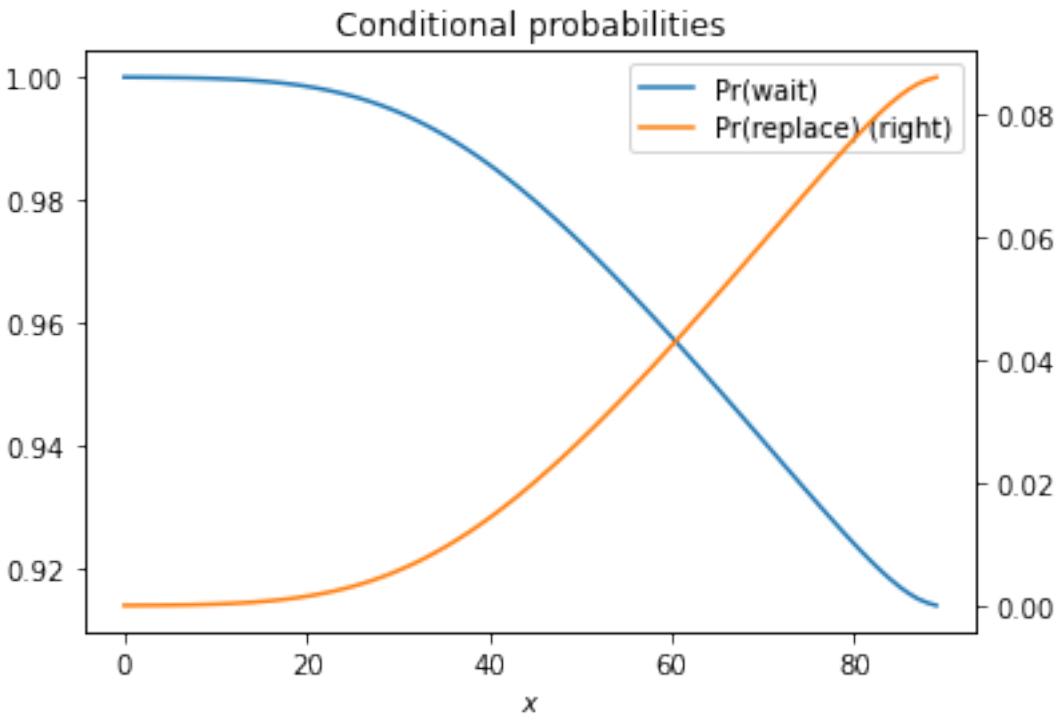
[26]:
```
# Use the solution to set up a new model, set lbound for theta_1at 0

r_star = Rust(p=p, q=q,  _1= _star[0], RC= _star[1])
EV_star = solve_EV(r_star)

print (r_star)
print (EV_star)
```

```
<numba.experimental.jitclass.boxing.Rust object at 0x000002091F3D0610>
[-986.68125775 -986.9424142  -987.19802054 -987.4480799  -987.69259645
 -987.93157565 -988.16502447 -988.39295162 -988.61536786 -988.83228633
 -989.04372283 -989.24969619 -989.45022868 -989.64534633 -989.83507934
 -990.01946244 -990.19853524 -990.37234259 -990.54093484 -990.7043681
 -990.86270443 -991.01601196 -991.16436494 -991.30784374 -991.44653468
 -991.58052991 -991.7099271  -991.83482908 -991.95534345 -992.07158209
 -992.18366061 -992.2916978  -992.39581503 -992.49613562 -992.59278421
 -992.6858862  -992.77556708 -992.86195196 -992.94516494 -993.02532872
 -993.10256408 -993.17698952 -993.24872093 -993.31787126 -993.38455032
 -993.44886453 -993.51091682 -993.57080648 -993.62862909 -993.68447649
 -993.73843677 -993.79059426 -993.84102959 -993.88981973 -993.93703806
 -993.98275442 -994.02703525 -994.06994364 -994.1115394  -994.1518792
 -994.19101663 -994.22900226 -994.26588373 -994.30170576 -994.33651025
 -994.37033623 -994.40321987 -994.43519439 -994.46629    -994.49653375
 -994.52594924 -994.55455638 -994.58237093 -994.60940398 -994.63566113
 -994.66114172 -994.68583729 -994.70973058 -994.73279226 -994.75498093
 -994.77623429 -994.79647611 -994.81558753 -994.83344799 -994.84982704
 -994.86457933 -994.87720138 -994.88770884 -994.89474677 -994.89947044]
```

[27]:
```
plt.plot(r_star.x, EV_star)
plt.title('Expected value at solution')
plt.xlabel('$x$')
plt.show()
```



[28]:
```
P_star = pd.DataFrame(conditional_probability(r_star)).T
P_star.iloc[:, 0].plot(label='Pr(wait)', legend=True, title='Conditional␣
 ↪probabilities', xlabel='$x$')
P_star.iloc[:, 1].plot(secondary_y=True, label='Pr(replace)', legend=True)
plt.show()
```


Conditional probabilities
Pr(wait)
Pr(replace) (right)
1.00
0.98
0.96
0.94
0.92
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
x

[ ]: