The Mesoscopic Membrane Manifold Stability Theorem Based on the Microscopic-Mesoscopic-Macroscopic Multi-Scale Dynamic System

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Abstract

This paper introduces the concept of a microscopic-mesoscopic-macroscopic multi-scale dynamic system for the first time, addressing chaotic phenomena such as turbulence, combustion, and axial compressor. The limitations of applying Lyapunov stability theory to such multi-scale dynamic systems are analyzed. Based on this analysis, the concept of the mesoscopic membrane manifold and the corresponding stability theorem are proposed. The paper posits that instability in a microscopic-mesoscopic-macroscopic multi-scale dynamic system arises from the transmission of microscopic disorder to the macroscopic level, with the mesoscopic force field and mesoscopic membrane manifold being decisive factors in this process. A mesoscopic membrane stability analysis theory for a class of microscopic-mesoscopic-macroscopic multi-scale dynamic systems is presented. This theory addresses the limitations of Lyapunov stability theory in the context of multi-scale dynamic systems, effectively expanding the methods available for stability analysis of complex mesoscopic energy systems. Additionally, it provides new perspectives for stability analysis, mechanical calculations, and the design of controllers for active stability control in aeroengine compressors.

Keywords: mesoscopic membrane manifold stability theorem, microscopic-mesoscopic-macroscopic multi-scale dynamic system, Lyapunov stability theory, axial compressor

1 Introduction

In nature and engineering, there exists a class of nonlinear dynamic systems whose internal mechanisms are determined by multiple scales: microscopic, mesoscopic, and macroscopic. This paper defines such systems as microscopic-mesoscopic-macroscopic multi-scale dynamic systems. The paper analyzes and summarizes the three typical microscopic-mesoscopic-macroscopic multi-scale dynamic systems.

The Introduction section, of referenced text [**?**] expands on the background of the work (some overlap with the Abstract is acceptable). The introduction should not include subheadings.

1.1 Turbulence

Turbulence is a chaotic, random, multi-scale, and disordered system that is unpredictable. The randomness of fluid trajectories and the description of its complex patterns in turbulence have always been a challenging problem in scientific research [1],[2]. The difficulty in understanding turbulence stems from the inadequacy of traditional mathematical methods, particularly linear approaches and perspectives, which fail to make significant progress when applied to such a highly nonlinear phenomenon[3].

Macroscopic fluid dynamics are described by the Navier-Stokes equations to understand fluid and turbulent motion[4]. The applicability of these equations under microscopic conditions is influenced by the subdivision of microscopic scale levels and various factors[5],[6].It can be theoretically hypothesized that variations in friction coefficients stem from different intermolecular forces, primarily including van der Waals forces, electrostatic forces, and spatial configuration forces[7],[8]. While these intermolecular forces are inherently short-range (<1nm), their cumulative effects can extend to long-range effects exceeding 1m. Within the range of 1 μm to 1mm, boundary layers significantly contribute to fluid losses along the flow path due to overlapping and squeezing at microscopic scales[9]. Additionally, the strong nonlinearity of turbulence allows effects at the microscopic scale to propagate to the macroscopic scale through cumulative effects. Consequently, in studying turbulent dynamic systems, it is imperative to comprehensively consider the influence of multiple scales: microscopic, mesoscopic, and macroscopic. Hence, this paper regards turbulence as a typical microscopic-mesoscopic-macroscopic multi-scale dynamic system.

1.2 Combustion System

The combustion process is characterized by a complex interplay of fluid mechanics, molecular thermal motion, and chemical reactions, encompassing both microscopic and macroscopic scales[10],[11]. Consequently, for power systems involving combustion processes, existing research predominantly relies on mechanistic model numerical simulations and combustion experiments to analyze their related characteristics[12],[13]. It is challenging to comprehensively simulate and analyze the combustion process solely through nonlinear dynamics. The complexity arising from multi-scale and multi-physical field coupling in the combustion process presents numerous challenges that remain to be addressed[14].

1.3 Axial Compressor

The instability phenomena, such as rotating stall and surge, in axial flow compressors are quintessential examples of chaotic vibrations[15]. These instabilities are highly sensitive to initial conditions, and multi-stage axial flow compressor systems exhibit inherent randomness[16]. As the instability develops, it results in the reciprocating aperiodic motion characteristic of rotating stall or surge. Consequently, studying the flow stability of axial flow compressors is crucial for understanding these phenomena.

Currently, research on the flow stability of axial compressors predominantly relies on the Navier-Stokes equations, aerodynamic testing, and computational fluid dynamics (CFD) models for compressor flow stability.[17],[18],[19],[20]. These models generally consider only the macroscopic fluid dynamics of the compressor system, without addressing the potential impacts of microscopic and mesoscopic scales on the flow field and stability. Given the significance of microscopic and mesoscopic effects in fluid dynamics, this paper also considers the compressor as a micro-meso-macro multi-scale dynamic system.

1.4 Motivation

Current research in classical nonlinear dynamics predominantly focuses on simple low-dimensional nonlinear dynamic systems, largely due to its lack of consideration for micro, meso, and macro scale influences. Consequently, there are significant shortcomings in the existing studies on complex nonlinear systems, such as turbulence and axial flow compressors, which are subject to multi-scale influences spanning microscopic, mesoscopic, and macroscopic levels.

The micro-meso-macro multi-scale dynamical system proposed in this paper aims to address the following issues:

(1) The vast majority of research on nonlinear dynamical systems focuses on simple low-dimensional nonlinear dynamical systems, whereas micro-meso-macro multiscale dynamical systems often exhibit infinite-dimensional characteristics.

(2) The current research on the stability of nonlinear systems often focuses solely on the macroscopic scale of dynamic effects, neglecting the potential influence of system operating mechanisms at the microscopic and mesoscopic scales on dynamic characteristics. For instance, in fluid mechanics, Stokes introduced viscosity (μ) in 1845 to describe fluid viscosity and eventually established the Navier-Stokes equations, providing an accurate quantitative representation of fluid viscosity. Under macroscopic conditions, fluid viscosity remains constant and is solely dependent on the properties of the fluid itself. However, under microscopic conditions, fluid viscosity is subject to various influencing factors. Additionally, processes such as combustion and vibration, particularly the occurrence of chaotic phenomena, may be influenced by the system's states at the microscopic and mesoscopic scales.

(3) Chaos represents a typical characteristic of micro-meso-macro multiscale dynamical systems. However, classical nonlinear dynamics often neglects the mechanistic study of chaos and instability mechanisms stemming from micro and meso-scale effects within the system, particularly in research concerning system chaos and stability.

Therefore, this paper proposes the concept of micro-meso-macro multiscale dynamical systems and defines the basic model for complex dynamic systems such as turbulence, combustion, and axial flow compressors. Building upon this foundation, the theoretical conception of mesoscale membrane manifold stability in micro-meso-macro multiscale dynamical systems is introduced in this paper.

2 Micro-Meso-Macro Multi-Scale Dynamical Systems

This paper presents a comprehensive investigation into chaotic phenomena such as turbulence, combustion, and instability in axial flow compressors. It examines the system's states across various scales, encompassing microscopic, mesoscopic, and macroscopic levels. Regarding the chaotic dynamics across multiple scales, the paper proposes the theory of mesoscale membrane manifold symmetry breaking, which leads to chaotic fractal phenomena in such systems. Furthermore, it introduces the theory of mesoscale membrane manifold stability for multi-scale dynamic systems at the microscopic, mesoscopic, and macroscopic levels.

2.1 Microscopic-Mesoscopic-Macroscopic Multi-Scale Dynamic System

2.1.1 Model Definition

In engineering applications, dynamical systems are frequently examined and assessed based on macroscopic variables. However, in specific dynamical systems, such as those involving fluids, changes occur across micro-, meso-, and macro-scale perspectives. In fluid-related systems, inferring corresponding macroscopic structures and dynamical characteristics from known microscopic structures is often straightforward. Conversely, deducing the mechanisms and reasons for changes in dynamical characteristics at microscopic and mesoscopic scales from macroscopic structures and dynamics presents significant challenges. This article utilizes the Navier-Stokes equations and fluid mechanics (compressors) as exemplars to delineate a category of dynamical systems wherein micro- and meso-scale structures and characteristics exert influence on macroscopic structures and dynamical properties. This category is termed as micro-meso-macro multi-scale dynamical systems.

$$\dot{x} = F\left(\omega\left(x, \Lambda, \sigma, \kappa, t, \tilde{t}\right), f\left(x, t\right)\right) \tag{1}$$

$$\omega\left(x, \Lambda, \sigma, \kappa, t, \tilde{t}\right) = \chi\left(M\left(\Lambda, x, t\right), \Gamma\left(x, \sigma, t\right), H\left(\kappa, \tilde{t}\right)\right) \tag{2}$$

In this context, x represents the state variable of the micro-meso-macro multi-scale dynamic system, where $x \in R^n$, t denotes the macroscopic time of the system,$t \in [0, \infty]$. . The function $f\left(x, t\right)$ is continuously differentiable with respect to both x and t. Specifically, $f : [t_0, \infty] \times D \to R^n$, where f is locally Lipschitz with respect to x.

The variable $\tilde{t}$ represents a microscopic time scale, used to denote the temporal scale of local microscopic states within the system, where $\tilde{t} \in [0, \infty]$.

Λ denotes the macroscopic spatial geometric configuration function of the micro-meso-macro multi-scale dynamic system. Given the complexity of the system's configuration, it can be simplified to a description in three-dimensional coordinates. Thus, the geometric configuration function is expressed as $\Lambda = \Lambda(\alpha, \beta, \gamma)$. α, β, γ represent the three-dimensional coordinates of the system's spatial geometry.

In a micro-meso-macro multi-scale dynamic system containing fluids, $H\left(\kappa, \tilde{t}\right)$

2.1.2 NS

$$\rho\left(\frac{\partial V}{\partial t} + V \cdot \nabla V\right) = \nabla P + \rho g + \mu \nabla^2 V \tag{3}$$

In the context of fluid dynamics within pipelines of varying cross-sectional shapes, viscosity exhibits variability. Furthermore, the viscosity of a fluid is subject to modulation by temperature and pressure. Presently, the precise quantification of the relationship between viscosity and diverse factors remains elusive. Due to viscosity being contingent upon pipeline dimensions, cross-sectional geometry, temperature, pressure, and other variables, it cannot be regarded as a constant within the framework of the Navier-Stokes equations.

Rigorous control over application conditions becomes imperative when utilizing the Navier-Stokes equations to elucidate microscopic fluid characteristics. This article contends that the current Navier-Stokes equations adequately capture macroscopic fluid motion. However, given the persistent influences of microscopic and mesoscopic scales on macroscopic fluid dynamics, there arises the opportunity to extend the Navier-Stokes equations in accordance with the definition of micro-meso-macro multi-scale dynamical systems posited in this article.

$$\rho\left(\frac{\partial V}{\partial t} + V \cdot \nabla V\right) = \nabla P + \rho g + \mu(\omega) \nabla^2 V \tag{4}$$

$$\omega = \chi\left(M(\Lambda, x, t), \Gamma(x, \sigma, t), H\left(\kappa, \tilde{t}\right)\right) \tag{5}$$

2.1.3 Axial Compressor MG

The basic structure of the compressor system is shown in the figure. A complete compressor system mainly includes several parts: compressor inlet duct, compressor, compressor outlet duct, plenum, throttle valve, and piping. Corresponding to an aircraft engine, the plenum can represent the combustion chamber, while the turbine can be equivalent to the throttle valve. The dimensional parameters of each component of the compressor model and the system coordinates are marked in Figure 1.

Due to the intricate nature of the compressor system's structure, the instability process falls within the realm of a complex gas dynamic phenomenon, rendering it exceedingly challenging to precisely characterize the airflow state at each location and instance within the compressor system. Consequently, the Moore-Greitzer model offers a degree of simplification to the compressor system. Initially, the airflow within the compressor system constitutes a complex three-dimensional flow, delineated in cylindrical coordinates as axial, radial, and tangential. By disregarding the radial fluctuation of airflow, the three-dimensional flow can be streamlined into two-dimensional

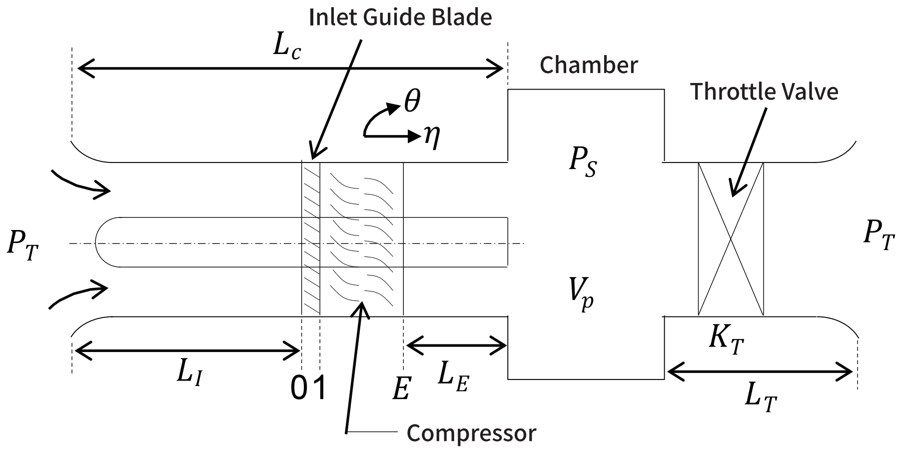

Fig. 1 Schematic diagram of basic compressor system structure

flow, focusing solely on alterations in the axial and tangential airflow within the compressor.

Moreover, the actual gas present in the compressor system is compressible and viscous, presenting formidable obstacles to modeling. Hence, the foundational MG model postulates the gas as incompressible and inviscid, streamlining the modeling process. It's crucial to acknowledge that overlooking gas viscosity may not be viable under all compressor operating conditions. From the standpoint of the micro-meso-macro multiscale dynamic system, the partial differential form of the Moore-Greitzer model can be reformulated as follows:

$$\Psi = \Psi_c\left(\Phi + \delta\right) - l_c\frac{d\Phi}{d\xi} - \left(m + \frac{1}{a}\right)\frac{\partial \delta}{\partial \theta} - \frac{1}{2a}\frac{\partial \delta}{\partial \theta} \tag{6}$$

$$\Psi + l_c\frac{d\Phi}{d\xi} = \frac{1}{2\pi}\int_0^{2\pi}\Psi_c\left(\Phi + \delta\right)d\theta \tag{7}$$

$$\frac{d\Psi}{d\xi} = \frac{1}{4B^2 l_c}\left(\Phi\left(\xi\right) - \Phi_T\left(\xi\right)\right) \tag{8}$$

Where Ψ symbolizes the total static pressure rise coefficient, Φ represents the axial mean velocity coefficient, δ denotes the axial perturbation velocity coefficient, and m serves as a parameter describing the length of the outlet duct. In cases of lengthy ducts, m is assigned a value of 2, whereas for sufficiently short outlet ducts, m is specified as 1. The parameters encapsulated within the parentheses within the second term on the right-hand side signify the effective length of both the compressor and its adjoining upstream and downstream ducts, conventionally defined as l_c.

Therefore, the MG model from the perspective of micro-meso-macro can be expressed in the following form:

$$\Psi = \Psi_c\left(\Phi + \delta\right) - l_c\frac{d\Phi}{d\xi} - \left(m + \frac{1}{a}\right)\frac{\partial \delta}{\partial \theta} - \frac{1}{2a}\frac{\partial \delta\left(\omega\right)}{\partial \theta} \tag{9}$$

$$\Psi + l_c \frac{d\Phi}{d\xi} = \frac{1}{2\pi} \int_0^{2\pi} \Psi_c \left(\Phi + \delta\left(\omega\right) \right) d\theta \tag{10}$$

$$\frac{d\Psi}{d\xi} = \frac{1}{4B^2 l_c} \left(\Phi\left(\xi\right) - \Phi_T\left(\xi\right) \right) \tag{11}$$

$$\delta\left(\omega\right) = \delta\left(\chi\left(M\left(\Lambda, x, t\right), \Gamma\left(x, \sigma, t\right), H\left(\kappa, \tilde{t}\right)\right)\right) \tag{12}$$

In the formula, $\delta\left(\omega\right)$ represents the circumferential disturbances occurring within the compressor, which are considered as the primary factors leading to compressor instability. The generation and development of circumferential disturbances are among the decisive factors causing compressor instability. In existing research, there is still no consensus on the causes of axial disturbances preceding compressor instability. This paper posits that circumferential disturbances result from instability at microscopic and mesoscopic scales, leading to macroscopic aerodynamic instability phenomena. Therefore, building upon the two-dimensional Moore-Greitzer dynamic model and considering the compressor as a multi-scale dynamic system at microscopic, mesoscopic, and macroscopic levels, the MG model is modified as described above.

2.2 The fundamental characteristics of MMM-MSDS

The fundamental characteristics of Microscopic-Mesoscopic-Macroscopic Multi-Scale Dynamic System is followed as:

(1) Multi-scale characteristics: The system exhibits multiple orders of magnitude in time and spatial scales, concurrently manifesting mechanical features at microscopic, mesoscopic, and macroscopic levels in both time and space. The system encompasses both macroscopic dynamic characteristics and the mechanical properties of continuous media.

(2) Strong nonlinearity, intrinsic randomness, and chaos: The system exhibits strong nonlinear characteristics and intrinsic randomness, particularly when energy interactions occur between microscopic, mesoscopic, and macroscopic scales, making it highly susceptible to phenomena such as bifurcation and chaos. The system's intrinsic randomness mode is formed by the combined effects of its microscopic, mesoscopic, and macroscopic scales.

(3) The presence of mesoscale effects: The mechanical properties of continuous media within the system will affect its dynamic behavior.

(4) Spatial geometric configurations influence the dynamic characteristics of the system: The spatial geometric configuration of the system has a certain impact on its nonlinear dynamic characteristics.

(5) Due to the presence of continuous media and motion across multiple spatiotemporal scales, the movement of the system is essentially infinite-dimensional, as exemplified by the 3D Navier-Stokes equations.

2.3 The Relationship between MMM-MSDS and Classical Dynamic Systems

The intrinsic randomness mode function ω of the system arises from the interaction between the macroscopic mechanical field $\Gamma(x,\sigma,t)$ and the microscopic particle spatial probability distribution $H\left(\kappa,\tilde{t}\right)$. In instances where there is energy interaction between the microscopic particle spatial probability distribution field $H\left(\kappa,\tilde{t}\right)$ and the macroscopic mechanical field $\Gamma(x,\sigma,t)$ within the system, intrinsic randomness is generated.

$$\chi\left(M(\Lambda,x,t),\Gamma(x,\sigma,t),H\left(\kappa,\tilde{t}\right)\right)\neq 0 \tag{13}$$

If there is no energy interaction between the microscopic and macroscopic levels, the system will exhibit stability characteristics of deterministic nonlinear dynamical systems:

$$\chi\left(M(\Lambda,x,t),\Gamma(x,\sigma,t),H\left(\kappa,\tilde{t}\right)\right)=0 \tag{14}$$

Therefore, when the intrinsic randomness coefficient $\omega=0$, the multi-scale dynamic system at microscopic, mesoscopic, and macroscopic levels is equivalent to traditional nonlinear dynamic systems:

$$\dot{x}=f(x,t) \tag{15}$$

That is, when $\omega=0$, the multi-scale dynamic system at microscopic, mesoscopic, and macroscopic levels is topologically conjugate to classical dynamic systems.

2.4 The Mesoscopic Membrane Manifold

Based on the analysis of micro-meso-macro multi-scale dynamic systems, as described above, it is evident that interactions of mechanics and energy occur across micro, meso, and macro scales within such systems. Drawing from principles in nanomechanics and mesoscale mechanics, where continuum mechanics remains applicable at the mesoscale, the concept of the mesoscopic membrane manifold is introduced in this paper. Expanding upon this concept, the paper proposes a conjecture for the stability theory of the mesoscopic membrane manifold in micro-meso-macro multi-scale dynamic systems.

2.4.1 Theoretical Foundation of Mesoscopic Membrane Manifold: Continuum Mechanics Remains Applicable at the Mesoscale

Nanomechanics and mesoscale mechanics occupy a position between quantum mechanics and macroscopic continuum mechanics. A fundamental challenge in these fields is delineating the transitional region between quantum mechanics and continuum mechanics. It's essential to recognize that the boundary separating nanomechanics and mesoscale mechanics from both quantum mechanics and continuum mechanics is not fixed; instead, it encompasses a range of possibilities.

In macroscopic continuum mechanics, it is generally assumed that material properties such as elastic modulus, Poisson's ratio, and yield stress are not influenced by size or surface effects. Hence, the scale at which these material properties begin to exhibit size or surface effects can be considered the boundary between nanomechanics/mesomechanics and continuum mechanics. As a result, nanomechanics and mesomechanics must account for the modifications in material properties due to size and surface effects. Nonetheless, the principles of continuum mechanics theory remain applicable at the nanoscale.

2.4.2 Definition of Mesoscopic Membrane Manifold

For flows at the mesoscale, considerations must be made for phenomena such as slip at fluid-solid boundaries, which are not typically accounted for in macroscopic fluid mechanics. This paper introduces the concept of a mesoscopic membrane manifold, denoted as $M(\Lambda, x, t)$, which represents the continuum medium at the mesoscale situated between the fluid-solid boundary. As the characteristic scale of the system further decreases, quantum effects begin to manifest and become significant. Therefore, the scale at which quantum effects start to appear can be regarded as the boundary between nanomechanics, mesoscale mechanics, and quantum mechanics, with the mesoscopic membrane manifold serving as the interface between mesoscale mechanics and quantum mechanics.

Therefore, the mesoscopic membrane manifold within the micro-meso-macro multiscale dynamic system is topologically homeomorphic to the system's geometric configuration Λ, and the mesoscopic membrane manifold $M(\Lambda, x, t)$ on the interface between the two phases enveloping the geometric configuration Λ is crucial in distinguishing mechanical properties at the micro and macro scales. This paper introduces the concept of the mesoscopic membrane manifold based on the continuum properties at the mesoscale, asserting the existence of a manifold at the mesoscale within the micro-meso-macro multi-scale dynamic system. This manifold exhibits continuum properties at the mesoscale, serving as the foundation for proposing the mesoscopic membrane manifold stability theorem for multi-scale dynamic systems at micro-meso-macro levels.

3 The Limitations of Lyapunov Stability in Micro-Meso-Macro Multiscale Dynamical Systems

The characteristics of microscopic-mesoscopic-macroscopic multi-scale dynamic system are addressed in this paper, and the limitations of the Lyapunov stability theorem are analyzed in the stability analysis of such systems, as follows:

(1) Multiple Spatiotemporal Scales: microscopic-mesoscopic-macroscopic multiscale dynamic system cannot be fully described by traditional dynamic equations, particularly systems such as axial flow compressors with flow field crystallization, which exhibit characteristics of multiple spatiotemporal scales and intrinsic randomness.

(2) Intrinsic Randomness: In multi-scale dynamic systems at microscopic, mesoscopic, and macroscopic levels, Lyapunov analysis can stabilize their dynamic characteristics, but instability may still occur within the continuous media inside the system.

(3) Spatial Geometric Configuration of the System: In actual microscopic-mesoscopic-macroscopic multi-scale dynamic system, the geometric configuration of the system significantly influences its stability characteristics. However, the Lyapunov stability theorem does not fully account for the spatial geometric attributes of multi-scale dynamic systems. Consequently, two systems with identical dynamic characteristics but differing spatial geometric structures may display distinct stability features. For example, vibrations in a rotor at a specific speed cannot be analyzed using Lyapunov stability.

(4) Sufficiency Rather Than Necessity: Here, we present only the sufficient condition. Specifically, if a Lyapunov function V(x,t) is constructed, then the system is deemed asymptotically stable. However, the absence of such a Lyapunov function precludes drawing any conclusion. For instance, it cannot be inferred from this absence that the system is unstable.

(5) Lyapunov stability primarily addresses the energy and dynamic attributes of a system, overlooking deeper physical mechanisms, particularly in multi-scale dynamic systems across microscopic, mesoscopic, and macroscopic levels. These systems frequently involve continuous media with molecular-level movements influencing macroscopic dynamics at every tier. Relying solely on Lyapunov stability falls short of fully capturing essential system information and stability characteristics.

(6) Traditional nonlinear systems are inadequate for fully describing the state variables of multi-scale dynamic systems spanning microscopic, mesoscopic, and macroscopic levels. Take axial flow compressors, for example, where state variables include temperature, pressure, and flow velocity across the entire inlet interface, along with the system's three-dimensional geometric configuration. In reality, this constitutes an infinite-dimensional state dynamic system. Thus, fully describing the stability of multi-scale dynamic systems necessitates proposing an infinite number of Lyapunov functions for stability analysis. However, formulating Lyapunov functions that satisfy N nonlinear inequality equations is impractical and challenging. Therefore, traditional nonlinear system dynamics equations cannot entirely capture the stability of multi-scale dynamic systems spanning microscopic, mesoscopic, and macroscopic levels.

(7) The traditional Lyapunov method for nonlinear dynamical systems is only applicable to low-dimensional nonlinear dynamical systems. It can explain the general stability properties of nonlinear dynamical systems but cannot simulate or study the specific properties of fully developed turbulence (phase space dimension greater than 10^{10}) and other multi-scale dynamic systems spanning microscopic, mesoscopic, and macroscopic levels.

4 The Mesoscopic Membrane Manifold Stability Theorem in Micro-Meso-Macro Multiscale Dynamical Systems

The definition of micro-meso-macro multi-scale dynamic systems, and their distinctions and correlations with classical nonlinear dynamic systems, suggest limitations in employing the Lyapunov stability theorem for analyzing such systems. This study conducts an initial examination of these limitations within the context of multi-scale dynamic systems at micro-meso-macro levels. Additionally, leveraging the framework of micro-meso-macro multi-scale dynamic systems and the mesoscopic membrane manifold hypothesis, this paper introduces the mesoscopic membrane manifold stability theorem and conjecture tailored for multi-scale dynamic systems at micro-meso-macro levels.

4.1 The Mesoscopic Membrane Manifold and Chaos of the Micro-Meso-Macro Multiscale Dynamical Systems

Based on the definition and characteristics of micro-meso-macro multi-scale dynamic systems, it is possible to define and preliminarily analyze the phenomenon of chaos in such systems. This involves proposing the hypothetical conditions under which chaos analysis phenomena occur in this type of system. These assumptions provide a basis for the conjecture of the mesoscopic membrane manifold stability theory proposed in this paper for micro-meso-macro multi-scale dynamic systems.

4.1.1 Definition of Chaos in the Micro-Meso-Macro Multiscale Dynamical Systems

The mathematical description of the chaotic state in micro-meso-macro multi-scale dynamic systems from the perspective of mesoscopic scale manifolds is as follows:

For the micro-meso-macro multi-scale dynamic system $F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}} : R^n \longrightarrow R^n$, $t \in [0, \infty)$ and the set $A \subset R^n$, if there exists $\delta > 0$ such that for any $x \in A$ and any open ball $B^n(x, \varepsilon) \cap A$ and $t \in [0, \infty)$ satisfying:

$$\left| F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}}(y) - F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}}(x) \right| \tag{16}$$

Then, $F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}}$ is said to exhibit sensitive dependence on initial conditions on A, or equivalently. $F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}}$ is chaotic on A. lf the Lebesgue measure $L^{n(A)} > 0$, then $F^{t,\Lambda,x,\Lambda,\sigma,\kappa,t,\tilde{t}}$ is classified as a chaotic dynamic system. This implies that a chaotic dynamic system contains a significant chaotic set, which can be readily detected through experimental methods.

4.1.2 Chaos and Mesoscopic Membrane Manifolds in Micro-Meso-Macro Multi-Scale Dynamic Systems

The mechanical similarity and uncertainty symmetry between micro and macro spatiotemporal scales are key factors contributing to the emergence of chaos in micro-meso-macro multi-scale dynamic systems. Leveraging these principles, we can establish mechanical relationships and uncertainty transfer relationships across the micro, meso, and macro scales. Given the mechanical similarity and uncertainty relationships between the micro and macro spatiotemporal scales in these systems, it can be hypothesized that a mesoscopic membrane manifold exists within such systems, bridging the micro and macro scales. This hypothesis facilitates an effective analysis of the fundamental causes of chaos in these systems, offering a new approach for investigating chaotic phenomena in micro-meso-macro multi-scale dynamic systems.

A mesoscopic membrane manifold represents a manifold within a micro-meso-macro multi-scale system, operating at the mesoscale. It is topologically equivalent to the system's geometric configuration Λ and resides on the interface between two phases enclosing Λ. As per the definition of micro-meso-macro multi-scale dynamic systems, this mesoscopic membrane manifold can be denoted as $M(\Lambda, x, t)$, playing a pivotal role in the emergence of chaotic fractals within such systems.

The existence of this mesoscopic membrane manifold $M(\Lambda, x, t)$ disrupts the establishment of mechanical similarity and uncertainty symmetry between the micro and macro scales. However, disruptions such as disturbances within the system may compromise the global or local smoothness of $M(\Lambda, x, t)$, potentially resulting in its absence. In such cases, uncertainty symmetry and mechanical similarity between the micro and macro scales are established, leading to system instability and subsequent chaotic phenomena.

4.2 The Mesoscopic Membrane Manifold and Chaos of the Micro-Meso-Macro Multiscale Dynamical Systems

Proposition 1. *Stability Theorem Conjecture 3.1: For the Micro-Meso-Macro Scale Dynamic Systems, the equilibrium point $\dot{x}(\epsilon) = 0$, the satisfaction of both conditions outlined below is required:*

(1) The topological equivalence with the geometric configuration Λ. Furthermore, for any smooth manifold $M(\Lambda, x, t)$ at the mesoscopic scale encompassingthe surface of Λ, energy interaction between the spatiotemporal probability distribution field H of microscopic particles and the macroscopic mechanical field does not occur on the mesoscopic membrance manifold $M(\Lambda, x, t)$.

(2) The classical nonlinear dynamical system equation corresponding to the system is $\dot{x} = f(x, t)$, where there exists a Lyapunov function $V(x, t)$, statisfying:

$$\dot{V}(x, t) \leq 0 \tag{17}$$

Then, the micro-meso-macro multi-scale dynamic system satisfies uniform stability.

Theorem 2 (Stability theorem of Micro-Meso-Macro Multiscale Dynamical Systems)**.** *For micro-meso-macro multiscale dynamical systems (1), at the equilibrium point $\dot{x}(\epsilon) = 0$, the following three conditions must be simultaneously satisfied:*

(1) For any mesoscopic manifold $M(\Lambda, x, t)$, which is homeomorphic to the spatial geometricconception of the system and encloses the geometric configuration surface at themesoscopic scale, the Stokes integral of the macroscopic mechanical field on the manifold $M(\Lambda, x, t)$ converges topologically to a finite value within a finite time.

$$\lim_{t \to t+\Delta t} \int\limits_{\partial M} \Gamma = \lim_{t \to t+\Delta t} \iint\limits_{M} d\Gamma = 0 \tag{18}$$

(2) The expected value of the probability distribution of the microscopic particle spacetimeprobability distribution field I on the mesoscopic membrane manifold is zero within themicroscopic timescale $\tilde{t}$:

$$\lim_{0\to\tilde{t}} \int_0^{\tilde{t}} E_M\left(H\right) d\tilde{t} = 0 \tag{19}$$

(3) For the traditional nonlinear dynamical system equation $\dot{x} = f\left(x,t\right)$ corresonding to the system, there exists a Lyapunuov function $V\left(x,t\right)$ that satisfies:

$$\dot{V}\left(x,t\right) \leq 0 \tag{20}$$

If the three conditions mentioned above are simultaneously satisfied, then the micro-meso-macro multiscale dynamic system is uniformly stable.

Proof of Theorem 2. Given $\epsilon > 0$, choose $r \in (0,\epsilon]$, such that:

$$B_r = \{x \in R^n \mid \|x\| \leq r\} \subset D \tag{21}$$

Let $\alpha = \min_{\|x\|=r} V\left(x\right)$.

Since $V(0) > 0$ and $V(0) > 0$ in $D-\{0\}$, $\alpha > 0$.is got. Choose $\beta \in (0,\alpha)$, and define

$$B_r = \{x \in R^n \mid \|x\| \leq r\} \subset D \tag{22}$$

Then, Ω_β is within B_r (Figure 3.1). The set Ω_β has the following property at $t = 0$, any trajectory starting within Ω_β remains within Ω_β for all $t \geq 0$, because

$$\dot{V}\left(x\left(t\right)\right) \leq 0 \Rightarrow V\left(x\left(t\right)\right) \leq \beta, \forall t \geq 0 \tag{23}$$

Since Ω_β is compact, we can state that for all $t \geq t_0$ and any region $D \subset R^n$, the function $f(t,x)$ is piecewise continuous in t, locally Lipschitz in x on D. Let W be a compact subset of D, and $x_0 \in W$. Assume that

$$\dot{x} = f\left(t,x\right), x\left(t_0\right) = x_0 \tag{24}$$

Every solution stays within W, so for all $t \geq t_0$, the system has a unique solution. Based on the above conclusions, we obtain: for $x\left(0\right) \in \Omega_\beta$, the equation $\dot{x} = f\left(t,x\right)$ has a unique solution for all $t > t_0$. Since $V\left(x\right)$ is continuous and $V\left(0\right) = 0$, there exists $\delta > 0$ such that

$$\|x\| \leq \delta \Rightarrow V\left(x\right) < \beta \tag{25}$$

Thus,

$$B_\delta \subset \Omega_\beta \subset B_r \tag{26}$$

and $x\left(0\right) \in B_\delta \Rightarrow x\left(0\right) \in \Omega_\beta \Rightarrow x\left(t\right) \Rightarrow \Omega_\beta \Rightarrow x\left(t\right) \in B_r$. Therefore,

$$\|x\left(0\right)\| < \delta \Rightarrow \|x\left(t\right)\| < \epsilon, \forall t \geq 0 \tag{27}$$

This implies that the equilibrium $x = 0$ is stable.

□

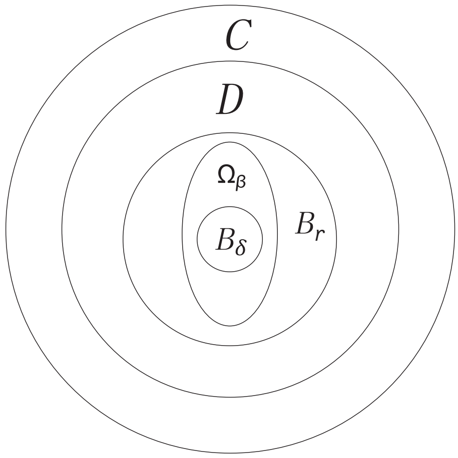

Fig. 2 The geometric representation of the sets in the theorem

5 Conclusion

This paper proposes a mesoscopic membrane stability analysis theory for a class of micro-meso-macro multi-scale dynamic systems. This theory can address the limitations of the Lyapunov stability theory in stability analysis of micro-meso-macro multi-scale dynamic systems, effectively expanding the stability analysis methods for a class of complex meso-scale systems. It also provides new insights for stability analysis, mechanical computation, and controller design for active stability control of aircraft engine compressors within this class of systems.

The mesoscopic membrane manifold stability theorem, meanwhile, can better approximate the actual physical characteristics of the system and fully reflect the multiscale dynamics relationship among micro, meso, and macro levels. The mesoscopic membrane manifold stability theory for micro-meso-macro multi-scale dynamic systems can be extended to general nonlinear systems. As long as such systems possess microstructures and continuous media, stability analysis can be conducted using the mesoscopic membrane manifold stability theory.

The micro-meso-macro multi-scale dynamic systems provide enhanced capabilities for addressing stability challenges across diverse spatiotemporal scales, including turbulence, and offer insights into the origins of chaos within these systems from a physical mechanism perspective. The mesoscopic membrane manifold stability theory demonstrates considerable potential for various applications, including advancements in computational fluid dynamics (CFD) simulation methods, finite element analysis, and the aerodynamic stability of axial flow compressors.

Appendix A Section title of first appendix

An appendix contains supplementary information that is not an essential part of the text itself but which may be helpful in providing a more comprehensive understanding of the research problem or it is information that is too cumbersome to be included in the body of the paper.

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