Midterm Exam

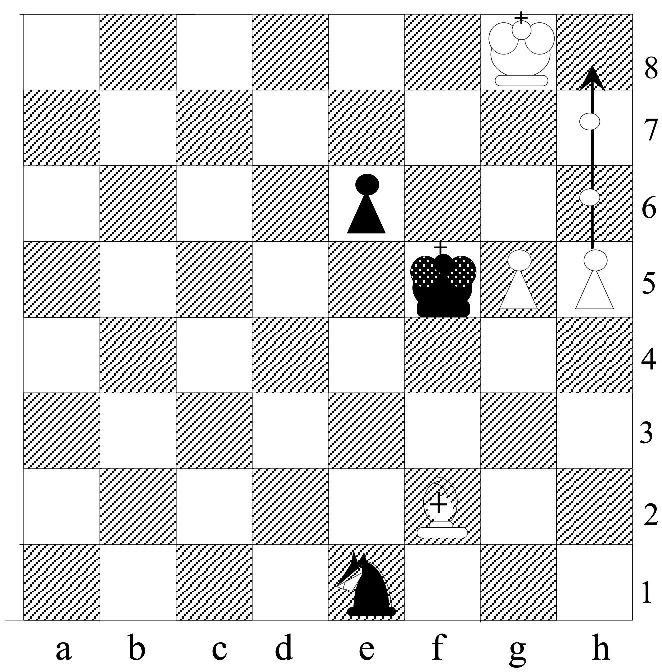
March 21, 8:00 am - 12:00 pm, LW-844

Midterm Review

1. **Represent the system shown below as an ABG.**

2. **Generate Zone with the main trajectory $a(h5)a(h6)a(h7)a(h8)$. Use grammars of Trajectories and Zones, show the generations with *important* values of the functions and sets including**
   DOCK, med_i, $lmed_i$, SUM, ST_k, MOVE, $next_i$,
   f, h_i^o, h_j, g, v, w, TIME, and NEXTTIME.

3. **Show your understanding of Translations. Show a translation table (analogous to table in handout No. 14) for the main variation (your choice) assuming that Black moves first. Include only the most important trajectories.**



Note. On the exam it is going to be a meaningful ABG (which may include non-chess-like pieces). It will be stated as a gaming problem so that generating a zone and moving in it would allow you to solve this ABG.

Class of Problems

Abstract Board Game

is the following eight-tuple

$< X, P, R_p, \{ON\}, v, S_i, S_t, TR>$

$X = \{x_i\}$ is a finite set of ***points***;

$P = \{p_i\}$ is a finite set of ***elements***; **$P = P_1 \cup P_2$, $P_1 \cap P_2 = \varnothing$**;

$R_p(x, y)$ is a family of binary relations of ***reachability*** in X ($\mathbf{x} \in X$, $\mathbf{y} \in X$, $\mathbf{p} \in P$); y is ***reachable*** from x for p;

ON(p)=x is a partial function of ***placement*** of elements P into X;

v > 0 is a real function, **$v(p_i)$** are the ***values*** of elements;

S_i is a set of ***initial*** states of the system, a certain set of formulas $\{\mathbf{ON(p_i) = x_i}\}$;

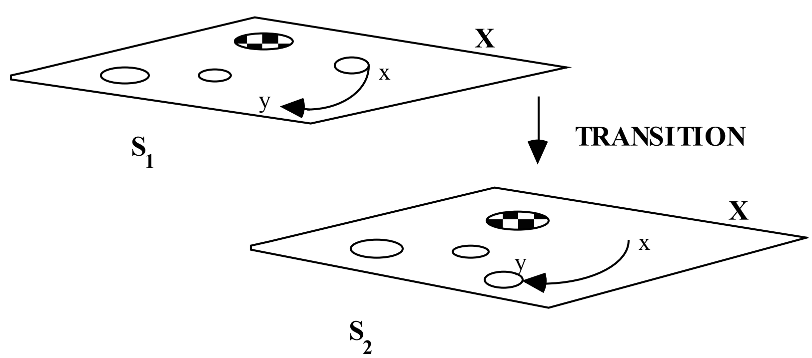
S_t is a set ***target*** states of the system (as **S_i**);

TR is a set of operators **TRANSITION(p, x, y)** for transition of the system from one state to another described as follows

precondition: $ON(p) = x \wedge R_p(x, y)$

delete: $ON(p) = x$, $ON(q) = y$

add: $ON(p) = y$



1	$\boldsymbol{Q_1}$	$\boldsymbol{S}(u, v, w) \to \boldsymbol{A}(u, v, w)$		*two*	ø
2_i	$\boldsymbol{Q_2}$	$\boldsymbol{A}(u, v, w) \to \boldsymbol{t}(\boldsymbol{h_i^o}(u), l_o+1)$ $\boldsymbol{A}((0, 0, 0), \boldsymbol{g}(\boldsymbol{h_i^o}(u), w), zero)$	$\mathrm{TIME}(z) = \boldsymbol{DIST}(z, \boldsymbol{h_i^o}(u))$	3	ø
3	$\boldsymbol{Q_3}$	$\boldsymbol{A}(u, v, w) \to \boldsymbol{A}(\boldsymbol{f}(u, v), v, w)$	$\mathrm{NEXTTIME}(z) = \boldsymbol{init}(u, \mathrm{NEXTTIME}(z))$	*four*	5
4_j	$\boldsymbol{Q_4}$	$\boldsymbol{A}(u, v, w) \to \boldsymbol{t}(\boldsymbol{h_j}(u), \mathrm{TIME}(y)))$ $\boldsymbol{A}(u, v, \boldsymbol{g}(\boldsymbol{h_j}(u), w))$	$\mathrm{NEXTTIME}(z) = \boldsymbol{ALPHA}(z, \boldsymbol{h_j}(u), \mathrm{TIME}(y) - l_j+1)$	3	3
5	$\boldsymbol{Q_5}$	$\boldsymbol{A}(u, v, w) \to \boldsymbol{A}((0, 0, 0), w, zero)$	$\mathrm{TIME}(z) = \mathrm{NEXTTIME}(z)$	3	6
6	$\boldsymbol{Q_6}$	$\boldsymbol{A}(u, v, w) \to \boldsymbol{e}$		ø	ø

$\boldsymbol{Q_1}(u) = (\mathrm{ON}(p_o) = x) \wedge (\mathrm{MAP}_{x,p_o}(y) \le l \le l_o) \wedge (\exists q\, ((\mathrm{ON}(q) = y) \wedge (\mathrm{OPPOSE}(p_o, q))))$

$\boldsymbol{Q_2}(u) = T$; $\quad \boldsymbol{Q_3}(u) = (x \ne n) \vee (y \ne n)$ $\quad \boldsymbol{Q_5}(w) = (w \ne zero)$; $\quad \boldsymbol{Q_6} = T$;

$\boldsymbol{Q_4}(u) = \exists p\, ((\mathrm{ON}(p) = x) \wedge (l > 0) \wedge (x \ne x_o) \wedge (x \ne y_o)) \wedge [(\neg \mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) = 1)) \vee (\mathrm{OPPOSE}(p_o, p) \wedge (\mathrm{MAP}_{x,p}(y) \le l)]$

$$\boldsymbol{init}(u, r) = \begin{cases} 2n, & \text{if } u = (0,0,0), \\ r, & \text{if } u \ne (0,0,0). \end{cases}$$

$$f(u, v) = \begin{cases} (x+1, y, l), & \text{if } (x \ne n) \wedge (l > 0)) \vee ((y = n) \wedge (l \le 0)) \\ (1, y+1, TIME(y+1) \times v_{y+1}), & \text{if } (x = n) \vee ((l \le 0) \wedge (y \ne n)). \end{cases}$$

Let $t_o \in \mathbf{L_t}^{l_o}(S)$, $t_o = \boldsymbol{a}(z_o)\boldsymbol{a}(z_1)\ldots\boldsymbol{a}(z_m)$, $t_o \in t_{p_o}(z_o, z_m, m)$;

If $((z_m = y_o) \wedge (p = p_o) \wedge (\exists\, k\, (1 \le k \le m) \wedge (x = z_k))) \vee$
$(((z_m \ne y_o) \vee (p \ne p_o)) \wedge (\exists\, k\, (1 \le k \le m-1) \wedge (x = z_k)))$
then $\boldsymbol{DIST}(x, p_o, t_o) = k+1$ **else** $\boldsymbol{DIST}(x, p_o, t_o) = 2n$

$$ALPHA(x, p_o, t_o, k) = \begin{cases} max\,(NEXTTIME(x), k), & \text{if}(DIST(x, p_o, t_o) \ne 2n) \wedge (NEXTTIME(x) \ne 2n); \\ k, & \text{if } (DIST(x, p_o, t_o) \ne 2n) \wedge (NEXTTIME(x) = 2n); \\ NEXTTIME(x), & \text{if } DIST(x, p_o, t_o) = 2n). \end{cases}$$

$$\boldsymbol{g_r}(p_o, t_o, w) = \begin{cases} 1, & \text{if } \boldsymbol{DIST}(r, p_o, t_o) < 2n, \\ w_r, & \text{if } \boldsymbol{DIST}(r, p_o, t_o) = 2n. \end{cases} \qquad \mathrm{TRACKS}_{p_o} = \{p_o\} \times \Big(\bigcup_{1 \le k \le l} L[\,\mathbf{G_t^{(2)}}(x, y, k, p_o)]$$

If $\mathrm{TRACKS}_{p_o} = e$
then $\boldsymbol{h_i^o}(u) = e$

else $\mathrm{TRACKS}_{p_o} = \{(p_o, t_1), (p_o, t_2), \ldots, (p_o, t_b)\}, (b \le M)$ **and** $\boldsymbol{h_i^o}(u) = \begin{cases} (p_o, t_i), & \text{if } i \le b, \\ (p_o, t_b), & \text{if } i > b. \end{cases}$

$\mathrm{TRACKS} = \bigcup_{\mathrm{ON}(p)=x} \mathrm{TRACKS}_p$, where TRACKS_p is the same as for $\boldsymbol{h_i}^o$

If $\mathrm{TRACKS} = e$
then $\boldsymbol{h_i}(u) = e$

else $\mathrm{TRACKS} = \{(p_1, t_1), (p_1, t_2), \ldots, (p_m, t_m)\}$, $(m \le M)$ **and** $\boldsymbol{h_i}(u) = \begin{cases} (p_i, t_i), & \text{if } i \le m, \\ (p_m, t_m), & \text{if } i > m. \end{cases}$

At the beginning: $u = (x_o, y_o, l_o)$, $w = zero$, $v = zero$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$, $p_o \in P$, and $\mathrm{TIME}(z) = 2n$, $\mathrm{NEXTTIME}(z) = 2n$ for all z from X.

Grammar $G_t^{(2)}$ of shortest and admissible trajectories.

L	*Q*	Kernel, π_k	π_n	F_T	F_F
1	Q_1	$S(x,y,l) \to A(x,y,l)$		*two*	ø
2_i	Q_2	$A(x,y,l) \to A(x,\ med_i(x,y,l),\ lmed_i(x,y,l))$ $A(med_i(x,y,l),\ y,\ l - lmed_i(x,y,l))$		*three*	*three*
3_j	Q_3	$A(x,y,l) \to a(x)A(next_j(x,l),y,f(l))$		*three*	4
4	Q_4	$A(x,y,l) \to a(y)$		*three*	5
5	Q_5	$A(x,y,l) \to e$		*three*	ø

$V_T = \{a\}$,

$V_N = \{S, A\}$,

V_{PR}

Pred $= \{Q_1, Q_2, Q_3, Q_4, Q_5\}$

$Q_1(x, y, l) = (MAP_{x,p}(y) \le l < 2MAP_{x,p}(y)) \wedge (l < 2n)$

$Q_2(x, y, l) = (MAP_{x,p}(y) \ne l)$

$Q_3(x, y, l) = (MAP_{x,p}(y) = l) \wedge (l \ge 1)$

$Q_4(y) = (y = y_o)$

$Q_5(y) = (y \ne y_o)$

Var $= \{x, y, l\}$;

Con $= \{x_o, y_o, l_o, p\}$;

Func $=$ *Fcon* $\cup$ *Fvar*;

Fcon $= \{f, next_1, \ldots, next_n,\ med_1, \ldots, med_n,$
$lmed_1, \ldots, lmed_n\}$ $(n = |X|)$,

$f(l) = l - 1$, $D(f) = \mathbf{Z}_+ \setminus \{0\}$

functions $next_i$, med_i *and* $lmed_i$ are defined below.

Fvar $= \{x_o, y_o, l_o, p\}$

$E = \mathbf{Z}_+ \cup X \cup P$ is the subject domain;

Parm: $S \to Var$, $A \to Var$, $a \to \{x\}$;

$L = \{1, 4\} \cup two \cup three$, $two = \{2_1, 2_2, \ldots, 2_n\}$, $three = \{3_1, 3_2, \ldots, 3_n\}$

At the beginning of derivation: $x = x_o$, $y = y_o$, $l = l_o$, $x_o \in X$, $y_o \in X$, $l_o \in \mathbf{Z}_+$, $p \in P$.

Definition of functions *med*, *lmed*, *next*

med_i is defined as follows:

$D(med_i) = X \times X \times \mathbf{Z}_+ \times P$

DOCK = {v | v from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v) = l$},

If

$DOCK_l(x) = \{v_1, v_2, ..., v_m\} \neq \emptyset$

then

$med_i(x, y, l) = v_i$ for $1 \leq i \leq m$ and

$med_i(x, y, l) = v_m$ for $m < i \leq n$,

otherwise

$med_i(x, y, l) = x$.

$lmed_i$ is defined as follows:

$D(med_i) = X \times X \times \mathbf{Z}_+ \times P$

$lmed_i(x, y, l) = MAP_{x,p}(med_i(x, y, l))$

$next_i$ is defined as follows:

$D(next_i) = X \times \mathbf{Z}_+ \times X^2 \times \mathbf{Z}_+ \times P$

SUM = {v | v from X, $MAP_{x_o,p}(v) + MAP_{y_o,p}(v) = l_o$},

$ST_k(x)$ = {v | v from X, $MAP_{x,p}(v) = k$},

$MOVE_l(x)$ is an intersection of the following sets:

$ST_1(x)$, $ST_{l_o-l+1}(x_o)$ and SUM.

If

$MOVE_l(x) = \{m_1, m_2, ..., m_r\} \neq \emptyset$

then

$next_i(x, l) = m_i$ for $i \leq r$ and

$next_i(x, l) = m_r$ for $r < i \leq n$,

otherwise

$next_i(x, l) = x$.