William Daniels
Linguistic Geometry, HW1

Problem 1: In this problem, the trick is to simply change the false condition (4 instead of 3), and then the rest simply falls out. There are a few special cases to worry about, but they're few and far between.

L	Q	Kernel	F_T	F_F
1	Q_1	$S(c, m) \longrightarrow A(c + 1, m + 1)$	2	∅
2	Q_2	$A(c, m) \longrightarrow p(c, m)A(f_1(c, m), f_2(m))$	2	3
3	Q_3	$A(c, m) \longrightarrow p(c, m)$	∅	∅

All steps of this grammar take place when the boat is on side B of the river.

Pred = { Q_1, Q_2, Q_3 }

$Q_1 = \mathbf{T}$

$Q_2(c, m) = \mathbf{T}$ if $c > 0$ or $m > 0$; $Q_2(c, m) = \mathbf{F}$ if $((c=4) \wedge (m = 4))$ or $(c > m) \vee ((4 - c) > (4 - m))$

$Q_3(c, m) = \mathbf{T}$ if $((c = 3) \wedge (m = 3))$

Var = {c, m}

Fcon = $\{f_1, f_2\}$

$f_1(c, m)$ is defined as follows:

If $(c = 1) \wedge (m = 1)$ $f_1(c, m) = c + 2$
else
if $(c \neq 1) \wedge (m \neq 3)$
$f_1(c, m) = c - 1$ else
$f_1(c, m) = c + 1$.

$f_2(m)$ is defined as follows:

If $m = 1$
$f_2(m) = m - 1$ else
if $m = 0$
$f_2(m) = m + 2$ else
if $m = 4$
$f_2(m) =$

m else

$f_2(m) = m + 1$

At the beginning of derivation: c = 0, m = 0

$\boldsymbol{S}(0, 0) \overset{1}{\Rightarrow} \boldsymbol{A}(1, 1)$

$\overset{2}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{A}(3, 0)$

$\overset{2}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{p}(3, 0)\boldsymbol{A}(2, 2)$

$\overset{2}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{p}(3, 0)\boldsymbol{p}(2, 2)\boldsymbol{A}(1, 3)$

$\overset{2}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{p}(3, 0)\boldsymbol{p}(2, 2)\boldsymbol{p}(1, 3)\boldsymbol{A}(2, 3)$

$\overset{2}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{p}(3, 0)\boldsymbol{p}(2, 2)\boldsymbol{p}(1, 3)\boldsymbol{p}(2, 3)\boldsymbol{A}(3, 3)$

$\overset{3}{\Rightarrow} \boldsymbol{p}(1, 1)\boldsymbol{p}(3, 0)\boldsymbol{p}(2, 2)\boldsymbol{p}(1, 3)\boldsymbol{p}(2, 3)\boldsymbol{p}(3, 3)$

Problem 1, part b: The same logic can be applied to the general case, if n is odd, then you can simply use the original controlled grammar, just with all the 3's replaced by 'n's. In the even case, you simply use the aforementioned grammar, and then replace all the4's with 'n's. Very Straightforward. Assume the boat can hold n-1 people.

Problem 2: The towers of Hanoi problem :

Controlled grammar generating solutions
to the Tower of Hanoi Problem

L	Q	Kernel, πk	πn	F_T	F_F
1	Q_1	$S(n, x, y) \to A(n, x, y)$		2	ø
2	Q_2	$A(n, x, y) \to A(f_1(n), x, f_2(x, y))$ $p(n, x, y)$ $A(f_1(n), f_2(x, y), y)$		2	3
3	Q_3	$A(n, x, y) \to p(n, x, y)$		2	ø

Here $V_T = \{p\}$

$V_N = \{S, A\}$
V_{PR}

Pred = $\{Q_1, Q_2, Q_3\}$,

$Q_1 = T$

$Q_2(n) = T$, if n > 1; $Q_2(n) = F$, if n = 1.

$Q_3(n) = T$, if n = 1; $Q_3(n) = F$, if n > 1.

Var = {n, x, y}

$F = Fcon \cup Fvar$,

$Fcon = \{f_1, f_2\}$

$f_1(n) = n-1$, n = 2, 3,...

$f_2(x, y)$ yields the value from $\{a, b, c\} - \{x, y\}$, where values of x, y are from $\{a, b,$

$c\}$ $Fvar = \{3, a, c\}$

$E = \mathbf{Z+} \cup \{a, b, c\}$

Parm: $S \to Var$, $A \to Var$, $p \to Var$

$L = \{1, 2, 3\}$

At the beginning of derivation: x = a, y = c, n = 3.