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CSCI 3630

Linguistic Geometry

Homework #5 02/28/15

1. Generate Zone with the main trajectory shown in the fugure below. Use the grammar of zones G_z, show values of the key functions and sets:

   Here is the grammar generation..:

   $u = (13, 39, 2), l = l_0 = 2$ Because $Q_1(u) = (ON(BISHOP) = 13) \wedge (MAP_{h5,BISHOP}(39) \leq 2\, leq 2) \wedge ((ON(F$

   $$S(u, zero, zero)Q1 \rightarrow A(u, zero, zero)$$

   $$A(u, zero, zero)Q2_{i=1} \rightarrow t(h_i^0(u), 3)A((0, 0, 0), g(h_1^0(u), zero), zero)$$

   Computation of $h_i^0(u)(l = 2)$

   $$\text{TRACKS}_{\text{BISHOP}} = \{\text{BISHOP}\}\text{X} \cup L\left[G_t^{(2)}(f2, e5, k, \text{BISHOP})\right]$$

   The trajectory requested in the problem is the trajectory:

   $$T_B = a(d3)a(e5)$$

   Thus TRACKS $= \{(\text{BISHOP}, t_B)\}$, the number of trajectories $b = 1$
   and $h_1^0(u) = (BISHOP, t_B)t(BISHOP, t_B, 3)$ Computation of $g(h_1^0(u), zero)$

   $$g(h_1^0(u), zero) = g(BISHOP, t_B, zero)$$

   $$g_r\,(\text{BISHOP}, t_B, zero) = \left\{\frac{1, if\ \text{DIST}(rtextif BISHOP, t_B) < 2n}{0, \text{if DIST}\,(r, \text{BISHOP}, t_B) = 2n}\right\}$$

   $$DIST(x, BISHOP, t_B) = k + 1$$

   where k is the number of symbols of the trajectory t_B, whose parameter value is equal to x

   $$DIST(d3, BISHOP, t_B) = 2$$

   $$DIST(e5, BISHOP, t_B) = 3$$

   For the rest of x from $XDIST(x, BISHOP, T_B) = 2X63 = 128$ For $r \in \{e5, d3\} = \{39, 30, 23\} g_r(BISHOP, t_B, zero) = 1$, for the rest of $rg_r = 0$

   $$A(u, zero, zero) \rightarrow t(BISHOP, t_B, 3)$$

   $$A((0, 0, 0), g(BISHOP, t_B, zero), zero)$$

   $$TIME(z) = DIST(z, BISHOP, t_B)$$

   $TIME(z)$ is equal to 128 for all $z \in X$ except $\{e5, d3\}$, where $TIME(z)$ is equal to $3, 2$ respectively

   $$Q3((0, 0, 0)) = (0 \neq 63) \wedge (0 \neq 63)$$

$$t(BISHOP, t_1, 3)A((0,0,0), v, zero)Q3 \to t(BISHOP, t_1, 3)A(f((0,0,0), v), v, zero)$$

computation of f

$u = (x, y, l) = (0,0,0)$ and $v_{y+1} = v_1 = 0 : f(u, v) = (1, y+1, TIME(y+1)Xv_{y+1}) = (1,1,0)$

$Q3 \to t(BISHOP, t_B, 3)A((1,1,0), v, zero)NEXTTIME(z) = init(0,0,0), NEXTTIME(z)) = 2n = 128$ f

try $4ju = (x, y, l) = (1,1,0)$, i.e. $, l = 0$ and $Q4 = F$ Try 3 $u = (x, y, l) = (1,1,0), v$ is in Table, and $w = zero$

$$Q3(1,1,0) = T$$

$$Q3 \to t(BISHOP, t_B, 3)A(f((1,1,0), v), v, zero).$$

computation of f
As far as $(l = 0) \wedge (y = 1)$ and $v_{y+1} = v_2 = 0, f(u, v) = (1, y+1, TIME(y+1)Xv_{y_1}) = (1,2,0)$

$$Q3 \to t(BISHOP, t_B, 3)A((1,2,0), v, zero)$$

$$NEXTTIME(z) = init((1,1,0), NEXTTIME(z))$$

$$NEXTTIME(z) = 128 \text{ for all } z \text{ from} X$$

Try $4jQ4(1,2,0) = F$ Try 3 $Q3(1,2,0) = T$ Try 4j $Q4(1,3,0) = F$ Loop continues until u changes either way: Note, that we encounter a number of white pieces, but none of them are opposing, so we continue.

$$l = TIME(y+1)Xv_{y+1} \neq 0$$

or

$$y = 128$$

In our case $v_{38+1} = 1 (\neq 0)$ At 38'th application of prouction 3 will result:

$$Q3 \to t(BISHOP, t_B, 3)A((1,37,3), v, zero)$$

because

$$u = (1,38,0))$$

$y+1$ corresponds to $h1, TIME(y+1)Xv_{y_1} = TIME(h1)X1 = 3$ Try 4j $Q4(1,39,3) = F$ Try 3 with $l > 0$ and $x \neq 63$

This means the beginning of a new loop which consists of multiple application of production 3 after failures of attemptes to apply one of productions $4j$

$$Q3 \to t(BISHOP, t_B, 3)A((2,39,3), v, zero)$$

$$Q3 \to t(BISHOP, t_B, 3)A((3,39,3), v, zero)$$

$$......$$

$$Q3 \to t(BISHOP, t_B, 3)A(50,38,3), v, zero)$$

With $u = (50,39,3)$ this loop will terminate because

$Q4(50,39,3) = (ON(KING) = 50) \wedge (3 > 0) \wedge (\phi(BISHOP, KING) = 0) \wedge (MAP_{b7,KING}(e5) = 3) = T$

which means that productions $4j$ are applicable. These productions will generate intercepting trajectories from $b7$ to $e5$

$$4j \to t(BISHOP, t_b, 3)t(h_j(50,39,3), TIME(39))A((50,39,3), v, g(h_j(50,39,3), zero))$$

$$4j \to t(BISHOP, t_B, 3)t(h_j(50,39,3), TIME(8))A((50,39,3), v, g(h_j(50,39,3), zero))$$

Computation of $h_j(50,39,3)$ We have to generate all the shortest trajectories from point $b7$ to $e5$ for the black KING. The length of these trajectories should be less or equal to 3

$$TRACKS_{KING} = \{KING\}\, X \cup L\left[G_t^{(2)}(b7, e5, k, KING)\right]$$

$$TRACKS = \{(KING, t_1), (KING, t_2)\}, m = 2$$

and

$$h_1(50,39,3) = (KING, t_1)$$
$$h_2(50,39,3) = (KING, t_2)$$

There are two such trajectories $t1, t2$ and they are generated by the grammar $G_t^{(2)}$ Taking into account that $TIME(39) = 3$ we have:

$$3_1 \to t(BISHOP, t_B, 3)t((KING, t_K), 3)A((50,39,3), v, g(KING, t_K, zero))$$

$$3_1 \to t(BISHOP, t_B, 3)t((KING, t_K), 3)A((50,39,3), v, g(KING, t_K, zero))$$

For all $r \in X$ the r-th component of functiong g is as follows:

$$g_r\,(\text{KING}, t_K, zero) = \left\{\frac{1, if\ \text{DIST}(rtextif KING, t_K) < 2n}{0, \text{if DIST}\,(r, \text{KING}, t_K) = 2n}\right\}$$

$$DIST(x, KING, t_K) = k + 1$$

where k is the number of symbols of the trajectory t_B, whose parameter value is equal to x

$$DIST(c6, KING, t_K) = 2$$
$$DIST(d5, KING, t_K) = 3$$
$$DIST(e5, KING, t_K) = 3$$

For the rest of x from $XDIST(x, KING, T_K) = 2X63 = 128$ For $r \in \{e5, d5, c6\} = \{39, 37, 33\} g_r(KING, t_K, zero) = 1$, for the rest of $rg_r = 0$ Computation of NEXTTIME

$$NEXTTIME(z) = ALPHA(z, (KING, t_K), 3 - 3 + 1)$$

From previous steps $NEXTTIME(x) = 128$ for all x from X

$$ALPHA(x, p_0, t_0, k) = \begin{cases} max & (NEXTTIME(x), k),\ if\ (DIST(x, p_0, t_0) \neq 2n) \\ & \wedge(NEXTTIME(x) \neq 2n); \\ k, & if\ DIST(x, p_0, t_0) \neq 2n) \\ & \wedge(NEXTTIME(x) = 2n); \\ NEXTTIME(x) & if\ DIST(x, p_0, t_0) = 2n). \end{cases}$$

Try 3 with $u = (50, 39, 3)$, i.e. with $l > 0$ and $x \neq 63$

New lop consists of multiple application of production 3 after failures of attemptes to apply one of productions $4j$

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)A((50, 39, 3), v, w)$$

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)A((51, 39, 3), v, w)$$

......

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)A(55, 38, 3), v, w)$$

This will terminate at x = 64 since there are no more pieces that can intercept at location $TIME(z) = 3$, we then decrement l, and start over. This leads to the same loop repeating. With $u = (55, 28, 2)$ this loop will terminate because

$$Q4(55, 28, 2) = (ON(Knight) = 54) \wedge (2 > 0) \wedge (\phi(BISHOP, KNIGHT) = 0) \wedge (MAP_{b7,KING}(e5) = 2) = T$$

which means that productions $4j$ are applicable. These productions will generate intercepting trajectories from $g7$ to $e5$

$$4j \to t(BISHOP, t_b, 3)t(h_j(55, 28, 2), TIME(39))A((55, 28, 2), v, g(h_j(55, 28, 2), zero))$$

$$4j \to t(BISHOP, t_B, 3)t(h_j(55, 28, 2), TIME(8))A((55, 28, 2), v, g(h_j(55, 28, 2), zero))$$

Computation of $h_j(55, 28, 2)$ We have to generate all the shortest trajectories from point $g7$ to $e5$ for the black KNIGHT. The length of these trajectories should be less or equal to 2

$$TRACKS_{KNIGHT} = \{KNIGHT\}\, X \cup L\left[G_t^{(2)}(g7, e5, k, KNIGHT)\right]$$

$$TRACKS = \{(KNIGHT, t_1), (KNIGHT, t_2)\}, m = 2$$

and

$$h_1(55, 28, 2) = (KING, t_1)$$

$$h_2(55, 28, 2) = (KING, t_2)$$

There are two such trajectories $t1, t2$ and they are generated by the grammar $G_t^{(2)}$ Taking into account that $TIME(28) = 2$ we have:

$$3_1 \to t(BISHOP, t_B, 3)t((KING, t_K), 3)t((KNIGHT, t_K), 2)A((55, 28, 2), v, g(KNIGHT, t_K, zero))$$

$$3_1 \to t(BISHOP, t_B, 3)t((KING, t_K), 3)t((KNIGHT, t_K), 2)A((55, 28, 2), v, g(KNIGHT, t_K, zero))$$

For all $r \in X$ the r-th component of functiong g is as follows:

$$g_r\,(\text{KNIGHT}, t_K, zero) = \left\{\frac{1, if\ \text{DIST}(rtextif KNIGHT, t_K) < 2n}{0, \text{if DIST}\,(r, \text{KNIGHT}, t_K) = 2n}\right\}$$

$$DIST(x, KNIGHT, t_K) = k + 1$$

where k is the number of symbols of the trajectory t_B, whose parameter value is equal to x

$$DIST(e6, KNIGHT, t_K) = 1$$

$$DIST(e5, KNIGHT, t_K) = 2$$

For the rest of x from $XDIST(x, KING, T_K) = 2X63 = 128$ For $r \in \{e5, c6\} = \{39, 45\} g_r(KNIGHT, t_K, zero) = 1$, for the rest of $r g_r = 0$ Computation of NEXTTIME

$$NEXTTIME(z) = ALPHA(z, (KNIGHT, t_K), 2 - 2 + 1)$$

From previous steps $NEXTTIME(x) = 128$ for all x from X

$$ALPHA(x, p_0, t_0, k) = \begin{cases} max & (NEXTTIME(x), k), \ if \ (DIST(x, p_0, t_0) \neq 2n) \\ & \wedge(NEXTTIME(x) \neq 2n); \\ k, & if \ DIST(x, p_0, t_0) \neq 2n) \\ & \wedge(NEXTTIME(x) = 2n); \\ NEXTTIME(x) & if \ DIST(x, p_0, t_0) = 2n). \end{cases}$$

Try 3 with $u = (55, 28, 2)$, i.e. with $l > 0$ and $x \neq 63$ With $u = (64, 28, 2)$ this loop will terminate which means no other starting points are found. New loop begins. The grammar changes ending point of prospective trajectories

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)t((KNIGHT, t_K), 2)A((50, 30, 0), v, w)$$

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)t((KNIGHT, t_K), 2)A((50, 31, 0), v, w)$$

......

$$Q3 \to t(BISHOP, t_B, 3)t(KING, t_F, 3)t((KNIGHT, t_K), 2)A(50, 64, 0), v, w)$$

It is determined that there are no more valid ending points, and thus the loop terminates. Try 5: $Q5(w) = (w \neq 0) = T$

$$Q5 \to t(BISHOP, t_B, 3)t(KING, t_K, 3)t((KNIGHT, t_K), 2)A((0, 0, 0), w, zero)$$

$$TIME(Z) = NEXTTIME(z)$$

All the steps, 3 and 4j, which have been executed (or tried) for generating 1-st negation trajectroeis will be repeated for generated 2-nd negation, same as the 1-st, just using the trajectories found in the first steps instead of the original trajectory. There are two such trajectories that get found, both from the pawns on e5, and d5 respectively. The white pawn on d5 gets there with a time of 2, the black pawn gets there with a time of 1. There are no third negation zones. We will then repeat everything and try for a third negation, of which there are none. The next return to produciton 5 will happen with $w = zero$ (nothing is found) that means we move to production 6:

$$Q6 \to t(BISHOP, t_B, 3)t(King, t_K, 3)t((KNIGHT, t_K), 2)t((BPAWN, t_B P), 2)t((WPAWN, t_W P), 3)$$

And we are done. Whew.

2. Modify the grammar of Zones G_z in order to generate all the attack Zones within horizon H in any given position by ONE application of the grammar.