Notes

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Commutative Laws
$p \wedge q \equiv q \wedge p$ ($\wedge$-Commutative)
$p \vee q \equiv q \vee p$ ($\vee$-Commutative)

Association Laws
$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
$p \vee (q \vee r) \equiv (p \vee q) \vee r$

DeMorgan's Laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Distributive Laws
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Double Negation
$\neg\neg p \equiv p$

Idempotence Laws
$p \wedge p \equiv p$
$p \vee p \equiv p$

Absorption
$p \wedge (q \vee p) \equiv p$

Implication or Material Implication
$p \rightarrow q \equiv \neg p \vee q$

Bi-Implication
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

Exclusive Disjunction
$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$

Equality is <u>transitive</u> meaning that if $x = y$ and $y = z$, then $z = y$. Because + is transitive, these chain proofs make sense.

Claim: $(A \wedge B) \vee (C \wedge D) \equiv (D \wedge C) \vee (B \wedge A)$

Proof:
$(A \wedge B) \vee (C \wedge D) \equiv (C \wedge D) \vee (A \wedge B)$ ($\vee$-Commutative)
$\equiv (D \wedge C) \vee (A \wedge B)$ ($\wedge$-Commutative)
$\equiv (D \wedge C) \vee (B \wedge A)$ ($\wedge$-Commutative)$\square$

Claim: $X \rightarrow Y \equiv \neg Y \rightarrow \neg X$

Proof:
$X \rightarrow Y \equiv \neg X \vee Y$ (Material implication)
$\equiv Y \vee \neg X$ ($\vee$-Comm.)

$\neg Y \rightarrow \neg X \equiv \neg\neg Y \vee \neg X$ (Mat. Imp)
$\equiv Y \vee \neg X$ (double negation) $\square$

DO NOT DO THIS:
Pf:
$X \rightarrow Y \equiv \neg Y \rightarrow \neg X$
$\neg X \vee Y \equiv \neg\neg Y \vee \neg X$
$\neg X \vee Y \equiv Y \vee \neg X$
$Y \vee \neg X \equiv Y \vee \neg X \square$
DO NOT START A PROOF BY STATING THAT YOUR CLAIM IS TRUE

Claim: $(A \rightarrow B) \rightarrow C \equiv A \rightarrow (B \rightarrow C)$

Proof:
$(A \rightarrow B) \rightarrow C \equiv \neg(A \rightarrow B) \vee C$ (imp.)

$\equiv \neg(\neg A \vee B)$ (imp.)$\vee C$
$\equiv (\neg\neg A \wedge \neg B) \vee C$ (demorgan's)
$\equiv (A \wedge \neg B) \vee C$ (double neg)

$A \rightarrow (B \rightarrow C) \equiv \neg A \vee (B \rightarrow C)$ (imp.)
$\equiv \neg A \vee (\neg B \vee C)$ (imp.)
$\equiv (\neg A \vee \neg B) \vee C$ (Assoc.)
Not equivalent! (A=F, B=T , C=F)