Homework 8

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Question 1
$H \to (T \land \neg R)$

Question 2
A) $\exists(P(\mathrm{x}) \land \neg E(\mathrm{x}))$
B) All positive numbers are even.
C) (1,3,5,9)

Question 3
A) fml2 = A∧B, fml1 = A ∨ B
B) $\{2/3, 4/3, -2/3\} \subseteq \mathrm{A} \setminus \mathbb{Z}$
C) $\{-2\} \in P(\mathrm{B})$

Question 4
Claim: (R∧S)→¬Q, R→S ⊢ (R∧P)→¬(P∧Q)

Proof
Assume (R∧S)→¬Q and R→S
— Assume R∧P, and Q (to show a contradiction)
— From R∧P, R and P. (∧-elimination)
— From P and Q, P∧Q (∧-introduction)
— Due to R→S and R, S (→-elimination)
— From R and S, R∧S (∧-introduction)
— Due to (R∧S)→¬Q and R∧S, ¬Q (→-elimination)
Assuming R∧P and Q, we got both Q and ¬Q, as well as P∧Q, which contradict, so ¬(P∧Q); therefore (R∧P)→¬(P∧Q) (proof by contradiction/direct proof)□

Question 5
A) False. (A=T, C=F, B=F, D=T)

B) **Claim:** (A∧B)→(C∨D) ≡ (A→C)∨(B→D)

Proof

(A∧B)→(C∨D) ≡ ¬(A∧B) ∨ (C∨D) (Material implication)
≡ ¬A ∨ ¬B ∨ (C∨D) (Demorgans)
≡ ¬A ∨ C ∨ ¬B∨D (∨ associative)
≡ (A→C) ∨ (¬B∨D) (Implication)
≡ (A→C) ∨ (B→D) (Implication)□

Question 6

Claim: For all sets A, B, C, and D, if A∪C⊆D, then A\B⊆D\B
Proof:
Choose sets A, B, C, and D, and assume A∪C⊆D — Choose x∈A\B and A∪C.
— So x∈A and x∉B.
— From A∪C, x∈A or x∈D
—— Case 1: Suppose x∈A
—— From this, we can directly prove x∈D∩A
—— Case 2: Suppose x∈C
—— From this, we can directly prove x∈D∩A
— Either case, we proved x∈D∩A
Therefore A\B⊆D\B □