Notes

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April 23, 2019

$12 \in X$
$15 \in X$

Addition and subtraction same as homework

Claim: Every member of x is div. by 3.
This is a universal claim about the members of X, so we use the structure of the definition of X for our induction proof.

Base Case:
... prove $3 \mid 12$
... prove $3 \mid 15$

Ind Step:
Assume x, $y \in X$ and and $3 \div x$ and $3 \div y$
..prove $3 \div x+y$
Ind Step:
Assume $3 \div x$ and $3 \div y$ for some natural number $\in X$
... prove $3 \div x-y$

claim: for any natural number n, $3n \in x$
This is a universal claim about all of the natural numbers, so we use the definition of the natural numbers for our induction proof.
Base Case:
... prove $3*0 \in X$

Ind Step:
Assume $3k \in X$ for some natural number k
... prove $3(k+1) \in X$

Quiz this week is a "review" quiz. Thursday lecture is a review lecture. Might post a bonus assignment but not for points. Test is tuesday morning at 10 or something.

Define F on the set of intergers by F = {(n, m) | $2n + 3m$ is divisible by 5}
F(5, 5) because $2 * 5 + 3 * 5 = 10 + 15 = 25$ and $5|25$
F(-5, 10) because $2(-5) + 3(10) = -10 + 30 = 20$ and $5|20$
F(1, 6) beause $2 + 18 = 20$ and $5|20$
¬F(1, 2) because $2 * 1 + 3 * 2 = 2 + 6 = 8$ and $5 \neg| 8$ (not divisible)

Is F reflexive?
A relation R on A is reflexive iff for every a ∈ A, R(a, a)
If you have an integer a, does F(a, a) have to be true?
Is $2a + 3a$ always/sometimes/never divisible by 5?
Yes, $2a + 3a = 5a$ is always divisible by 5.

Claim:
F is transitive.
NOTE: the following will be an incorrect proof.
Proof:
Choose a, b, c that are integers and assume F(a. b) and F(b, c)
So $2a + 3b$ is divisible by 5 and $2b + 2c$ is divisible by 5.
$2a + 3b = 2 * n$ for some integer n
$2b + 3c = 5k$ for some integer k
NOTE: the following is purposely bad
$2b = 5k - 3c$
$b = (5k - 3c)/2$
$4a + 15k - 9c =$ 10n
$4a - 9c = 10n - 15k$
$4a - 9c = 5(2n - 3k)$
$2(2a) - 3(3c) = 5(2n - 3k)$
NOTE: Do not just conclude this because we did not get our goal of 2a+3c = 5n
Since 5, 2, n, 3, and k are integers
Therefore $2(2n) - 3(3c)$is div by 5...? not what we wanted to prove.