Homework 12

Will Boland

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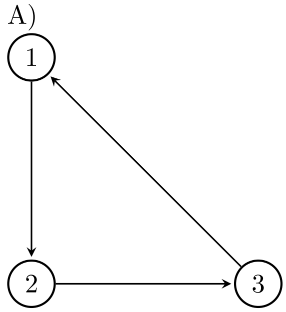
Question 1

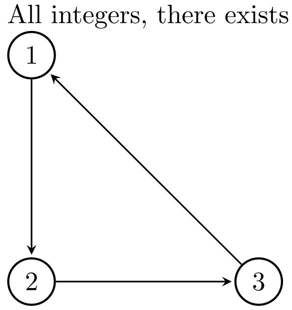
A) $\forall x \exists y((E(y) \land T(x)) \to R(x, y))$
B) $\neg \exists x \forall y(T(y) \to R(x, y))$
C) $\forall x(E(x) \to \neg \exists y(T(y) \land R(x, y)))$
D) $\exists x \forall y(E(y) \to \neg R(x, y))$
E) All textbooks are referenced by a book.
F) NOTE: T(x) was supposed to be T(y): There is a book that does not reference any textbook.
G) All textbooks have a book that they don't reference

Question 2

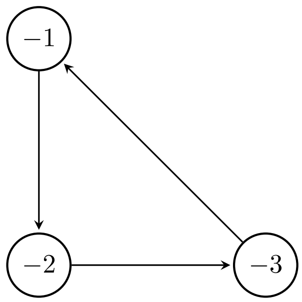
A)



B) All integers point to at least one real number
All integers, there exists a real number that is not pointed to by an integer.



C) All real numbers point to a negative number.
There does not exist a real number that points to all negative numbers.



Question 3
Example: L = {(”lol”, ”lol”), (”hello”, ”hi”), (”hi”, ”hello”), (”lol, ”hi”)}

Question 4
Claim: For any sets A, B, C, D, and E, if $D \cap B \subseteq A \setminus C$, then $D \cap E \subseteq E \setminus (B \cap C)$

Proof
Choose sets A, B, C, D, and E and assume Assume $D \cap B \subseteq A \setminus C$
— Choose $D \cap E$
— So, $x \in D$ and $x \in E$
— From $x \in D \cap B$, $x \in D$ and $x \in B$
— From $D \cap B \subseteq A \setminus C$, so $x \in A$ but $x \notin C$
— So, because $x \in B$ and $x \notin C$ and $x \in E$, $E \setminus (B \cap C)$
Therefore, if $D \cap B \subseteq A \backslash$, then $D \cap E \subseteq (B \cap C)$$\square$

Question 5
Claim: The cube of an odd # is odd

Proof
Choose an odd number x
So there exists an integer k with x = 2k + 1. (Side note: In class he said this would get half credit if stopped right here)
$x^3 = (2k+1)^3$
= (2k+1)(2k+1)(2k+1)
$= (4k^2 + 4k + 1)(2k + 1)$
$= 8k^3 + 12k^2 + 6k + 1$
$= 2(4k^3 + 6k^2 + 3k) + 1$
Since 4, k, 6, and 3 are integers, so is $4k^3 + 6k^2 + 3k$
Therefore x^3 is odd $\square$

Question 6

Claim: Prove R is transitive

Proof

Choose real numbers x, y, and z and assume R(x, y) and R(y, z)

So $x - y > 1$ and $y - z > 1$ (Side note: In class he said this would get half credit if stopped right here)

$(x - y) + (y - z) > 1 + 1$

$x - z > 2$

> 1

So $x - z > 1$ □