Homework 3

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Question 1

A) **Claim:** $(P \land Q) \to R$, $P \land S$, $\neg\neg Q \vdash R$ is valid.

Proof:
Assume $(P \land Q) \to R$, $P \land S$, and $\neg\neg Q$.
Because $\neg\neg Q$, we know Q. ($\neg$-Eliminiation)
Because $P \land S$, we know P. ($\land$-Elimination)
From P and Q, we know $P \land Q$ ($\land$-introduction)
Due to $(P \land Q) \to R$ from $P \land Q$, we know R. ($\to$-Elimination)$\square$

B) **Claim:** $X \land (X \to (Z \land Y)) \vdash X \land Y$ is valid.

Proof:
Assume $X \land (X \to (Z \land Y))$.
From $X \land (X \to (Z \land Y))$, we know X and $(X \to (Z \land Y))$. ($\land$-elimination)
Due to $X \to (Z \land Y)$ from X, we know $(Z \land Y)$ ($\to$-elimination)
From $Z \land Y$ we know Y. ($\land$-elimination)
From X and Y, we know $X \land Y$ ($\land$-introduction)$\square$

C) **Claim:** $A \land \neg\neg B \vdash B \lor (A \to \neg C)$ is valid

Proof:
Assume $A \land \neg\neg B$.
From $A \land \neg\neg B$, we know A and $\neg\neg B$. ($\land$-elimination)
From $\neg\neg B$, we know B. (Double negation)
From B, we know $B \lor (A \to \neg C)$ ($\lor$-introduction)$\square$

D) **Claim:** $(K \lor L) \to N$, $K \land M \vdash N \land M$ is valid

Proof:
Assume $(K \lor L) \to N$ and $K \land M$.

From $K \wedge M$, we know both K and M. ($\wedge$-elimination)
From K, we know $K \vee L$. ($\vee$-introduction)
Due to $(K \vee L) \rightarrow N$ from $K \vee L$ from K, we know N. ($\rightarrow$-elimination)
From M and N, we know $N \wedge M$ ($\wedge$-introduction)$\square$

E) **Claim:** $(A \wedge B) \rightarrow C, B, A \wedge \neg D \vdash C \wedge \neg D$ is valid

Proof:
Assume $(A \wedge B) \rightarrow C$, B, and $A \wedge \neg D$
From $A \wedge \neg D$, we know both A and $\neg D$ ($\wedge$-elimination)
From A and B, we know $A \wedge B$ ($\wedge$-introduction)
Due to $(A \wedge B) \rightarrow C$ from $A \wedge B$, we know C ($\rightarrow$-elimination)
From C and $\neg D$, we know $C \wedge \neg D$ ($\wedge$-introduction)$\square$

F) **Claim:** $(A \wedge B) \rightarrow C, B \vdash (A \wedge \neg D) \rightarrow (C \wedge \neg D)$ is valid

Proof:
Assume $(A \wedge B) \rightarrow C$ and B .
— Assume $A \wedge \neg D$
— From $A \wedge \neg D$, we have both $\neg D$ and A ($\wedge$-elimination)
— From A and B, we get $A \wedge B$ ($\wedge$-introduction)
— Due to $(A \wedge B) \rightarrow C$ from $A \wedge B$, we know C ($\rightarrow$-elimination)
— From C and $\neg D$, we know $C \wedge \neg D$ ($\wedge$-introduction)
Assuming $A \wedge \neg D$, we proved $(C \wedge \neg D)$ and therefore $(A \wedge \neg D) \rightarrow (C \wedge \neg D)$. (direct proof or $\rightarrow$-introduction)$\square$

G) **Claim:** $Z \rightarrow \neg X, Z \wedge \neg\neg Y \vdash \neg X \vee Y$ is valid

Proof:
Assume $Z \rightarrow \neg X$ and $Z \wedge \neg\neg Y$
From $Z \wedge \neg\neg Y$, we know Z and $\neg\neg Y$ ($\wedge$-elimination)
From $\neg\neg Y$ we get Y (double negation)
Due to $Z \rightarrow \neg X$ from Z, we get $\neg X$ ($\rightarrow$-elimination)
From $\neg X$ or Y, we know $\neg X \vee Y$ ($\vee$-introduction)$\square$

H) **Claim:** $\vdash ((Z \rightarrow \neg X) \wedge (Z \wedge \neg\neg Y)) \rightarrow (\neg X \vee Y)$ is valid

Proof:
— Assume $((Z \rightarrow \neg X) \wedge (Z \wedge \neg\neg Y))$
— From $((Z \rightarrow \neg X) \wedge (Z \wedge \neg\neg Y))$, we know $(Z \rightarrow \neg X)$ and $(Z \wedge \neg\neg Y)$ ($\wedge$-elimination)
— From $(Z \wedge \neg\neg Y)$, we know Z and $\neg\neg Y$ ($\wedge$-elimination)
— From $\neg\neg Y$, we know Y ($\neg$-elimination)

— Due to $(Z \rightarrow \neg X)$ from Z, we know $\neg X$ ($\rightarrow$-elimination)
— From $\neg X$ and Y, we know $\neg X \vee Y$ ($\vee$-introduction)
Assuming $((Z \rightarrow \neg X) \wedge (Z \wedge \neg\neg Y))$, we proved $\neg X \vee Y$; therefore, $((Z \rightarrow \neg X) \wedge (Z \wedge \neg\neg Y)) \rightarrow (\neg X \vee Y)$.
(Direct proof)$\square$