Mini-Homework 3

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April 22, 2019

Question 1

A) **Claim:** $(W \wedge X) \to \neg Y, X \vdash \neg(W \wedge Y)$.

Proof:
Let $(W \wedge X) \to \neg Y$ and X be true.
— Assume that $W \wedge Y$ is true (will show a contradiction)
— $W \wedge Y$ implies W and Y. ($\wedge$-elimination)
— From X and Y, $W \wedge X$ ($\wedge$-elimination)
— Since $(W \wedge X) \to \neg Y$ and $W \wedge X$, $\neg Y$. ($\to$-elimination)
Assuming $W \wedge Y$, we proved Y and $\neg Y$, which contradict eachother, and therefore $\neg(W \wedge Y)$ (Contradiciton) $\Box$

B) **Claim:** $\vdash (M \to \neg N) \to \neg(M \wedge N)$

Proof:
— Assume that $M \wedge N$ and $M \to \neg N$ is true (will show a contradiction)
— From $M \wedge N$, M and N. ($\wedge$-elimination)
— Since $M \to \neg N$ and M, $\neg N$. — Since $(M \to \neg N) \to \neg(M \wedge N)$ and $M \to \neg N$, $\neg(M \wedge N)$. ($\to$-elimination)
Asssuming $M \wedge N$ and $M \to \neg N$, we proved N and $\neg N$, which contradict eachother, and therefore $\neg(M \wedge N)$ (Contradiction)$\Box$

C) **Claim:** $Z \to A, \neg A \vdash \neg Z$

Proof:
Assume $Z \to A$, $\neg A$.
— Assume that Z is true (to show a contradiction)
— $Z \to A$ and Z, so A ($\to$-elimination)
Assuming Z, we proved A and had (under the assumption) $\neg A$, which contradict eachother, and therefore $\neg Z$. (Contadiction) $\Box$

D) **Claim:** $Z \to A, B \to A, \neg A \vdash \neg Z \wedge \neg B$

Proof:
Assume $Z \to A$, $B \to A$, $\neg A$.
— Assume that Z and B are true (to show a contradiction)
— From Z and B, $Z \wedge B$ — $Z \to A$ and Z, A. ($\to$-elimination)
— $B \to A$ and B, A. ($\to$-elimination)
Assuming Z and B, we proved A and had $\neg A$, which contradict eachother, and therefore $\neg Z \wedge \neg B$ (Contradiction) $\square$

E) **Claim:** $P \vee (Q \wedge \neg\neg R), P \to R \vdash R$

Proof:
Assume $P \vee (Q \wedge \neg\neg R)$, $P \to R$
— **Case 1:** Assume P
— $P \to R$ and P, R ($\to$-elimination)
— **Case 2:** Assume $Q \wedge \neg\neg R$
— From $Q \wedge \neg\neg R$, $\neg\neg R$ ($\wedge$-elimination)
— From $\neg\neg R$, R (Double-negation)
Under both assumptions (P and $Q \wedge \neg\neg R$) we proved R. Therefore R. (Cases)$\square$

F) **Claim:** $(F \wedge G) \vee (H \to I), H \vdash G \vee I$

Proof:
Assume $(F \wedge G) \vee (H \to I)$, H.
— **Case 1:** Assume $F \wedge G$
— $F \wedge G$, so F and G. ($\wedge$-elimination)
— From G, $G \vee I$ ($\vee$-introduction)
— **Case 2:** Assume $H \to I$
— $H \to I$ and H, I $\to$-elimination
— From I, $G \vee I$ ($\vee$-introduction)
Under both assumptions ($F \wedge G$ and $H \to I$) we proved $G \vee I$, therefore $G \vee I$ (Cases)$\square$

G) **Claim:** $J \to K \vdash (J \vee \neg\neg K) \to K$

Proof:
Assume $J \to K$.
— Assume $J \vee \neg\neg K$
—— **Case 1:** Assume J
—— $J \to K$ and J, K ($\to$-elimination)
—— **Case 2:** Assume $\neg\neg K$
—— $\neg\neg K$, K (Double negation)
— In either case of $J \vee \neg\neg K$, K is true. (cases)

From $J \to K$, we proved K, and therefore $J \to K \vdash (J \vee \neg\neg K) \to K$ (Direct proof)□

H) **Claim:** $(A \wedge C) \to D, A \wedge \neg B \vdash \neg(A \to B) \wedge (C \to D)$

Proof:
Assume $(A \wedge C) \to D$, $A \wedge \neg B$
From $A \wedge \neg B$, A and $\neg B$ ($\wedge$-elimination)
— Assume $A \to B$ (to show a contradiction)
— $A \to B$ and A, B. ($\to$-elimination)
Assuming $A \to B$ we proved B and $\neg B$, which contradict eachothere, and therefore $\neg(A \to B)$ (Contradiction)
— Assume $A \wedge C$
— $A \wedge C$, so C ($\wedge$-elimination)
— Due to $(A \wedge C) \to D$ and C from $A \wedge C$, $C \to D$ and C, so D. ($\to$-elimination)
$\neg(A \to B)$ and $C \to D$, so $\neg(A \to B) \wedge C \to D$ ($\wedge$-introduction)□

Question 2 (Assuming that word phrasing is correct)
A) Line 2 is wrong because you need to do proof by cases.
B) Correct
C) Correct
D) Correct
E) Line 2 is incorrect: Need to assume $A \vee \neg B$ before proof by cases due to it all connecting by $\to$.
F) Line 9 is incorrect: K was never used to prove J, it was the double negation after assuming $\neg\neg J$ that proved J.

Question 3:
A) **Claim:** $A \to (B \wedge C) \vdash (A \to B) \wedge (A \to C)$

Proof:
Assume $A \to (B \wedge C)$.
— Assume A
— Since $A \to (B \wedge C)$ and A, $B \wedge C$ ($\to$-elimination)
— Since $B \wedge C$, B and C ($\wedge$-elimination)
Assuming A, we proved B and C, so $A \to B$ and $A \to C$ (Direct Proof)
$A \to B$ and $A \to C$, so $A \to B \wedge A \to C$ ($\wedge$-introduction)□

Claim: $(A \to B) \wedge (A \to C) \vdash A \to (B \wedge C)$

Proof:
Assume $(A \to B) \wedge (A \to C)$.
From $(A \to B) \wedge (A \to C)$, $(A \to B)$ and $(A \to C)$.

— Assume A.
— (A→B) and A, B (→-elimination)
— (A→C) and A, C (→-elimination)
— From B and C, B∧C (∧-introduction)
Assuming A, we proved B∧C, therefore A→(B∧C) (Direct Proof)□

B) **Claim:** A→(B∧C) ≡ (A→B)∧(A→C)

Proof:
A→(B∧C) ≡ ¬A∨(B∧C) (Implication)
≡ (¬A∨B)∧(¬A∨C) (∨ Distributive)
≡ (A→B)∧(A¬∨C) (Implication)
≡ (A→B)∧(A→C) (Implication)□

Question 4
A) **Claim:** A→(B∧C) ≡ (A→B)∧(A→C)

Proof:
A→(B∧C) ≡ ¬A∨(B∧C) (Implication)
≡ (¬A∨B)∧(¬A∨C) (∨ Distributive)
≡ (A→B)∧(A¬∨C) (Implication)
≡ (A→B)∧(A→C) (Implication)□

B) Not Equal. (A=T, B=F, C=F)

C) **Claim:** (A→B)→C ≡ (¬A→C)∧(B→C)

Proof:
(A→B)→C ≡ (¬A∨B)→C (Implication)
≡ ¬(¬A∨B)∨C (Implication)
≡ (¬¬A∧¬B)∨C (DeMorgan's)
≡ C∨(¬¬A∧¬B) (∨-Commutative)
≡ C∨(A∧¬B) (Double negation)
≡ (C∨A)∧(C∨¬B) (∨-Distribution)
≡ (A∨C)∧(C∨¬B) (∨-commutative)
≡ (A∨C)∧(¬B∨C)(∨-commutative)
≡ (¬A→C)∧(¬B∨C) (Implication)
≡ (¬A→C)∧(B→C) (Implication)□

D) Not Equal. (W=T, X=F, Y=T, Z=F)

E) **Claim:** $\neg((W\wedge\neg X)\rightarrow(\neg Y\vee Z)) \equiv (Y\wedge\neg Z)\wedge(W\wedge\neg X)$

Proof:

$\neg((W\wedge\neg X)\rightarrow(\neg Y\vee Z)) \equiv \neg(\neg(W\wedge\neg X)\vee(\neg Y\vee Z))$ (implication)
$\equiv \neg\neg(W\wedge\neg X)\wedge(Y\wedge\neg Z)$ (DeMorgan's)
$\equiv (W\wedge\neg X)\wedge(Y\wedge\neg Z)$ (Double negation)
$\equiv (Y\wedge\neg Z)\wedge(W\wedge\neg X)$ ($\wedge$-commutative)$\square$

F)They are equivalent (truth table's are same); however, I can't seem to prove it using proof? Silly error on my part?
Claim: $P\leftrightarrow Q \equiv\neg(P\oplus Q)$

Proof:

$P\leftrightarrow Q \equiv (P\wedge Q)\vee(\neg P\wedge\neg Q)$ (Bi-Implication)
$\equiv \neg((\neg P\vee\neg Q)\wedge(P\vee Q))$ (DeMorgan's)
$\equiv \neg((P\rightarrow\neg Q))\wedge(P\vee Q))$ (Implication)
$\equiv \neg((P\rightarrow\neg Q))\wedge(\neg P\rightarrow Q))$ (implication)

$\neg(P\oplus Q) \equiv \neg((P\wedge\neg Q)\vee(\neg P\wedge Q))$ (Exclusive Disjunction)
$\equiv ((\neg P\vee Q)\wedge(P\vee\neg Q))$ (DeMorgan's)
$\equiv (P\rightarrow Q)\wedge(P\vee\neg Q)$ (implication)
$\equiv (P\rightarrow Q)\wedge(\neg P\rightarrow Q)\square$
