Homework

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A **direct proof** is a sequence of deductions starting by assuming the premises and eventually proving the conclusion.

Semi-formal proofs

- Limited set of rules
- One rule per step
- Always say what formulas you are using
- Use english sentences
- clearly write down the assumptions
- cite the names of the logical rules

Claim: this is valid

$\neg\neg A$
$A \rightarrow (B \wedge C)$
———————-
$A \wedge C$

This can also be read as $\neg\neg A$, $A \rightarrow (B \wedge C) \vdash A \wedge C$

The turnstile symbol $\vdash$ means there is a proof with assumptions p1,...pn, and conclusion c.

Pf:

Assume $\neg\neg A$ and $A \rightarrow (B \wedge C)$ are true.
Since $\neg\neg A$ is true, so is A. (Double Negation)
Because $A \rightarrow (B \wedge C)$ and A, we know $B \wedge C$. (Appl.)
From $B \wedge C$, we can conclude C. ($\wedge$-Elim.)
A and C together imply $A \wedge C$. ($\wedge$-Elim.) End the proof with a box like a period.

Natural Deduction Rules

All of the Natural Deduction rules are inference rules, meaning that they only work in one direction, and they only work aon entire formulas, not parts of forumulas.

$\wedge$-Elimination or simplification.
(If you know that $p\wedge q$ is true, then you can conclude p is true, or q is true, or both.)
$p\wedge q \vdash p$
$P\wedge q \vdash q$

$\wedge$-Introduction or conjuction.
$p,q \vdash q\wedge p$

$\rightarrow$-Elimination or Applicaiton or Modus Ponanus.
$p\rightarrow q$ by itself is useless. $p\rightarrow q$ together with p, implies q.
$p\rightarrow q,\ p\vdash q$
This DOES NOT mean that you can conclude that B == $(A\wedge B)\rightarrow C$ due to $\rightarrow$-Elimination!!!

$\neg$-Elimination or Double Negation.
$\neg\neg p \vdash p$

Weakening or $\vee$-Introduction.
$p\vdash p\vee q$
$p\vdash q\vee p$

Claim: $(A\vee B)\rightarrow C,\ A \vdash C)$
(Pf:)
Assume $(A\vee B)\rightarrow C$ and A. (Weak)
Apply $(A\vee B)\rightarrow$ C to $A\vee$ B to get C (Appl.)

Since X, we can conclude $\neg(Z\rightarrow\neg W)\vee X$. (Weak).