Notes

Will Boland

February 28, 2019

Claim: $(A \land B) \to C$, $A \vdash (B \land D) \to C$ is valid)

Proof:
Suppose $(A \land B) \to C$ and A.
— Assume $B \land D$
— From $B \land D$, we get B and D ($\land$-elim.)
— A and B together imply $A \land B$ ($\land$-intro)
— We have $(A \land B) \to C$ and $A \land B$, so C ($\to$-elim)
Assuming $B \land D$, we proved C and therefore $(B \land D) \to C$. (direct proof or $\to$-introduction)

Claim: $(X \lor Y) \land \neg\neg Z \vdash X \to Z$

Proof:
Assume $(X \lor Y) \to \neg\neg Z$
— Assume $X \lor Y$
— Apply $(X \lor Y) \to \neg\neg Z$, to $X \lor Y$, resulting in $\neg\neg Z$. (Appl.)
— $\neg\neg Z$ leads to Z (double neg)
Under the assumption $X \lor Y$, we orived Z, and si $(X \lor Y) \to Z$ (Dir. proof)

Do this instead of above!!!:

Assume $(X \lor Y) \to \neg\neg Z$
— Assume X
— From X, we know $X \lor Y$ (weak.)
— $(X \lor Y) \to \neg\neg Z$ and $X \lor Y$ imply $\neg\neg Z$ (appl.)
— Because $\neg\neg Z$, Z. (double neg)
We used X to prove Z, and hence $X \to Z$ (Dir. pf)$\square$

When you pick your assumption, think about what you will do after you have finished the subproof, <u>not</u> what you will do <u>inside</u> the subproof.