Homework 10

Will Boland

April 1, 2019

Question 1
A) i) No, (c, c) $\notin$ R
ii) No, (a, a) $\in$ R
iii) No, (a, b) $\in$ R, but (b, a) $\notin$ R
iv) Yes, every instance of (x, y) and (y, x) that is reflexive in R, it has x = y
v) Yes, everytime you have (x, y) and (y, z) in R, you have (x, z)

B) i) Yes, it includes all the required pairs (1,1), (2, 2), and (3, 3)
ii) No, (1, 1) $\in$ S
iii) No, (3, 2) $\notin$ S
iv) No, R(1, 2) and R(2, 1) but $1 \neq 2$
v) No, S(1, 2) and S(2, 4) but (1, 4) $\notin$ S

C) i) Yes, for any x $\in \mathbb{Z}$E, x + x is even, so E(x, x)
ii) No, for any x $\in \mathbb{Z}$, x + x is even, so E(x, x)
iii) Yes, for any x $\in \mathbb{Z}$, where x + y is even, so is y + x, so E(x, y) and E(y, x)
iv) No, for any x $\in \mathbb{Z}$, where x + y is even, so is y + x, so E(x, y) and E(y, x)
v) Yes, for any x $\in \mathbb{Z}$, where x + y is even, and y + z is even, there exists an x + z that is also even

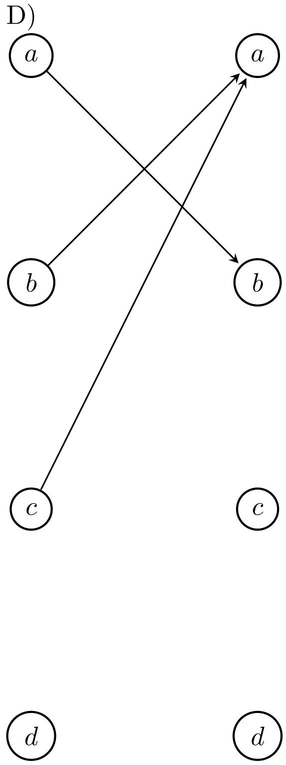
D) i) No, because (”ABC”, ”ABC”) $\notin$ C
ii) Yes, because no instance of members of C are of (x, x) for any substring due to *s* being a proper substring of *t*
iii) No, (”ABC”, ”ABCD”) $\in$ C but (”ABCD”, ”ABC”) $\notin$ C
iv) Yes, but only trivially as there are no instances where (x, y) $\in$ C and (y, x) $\in$ C.
v) Yes, because every (x, y) and (y, z) has a (x, z) due to y being based off of z and x being based off of x.

E) i) No, because for it to intersect it has to have members that are the same which conflict the requirement

ii) Yes, because (x, x) $\notin$ D because the intersection would not just be the empty set
iii) Yes becuase the order of sets doesn't matter for the intersection
iv) No because The intersection will be the empty set, so no two sets will be the same
v) Yes, because if D(x, y) then X and y dont intersect other than the empty set meaning D(y, z) that y and z dont intersect so D(x, z).

Question 2

A) The relation X = {(a, d), (d, a)}
B) The relation R = {(2, 1), (2, 2), (1, 3), (3, 3)}
C) The relation T = {(2, 4), (4, -6), (2, -6)}
D)



E) Cannot exist because there are a finite set of english words

F) The relation F = {(x, y) | where x = 2^y $\wedge$ y $\neq$ x}
G) The relation P = {(Will Boland, Will Boland), (Will Boland, Will Boland)} where "Will Boland" is the same person.
H) The relation (L) cannot exist because if it is anti-symmetric then L(x, y) and L(y, x) and x = y but if it is anti-reflexive x $\neq$ y
I) The relation S = {(Will Boland, Will Boland), (Will Boland, Augie), (Will

Boland, Jack), (Augie, Jack), (Augie, Augie), (Jack, Jack)}

Question 3

A) *Proof*
Choose an integer n, and m, where m = n
n = m * k where k is some integer
So, m = n * k
Since n = m and m|n then D(n, n)□

B) *Proof*
Choose numbers m and n and assume D(m, n) and D(n, m)
That means n is divisble by m and m is divisble by n
So there exists a k where n = m * k and m = n * k
thus n = (n * k) * k
K must be 1 otherwise n would not be divisble by m and vice versa □

C) *Proof*
Choose any number m, n, and z and assume D(m, n), D(n, z)
So, n is divisble by m and z is divisble by n.
Thus there exists a number k where n = m * k and z = n * k
So, m = n/k
So, z = (n/k)*k
So, z = m * k
So, D(m, z)□

Question 4

A) *Proof*
Have any number x and y where 5 | (x-y) and x = y
So, (x-y) is divisble by 5
So, (x-x) is divisible by 5
So, ≡(x, x)□

B) *Proof*
Have any number x and y and assume ≡(x, y)
So, (x-y) is divisible by 5
So, there exists a k such that x - y = 5 * k
y - x = -(x - y) = -5k = -5 * -(x - y)
So therefore y - x is divisble by 5
So, ≡(x, y) and ≡(y, x)□

C) *Proof*
Have any number x and y and z and assume ≡(x, y) and ≡(y, z)
So x - y is divisble by 5 and y - z is divisible by 5
Thus there exists a k and j such that x - y = 5 * k and y - z = 5 * j

so $(x - y) + (y - z) = 5k + 5j$
So $x - z = = 5k + 5j$
So, $x - z = 5(k + j)$
since k and j are integers so is $k + j$
Therefore $x - z$ is divisble by 5, meaning $\equiv(x, z)\Box$