Basic Proof Methods

Direct Proof: To prove an implication $P \Rightarrow Q$, assume P and derive Q.

Assume	Goal
P	Q

Proof by Cases: To prove $(P \text{ or } Q) \Rightarrow R$, prove $P \Rightarrow R$ and $Q \Rightarrow R$.

<u>case 1</u>:

Assume	Goal
P	R

<u>case 2</u>:

Assume	Goal
Q	R

Proving "and" statements: To prove $P \Rightarrow (Q \text{ and } R)$, prove $P \Rightarrow Q$ and $P \Rightarrow R$.

Assume	Goal
P	Q

Assume	Goal
P	R

Proving the Contraposotive: To prove $P \Rightarrow Q$, it is equivalent to prove the contrapositive $((\text{not } Q) \Rightarrow (\text{not } P))$.

Assume	Goal
not Q	not$P$

Proof by Contradiction: To prove P, assume not P and prove any contradiction (Q and (not Q)).

Assume	Goal
not P	contradiction

Proving "or" statements: To prove $P \Rightarrow (Q \text{ or } R)$, procede by contradiction. Assume P, not Q and not R and derive a contradiction.

Assume	Goal
P, not Q, not R	contradiction

Proofs of "if and only if"s: To prove $P \Leftrightarrow Q$. Prove both $P \Rightarrow Q$ and $Q \Rightarrow P$.

Assume	Goal
P	Q

Assume	Goal
Q	P

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