Problem Set 2

Due Date: October 28, 2013

a) Write a subroutine solving a general (non-symmetric) system of linear algebraic equations. Given A and y, we want to find x such that $Ax = y$. We do this by minimizing the function

$$f(x) = ||A\,x - y||^2$$

via the steepest descent method. For the initial approximation, choose $x_0 = 0$. The subroutine should iterate until either the specified precision *eps* is achieved or *numit* iterations have been performed, with *numit* a user-specified (integer) parameter.

In FORTRAN, your calling sequence should be

$$dumb\_solve(a, y, n, eps, numit, x, niter, discreps)$$

In C, your calling sequence should be

$void\,dumb\_solve(double\,*a,\ double\,*y,\ int\,n,\ double\,eps,\ int\,numit,\ double\,*x,\ int\,*niter,\ double\,*discreps)$

The input parameters are:

$a(n, n)$ is the $n \times n$ matrix for the system of equations, given as a real array of size n^2. Note a will be passed in as a one-dimensional array stored by column in FORTRAN and by row in C as in the previous problem set

$y(n)$ is the right-hand side for the system of equations, given as a real array

n is the size of the system

eps is the relative accuracy to which the system is to be solved

numit is the maximum number of iterations to be performed

The output parameters are:

$x(n)$ is the solution to the system, given as a real array

niter is the number of iterations actually performed

discreps is an array of length niter, containing the discrepancies after each iteration

b) After the code is tested to your satisfaction, apply it to the system

$$A\,x = y,$$

where $A(n,n)$ is a diagonal 6×6-matrix, with $a(i,i) = 1/i^2$, $y(i) = 1$ for all $i = 1,2,3,4,5,6$, $numit = 1000$, and $eps = 10^{-6}$.

Perform the same experiment with $n = 3, 4, 10$.

Analyze the performance of the scheme. Among other things, discuss the output *discreps* and *niter* (for every case).