

$$\text{sigmoid}(x) = \sigma(x) = \frac{1}{1 + e^{-x}}$$

for very positive values of x , $\sigma(x)$ approaches 1
 for very negative values of x , $\sigma(x)$ approaches 0
 for x value close to 0, $\sigma(x)$ approaches $\frac{1}{2}$
 " $\frac{1}{1+e^0} = \frac{1}{1+1} = \frac{1}{2}$ "

$$\frac{d}{dx} \sigma(x) = \frac{d}{dx} \frac{1}{1 + e^{-x}}$$

* $\frac{1}{x} = x^{-1}$

$$= \frac{d}{dx} (1 + e^{-x})^{-1}$$

* power rule + chain rule

$$= -(1 + e^{-x})^{-2} (e^{-x})$$

derivative of the "inside" i.e. $(1 + e^{-x})$
 * remember, $\frac{d}{dx} e^x = e^x$:)

$$= \frac{e^{-x}}{(1 + e^{-x})^2}$$

reverting the negative exponent back into a fraction.

* $(1 + e^{-x})^2 = (1 + e^{-x}) \cdot (1 + e^{-x})$

* $e^{-x} = 1 \cdot e^{-x}$

$$= \frac{1}{1 + e^{-x}} \cdot \frac{e^{-x}}{1 + e^{-x}}$$

splitting up the exponent in the denominator

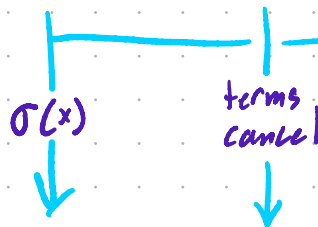
$$= \frac{1}{1 + e^{-x}} \cdot \frac{1 + e^{-x} - 1}{1 + e^{-x}}$$

strategically subtracting and adding 1

* $\frac{1 - e^{-x}}{1 - e^{-x}} = 1$

* $\frac{1}{1 + e^{-x}} = \sigma(x)$

$$= \frac{1}{1 + e^{-x}} \cdot \left(\frac{1 + e^{-x}}{1 + e^{-x}} - \frac{1}{1 + e^{-x}} \right)$$



$$\frac{d}{dx} \sigma(x) = \sigma(x) \cdot (1 - \sigma(x))$$

Therefore, we can just define the derivative of $\text{sigmoid}(x)$ to be simply:
 $\text{sigmoid}(x) \cdot (1 - \text{sigmoid}(x))$