$$\text{Sigmoid}(x) = \sigma(x) = \frac{1}{1+e^{-x}}$$

for very positive values of x, $\sigma(x)$ approaches 1
for very negative values of x, $\sigma(x)$ approaches 0
for x value close to 0, $\sigma(x)$ approaches $1/2$
" $\frac{1}{1+e^{-0}} = \frac{1}{1+1} = \frac{1}{2}$ "

$$\frac{d}{dx}\sigma(x) = \frac{d}{dx}\frac{1}{1+e^{-x}}$$

* $\frac{1}{x} = x^{-1}$

$$= \frac{d}{dx}(1+e^{-x})^{-1}$$

* power rule + chain rule

$$= -(1+e^{-x})^{-2}(e^{-x})$$

-1

derivative of the "inside" i.e. $(1+e^{-x})$
* remember, $\frac{d}{dx}e^x = e^x$:)

$$= \frac{e^{-x}}{(1+e^{-x})^2}$$

reverting the negative exponent back into a fraction.

* $(1+e^{-x})^2 = (1+e^{-x})\cdot(1+e^{-x})$
* $e^{-x} = 1 \cdot e^{-x}$

$$= \frac{1}{1+e^{-x}} \cdot \frac{e^{-x}}{1+e^{-x}}$$

splitting up the exponent in the denominator

$$= \frac{1}{1+e^{-x}} \cdot \frac{1+e^{-x}-1}{1+e^{-x}}$$

strategically subtracting and adding 1

* $\frac{1-e^{-x}}{1-e^{-x}} = 1$
* $\frac{1}{1+e^{-x}} = \sigma(x)$

$$= \underbrace{\frac{1}{1+e^{-x}}}_{\sigma(x)} \cdot \underbrace{\left(\frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}\right)}_{\text{terms cancel}}$$

$$\frac{d}{dx}\sigma(x) = \sigma(x)\cdot(1-\sigma(x))$$

$$\boxed{\text{Therefore, we can just define the derivative of sigmoid}(x) \text{ to be simply: sigmoid}(x)\cdot(1-\text{sigmoid}(x))}$$