Physics 129: Particle Physics
Lecture 9: Isospin, Strangeness and SU(3)

Sept 24, 2020

- Suggested Reading:
  - Thomson Sections 9.2-9.4
  - Griffiths 5.8-5.9
  - Perkins Sections 3.12-3.14, 4.3-4.7

Reminder: Why Isospin?

- Proton and neutron have almost the same mass and see the same nuclear force
  - Very similar masses: $(m_p = 938.205$ MeV, $m_n = 939.57$ MeV
- Natural to call them two "states" of the nucleon

$$N \equiv \begin{pmatrix} p \\ n \end{pmatrix}$$

- Similar situation arises for other hadrons
  - $m_{\pi^\pm} = 139.57$ Mev, $m_{\pi^0} = 134.97$ MeV

$$\Pi \equiv \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

- Seems natural to postulate that these particles are related by a continuous symmetery

  Rotations in the space of this symmetry transforms particles of one charge to the same species but with a different charge

Our postulates

- We already know one symmetry that puts states into multiplets: Angular Momentum (orbital or spin)
  - Postulate that our new symmetry follows the same math as angular momentum

$$\begin{aligned}[I_i, I_j] &= i\epsilon_{ijk} I_k \\ \left[I^2, I_i\right] &= 0\end{aligned}$$

  - Call the new symmetry isospin
    - Nothing to do with spin except that the commuation relations are the same
  - Since nucleon is a doublet, it would have $I = \frac{1}{2}$
  - π are a triplet, so they have $I = 1$

  Note: Until we test this hypothesis it is only a guess!

- Nuclear force between nucleons same for p and n
  - Postulate that the strong interactions are isospin symmetric

$$\begin{aligned}\left[H_{int}^{strong}, I^2\right] &= 0 \\ \left[H_{int}^{strong}, I_3\right] &= 0\end{aligned}$$

Testing our isospin postulates

- If same commutation rules as angular momentum, addition of isospin would follow the same rules as addition of angular momentum
- If isospin is a good symmetry of the strong interactions, then it is conserved
  - If we know tisospin of the inital state, we know isospin of the final state
  - Two initial particles with isospin I_1 amd I_2, have total inital between $I_1 + I_2$ and $|I_1 - I_2|$
- Which values are permitted depend on the values of I_3
- Example:
  - $\pi^+\pi^+$ scattering:
  $$|1,1\rangle\,|1,1\rangle = |2,2\rangle$$
  - $\pi^+\pi^0$ scattering:
  $$|1,1\rangle\,|1,0\rangle = a\,|2,1\rangle + b\,|1,1\rangle$$

where a and b are called the Clebsch-Gordon coefficients and satisfy $\sqrt{a^2 + b^2} = 1$

Reminder: From the uncoupled to the coupled basis

- Language comes from atomic physics ($\vec{L} \cdot S$ coupling)
  - Uncoupled basis: Specify I and I_3 for each particle:

$$\left|I^{(1)}, I_3^{(1)}\right\rangle \left|I^{(2)}, I_3^{(2)}\right\rangle$$

  - Coupled basis: $I^{(1)}$ and $I^{(2)}$ implicit, specify I^{tot}, I_3^{tot}:

$$\left|I^{tot}, I_3^{tot}\right\rangle$$

- Can construct the coupled basis using raising and lowering operators
- Eg: Two $I = 1/2$ particles:
  - Begin with "stretch state" where we know I^{tot}:

$$\left|\frac{1}{2}\frac{1}{2}\right\rangle \left|\frac{1}{2}\frac{1}{2}\right\rangle = |1, 1\rangle$$

- Apply lowering operator $I_- = I_-^{(1)} + I_-^{(2)}$:

$$S_- \left|\frac{1}{2}\frac{1}{2}\right\rangle \left|\frac{1}{2}\frac{1}{2}\right\rangle = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle + \left|\frac{1}{2}, \frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle$$

- But in the coupled basis this is

$$\begin{aligned} S_- |1, 1\rangle &= \sqrt{(1)(2) - (1)(0)}\, |1, 0\rangle \\ &= \sqrt{2}\, |1, 0\rangle \end{aligned}$$

- Putting this together

$$|1, 0\rangle = \frac{1}{\sqrt{2}} \left(\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle + \left|\frac{1}{2}, \frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle \right)$$

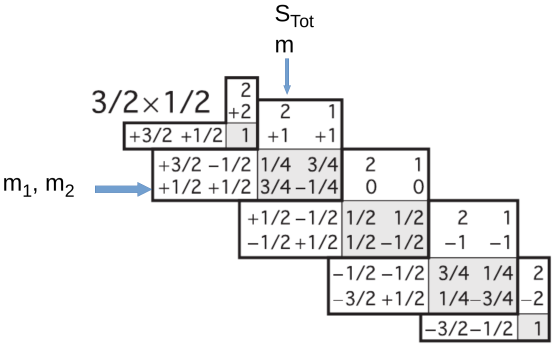
- Do it again

$$|1, -1\rangle = \left|\frac{1}{2}\frac{1}{2}\right\rangle \left|\frac{1}{2}\frac{1}{2}\right\rangle$$

- Final state must be orthognal to the other three

$$|0, 0\rangle = \frac{1}{\sqrt{2}} \left(\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle - \left|\frac{1}{2}, \frac{1}{2}\right\rangle \left|\frac{1}{2}, \frac{1}{2}\right\rangle \right)$$

Reading Clebsch-Gordon Tables



- Tedious to construct coupled basis
- Don't need to keep doing the same calculation
  - Results tabulated in "Clebsch-Gordon" tables
  - See, eg, PDG web site reviews under *Mathematical Tools*
- Example above for combining spin-3/2 with spin-1/2
  - Implied $\surd$ for all entries
  - Minus sign outside $\surd$

Back to our story: Using Isospin to relate rates

- Decays: Fermi's Golden Rule

$$\frac{d}{dt}\left(P_{a\to k}\right) \equiv W_{ka} = 2\pi\lambda^2 \left|H'_{ka}\right|^2 \mathcal{D}(E_k)$$

- Cross Sections: We'll see next week that scattering cross sections follow a similar rule

$$\sigma \propto \int |M_{if}|\, d\Omega$$

where $M_{if} = \langle\psi|\, H_{int}\, |\psi\rangle$

- Strong interactions are invariant under isospin and are independent of I_3
- If I and I_3 are symmetries of H_{int} (as for the strong interaction), then $|H'_{ka}|$ and M_{if} depend only on I and not on I_3
  - Number of independent matrix elements depends only on how many values of I are possible

Example: πN scattering

- $\left|\frac{1}{2},\frac{1}{2}\right\rangle|1,1\rangle \Rightarrow I_{Tot}=\frac{3}{2}$ or $\frac{1}{2}$ so two indep matrix elements:

$$\mathcal{M}_{\frac{1}{2}} \equiv \left\langle \frac{1}{2}\right| H \left|\frac{1}{2}\right\rangle \quad \mathcal{M}_{\frac{3}{2}} \equiv \left\langle \frac{3}{2}\right| H \left|\frac{3}{2}\right\rangle$$

(Since I conserved $\left\langle \frac{1}{2}\right| H \left|\frac{3}{2}\right\rangle = \left\langle \frac{3}{2}\right| H \left|\frac{1}{2}\right\rangle = 0$)

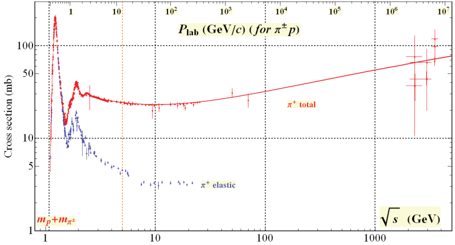
- Examples of decomposition

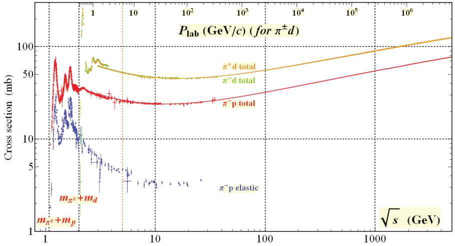
$$\begin{aligned} p\pi^+ &= \left|\frac{3}{2}\,\frac{3}{2}\right\rangle \\ p\pi^0 &= \sqrt{\frac{2}{3}}\left|\frac{3}{2}\,\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}\left|\frac{1}{2}\,\frac{1}{2}\right\rangle \end{aligned}$$

- From Clebsch tables (you will show this on next week's homework):

$$\begin{aligned} \sigma(\pi^+ p \to \pi^+ p) &\sim \left|\mathcal{M}_{\frac{3}{2}}\right|^2 \\ \sigma(\pi^+ n \to \pi^+ n) &\sim \left|\frac{1}{3}\mathcal{M}_{\frac{3}{2}} + \frac{2}{3}\mathcal{M}_{\frac{1}{2}}\right|^2 \\ \sigma(\pi^- p \to \pi^0 n) &\sim \left|\frac{\sqrt{2}}{3}\mathcal{M}_{\frac{3}{2}} - \frac{\sqrt{2}}{3}\mathcal{M}_{\frac{1}{2}}\right|^2 \end{aligned}$$

πN scattering (continues)

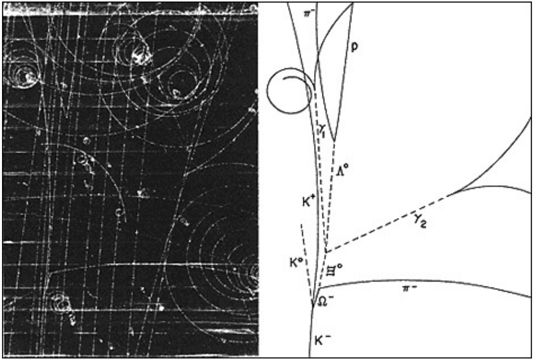




- Large bumps: "resonances"
- Eg: in $p\pi^+$ scattering near 1236 MeV
- This is the Δ responance
  - Since we see it in $p\pi^+$ it must have $I = 3/2$
  - Four states : Δ^{++},Δ^{+}, Δ^{0},Δ^{-}
  - There is NO Δ^{--}
- Size in $p\pi^-$ smaller by ratop expected from Clebsch Gordon coefficients
- Can also look at the other $N\Pi$ scattering rates

  Evidence that isospin is a good symmetry of strong interactions
- Many additional checks using both scattering and decays lead to the same conclusion

Strangeness (I)



- In 1950's a new class of hadrons seen
  - Produced in πp interaction via Strong Interaction
  - But travel measureable distance before decay, so decay is weak

Why should this happen? There must be quantum number conserved in multihadron production that cannot be conserved in single hadron decay.

Strangeness (II)

- Example:

$$\pi^- p \to \Lambda^0 K^0$$

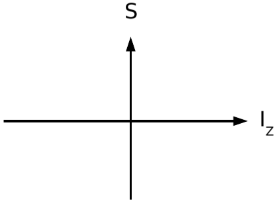
  - $\Lambda^0 \to p\pi^-$ with lifetime $\tau = 2.6 \times 10^{-10}$ sec
  - $K^0 \to \pi^+\pi^-$ with lifetime $\tau = 0.8958 \times 10^{-10}$ sec
  - Thus both decay weakly
- But production rate tells us that production process is strong
- Assign a new quantum number called strangeness to the Λ and K^0
  - Λ^0 and K^0 must have opposite strangeness
- By convention Λ has $S = -1$ and K^0 has $S = 1$ (an unfortunate choice, but we are stuck with it)
- Strangeness is an additive quantum number
  - Corresponds to a continuous symmetry

Putting Strangeness and Isospin together

- Look for particles with similar mass and the same spin and parity
- Strange hadrons tend to be heavier than non-strange ones but we can still find patterns

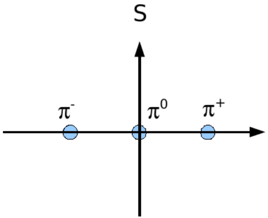
J^P	Name	Mass (MeV)
0^-	$\pi^\pm$	140
	$K^\pm$	494
1^-	$\rho^\pm$	775
	$K^{*\pm}$	892

- Associate strange particles with the isospin multiplets

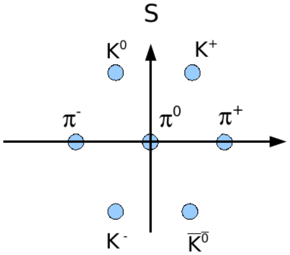


Adding Particles to the Axes: Pseudoscalar Mesons

- Pions have $S = 0$
- Three charge states $\Rightarrow I = 1$
- Draw the isotriplet:



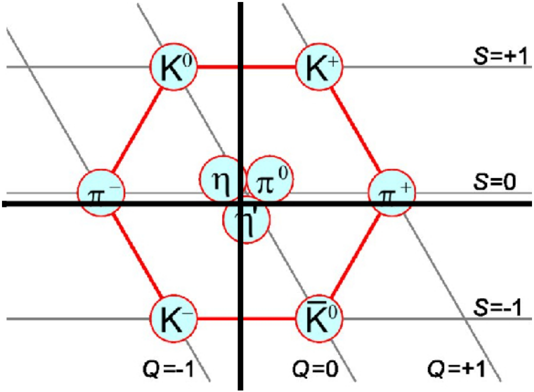
- From $\pi^- p \to \Lambda^0 K^0$ define K^0 to have $S = 1$
- If strangeness an additive quantum number, $\exists$ anti-K^0 with $S = -1$
- Also, K^+ and K^- must be particle-antiparticle pair: (eg from $\phi \to K^+ K^-$)



But this is not the whole story

There are 9 pseudoscalar mesons (not 7)!

The Pseudoscalar Mesons



- Will try to explain this using group theory

Introduction to Group Theory (via SU(2) Isospin)

- The algebra we are familiar with from angular momentum described mathematically by the group SU(2)
- Fundamental SU(2) representation: a doublet

$$\chi = \begin{pmatrix} u \\ d \end{pmatrix} \quad \text{so} \quad u = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad d = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Infinitesmal generators of isospin rotations:

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- τ matrices satisfy commutation relations

$$\left[\frac{1}{2}\tau_i, \frac{1}{2}\tau_j\right] = \frac{1}{2} i\epsilon_{ijk}\tau_k$$

  Commutation relations define the SU(2) algebra
- $\exists$ higher representations: $N \times N$ matrices with $N = 2I + 1$
- Also, there is an operator that commutes with all the τ's:

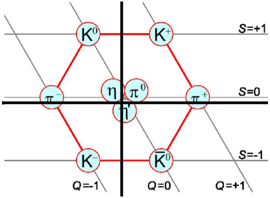
$$I^2 = \left(\frac{1}{2}\vec{\tau}\right)^2 = \frac{1}{4}\Sigma_i \; \tau_i^2$$

  and there are raising and lowering operators

$$\tau_\pm = \frac{1}{2}\left(\tau_1 \pm i\tau_2\right)$$

Raising and Lowering Operators

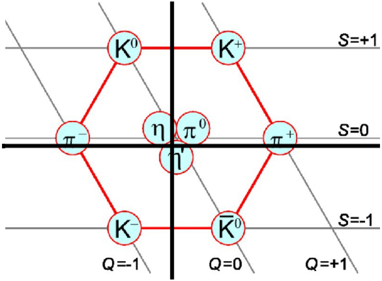
- In Quantum Mechanics, can start with state $|J, J_z = J\rangle$ and construct all other states of same J using lowering operator J_-
- Similarly with Isopsin, if start with $\pi^+ = |I = 1, I_z = 1\rangle$ and construct π^0 and π^- using lowering operator τ_-
- If we introduce strangeness, can we navigate among all the mesons in the same way?



- Need second lowering operator to navigate in the S direction

$\Rightarrow$ Extend group from SU(2) to SU(3)

Group Theory Interpretation of Meson Spectrum



- Particles with same spin, parity and charge conjugation symmetry described as multiplet
  - Different I_z and S
  - Sometimes instead use y-axis that is $Y = B + S$ (see slide 21 for why)

- Raising and lowering operators to navigate around the multiplet
- Gell Man and Zweig: Patterns of multiplets explained if all hadrons were made of quarks
  - Mesons: $q\bar{q}$ $\quad 3 \otimes \bar{3} = 1 \oplus 8$
  - Baryons: qqq $\quad 3 \otimes 3 \otimes 3 = 1 \oplus 8 \oplus 8 \oplus 10$
- In those days, 3 flavors (extension to more flavors discussed later this semester)
- Are the quarks real or just constructs? More on that in a couple of weeks

Defining SU(3)

- SU(3): All unitary transformations on 3 component complex vectors without the overall phase rotation (U(1))

$$U^\dagger U = UU^\dagger = 1 \qquad \det U = 1$$

$$U = exp\left(i \sum_{a=1}^{8} \lambda_a \theta_a / 2\right)$$

- The fundamental representation of SU(3) are 3×3 matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- Commutation relations:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$$

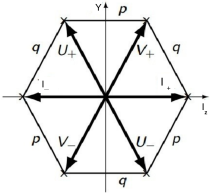
where $f_{123} = 1$, $f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$, $f_{156} = f_{367} = -\frac{1}{2}$ and $f_{458} = f_{678} = \sqrt{3}/2$.

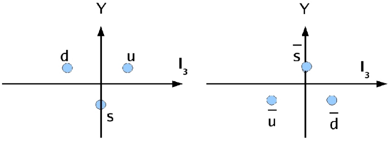
SU(3) Raising and Lowering Operators

- SU(3) contains 3 SU(2) subgroups embedded in it

$$\begin{array}{llll} \text{Isospin}: & \lambda_1 & \lambda_2 & \lambda_3 \\ \text{U}-\text{spin}: & \lambda_6 & \lambda_7 & \sqrt{3}\lambda_8 - \lambda_3 \\ \text{V}-\text{spin}: & \lambda_4 & \lambda_5 & \sqrt{3}\lambda_8 + \lambda_3 \end{array}$$

- For each subgroup, can form raising and lowering operators
- Any two subgroups enough to navigate through multiplet



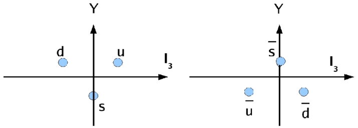


- Fundamental representation: A triplet (our quarks)
- Define group structure starting at one corner and using raising and lowering operators
- Define "highest weight state" ψ as state where both $I_+\psi = 0$ and $V_+\psi = 0$
- Quarks (u, d, s) have $p = 1,\ q = 0$ while antiquarks $(\overline{u}, \overline{d}, \overline{s})$ have $p = 0,\ q = 1$

$$\begin{array}{lcl} (V_-)^{p+1}\phi_{max} & = & 0 \\ (I_-)^{q+1}\phi_{max} & = & 0 \\ \text{structure}: & (p, q) & \end{array}$$

Some comments on quarks

- Group theory interpretation is clear



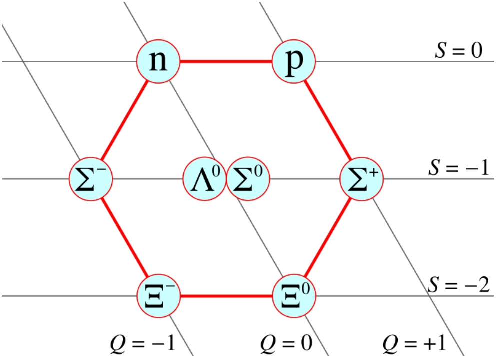
- They are the fundamental representation of our group
- But their properties as particles are a bit odd
  - Spin-1/2 (that's fine)
  - Charge (very wierd):

quark	charge
u	2/3
d	-1/3
s	-1/3

  - Baryon number 1/3 (very wierd)
- Next Tuesday, we'll see how to "build" the hadrons from the quarks
- But for now, let's continue our review of the hadrons

Baryon Octet ($J = \frac{1}{2}^-$)

- Begin with the lightest baryons: the proton and neutron
- Construct the remainder using SU(3) raising and lowering operators



- Note: because they are the lightest baryons, they decay weakly (except proton which is stable)

Strangeness and I_Z

- We've already seen that within an isospin multiplet, different I_z have different charge
- Can generalize this observation for all light quark (u,d,s) multiplets:

$$Q = I_z + \frac{B+S}{2}$$

  Define hypercharge $Y \equiv B + S$

- This is called the Gell Mann-Nishijima Eq
- Note: Because Q depends on I_3, EM interactions cannot conserve isospin, but do conserve I_3
  - Analogous to the Zeeman effect, where a B field in z direction destroys conservation of angular momentum but leaves J_z as a good quantum number
- $\alpha = 1/137$ while $\alpha_S \approx 1$.
  - Effects of isospin non-conservation are small and can be treated as perturbative correction to strong interaction