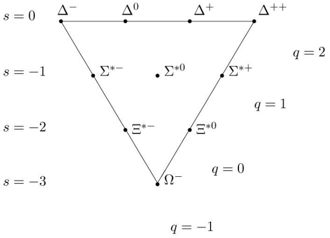
Physics 129: Particle Physics
Lecture 19: Introduction to QCD:
$e^+e^- \to hadrons$

Oct 29, 2020

- Suggested Reading:
  - Thomson 10.1-10.7
  - Griffiths 8.2, 9.1-9.5

Why Color (I)

Imposition of Fermi Statistics



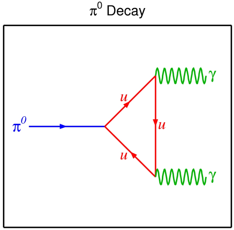
$\Delta^{++} = uuu$: Identical particles

- spin$=3/2$: Symmetric under interchange
- s-wave ($\ell = 0$): Symmetric under interchange
- Need another degree of freedom to antisymmetrize

Need at least 3 possible states

to antisymmetrize 3 objects

$\pi^0 \to \gamma\gamma$

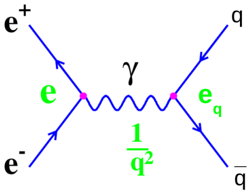


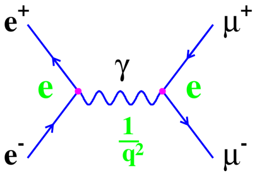
Decay process through an internal quark loop

$$\Gamma \propto N_C^2 (Q_U^2 - Q_d^2)^2$$

Consistent with 3 colors

Why Color (II): $e^+e^- \rightarrow hadrons$



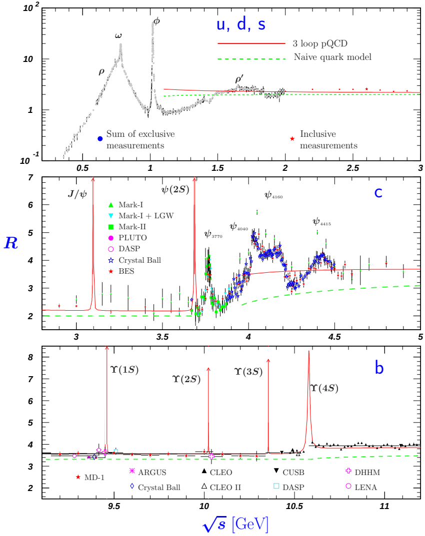


- Describe process $e^+e^- \rightarrow hadrons$ as $e^+e^- \rightarrow q\overline{q}$ where q and $\overline{q}$ turn into hadrons with probability=1
- Same Feynman diagram as $e^+e^- \rightarrow \mu^+\mu^-$ except for charge. To lowest order (no QCD corrections)

$$R = \frac{\sigma(e^+e^- \rightarrow hadrons)}{e^+e^- \rightarrow \mu^+\mu^-} = N_C \sum_q e_q^2$$

where N_C counts number of color degrees of freedom and sum is over all quark species kinematically allowed

Reminder: $e^+e^- \to hadrons$ Measurement of R



$$R \equiv \frac{\sigma(e^+e^- \to hadrons)}{e^+e^- \to \mu^+\mu^-} = N_C \sum_q e_q^2$$

where N_C is number of colors

- Below $\sqrt{s} \sim 3.1$ GeV, $R = 2$

  Only u, d, s quark-antiquark pairs can be created

$$\sum_q e_q^2 = \left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{6}{9} = \frac{2}{3} \Rightarrow N_c = 3$$

- Above 3.1 GeV, R increases by $3(\frac{2}{3})^2 = \frac{4}{3}$
  - Pass threshold for producing a new quark with $e_1 = \frac{2}{3}$: Charm
- Above 9.4 GeV, R increases by $3(\frac{-1}{3})^2 = \frac{1}{3}$
  - Pass threshold for producing a new quark with $e_1 = \frac{1}{3}$: Bottom

From Color To QCD (I)

- R tell us $\exists$ 3 colors, but doesn't tell us anything about the color force.
- Theory of Strong Interactions QCD developed in analogy with QED:
  - Assume color is a continuous rather than a discrete symmetry
  - Describe our fundamental fermion fields as a 3-vector in color space

$$\psi = \begin{pmatrix} \psi_r \\ \psi_b \\ \psi_g \end{pmatrix}$$

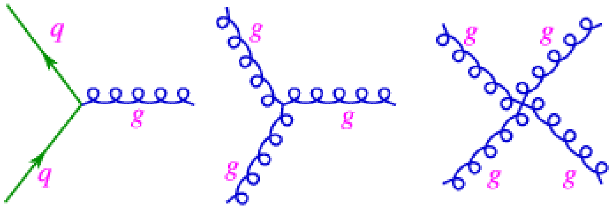
  - Let's take SU(3) as our the candiate for the group defining this 3-space

$$\psi'(x) = e^{i\lambda^i \alpha_i / 2}$$

  where the λ^i are the 8 SU(3) matrices we already know

- QCD constructed in anaology with QED
  - But the color charge respresented by a 3-component matrix while electric charge just a number
  - That means some commutators that were zero for QED are not zero for QCD leading to more complex interactions
  - One example: gluons have color while photons don't have charge

The QCD Feynman Diagrams



- qqg vertex looks just like $qq\gamma$ with $e \to g$
  - Strength of coupling at vertex has color factor g rather than EM factor e
- Three and four gluon vertices
  - Three gluon coupling strength gf^{abc}
  - Four gluon coupling strength $g^2 f^{xac} f^{xbd}$

where

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2}\right] = i f_{abc} \frac{\lambda_c}{2}$$

and $f_{123} = 1$, $f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$, $f_{156} = f_{367} = -\frac{1}{2}$ and $f_{458} = f_{678} = \sqrt{3}/2$.

The Running of α_s (I)

- In calculating R, assumed that strong interactions didn't significantly affect the cross section; derived this using impulse approximation
  - Quarks act as if they are free during the EM interaction
- Seems odd since α_s is large, as measured via the decay widths of strong decays
- Great success of QCD is ability to explain why strong interactions are strong at low q^2 but quarks act like free particles at high q^2
- Coupling constant α_s *runs*; It is a function of q^2

Low q^2	α_s large	"confinement"
High q^2	α_s small	"asympotic freedom"

- This running is not unique to QCD; Same phenomenon in QED
  - But α runs more slowly and in opposite direction
  - eg at $q^2 = M_z^2$, $\alpha(M_Z^2) \sim 1/129$
- Running of the coupling constant is a consequence of *renormalization*
- Incorporation of infinities of the theory into the definitions of physical observables such as charge, mass

The Running of α_s (II)

- QED and QCD relate the value of the coupling constant at one q^2 to that at another through renormalization procedure

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\left(\frac{Q^2}{\mu^2}\right)}$$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \frac{\alpha_s(\mu^2)}{12\pi}(33 - 2n_f)\log\left(\frac{Q^2}{\mu^2}\right)}$$

- In the case of QED, the natural place to measure α is clear: $Q^2 \to 0$
- Since α_s is large at low Q^2, no obvious μ^2 to choose
- It is customary (although a bit bizarre) to define things in terms of the point where α_s becomes large

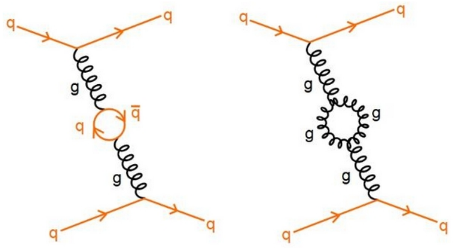
$$\Lambda^2 \equiv \mu^2 \exp\left[\frac{-12\pi}{(33 - 2n_f)\,\alpha_s(\mu^2)}\right]$$

- With this definition

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\log(Q^2/\Lambda^2)}$$

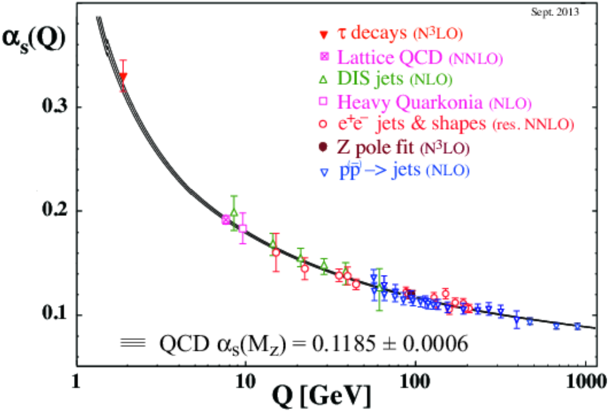
  - For $Q^2 \gg \Lambda^2$, coupling is small and perturbation theory works
  - For $Q^2 \sim \Lambda^2$, physics is non-perturbative
- Experimentally, $\Lambda \sim$ few hundred MeV

Why do coupling constants run?



- Higher order loop corrections in propagator
  - Photon propagator only has fermion loops
  - Gluon propagator also has gluon loops
  - Fermion and gluon loop terms opposite have opposite sign
  - Hence running depends on number of flavors
- Must perform renormalization to remove unphysical infinities

Measurements of α_s



We'll talk more about the e^+e^- measurements later today

Implications of the Running of α_s

- α_s small at high q^2:

  High q^2 processes can be described perturbatively

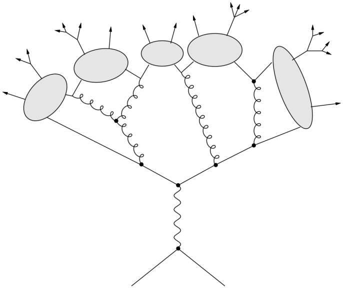
  - For DIS and $e^+e^- \to hadrons$, the lowest order process is electromagnetic or weak
  - Higher order perturbative QCD corrections can be added to the basic process
  - Processes we will discuss later (such as pp collisions), the lowest order process will be QCD
  - Again, can include perturbative corrections

- α_s large at low q^2:

  Quarks dress themselves as hadrons with probability=1 and on a time scale long compared to the hard scattering

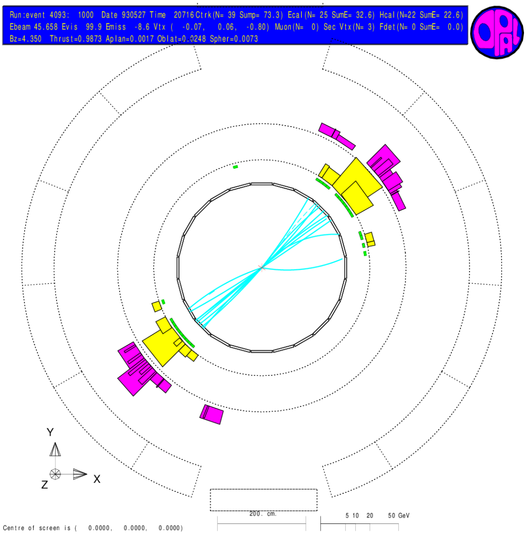
  - Describe dressing of final quark and antiquark (and gluons if we consider higher order corrections) into a "Fragmentation Function"
  - Process of quarks and gluons turning into hadrons is called *hadronization*

Hadronization as a Showering Process



- Similar description to the EM shower we talked about in lecture 5
  - Quarks radiate gluons
  - Gluons make $q\overline{q}$ pairs, and can also radiate gluons
- Must in the end produce color singlets
  - Nearby q and $\overline{q}$ combine to form clusters or hadrons
  - Clusters or hadrons then can decay
- Warning: Picture does not make topology of the production clear
  - Gluon radiation peaked in direction of initial partons
  - Expect collimated "jets" of particles following initial partons

What do Jets Look Like?



Opal Experiment at LEP

Discovery of Jet Structure: Strategy

- While jets are clearly visible by eye at high energy, not the case for original experiments at low $\sqrt{s}$
- Discovery of jet structure required a statistical analysis using a global metric
  - Is the event spherical (as phase space would predict) or does it have a defined axis (the directions of the initial quark and anti-quark)?
- Define Sphericity Tensor

$$S_{ab} = \frac{\sum_{i=1}^{N} p_{ia} p_{ib}}{\sum_{i=1}^{N} p_i^2}$$

where a and b label x, y and z axes and the sum over i is a sum over all the (charged) particles in the event

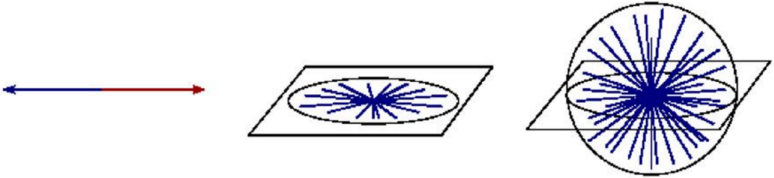
- This looks just like a moment of inertia tensor
  - The relative value of the 3 eigenvalues tell us about the shape
    - Spherical events have 3 eigenvalues of similar size
    - Pancakes have 2 eigenvalues of moderate size and 1 small one
    - Cigars have 2 small eigenvalues and 1 large one

Eigenvalues of the Sphericity Tensor (I)

- From previous page: Sphericity Tensor

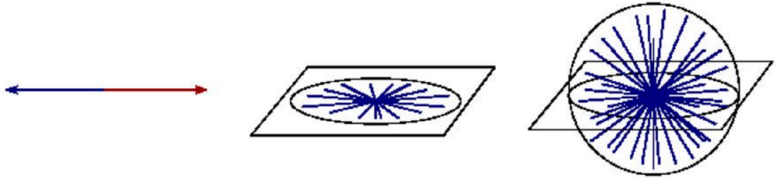
$$S_{ab} = \frac{\sum_{i=1}^{N} p_{ia} p_{ib}}{\sum_{i=1}^{N} p_i^2}$$

- S_{ab} has 3 eigenvalues $\lambda_1 \geq \lambda_2 \geq \lambda_3$ with $\lambda_1 + \lambda_2 + \lambda_3 = 1$



$\lambda_2 \sim \lambda_3 \sim 0; \lambda_1 \sim 1$ $\lambda_3 \sim 0; \lambda_2 \sim \lambda_1$ $\lambda_1 \sim \lambda_2 \sim \lambda_3$

Eigenvalues of the Sphericity Tensor (II)



$\lambda_2 \sim \lambda_3 \sim 0; \lambda_1 \sim 1$ $\quad$ $\lambda_3 \sim 0; \lambda_2 \sim \lambda_1$ $\quad$ $\lambda_1 \sim \lambda_2 \sim \lambda_3$

- Define the sphericity S

$$S = \frac{3}{2}(\lambda_2 + \lambda_3) = \quad 0 \leq S \leq 1$$

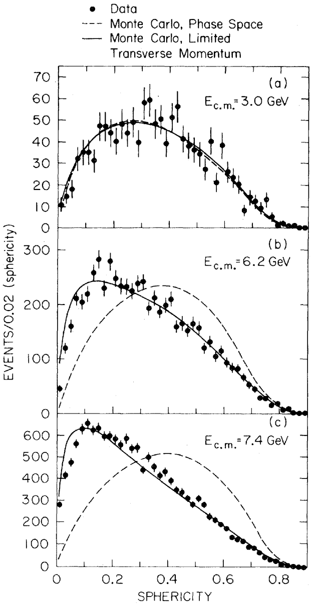
$S = 0 \Rightarrow$back to back jets; $S = 1 \Rightarrow$spherical distribution

- And the aplanarity

$$A = \frac{3}{2}\lambda_3 \quad 0 \leq A \leq \frac{1}{2}$$

$A = 0 \Rightarrow$all particvles in one plane; $A = \frac{1}{2} \Rightarrow$like $S = 1$

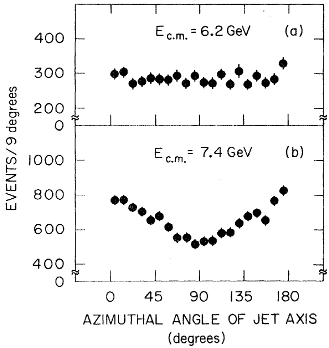
Emergence of Jets



Phys. Rev. Lett. **35**, 1609 (1975)

- Data collected by Mark-I experiment at SPEAR e^+e^- collider
- Study sphericity distribution for different E_{cm}
- Compare to a jet model and a phase space model
- As E_{cm} increase, data becomes consistent with jet model
  - Not consistent with phase space

Angular dependence of jet axis (same paper)



- Assume jet axis provides estimate of direction of outgoing quarks
- Since quarks have spin-$\frac{1}{2}$, distribution in polar angle

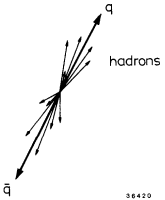
$$\frac{dN}{d\cos\theta} = 1 + \cos^2\theta$$

- But Mark-I had limited $\cos\theta$ coverage!
- But, if incoming beams transversely polarized, there is also a ϕ dependence

$$\frac{dN}{d\cos\theta} = 1 + \cos^2\theta + P_+P_-\sin^2\phi\cos 2\phi$$

- Turns out that beams at SLAC were transversely polarized with polarization dependent on E_{cm}
- Angular dependence consistent with expectations for spin-1/2 Dirac particles

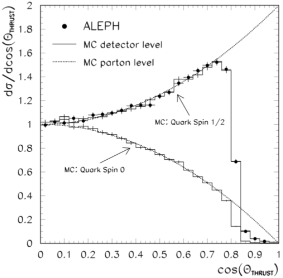
An alternative event shape variable: Thrust

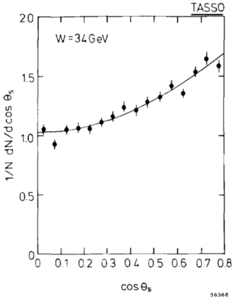


- Sphericity quadratic in p
  - Sensitive to hadronization details
- Linear alternative: Thrust axis

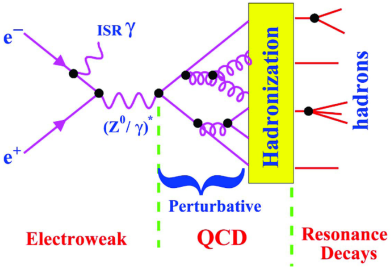
$$T = \max \frac{\sum |\vec{p_i}| \cdot \hat{n_T}}{\sum |\vec{p_i}|}$$

- Both choices appear to track quark direction well
  - Again, clear evidence for spin-$\frac{1}{2}$ quarks





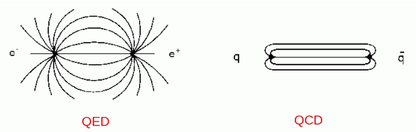
QCD at many scales



- Impulse approximation
  - Short time scale hard scattering (EM interaction in this case)
  - Perturbative QCD corrections (will discuss next time)
  - Long time scale hadronization process
- Approach to the hadronization:
  - Describe distributions individual hadrons statistically
  - Collect hadrons together to approximate the properties of the quarks and gluons they came from

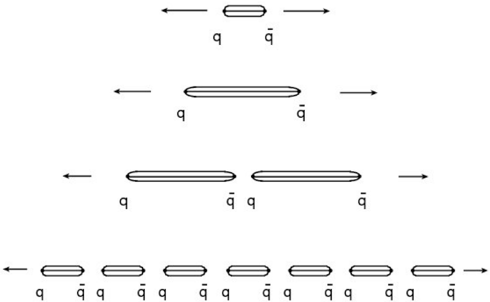
Describe non-perturbative effects using a phenomonological model

Another Way of Thinking About Hadronization



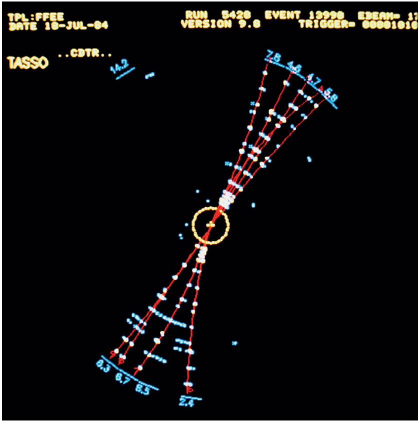
- q and $\overline{q}$ move in opposite directions, creating a color dipole field
- Color dipole looks different from familiar electric dipole:
  - Confinement: At low q^2 quarks become confined to hadrons
  - Scale for this confinement, hadronic mass scale: $\Lambda =$ few 100 MeV
  - Coherent effects from multiple gluon emission shield color field far from the colored q and $\overline{q}$
  - Instead of extending through all space, color dipole field is flux tube with limited transverse extent
- Gauss's law in one dimensional field: E independent of x and thus $V(x_1 - x_2) = k(x_1 - x_2)$ where k is a property of the QCD field (often called the "string tension")
  - Experimentally, $k = 1$ GeV/fm = 0.2 GeV^2
  - As the q and $\overline{q}$ separate, the energy in the color field becomes large enough that $q\overline{q}$ pair production can occur
  - This process continues multiple times
  - Neighboring $q\overline{q}$ pairs combine to form hadrons

Color Flux Tubes



- Particle production is a stocastic process: the pair production can occur anywhere along the color field
- Quantum numbers are conserved locally in the pair production
- Appearence of the q and $\overline{q}$ is a quantum tunneling phenomenon: $q\overline{q}$ separate eating the color field and appear as physical particles

Jet Production



- Probability for producing pair depends quark masses

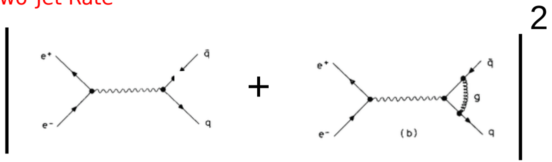
$$\mathrm{Prob} \propto e^{-m^2/k}$$

  relative rates of popping different flavors from the field are
  $u : d : s : c = 1 : 1 : 0.37 : 10^{-10}$

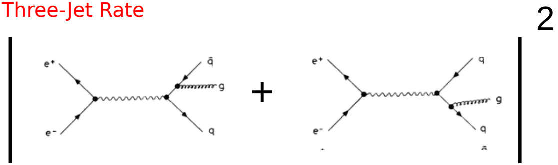
- Limited momentum transverse to $q\overline{q}$ axis
  - If q and $\overline{q}$ each have tranverse momentum $\sim \Lambda$ (think of this as the sigma) the mesons will have $\sim \sqrt{2}\Lambda$
  - Meson transverse momentum (at lowest order) independent of qq center of mass energy
  - As E_{cm} increases, the hadrons collimate: “jets”

QCD corrections to $e^+e^- \rightarrow hadrons$

Two-Jet Rate



Three-Jet Rate



- Two- and Three-jet rates separately diverge
- Sum of the two converge (see next page)
- Can only define sensible three-jet rate with a cutoff in 3^{rd} jet energy

First Order QCD: Jet rates

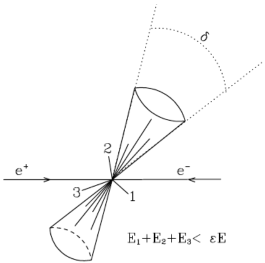
- Using gluon mass to regularize:

$$
\begin{aligned}
&\text{2 jet}: \quad \sigma_0 (1 + \tfrac{\alpha_s}{2\pi}\tfrac{4}{3}\left\{-\ln^2\left(\tfrac{m_g}{Q}\right) - 3\ln\left(\tfrac{m_g}{Q}\right) + \tfrac{\pi^2}{3} - \tfrac{7}{2}\right\} \\
&\text{3 jet}: \quad \sigma_0 \tfrac{\alpha_s}{2\pi}\tfrac{4}{3}\left\{\ln^2\left(\tfrac{m_g}{Q}\right) + 3\ln\left(\tfrac{m_g}{Q}\right) - \tfrac{\pi^2}{3} + 5\right\} \\
&\text{Sum}: \quad \sigma_0 \left(1 + \tfrac{\alpha_s}{\pi}\right)
\end{aligned}
$$

(see Halzen & Martin pg 244)

- Cancellation of divergences not an accident
- Occurs throughout gauge theories (QED as well as QCD)
- Cancellation of infrared divergences described using general theorm by Kinoshita, Lee and Nauenberg
- In practice, divergences in 2 and 3 jet rates NOT a problem
  - Can only distinguish two jets if they are separated in angle and both jets have measurable energy.

How to define a 2-jet event: Sterman-Weinberg



- Classify as a two-jet event if we can find two cones of opening angle δ that contain all but at most a fraction ϵ of the total energy of the event
  - So, the classification depends on the values of δ and ϵ chosen
- In QCD theory, the jets are defined in terms of the partons of the calculation.
- In experiment, defined in terms of final state particles
  - Or in terms of proxies for these particles (eg energy clusters in a calorimeter)

Calculating the 3-jet rate in region away from singularity

- Define the energy fractions of the 3 jets

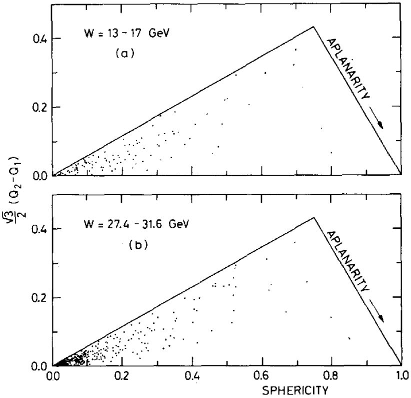
$$x_q = \frac{2E_q}{\sqrt{s}}; \quad x_{\overline{q}} = \frac{2E_{\overline{q}}}{\sqrt{s}}; \quad x_g = \frac{2E_g}{\sqrt{s}};$$

- Conservation of energy: $x_q + x_{\overline{q}} + x_g = 2$
- In practice, don't know which is the q, $\overline{q}$, g
- Order them in momentum

$$\frac{d\sigma_{3\ jet}}{dx_1 dx_2} = \sigma_0 \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

- Note: σ diverges if $x_1 \to 1$ or $x_2 \to 1$
  - $\vec{p_3} \mid\mid \vec{p_1} \Rightarrow x_2 \to 1$: Collinear divergence
  - $x_3 \to 0 \Rightarrow x_1, x_2 \to 1$: Soft Divergence

Searching for 3 jet events using the Sphericity Tensor



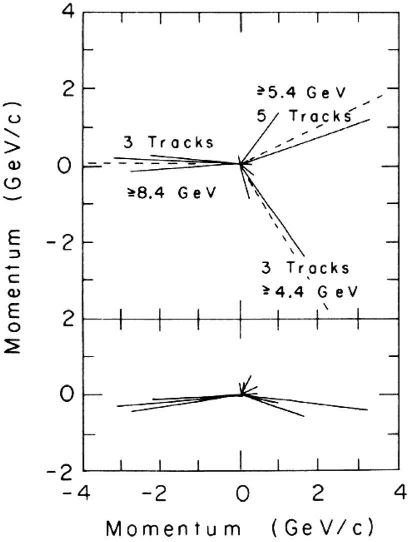
$$\lambda_1 + \lambda_2 + \lambda_3 = 1$$

Sphericity $S = \frac{3}{2}(\lambda + 2 + \lambda_3)$

Aplanarity $A = \frac{3}{2}\lambda_3$

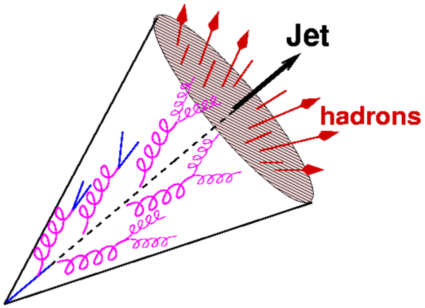
- As the energy increases, the narrowing of the jets allows us to look for cases of wide angle gluon emission (3-jet events)
- QCD brem cross section diverges for colinear gluons or when the gluon momentum goes to zero
  - But that is the case where we can't distinguish 2 and 3 jet events anyway
  - Total cross section is finite (QCD corrections to R)
- Can use the sphericity tensor to search for 3-jet events
- Data on right was first experiment evidence for gluon bremstrahlung off quarks
  - Discovery of the gluon

Example of a 3-Jet event



You will get to try this out with some simulated data on this week's HW

Jets in high energy collisions

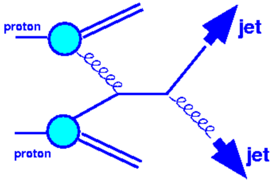


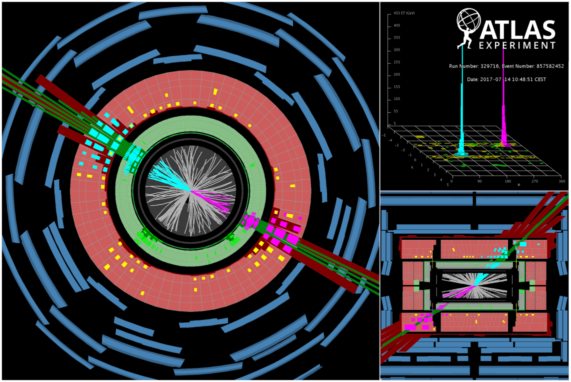
- Final states with quarks or gluons will always result in jets
- As jet energy increases, these jets are easier to "see"
- Can reconstruct the direction and energy of the intial quark or gluon through its hadron products
- This strategy is calle "jet finding"

Jet Finding Algorithms

- Shape variables like Thrust have advantage that they allow tests with minimal sensitivity to hadronization
- But don't allow us to study multijets well
- Need an algorithm to decide how many jets we have and associate particles with the jets
  - Algorithm will have some parameter to handle the infrared divergence (eg a cut-off)
- Two basic types of algorithm:
  - Geometric algorithms (aka cone algorithms):
    - Cluster based on angular separation. Define in terms of a cone-size (eg the δ of Sterman-Weinberg)
  - Recombination algorithms:
    - Find particles close together in a momentum-based metric and replace them with the sum of their four-momenta
- This remains an active area of study

An example from proton-proton collisions





- Parton from one proton collides with parton from other proton
  - Outgoing partons turn into jets of hadrons at wide angle with beamline
  - Remains of the two protons will turn into streams of particles heading largely in beam direction