# Physics 129: Particle Physics
# Lecture 16: The Structure of the Proton (I)

Oct 20, 2020

- Suggested Reading:
  - Thomson 7.1-7.4, 8.2
  - Griffiths 8.3-8.5
- I've changed the order of some topics in syllabus: bCourses has been updated to reflect this

  Reminder: Quiz #2 in progress

# The Proton is Not a Point-like Particle

- Quark model says $p$ consists of 3 quarks
  - Are they real?
- Gyromagnetic moment $g_p = 5.586$ is far from the Dirac value of 2 that holds for pointlike spin-$\frac{1}{2}$ particles
  - Pattern of baryon magnetic moments can be explained using quark model with fraction charges, fitting for quark masses
- Size of nucleus consistent with nucleons of size $\sim 0.8$ fm

To study structure of the proton, will use scattering techniques

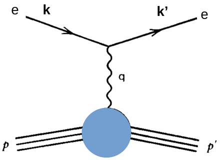
Similar idea to Rutherford's initial discover of the nucleus

# Choice of probe

- Want to study proton structure through measurements of scattering cross section
- Easiest to interpret if scattered particle ("the probe") is pointlike
- That means leptons
  - Initial measurements made using $e^-$
    - Electron beam incident on stationary target
  - Use of $\nu$ probes provides additional useful information
    - We'll talk about that on Thursday
  - High statistics measurements used the $ep$ collider Hera in Germany
    - Collider allows higher center of mass energy and hence can probe smaller length-scales

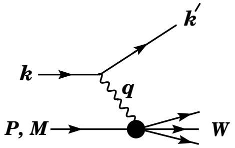
# What do we measure?

## Elastic Scattering




- $e^-$ with initial 4-monentum $k^\mu$ scatters to final moment $k'^\mu$
- Proton stays together: 4 momenta satisfy $P^2 = P'^2 = m_p^2$
- Cross section becomes small as $q^2$ becomes large

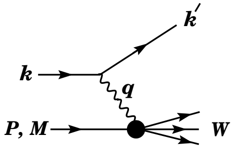
## Inelastic Scattering




- $e^-$ with initial 4-monentum $k^\mu$ scatters to final moment $k'^\mu$
- Proton breaks up: multiple particles in final state
- Invariant mass of outgoing state larger than that of proton (energy-momentum transferred from electron)

- Kinematics determined from quantities associated with electron alone:
  - Elastic: incoming $E$ and direction, outgoing angle enough
  - Inelastic: need outgoing energy as well
- Electron is a Dirac particle, so the current is $\overline{\psi}\gamma^\mu\psi$
- Photon proagator remains $-i\frac{g_{\mu\nu}}{q^2}$
- Proton *not* a Dirac particle so we can't calculate its current
  - But we know it must be a Lorentz tensor

# Writing Lorentz invariants as functions of lab frame variables




- $W$ is the invariant mass of the hadronic system
  - $W \equiv m_p$ for elastic scattering
- In lab frame:

$$
\begin{aligned}
P &= (M, 0, 0, 0) \\
k &= (E, 0, 0, E) \\
k' &= \left(E', E' \sin\theta, 0, E' \cos\theta\right)
\end{aligned}
$$

  where $M = m_p$ for $e^- p$ scatttering
- In any frame, $k = k' + q$, $W = p + q$

- Invariants of the problem:

$$
\begin{aligned}
q^2 &= (k - k')^2 \\
&= -2EE'(1 - \cos\theta) \;\; \text{[in lab]} \\
&= -4EE' \sin^2\left(\frac{\theta}{2}\right)) \;\; \text{[in lab]}
\end{aligned}
$$

$$
\begin{aligned}
P \cdot q &= P \cdot (k - k') \\
&= M(E - E') \;\; \text{[in lab]}
\end{aligned}
$$

  where the dot products are over a four momenta and where we used the formula $\sin^2\phi = \frac{1}{2}\left(1 - \cos(2\phi)\right)$
- In case of elasitic scattering:

$$
\begin{aligned}
P^2 &= W^2 \\
&= (P + q)^2 \\
&= P^2 + 2P \cdot q + q^2 \\
\Rightarrow P \cdot q &= -\frac{q^2}{2}
\end{aligned}
$$

  So indeed there is only one Lortenz invariant parameter

# Scattering of Pointlike Particles

- Rutherford Scattering (spin averaged electron scattering from a static point charge) in lab frame (non-relativistic limit)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4\left(\frac{\theta}{2}\right)}$$

here $E$ is energy of incident electron and $\theta$ is scattering angle in the lab frame

- For relativistic electron, still ignoring nucelar recoil:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \cos^2\left(\frac{\theta}{2}\right)}{4E^2 \sin^4\left(\frac{\theta}{2}\right)}$$

This is called Mott Scatting

# Adding nuclear recoil

- Elastic Scattering of a spin-$\frac{1}{2}$ electron from a pointlike spin-$\frac{1}{2}$ particle of mass $M$:
  - Elastic scattering of electron from infinite mass target changes angle but not energy
  - For target of finite mass $M$, final electron energy is

$$E' = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta}{2}\right)}$$

and the four-momentum transfer is

$$q^2 = -4EE'\sin^2\left(\frac{\theta}{2}\right)$$

- The elastic scattering cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\cos^2\left(\frac{1}{2}\theta\right)}{4E^2\sin^4\left(\frac{\theta}{2}\right)}\frac{E'}{E}\left[1 - \frac{q^2}{2M^2}\tan^2\left(\frac{\theta}{2}\right)\right]$$

This expression holds only for elastic scattering of Dirac particles

# Correcting for Finite Target Size: Form Factors (I)

- Reminder from Lecture 11: The Born Aprroximation

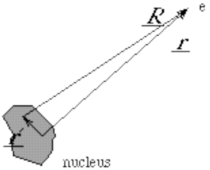
$$\frac{d\sigma}{d\Omega} = |f(\theta,\phi)|^2$$

- First term in PT for central potential gives:

$$\begin{aligned} f_{Born}(\theta,\phi) &= -\frac{1}{4\pi}\int d^3r'\; e^{i((\vec{k}-\vec{k}')\cdot\vec{r}')}V(r') \\ &= -\frac{1}{4\pi}\int d^3r'\; e^{i(\vec{q}\cdot\vec{r}')}V(r') \end{aligned}$$

- But here $V(r)$ is the Coulomb potential for finite size potential

$$\begin{aligned} f_{Born} &= Ze^2\int d^3R\, e^{iq\cdot R}\left[\int d^3r'\; e^{i\vec{q}\cdot r'}\;\frac{\rho(r')}{|R|}\right] \\ &= Ze^2\int d^3R\, e^{iq\cdot R}\; F(q^2) \end{aligned}$$




where $\vec{R} \equiv \vec{r} - \vec{r}'$

# Correcting for Finite Target Size: Form Factors (II)

- Charge distribution $\rho(r)$: $\int \rho(r) d^3r = 1$
- Scattering amplitude modified by a "Form Factor"

$$F(q^2) = \int d^3r e^{i\vec{q}\cdot\vec{r}} \rho(r)$$

  So that the cross section is modified by a factor of $|F(q^2)|^2$

- Note: As $q^2 \to 0$, $F(q^2) \to 1$
- We therefore can Taylor expand

$$F(q^2) = \int d^3r \left(1 + i\vec{q}\cdot\vec{r} - \frac{1}{2}(\vec{q}\cdot\vec{r})^2 + ...\right) \rho(r)$$

# Form Factors

- The first $\vec{q} \cdot \vec{r}$ term vanishes when we integrate

$$\begin{aligned}
F(q^2) &= 1 - \frac{1}{2} \int r^2 dr d\cos\theta d\phi \; \rho(r)(qr)^2 \cos^2\theta \\
&= 1 - \frac{2\pi}{2} \int dr d\cos\theta \; q^2 r^4 \cos^2\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \int \cos^2\theta \; d\cos\theta \\
&= 1 - \frac{< r^2 >}{4} q^2 \left[ \frac{\cos^3\theta}{3} \right]_{-1}^{1} \\
&= 1 - \frac{< r^2 >}{6} q^2
\end{aligned}$$

- For elastic scattering, can relate $q$ to the outgoing angle

$$q = \frac{2p \sin(\theta/2)}{\left[1 + (2E/M_p)\sin^2(\theta/2)\right]^{\frac{1}{2}}}$$

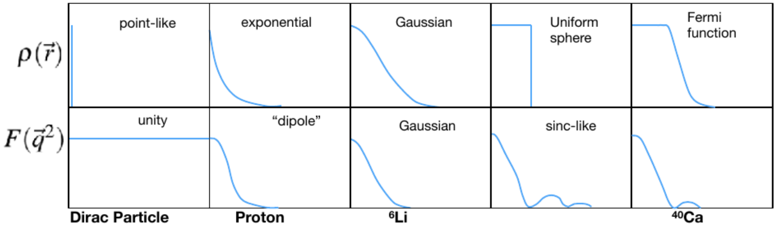
where $p$ and $E$ are the momentum and energy of the incident electron in the lab frame

# Interpreting Form Factors

- If proton is not pointlike cross section modified

$$\frac{d\sigma}{d\Omega} \longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{pointlike} |F(q^2)|^2$$

- Finite size of scattering center introduces a phase difference between plane waves scattered from different points in space




From Thomson

# Hoffstader and McAllister (1956)

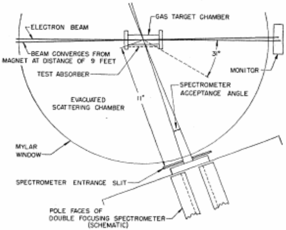



FIG. 2. Arrangement of parts in experiments on electron scattering from a gas target.

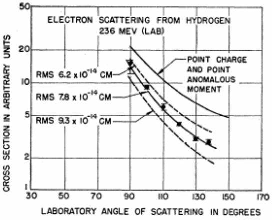



FIG. 6. This figure shows the experimental points at 236 Mev and the attempts to fit the shape of the experimental curve. The best fit lies near $0.78\times10^{-13}$ cm.

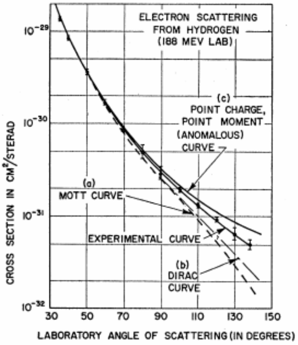



FIG. 5. Curve (a) shows the theoretical Mott curve for a spinless point proton. Curve (b) shows the theoretical curve for a point proton with the Dirac magnetic moment, curve (c) the theoretical curve for a point proton having the anomalous contribution in addition to the Dirac value of magnetic moment. The theoretical curves (b) and (c) are due to Rosenbluth.[8] The experimental curve falls between curves (b) and (c). This deviation from the theoretical curves represents the effect of a form factor for the proton and indicates structure within the proton, or alternatively, a breakdown of the Coulomb law. The best fit indicates a size of $0.70\times10^{-13}$ cm.

$$< r^2 >^{\frac{1}{2}} = 0.74 \pm 0.24 \times 10^{-13} \text{ cm} \sim 0.7 \text{ fm}$$

# What is the proton made of?

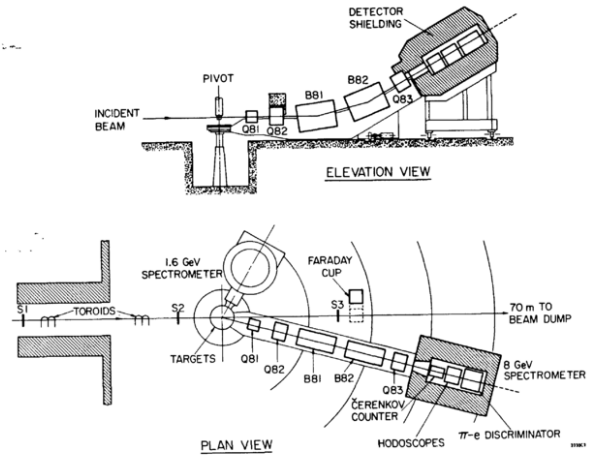
- Is the proton a soft mush or does it have hard composite objects inside?
- Need a high energy probe to resolve distances well below proton size
- Elastic cross section falls rapidly with $q^2$
- Inelastic cross section where proton breaks dominates rate at large $q^2$
  - "Deep inelastic scattering"
- Study energy and angle of outgoing electron
  - For inelastic scattering these are independent variables
    (subject to kinematic bounds of energy and momentum conservation)

# Need High Energy Lepton Probe

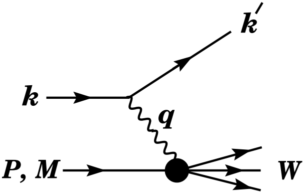
Stanford Linear Collider (SLAC)

- Two mile linear accelerator ($e^-$)
- Initial phase: energy = 20 GeV
- (Later, upgrade to 50 GeV)
- "End Station A" hall for fixed target experiments
- Study high momentum transfer
  - Need four-momentum transfer large enough to probe structure
  - Proton breaks apart
  - Deep Inelastic Scattering (DIS)

# The SLAC-MIT DIS Experiment (1968)



# Deep Inelastic Scattering: Kinematics




- $W$ is the invariant mass of the hadronic system
- In lab frame: $P = (M, 0)$
- In any frame, $k = k' + q$, $W = p + q$
- Invariants of the problem:

$$
\begin{aligned}
Q^2 &= -q^2 = -(k-k')^2 \\
&= 2EE'(1-\cos\theta) \quad \text{[in lab]} \\
P\cdot q &= P\cdot(k-k') \\
&= M(E-E') \quad \text{[in lab]}
\end{aligned}
$$

- Define $\nu \equiv E - E'$ (in lab frame) so $P \cdot q = M\nu$ and

$$
\begin{aligned}
W^2 &= (P+q)^2 \\
&= (P-Q)^2 \\
&= M^2 + 2P\cdot q - Q^2 \\
&= M^2 + 2M\nu - Q^2
\end{aligned}
$$

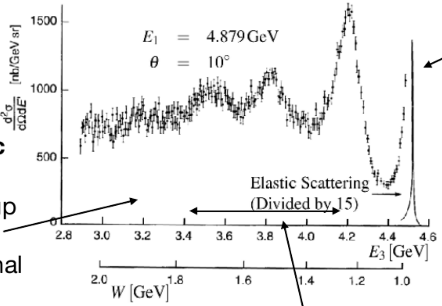
where $Q^2 = -q^2$

- Elastic scattering corresponds to $W^2 = P^2 = M^2$
  - ▶ $Q^2 = 2M\nu$ elastic scattering
- We can define 2 indep dimensionless parameters

$$
\begin{aligned}
x &\equiv Q^2/2M\nu; \quad (0 < x \le 1) \\
y &\equiv \frac{P\cdot q}{P\cdot k} = 1 - E'/E; \quad (0 < y \le 1)
\end{aligned}
$$

# Deep Inelastic Scattering: Observation

**Elastic Scattering**
proton is intact
W=M




**Deep inelastic scattering**
proton breaks up resulting in a many particle final state
DIS = large W

**Inelastic scattering**
produce excited states of the proton e.g. $\Delta^+(1232)$
$W=M_\Delta$

# The Most General Form of the Interaction

- Express cross section

$$d\sigma = L^{e}_{\mu\nu} W^{\mu\nu}$$

  where $W$ describes the proton current (allowing substructure)

- Most general Lorentz invarient form of $W^{\mu\nu}$
  - Constructed from $g^{\mu\nu}$, $p^{\mu}$ and $q^{\mu}$
  - Symmetric under interchange of $\mu$ and $\nu$ (otherwise vanishes when contracted with $L_{\mu\nu}$)

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^{\mu} p^{\nu} + \frac{W_4}{M^2} q^{\mu} q^{\nu} + \frac{W_5}{M^2} (p^{\mu} q^{\nu} + p^{\nu} q^{\mu})$$

- $W_3$ reserved for parity violating term (needed for $\nu$ scattering)
- Not all 4 terms are independent. Using $\partial_{\mu} J^{\mu} = 0$ can show

$$\begin{aligned}
W_5 &= -\frac{p \cdot q}{q^2} W_2 \\
W_4 &= \frac{p \cdot q}{q^2} W_2 + \frac{M^2}{q^2} W_1 \\
W^{\mu\nu} &= W_1 \left(-g^{\mu\nu} + \frac{q^{\mu} q^{\nu}}{q^2}\right) + W_2 \frac{1}{M^2} \left(p^{\mu} - \frac{p \cdot q}{q^2} q^{\mu}\right)\left(p^{\nu} - \frac{p \cdot q}{q^2} q^{\nu}\right)
\end{aligned}$$

# Structure Functions

- Using notation from previous page, we can express the x-section for DIS

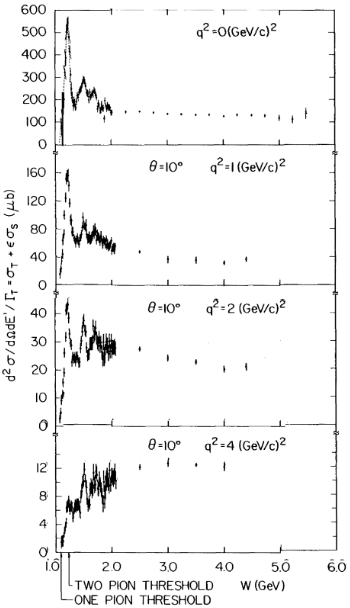
$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}\left[W_2(q^2,W) + 2W_1(q^2,W)\tan^2(\frac{1}{2}\theta)\right]$$

- These are the same two terms as for the elastic scattering
- $W_1$ and $W_2$ care called the *structure functions*
  - Angular dependence here comes from expressing covariant form on last page in lab frame variables
  - Two structure functions that each depend on $Q^2$ and $W$
  - Alternatively, can parameterize wrt dimensionless variables:

$$\begin{aligned} x &\equiv Q^2/2M\nu \\ y &\equiv \frac{P\cdot q}{P\cdot k} = 1 - E'/E \end{aligned}$$

- So our goal is to measure the two structure functions over a wide kinematic range and then to see if we can look at the $x$ and $q^2$ dependent (and the relationship betweeen $W_1$ and $W_2$) to learn about the constituents of the proton

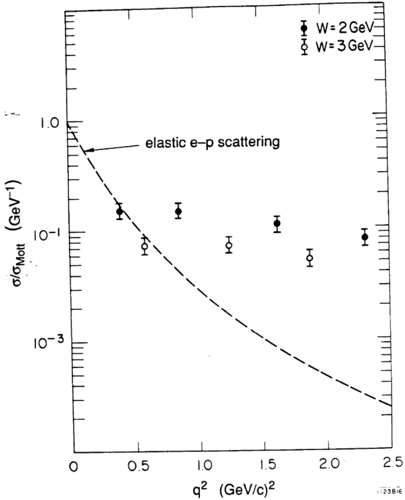
# Studying the Proton at Large Momentum Transfer




- SLAC-MIT group measured $d\sigma / dq^2 d\nu$ at 2 angles: $6^\circ$ and $10^\circ$
- For low $W$ dominated by production of resonances
- Surprise: Above the resonance region, $\sigma$ did *not* fall with $Q^2$
- Like Rutherford scattering, this is evidence for hard structure within the proton

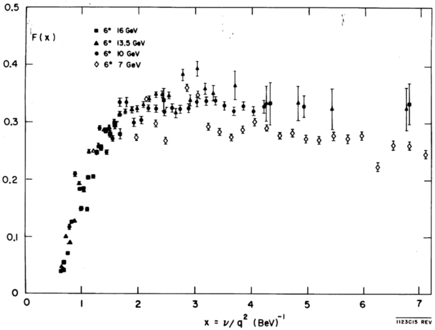
# Evidence for Hard Substructure

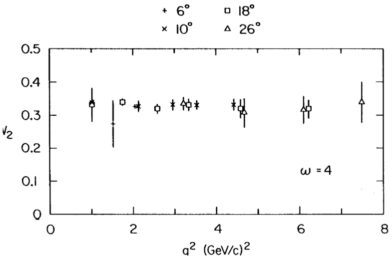
$$\frac{d\sigma}{d\Omega dE'} = \frac{\alpha^2}{4E^2}\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}\left[W_2(q^2,W) + 2W_1(q^2,W)\tan^2(\frac{1}{2}\theta)\right]$$




- How should we parameterize this deviation from behaviour predicted for pointlike proton?
  - To determine $W_1$ and $W_2$ separately, would need to measure at 2 values of $E'$ and of $\theta$ that give the same $q^2$ and $\nu$
  - The first exp couldn't do this: small angle where experiment ran, $W_2$ dominates so studied that
- Once $W$ and $Q^2$ large enough, cross section does not fall with $Q^2$
  - As with Rutherford, evidence for hard objects within the proton!

# SLAC-MIT Results: Scaling




- One more change of variables:

$$F_1(x, Q^2) \equiv MW_1(\nu, Q^2)$$
$$F_2(x, Q^2) \equiv \nu W_2(\nu, Q^2)$$

- Reminder: $x \equiv Q^2/2M\nu$
- Study $F_2$ for various energies and angles
- When low $Q^2$ data excluded, $F_2$ appears to depend only on dimensionless variable $x$ and not on $Q^2$
- This phenomenon is called "scaling"