(textbook definitions for these)

PSET 6

1) a) $\Omega^- = sss$, $S = -3$, $s = 3/2$

$\hat{U}_+ \Omega^- = \hat{U}_+ (sss)$

$= \hat{U}_+(s)ss + s\hat{U}_+(s)s + ss\hat{U}_+(s)$

$= N[dss + sds + ssd]$

$$\boxed{\hat{U}_+ \Omega^- = \frac{1}{\sqrt{3}}[dss + sds + ssd] = \Xi^{*-}}$$

$\hat{V}_+ \Omega^- = \hat{V}_+ (sss)$

$= N[\hat{V}_+(s)ss + s\hat{V}_+(s)s + ss\hat{V}_+(s)]$

$= N[uss + sus + ssu]$

$$\boxed{\hat{V}_+ \Omega^- = \frac{1}{\sqrt{3}}[uss + sus + ssu] = \Xi^{*0}}$$

$\hat{U}_+(\Xi^{*-}) = N[\hat{U}_+(d)ss + d\hat{U}_+(s)s + ds\hat{U}_+(s)$

$+ \hat{U}_+(s)ds + s\hat{U}_+(d)s + sd\hat{U}_+(s)$

$+ \hat{U}_+(s)sd + s\hat{U}_+(s)d + ss\hat{U}_+(d)]$

$= N[0 + dds + dsd + dds + 0 + sdd$

$+ dsd + sdd + 0]$

1(a), cont.

$$\hat{U}_+(\Xi^{*-}) = N(2\,dds + 2\,dsd + 2\,sdd)$$

$$\therefore \boxed{\hat{U}_+(\Xi^{*}) = \frac{1}{\sqrt{3}}(dds + dsd + sdd) = \Sigma^{*-}}$$

$$\hat{V}_+(\Xi^{*0}) = \hat{V}_+\left(\frac{1}{\sqrt{3}}(uss + sus + ssu)\right)$$

$$= N[\hat{V}_+ \quad \hat{V}_+(s) = u, \text{ zero otherwise}$$

$$\therefore \hat{V}_+(\Xi^{*0})$$

$$= N(0 + \underline{uus} + \underline{usu} + \underline{uus} + 0 + \underline{suu}$$

$$+ \underline{usu} + \underline{suu} + 0)$$

$$= N(2uus + 2usu + 2\,suu)$$

$$\therefore \boxed{\hat{V}_+(\Xi^{*0}) = \frac{1}{\sqrt{3}}(uus + usu + suu) = \Sigma^{*+}}$$

U

$\longrightarrow$

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(a), cont.

for Σ^{*0}:

$$\hat{U}_+(\Xi^{*0}) = \hat{U}_+\left[\frac{1}{\sqrt{3}}\cdot(uss + sus + ssu)\right]$$

$\hat{U}_+(s) = d$, zero otherwise

$$\therefore \hat{U}_+(\Xi^{*0}) = N[0 + uds + usd + dus + 0 + sud + dsu + sdu + 0]$$ all unique

$$= N[uds + usd + dus + sud + dsu + sdu]$$

$$\boxed{\therefore \hat{U}_+(\Xi^{*0}) = \frac{1}{\sqrt{6}}[uds + usd + dus + sud + dsu + sdu] = \Sigma^{*0}}$$

1b) given: $p = \frac{1}{\sqrt{2}}(ud - du)u = \frac{1}{\sqrt{2}}(udu - duu)$

$\hat{U}_-(p) =$ $\hat{U}_-(d) = s$

$$\therefore \hat{U}_-(p) = \hat{U}\left[\frac{1}{\sqrt{2}}(udu - duu)\right]$$

$$= N[0 + usu + 0 - suu + 0 + 0]$$

$$\boxed{\hat{U}_-(p) = \frac{1}{\sqrt{2}}(usu - suu) = \Sigma^+}$$

(1b, cont.)

$\circledast$ → (need this for later)

$$\hat{V}_-(u) = s$$

$$\cancel{\therefore \hat{V}_-(p) = \hat{V}_-\left(\tfrac{1}{\sqrt{2}}(udu - duu)\right)}$$

$$\cancel{= N\left[(sdu + 0 + uds) - (0 + dsu + dus)\right]}$$

$$= N\left[sdu + uds - dsu - dus\right] \quad \circledast$$

$\hookrightarrow = 1/2$

$$\hat{T}_-(u) = d$$

$$\hat{T}_-(p) = \hat{T}_-\left(\frac{1}{\sqrt{2}}(udu - duu)\right)$$

$$= N\left[ddu + 0 + udd - 0 - ddu - dud\right]$$

$$\hat{T}_-(p) = \frac{1}{\sqrt{2}}\left[udd - dud\right] = n$$

$$\hat{V}_-(u) = s$$

$$\therefore \hat{V}_-(n) = \hat{V}_-\left(\frac{1}{\sqrt{2}}(udd - dud)\right)$$

$$= N\left[sdd + 0 + 0 - 0 - dsd - 0\right]$$

$$\hat{V}_-(n) = \frac{1}{\sqrt{2}}(sdd - dsd) = \Sigma^-$$ } (global phase of -1)

1b), cont.

$\hat{T}_+(d) = u$

$\hat{T}_+(\Sigma^-) = \hat{T}_+\left(\frac{1}{\sqrt{2}}(dsd - sdd)\right)$

$= N[usd + 0 + dsu - 0 - sud - sdu]$

$\hat{T}_+(\Sigma^-) = N[usd + dsu - sud - sdu]$

$$\boxed{\hat{T}_+(\Sigma^-) = \frac{1}{2}[(us - su)d + (ds - sd)u] = \Lambda}$$

$\hat{T}_-(\Sigma^+) = \hat{T}_-\left(\frac{1}{\sqrt{2}}(usu - suu)\right)$, $\hat{T}_-(u) = d$

$= N[dsu + 0 + usd - 0 - sdu - sud]$

$= \underline{\Lambda}$ as well

$\hat{U}_-(n) = \hat{U}_-\left(\frac{1}{\sqrt{2}}(udd - dud)\right)$, $\hat{U}_-(d) = s$.

$\hat{U}_-(n) = N[0 + usd + uds - sud - 0 - dus]$

$= N[usd + uds - sud - dus]$ ⊛

$= 1/2$

$\frac{4}{12} + \frac{4}{12} +$

1b) (cont.)

from prev 2 pgs

i) $\hat{U}_-(n) = \frac{1}{2}[usd + uds - sud - dus]$

ii) $\hat{V}_-(p) = \frac{1}{2}[sdu + uds - dsu - dus]$

iii) $N(\hat{U}_-(n) + \hat{V}_-(p))$

$= N[2(uds - dus) + (us - su)d + (sd - ds)u$

$= \frac{N}{\sqrt{2^2 \cdot 3}}[2(\ldots$ $\frac{1}{\sqrt{2^2 + 2^2 + 1 + 1 + 1 + 1}} = \frac{1}{\sqrt{12}}$

$$\therefore \boxed{\Sigma^0 = \frac{1}{\sqrt{12}}[2(ud - du)s + (us - su)d + (ds - sd)u]}$$

2)

(correction): $Q^2 = |\langle \gamma | Q | \psi \rangle|^2$

$$= \left| \sum_i a_i \langle \gamma | Q | q_i \bar{q}_i \rangle \right|^2$$

given:

$$\Gamma(V \to \ell^+ \ell^-) \propto \frac{\alpha^2 Q^2}{M_V^2} |\psi(0)|^2$$

$$|\psi\rangle = \sum_i a_i |q_i \bar{q}_i\rangle$$

$\langle \gamma | Q | u\bar{u} \rangle = \frac{2}{3}$, $\langle \gamma | Q | d\bar{d} \rangle = \frac{1}{3}$, $\langle \gamma | Q | s\bar{s} \rangle = -\frac{1}{3}$

assume $|\psi(0)|^2$ the same for all 3 states.

mesons: $\rho^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$

$\omega^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

$\phi^0 = s\bar{s}$

$$Q^2(\rho^0) = \left| \frac{1}{\sqrt{2}} \langle \gamma | Q | u\bar{u} \rangle - \frac{1}{\sqrt{2}} \langle \gamma | Q | d\bar{d} \rangle \right|^2$$

$$= \frac{1}{2} \left| \frac{2}{3} - \left(-\frac{1}{3}\right) \right|^2 = 1/2 = Q^2(\rho^0)$$

$$Q^2(\omega^0) = \frac{1}{2} \left| \langle \gamma | Q | u\bar{u} \rangle + \langle \gamma | Q | d\bar{d} \rangle \right|^2$$

$$= (1/2) \left| \frac{2}{3} - \frac{1}{3} \right|^2 = (1/2)(1/9) = \frac{1}{18}$$

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2), cont.

$$Q^2(\phi^\circ) = |\langle\gamma|Q|s\bar{s}\rangle|^2 = \left|\frac{1}{3}\right|^2 = 1/9$$

$$\therefore \frac{\Gamma(\rho^\circ \to \ell^+\ell^-)}{\Gamma(\omega^\circ \to \ell^+\ell^-)} = \frac{\cancel{\alpha^2} Q^2(\rho^\circ)}{M_{\rho^\circ}^2} \cancel{|\psi(0)|^2} \frac{M_{\omega^\circ}^2}{\cancel{|\psi(0)|^2} \cancel{\alpha^2} Q^2(\omega^\circ)}$$

$$= \left(\frac{M_{\omega^\circ}}{M_{\rho^\circ}}\right)^2 \frac{Q^2(\rho^\circ)}{Q^2(\omega^\circ)}$$

$$= \left(\frac{M_{\omega^\circ}}{M_{\rho^\circ}}\right)^2 \frac{(1/2)}{(1/18)} = 9 \cdot \left(\frac{M_{\omega^\circ}}{M_{\rho^\circ}}\right)^2$$

pg 231 Thomson $\{ M_{\rho^\circ} = 775 \text{ MeV}, M_{\omega^\circ} = 783 \text{ MeV}, M_{\phi^\circ} = 1020 \text{ MeV}$

$$\therefore \frac{\Gamma(\rho^\circ \to \ell^+\ell^-)}{\Gamma(\omega^\circ \to \ell^+\ell^-)} = 9 \cdot \left(\frac{783}{775}\right)^2 = \boxed{9.187}$$

$$\frac{\Gamma(\rho^\circ \to \ell^+\ell^-)}{\Gamma(\phi^\circ \to \ell^+\ell^-)} = \left(\frac{M_{\phi^\circ}^2}{M_{\rho^\circ}^2}\right) \frac{Q^2(\rho^\circ)}{Q^2(\phi^\circ)}$$

$$= \left(\frac{1020}{775}\right)^2 \frac{(1/2)}{(1/9)} = \boxed{7.795}$$

→

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$$\frac{\Gamma(\omega^0 \to e^+ e^-)}{\Gamma(\phi^0 \to e^+ e^-)} = \left(\frac{M_{\phi^0}}{M_{\omega^0}}\right)^2 \left(\frac{Q^2(\omega^0)}{Q^2(\phi^0)}\right)$$

$$= \left(\frac{1020}{783}\right)^2 \times \frac{(1/18)}{(1/9)} = \boxed{0.848}$$

(remainding ratios can be found as reciprocals of these)

3) a) $m_{K^0} = 498$ MeV, $m_{K^{*0}} = 896$ MeV

$$\Delta M(\text{meson}) = A \frac{S_1 \cdot S_2}{m_1 m_2} |\Psi(0)|^2$$

$K^0 : d\bar{s}$, $K^{0*} : d\bar{s}$ } same quark composition

$$\Delta M = 896 - 498 \text{ [MeV]} = \frac{S_1 \cdot S_2}{m_d m_s} A|\Psi(0)|^2$$

$m_d = 336$ MeV

$m_s = 509$ MeV } (same quark comp)

$\langle S_1 \cdot S_2 \rangle = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1/4$

$$398 \text{ MeV} = \frac{(1/4)}{(336)(509) \text{ MeV}} A|\Psi(0)|^2 = \frac{0.0272 \text{ GeV}^3}{4}$$

$$\boxed{A|\Psi(0)|^2 = 4(398)(336)(509) = 2.72 \cdot 10^8 \text{ MeV}^3}$$

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$d\Omega = \sin(\theta)\, d\theta\, d\phi$

3b) from 3a), $A|\psi(0)|^2 = 2.72 \cdot 10^8\ MeV^3$

$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} + 2s\bar{s}), \quad \omega^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$\langle u\bar{u} | S_1 \cdot S_2 | u\bar{u} \rangle \quad m_1 = m_2 = m_u = 336\ MeV$

$\langle d\bar{d} | S_1 \cdot S_2 | d\bar{d} \rangle \quad m_1 = m_2 = m_d = 336\ MeV$

$\langle s\bar{s} | S_1 \cdot S_2 | s\bar{s} \rangle \quad m_1 = m_2 = m_s = 509\ MeV$

$$\Delta M\,(\text{meson}) = \underset{\text{pseudo}}{m_\eta} - \underset{\text{vector}}{m_{\omega^0}} = \sum_i (-1)^i \frac{\langle S_{1_i} \cdot S_{2_i} \rangle}{m_{1_i} m_{2_i}} A|\psi(0)|^2$$

$$\Delta M(\eta, \omega^0) = \sum_i (-1)^i \left(\frac{\langle \psi_i | S_1 \cdot S_2 | \psi_i \rangle}{m_{1_i} m_{2_i}} \right) (0.027\ GeV^3)$$

$$\Delta M = \left[\frac{\langle \eta | S_1 \cdot S_2 | \eta \rangle}{m_1 m_2} - \frac{\langle \omega^0 | S_1 \cdot S_2 | \omega^0 \rangle}{m_1 m_2} \right] A|\psi(0)|^2$$

from all matching $\langle S_1 \cdot S_2 \rangle$

$$= \frac{1}{4} \left[\frac{1}{6} \left(\frac{1}{m_u^2} + \frac{1}{m_d^2} + \frac{4}{m_s^2} \right) - \frac{1}{2} \left(\frac{1}{m_u^2} + \frac{1}{m_d^2} \right) \right] \cdot A|\psi(0)|^2$$

$$= (-8.32985 \cdot 10^{-7}\ MeV^2)(2.72 \cdot 10^8\ MeV^3)$$

$$= |-226.572\ MeV| \rightarrow \boxed{\Delta M(\eta, \omega^0) = 226.57\ MeV}$$

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