# Physics 129: Particle Physics
# Lecture 8: Symmetries and Conservation Laws (Part II)

Sept 22, 2020

- Suggested Reading:
  - Thomson Sections 1.1, 3.1, 9.1-9.2
  - Griffiths Chapter 4
  - Perkins Sections 3.1-3.10

# Review: Noether's Theorm

- There is a one-to-one correspondance between symmetries of nature and conservation law
- Every symmetry leads to a conservation law and every conservation law corresponds to a symmetery
  - In Quantum Mechanics, this is expresSed in terms of the Hamiltonia

$$\left[\hat{H}, \hat{S}\right] = 0 \Rightarrow \left\langle \hat{S} \right\rangle \text{ is conserved}$$

  - In Classical mechanics it is expressed as a symmetery of the action of the LaGrangian, but the correspondance with conservation laws is the same
- Examples for spatial symmetries:

| | | |
|---|---|---|
| Translation in time | $\Rightarrow$ | energy conservation |
| Translation in space | $\Rightarrow$ | momentum conservation |
| Rotations | $\Rightarrow$ | angular momentum conservation |

- Observing conservation laws tells us about the form of the interaction
- Alternately, invoking conservation laws can be used to constrain a problem without solving the equations of motion

# Review: Parity

- Parity operator defined as spatial inversion

$$\begin{aligned} (x, y, z) &\longrightarrow (-x, -y, -z) \\ P(\psi(\vec{r})) &= \psi(-\vec{r}) \end{aligned}$$

- Repetition of the operations gives $P^2 = 1$
  - $P$ is a unitary operator with eigenvalues $\pm 1$

- If system is an eigenstate of $P$, its eigenvalue is called the parity of the system
- $P^\dagger = P = P^{-1}$
- How do various operators transform under P?

$$\begin{aligned} P\,\vec{r}\,P^{-1} &= -\vec{r} \\ P\,\vec{p}\,P^{-1} &= -\vec{p} \\ P\,\vec{L}\,P^{-1} &= +\vec{L} \\ P\,\vec{S}\,P^{-1} &= +\vec{S} \end{aligned}$$

  First two are called vectors, last two are called axial vectors
- Parity is a multiplicative quantum number

# Review: Parity and Elementary Particles

- If parity is a good symmetry of $H_{int}$, all elementary particles must be eigenstates of $P$ with eigenvalues $\pm 1$.
- To determine if parity is a good symmetry, see if it's possible to define eigenstates for each elementary particle
  (independent of reaction)
  Note: It is not necessarily true that definition be *unique* as long as we can define it in a consistent one
- Experimental Facts:
  - Both Strong and EM interactions conserve parity
  - Weak interactions do not
- Elementary particles have intrinsic parity:
  - The Photon: $P(\gamma) = -1$
  - Spin-1/2 particles
    - Always pair produced
    - Particle and antiparticle have opposite parity
  - Pions: $P(\pi) = -1$
- Nomenclature:
  - Particles like the $\pi$ with spin-0 and $P = -1$ are called *pseudoscalar* particles
  - Particles like the $\rho$ with spin-1 and $P = -1$ are calle *vector* particles

# Some Perspective: Let's go back to Quantum Mechanics

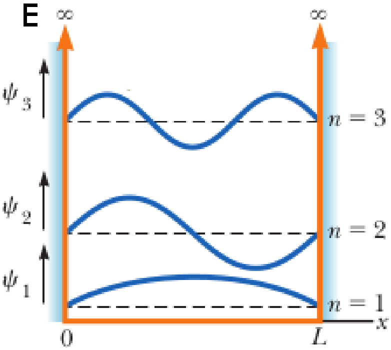
- $[H, P] = 0$
  - Can have simulateous eigenstates of $H$ and $P$
- Eigenstates alternate between even and odd partity

$$P\psi(x) = \pm\psi(x)$$

  - Any $\psi(x)$ with even parity can be constructed from the even eigenstates
  - Any $\psi(x)$ with odd parity can be constructed from the odd eigenstates
- Given a $\psi(x)$ of definite parity
  - If a physical observable $\hat{O}$ is odd under parity

$$\left\langle \hat{O} \right\rangle = 0$$

1 Dimensional Infinite Square Well



# Applying this idea to particle physics

- Orbital angular momentum state has parity $(-1)^{\ell}$
- Elementary particles have intrinsic parity
  - If decay occurs via strong or EM interaction, final state wave function of the decay products must have the same parity as the initial particle
  - Parity of the final wave function product of intrinsic parities and parity from orbital angular momentum
  - We implicitly used this last Thurs to determine the parity of the pion
- Often, in scattering problems, the intial state has definite partity that can be calculated. In that case, if scattering occurs via strong or EM interactions
  - Parity of final state must be same as initial state
  - $\left\langle \hat{O} \right\rangle = 0$ for any observable that is odd under parity
- For strong and EM interactions, when we have particle decays or when scattering occurs from state of definite parity:
  - Observables such as decay angular distributions must be even under parity
  - For example, a scattering cross section $d\sigma/d\Omega$ can only contain even powers of $\cos\theta$
  - By the same argument, the rates can be written as a function of scalar quantites **not** pseudoscalars

# Another discrete symmetry: Charge Conjugation (C) (I)

- In classical E&M, if sign of all charges is changed $q \Rightarrow -q$:
  - All $\vec{E}$ and $\vec{B}$ fields change direction
  - Force on a particle:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

  - Equations of motion invariant under transformation $q \Rightarrow -q$
- Define a transform C called *Charge Conjugation*
- C reverses the sign of the charge and magnetic moment and leaves spatial coordinates unchanged
- This is not a great name since this transform does more than change the sign of the charge
- It in fact changes particles into anti-particles
  - $e^- \Rightarrow e^+$ Lepton number changes sign
  - $p \Rightarrow \overline{p}$ Baryon number changes sign

# Another discrete symmetry: Charge Conjugation (C) (II)

- C is more difficult to study than P because elementary particles aren't in general eigenstates of C

$$C\left|\pi^{+}\right\rangle = \eta\left|\pi^{-}\right\rangle \neq \pm\left|\pi^{+}\right\rangle$$

- If C is a good symmetry of our iteraction, neutral particles will be eigenstates of C

$$C\left|\psi\right\rangle = \pm\left|\psi\right\rangle$$

  if the charge of $|\psi\rangle = 0$

- Just as elementary particles have intrinsic parity, neutral elementary particles have an intrinsic C quantum number
- The photon is an elementary particle and EM interaction is symmetric under C
- We can determine C for the photon from form of EM interaction
  - To find the sign, note that EM fields are produced from charges
  - Changing sign of charge changes direction of $\vec{E}$

The photon has $C = -1$

# C for the Neutral Pion

$$C\left|\pi^0\right\rangle = \eta\left|\pi^0\right\rangle$$

with $|\eta|^2 = 1$ so $\eta = \pm 1$

- Since $\pi^0 \to \gamma\gamma$, $\pi^0$ has $C = 1$
- Consequence: $\pi^0 \to 3\gamma$ is forbidden

$$\frac{\pi^0 \to 3\gamma}{\pi^0 \to 2\gamma} < 3.1 \times 10^{-8} \quad 90\% \text{ cl}$$

Note:
Although charged particles aren't eigenstates of C, C invariance is still useful for relating reaction rates

$$\mathcal{M}(ab \to cd) = \mathcal{M}(\overline{a}\overline{b} \to \overline{c}\overline{d})$$

# Time Reversal (T)

- Operator T turns $t \to -t$
- Tested experimentally for strong interactions by applying principle of detailed balance
- T is also a good symmetry of EM. Small T violations in Weak interaction (more in a few weeks)
- A good test of T symmetry in strong and EM interactions:

  Limit on electric dipole moment (EDM) of neutron
  - Electric dipole moment $\vec{d}$ would have to point along axis of spin $\vec{S}$ (no other preferred direction in center of mass)
  - $\vec{d}$ is a vector, but $\vec{S}$ is a pseudovector
  - An EDM for for neutron would violate P symmetry but would also violate T symmetry since $\vec{S}$ changes sign under T
  - Current limit:

$$|\vec{d^n}| \leq 1.8 \times 10^{-26} \ \text{e cm}$$

- Same argument holds for electron EDM

$$|\vec{d^3}| \leq 1.1 \times 10^{-29} \ \text{e cm}$$

# Flavor Symmetry Overview (I)

- 3 generations of leptons ($e$, $\mu$, $\tau$ and respective $\nu$)
  - Pointlike, as far as we can measure
  - Conserved quantum number
    - Total lepton number absolutely conserve (as far as we know)
    - Individual $e$, $\mu$, $\tau$ number conserved except in $\nu$ oscillations (more later this semester)
- Hundreds of hadrons:
  - Are they fundamental?
  - Look for patterns in mass, spin, charge
  - Rules to relate interaction and decay rates
- First classification:
  - Baryons (eg $p$ and $n$) have spin-1/2 (fermions)
    - Must be produced in pairs
    - Baryon number absolutely conserved for all known interactions
  - Mesons (eg $\pi$ and $K$) have integer spin (bosons)
    - Can be produced singly

# Flavor Symmetry Overview (II)

- Futher classification of hadrons
  - Hadrons are composite particles made of quarks
  - If we could calculate the hadron wave functions (as we do for the hydrogen atom), we could determine the mass of the hadrons
  - But even without knowin wave function, can use observed patterns of hadron masses to learn about the strong force
- In 1960's no one knew whether quarks were real or just mathematical constructs
  - We'll discuss how we learned that quarks were real in a few weeks
  - For now, take that as a postulate
- Since baryons and mesons are so different, we'll first discuss them separately and then see how to relate what we learn in the two cases
- First hadrons studied were the baryons, since they make up the nucleus

# The lightest baryons

- Earliest examples: $p$ and $n$
  - Both see same nuclear force
  - Masses almost the same
    - $m_p = 938.20$ MeV
    - $m_n = 939.57$ MeV
- Natural natural to of them as 2 states of same particle: the nucleon $N$

$$N \equiv \begin{pmatrix} p \\ n \end{pmatrix}$$

- Writing the two as a doublet is suggestive
  - There must be a transformation that turns on of these particles into the other
  - Think of this transformation as a "rotation" of some new quantum number
  - The rotation will have an infinitesmal operator associated with it
  - Commutation properties of the operator will define this new quantum number

# Introducing Isospin

- We will call this new quantum number isospin
- Postulate that it has the same algebra as spin
- Test this hypothesis against measurements
- If nucleon is a doublet, then $N$ has $I = \frac{1}{2}$:

$$
\begin{aligned}
N &\equiv \begin{pmatrix} p \\ n \end{pmatrix} \\
p &= \left| \frac{1}{2}\, \frac{1}{2} \right\rangle \\
n &= \left| \frac{1}{2} - \frac{1}{2} \right\rangle
\end{aligned}
$$

- Here the $z$ component of isospin is usually written $I_3$ instead of $I_z$
- Before testing is our postulate is correct, we'll need to introduce isospin for the pions

# Isospin for the pions

- Three charges $\pi^+$, $\pi^0$ ,$\pi^-$ so $I = 1$:

$$
\begin{aligned}
\Pi &\equiv \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \\
\pi^+ &= |11\rangle \\
\pi^0 &= |10\rangle \\
\pi^- &= |1-1\rangle
\end{aligned}
$$

- Pions are an isotriple
- Note: Both for the $N$ and for the $\Pi$, changing $I_3$ by 1 unit changes the charge of the particle by 1 unit
  - This is a general fact we will come back to next time

# Testing our isospin postulate

- We called our quantum number isospin hoping that the transforamtion would have the same properties as spin
- If that were true the isospin $\vec{I}$ would have 3 components that satisfy the commutation relation
$$[I_i, I_j] = i\epsilon_{ijk} I_k$$
- Addition of isospin would follow the same rules as addition of angular momentum
- If isospin is a good symmetry of the strong interactions, then it is conserved
  - If we know the isospin of the inital state, we know the isospin of the final state
- Given two initial particles with isospin $I_1$ amd $I_2$, the total isospin of the inital state can take any value from $I_1 + I_2$ to $|I_1 - I_2|$
  - If the interaction $H_{int}$ doesn't depend on $I_3$ but only on $I$, then $\langle\psi| H_{int} |\psi\rangle$ (aka the matrix element) does not depend on $I_3$
  - We have only one matrix element for each value of $I$
  - Can relate rates for processes with the same $I$ and different $I_3$ using Clebsh-Gordon coefficients

# Does the algebra of SU(2) hold? $\pi N$ scattering

- Use isospin to relate reaction rates
- One matrix element per value of $I_{Tot}$
- $\Rightarrow I_{Tot} = \frac{3}{2}$ or $\frac{1}{2}$ so two indep matrix elements:

$$\mathcal{M}_{\frac{1}{2}} \equiv \left\langle \frac{1}{2} \middle| H \middle| \frac{1}{2} \right\rangle \quad \mathcal{M}_{\frac{3}{2}} \equiv \left\langle \frac{3}{2} \middle| H \middle| \frac{3}{2} \right\rangle$$

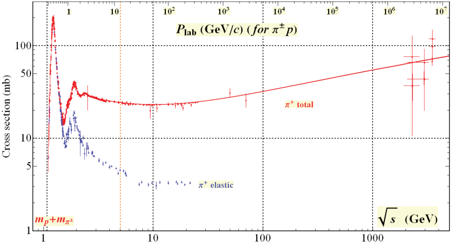
- Examples of decomposition

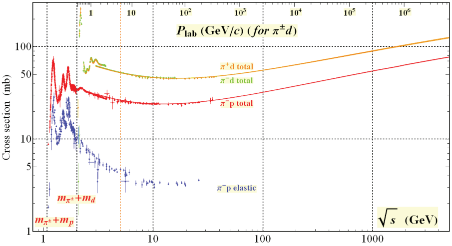
$$\begin{aligned} p\pi^+ &= \left| \frac{3}{2} \frac{3}{2} \right\rangle \\ p\pi^0 &= \sqrt{\frac{2}{3}} \left| \frac{3}{2} \frac{1}{2} \right\rangle - \sqrt{\frac{1}{3}} \left| \frac{1}{2} \frac{1}{2} \right\rangle \end{aligned}$$

- Working through the math:

$$\begin{aligned} \sigma(\pi^+ p \to \pi^+ p) &\sim \left| \mathcal{M}_{\frac{3}{2}} \right|^2 \\ \sigma(\pi^+ n \to \pi^+ n) &\sim \left| \frac{1}{3}\mathcal{M}_{\frac{3}{2}} + \frac{2}{3}\mathcal{M}_{\frac{1}{2}} \right|^2 \\ \sigma(\pi^- p \to \pi^0 n) &\sim \left| \frac{\sqrt{2}}{3}\mathcal{M}_{\frac{3}{2}} - \frac{\sqrt{2}}{3}\mathcal{M}_{\frac{1}{2}} \right|^2 \end{aligned}$$

# More on $\pi N$ scattering




- Large bumps: "resonances"
- Eg: near 1236 MeV
  - Width $\sim$ 120 MeV $\Rightarrow$ short lifetime: $\Delta E \Delta t \sim \hbar$:

$$\begin{aligned}\Delta t &\sim \frac{\hbar}{\Delta E} \\ &\sim \frac{6.58 \times 10^{-22} \text{ MeV s}}{120 \text{ MeV}} \\ &\sim 5 \times 10^{-24} \text{ s}\end{aligned}$$

- This is the $\Delta$
  - Four states: $I = 3/2$: $\Delta^{++}$,$\Delta^{+}$, $\Delta^{0}$,$\Delta^{-}$
  - There is NO $\Delta^{--}$