

Physics 129: Particle Physics

Lecture 20: Weak Interactions (I): Beta decay, Neutrinos and Parity Violation

Nov 3, 2020

- Suggested Reading:
 - ▶ Thomson Chapter 11
 - ▶ Griffiths 10.1-10.4

Nuclear β -Decay

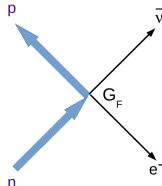
- First observed weak decay

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

- Existence of ν first proposed by Pauli:
 - ▶ Trajectory of e^{-} not co-linear with recoiling nucleus and no additional particle seen
 - Conservation of momentum \rightarrow additional unobserved decay product (the ν)
 - ▶ Electron does not have a discrete energy (3-body decay)
 - ▶ Endpoint of e energy spectrum close to maximum allowed for 2-body decay: $m_{\nu} \sim 0$
 - ▶ Change in nuclear spin is 0 or ± 1 *never* ± 2
 - Since e has spin- $\frac{1}{2}$, angular momentum conservation tells us the ν also has spin- $\frac{1}{2}$

Four-Fermi Interaction

- Fermi assumed weak decay occurs via hadronic weak current $\langle p | J_\mu^{wk} | n \rangle$ and leptonic weak current $\langle e\nu | J_\mu^{wk} | 0 \rangle$



- The complete matrix element was written

$$M_{if} = \langle p | J_\mu^{wk} | n \rangle \langle e\nu | J_\mu^{wk} | 0 \rangle$$

- Current-current form implies there are also purely leptonic processes, eg

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

and purely hadronic weak processes, eg

$$\Lambda \rightarrow p\pi^-$$

- Strength of interaction set by a constant G_F , assumed to be universal

Decay Rates and Fermi's Golden Rule

- Transition rate W :

$$W_{fi} = 2\pi G_F^2 |M_{if}|^2 \mathcal{D}(E_f)$$

where $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$ and \mathcal{D} is the density of states

- Note: the fact that G_F is not a dimensionless coupling constant tells us that something is going on. We'll talk about this in a few minutes

- The density of states

$$d^2 N = p_e^2 dp_e p_\nu^2 dp_\nu$$

For a massless neutrino (and ignoring small nuclear recoil)

$$p_\nu = (E_f - E_e); \quad dp_\nu = dE_f$$

Thus

$$\frac{dN}{dE_f} = p_e^2 (E_f - E_e)^2 dp_e$$

- Assume for now that $|M|^2$ is constant. Electron spectrum is

$$N(p_e) dp_e \propto p_e^2 (E_f - E_e)^2 dp_e$$

- Modification for non-zero neutrino mass

$$N(p_e) \propto p_e^2 (E_f - E_e)^2 \left[1 - \frac{m_\nu}{(E_f - E_e)} \right] dp_e$$

The Kurie Plot and ν Mass

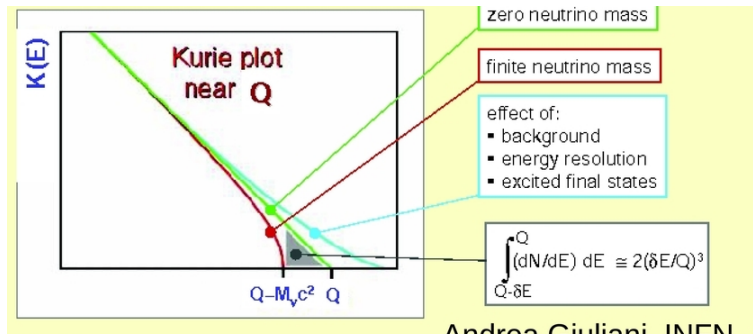
- From previous page:

$$N(p_e)dp_e \propto p_e^2(E_f - E_e)^2 \left[1 - \frac{m_\nu}{(E_f - E_e)} \right] dp_e$$

Thus

$$\sqrt{N(p_e)/p_e^2} \propto (E_f - E_e) \sqrt{1 - \frac{m_\nu}{E_f - E_e}}$$

- This is called a Kurie plot



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Issues in Direct Measurement of ν Mass

- Counting rate near endpoint is only a small fraction of total decay rate
- Integrating the β -spectra over interval ΔE from the endpoint, rate

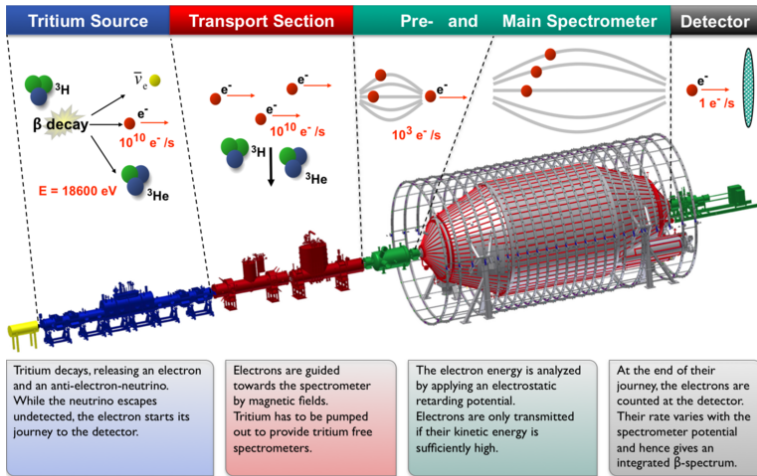
$$R \propto (1 - [m_\nu/\Delta E]^2)^{3/2}$$

Assuming a spectrometer resolution of 10 eV, number of events has to be increased by factor of ~ 15 to improve sensitivity from $m_\nu = 10$ eV to $m_\nu = 5$ eV

- The thickness of the source must be accounted for very accurately: energy loss of the electrons for a dense source
- Binding energy corrections for the nuclei can be important: This is why many modern experiments go to Tritium

PDG Limit: $m_{\nu_e} < 2$ eV (90% CL)

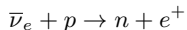
The Next Generation of Direct Mass Measurement: Katrin



- β -decay from Tritium gas
- Large volume for high rate
- Low temperature to (30K) to reduce thermal motion

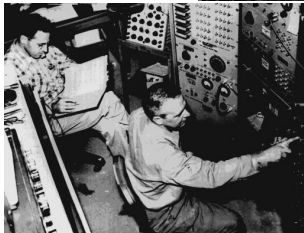
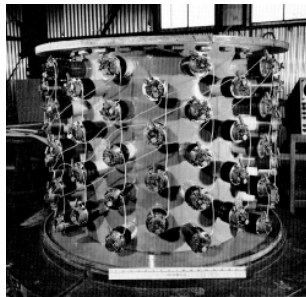
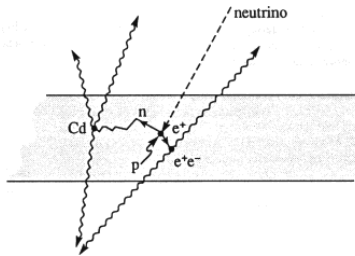
Inverse β -Decay

- Pauli and Fermi's explanation of β -decay postulated the existence of the ν , but it wasn't until 1959 that the particle nature was observed
- Inverse β -decay



- Reines and Cowan (Phys Rev 113 91959) 272) use Savannah River reactor as a source of $\bar{\nu}$ and a $CdCl_2 + H_2O$ target/detector
 - ▶ e^+ comes to rest (ionization energy loss) and forms positronium
 - ▶ Positronium decay to 2γ which produces electrons by Compton effect (10^{-9} sec)
 - ▶ Cd captures the neutron after it has moderated in H_2O . Radiative γ rays from neutron capture (μ sec time scale)
- Signal consisted of 2 pulses separated in time by a few μ sec.
- Rate can be estimated assuming matrix element related to β -decay
- Observations:
 - ▶ The $\bar{\nu}_e$ is a real massless or nearly massless particle
 - ▶ Rate consistent with predictions from Fermi theory

Pictures of the Reines and Cowan Experiment



From Four-Fermion Coupling to Intermediate Vector Bosons

- Why does G_F has dimensions of GeV^{-2} ?
 - ▶ Four-fermion coupling does not include a $1/q^2$ propagator
- Replace 4-point interaction with the exchange of W^\pm boson with mass M_W
 - ▶ Replace the QED propagator with weak propagator with massive W boson

$$e \frac{-g^{\mu\nu}}{q^2} \Rightarrow g_{wk} \frac{-g^{\mu\nu} + q^\mu q^\nu / M_W^2}{q^2 - M_W^2}$$

- ▶ Matrix element becomes

$$M \sim g_{wk} j_\mu^{wk} \left(\frac{-g^{\mu\nu} + q^\mu q^\nu / M_W^2}{q^2 - M_W^2} \right) j_\nu^{wk}$$

- ▶ For small q^2 , we get

$$M \sim g_{wk} (q^2 \rightarrow 0) j_\mu^{wk} \left(\frac{g^{\mu\nu}}{M_W^2} \right) j_\nu^{wk}$$

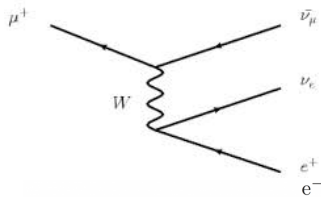
- ▶ Can identify

$$\frac{G_F}{\sqrt{2}} = \frac{g_{wk}^2}{8M_W^2}$$

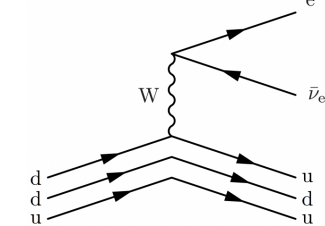
where numerical factors are partly of historical origin

Example Feynman Diagrams for Weak Decays

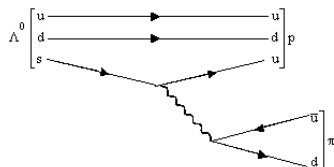
μ -decay



β -decay



Λ -decay



A Unitarity Argument for the Vector Boson Theory

- Suppose the four-fermion theory were right:
 - ▶ Using dimensional analysis: $e\nu$ scattering cross section

$$\sigma(e\nu \rightarrow e\nu) \propto G_F^2 s$$

- ▶ Similar expression for νp scattering, except with convolution over pdf's
 - ▶ Low energy ν scattering data agrees with this result
- If this formula holds to all energies, we have a problem
 - ▶ No cross section can exceed the unitarity bound
 - ▶ In plain English that means that the scattering probability cannot exceed 1
 - ▶ At $\sqrt{s} \sim 500$ GeV, unitarity is violated
- This argument told physicists that four point theory would fail at high energies and argued for the intermediate boson theory
- Note: We can estimate m_W if we assume $g_{wk} \approx e$:

$$G_F \sim g_{wk}^2 / M_W^2 \Rightarrow M_W \sim e / \sqrt{G_F} \approx 100 \text{ GeV}$$

The W was first observed in 1982. We'll come back to that part of the story in a few weeks

What We Have Learned So Far

- QED is a remarkably successful theory that describes EM interaction of charged leptons and photons
 - ▶ Want to build weak interaction theory using QED as a guide
- Neutral leptons (neutrinos) exist and are produced in β -decay
- β -decay is not a QED process. Fermi described it with a 4-fermion interaction. This explained:
 - ▶ The β -decay spectrum
 - ▶ Inverse β -decay
 - ▶ Existence of both purely leptonic and purely hadronic weak decays
- In analogy with QED, we can replace this 4-point interaction with exchange of a massive charged vector boson, the W :
 - ▶ Avoids unitarity crisis
 - ▶ Explains why weak interactions are weak
 - ▶ If $g = e$, $M_W \sim 100 \text{ GeV}$

Reminder: How we built QCD Feynman rules in analogy with QED

- When we talked about QCD, at this point in the story we drew the relevant vertex Feynman diagrams and quoted the Feynman rules
- I didn't give them to you explicitly in Lecture 19 but:
 - ▶ Quark-gluon vertex:

$$i\sqrt{4\pi}e\gamma^\mu \Rightarrow \frac{-ig_s}{2}\lambda^a\gamma^\mu$$

where a runs from 1 to 8

- ▶ Propagators look like QED, except gluon needs color indices

$$\frac{-ig_{\mu\nu}}{q^2} \Rightarrow \frac{-ig_{\mu\nu}}{q^2}\delta^{ab}$$

- ▶ Since gluons have charge we also had 3-gluon and 4-gluon vertices (gluons interact with other gluons)
- Can we use the same analogies to write down rules for weak interactions?

There are several complications that we need to address first

What makes weak theory different?

- Unlike strong and EM, weak interactions don't conserve parity
 - ▶ Vertex factor for W^\pm not just $\frac{-ig_s}{2}\lambda^a\gamma^\mu$
 - ▶ Includes factor that picks out left-handed state for particles (and right handed state for anti-particles)
 - We'll spend rest of today discussing this
- W^\pm coupling to leptons respect flavor families (e, μ, τ) but coupling to quarks do not
 - ▶ Coupling not diagonal in quark flavor: Need to change basis
 - This will be the topic for Thursday
 - ▶ Introduction of this change in basis gives new phenomenology, including mixing and CP violation
 - These will be discussed next week
- W^\pm has charge, so it couples to photon
 - ▶ Cannot write down a weak theory independent of QED
 - ▶ Unified electroweak theory includes Z^0 as well as W^\pm and γ
 - We'll discuss this the week of Nov 17
- Need mechanism to give W^\pm and Z^0 mass
 - ▶ This is the Higgs mechanism
 - We'll discuss this after Thanksgiving

Review: Parity

- Parity operator defined as spatial inversion

$$(x, y, z) \longrightarrow (-x, -y, -z)$$

$$P(\psi(\vec{r})) = \psi(-\vec{r})$$

- Parity conserved in strong and EM interactions
- Can classify parity of different operators:

Name	Form	Parity	Example
Scalar	$\bar{\psi}' \psi$	+1	Temperature
Pseudoscalar	$\bar{\psi}' \gamma^5 \psi$	-1	Helicity
Vector	$\bar{\psi}' \gamma^\mu \psi$	-1	Momentum
Axial Vector	$\bar{\psi}' \gamma^\mu \gamma^5 \psi$	+1	Angular Momentum
Tensor	$\bar{\psi}' (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) \psi$	+1	$F^{\mu\nu}$

where the prime on $\bar{\psi}'$ indicates that it can be a different particle than ψ

The θ - τ Puzzle

- In 1950's, bubble chamber measurements resulted discovery of many hadrons
- Among them, the (then called) θ^+ and τ^+ (Warning: this has nothing to do with the τ lepton)
- Properties of θ and τ :
 - ▶ Strong production
 - ▶ Same mass: 493 MeV
 - ▶ Same Lifetime: 1.2×10^{-8} sec: weak decay (strange particles)
 - ▶ Spin 0
 - ▶ Different decay modes:

$$\theta^+ \rightarrow \pi^+ \pi^0 \quad P = +1$$

$$\tau^+ \rightarrow \pi^+ \pi^+ \pi^- \quad P = -1$$

- If P conserved, these must be different particles

Why do they have the same mass and lifetime?

An aside: How do we know the parity of the final states?

- $\theta^+ \rightarrow \pi^+ \pi^0$

▶ Spin 0 particle decays to two spin 0 particles:

$$\ell = 0$$

▶ Parity from angular momentum and intrinsic parity:

$$P = (-1)^\ell (-1)^2 = 1$$

- $\tau^+ \rightarrow \pi^+ \pi^+ \pi^-$

▶ $\pi^+ \pi^+$ must have even ℓ (Bose Statistics)

▶ If $\ell(\pi^+ \pi^+) = 0$, angular momentum of π^- wrt this system also 0 and

$$P = (-1)^3 = -1$$

▶ If $\ell(\pi^+ \pi^+) = 2$, more possibilities but suppressed since WF at origin=0

Lee and Yang's Suggestion

- In 1956, question raised whether θ and τ could be the same particle
- Lee and Yang did extensive analysis of existing tests of P conservation.
Conclusion:
 - ▶ Stringent tests of P conservation for strong and EM interactions
 - ▶ No evidence for P conservation in weak decays
- Suggested tests of P conservation in weak decays:
 - ▶ Look for interactions that differentiate that left and right handed amplitudes
 - ▶ Since decay rate $\propto |\mathcal{M}|^2$, must look for interference between amplitudes of opposite parity
 - ▶ Express decay rate as sum of scalar and pseudoscalar terms
 - ▶ Identify possible pseudoscalars constructed from observables in decay of particle $P \rightarrow P_1 + P_2 + P_3$:
 - $\vec{p}_1 \cdot (\vec{p}_2 \times \vec{p}_3)$
 - $\vec{p}_1 \cdot \vec{S}$ (if P_1 has spin \vec{S})
- If weak decays include terms that violate parity conservation, then θ and τ can be the same particle
 - ▶ We know now that this is the case and call that particle the K^+

CS Wu's Discovery of Parity Violation (I)

- Look at relative β -decay rate \parallel and anti- \parallel to direction of polarization for a polarized nucleus
- Worked with Co^{60} ($J^P = 5^+$) (half-life: 5 years)
- Decay product: Ni^{60} ($J^P = 4^+$)
- Change of angular momentum without change in parity
 - e and ν_e must have $J = 1$
- To polarize Co^{60} need B field and low temperature
- Cool to $0.1^\circ K$
- Need state-of-the art (for then) refrigeration
- Experiment done at Bureau of Standards in Maryland

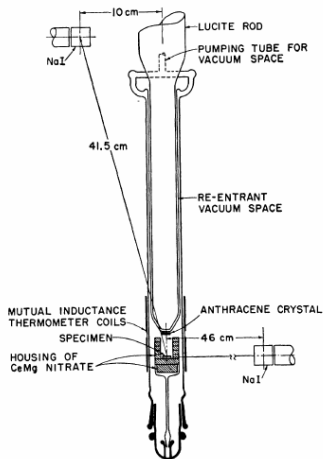


FIG. 1. Schematic drawing of the lower part of the cryostat.

CS Wu's Discovery of Parity Violation (II)

- Monitor level of polarization by studying photons produced in Ni decay
 - Two NaI crystals in polar and equatorial plane used to measure anisotropy
- Co^{60} source allowed to warm up: polarization disappears
- Also can change sign of B field
- Result shows β intensity

$$I(\theta) = 1 + \alpha \frac{\sigma \cdot p}{E}$$

with α negative

- Can't measure α but it is large (consistent with -1)
- Later work by Fraenfelder: $\alpha = -1$

The ν has a red single handedness!

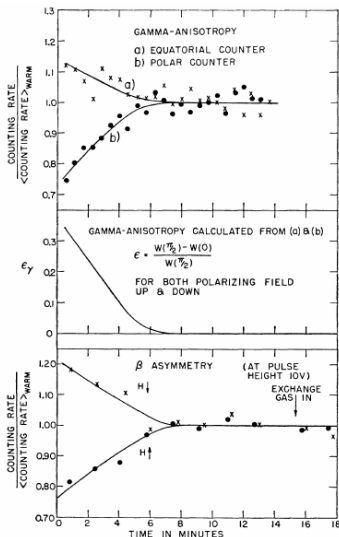
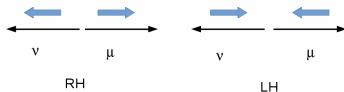


FIG. 2. Gamma anisotropy and beta asymmetry for polarizing field pointing up and pointing down.

Garwin and Lederman: Confirmation of Parity Violation (I)

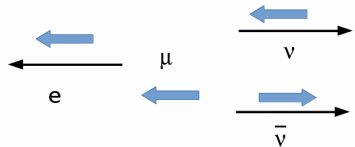
- Results of Wu et al led to flood of experiments
- First appeared in same Phys Rev issue as Wu
- Study $\pi^+ \rightarrow \mu^+ \nu_\mu$, $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$
- Since π^+ has spin 0, μ and ν must have $S = 0$



- If parity is conserved two possibilities need present equally
- If parity not conserved, one possibility can have higher rate

► Thus, μ will be polarized

- When μ decays, polarization results in asymmetry in direction of emission of electron
- For the case where ν is left-handed:



If ν were right-handed, just reverse ν and $\bar{\nu}$ labels

- In either case, electron will exhibit an asymmetry

$$I(\cos \theta) = 1 + \alpha \cos \theta$$

Garwin and Lederman: Confirmation of Parity Violation (II)

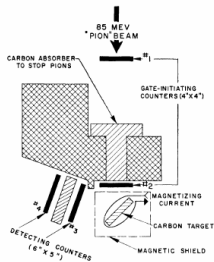


FIG. 1. Experimental arrangement. The magnetizing coil was close wound directly on the carbon to provide a uniform vertical field of 79 gauss per ampere.

- Apply small vertical B field to allow μ to precess
- Rate at fixed angle depends on precession speed and on polarization
- Possible to map complete distribution with one fixed counter

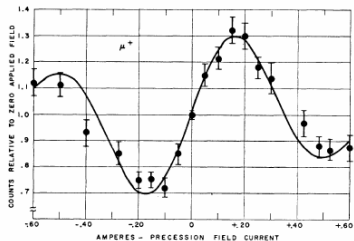


Fig. 2. Variation of gated 3-4 counting rate with magnetizing current. The solid curve is computed from an assumed electron angular distribution $1 - \frac{1}{2} \cos \theta$, with counter and gate-width resolution folded in.

- First measurement of g for the μ : $g = 2$ as expected
- Clear evidence for parity violation
- Repeated experiment with π^- and saw asym change sign

The Helicity of the Neutrino (Goldhaber et al)

- Begin with Eu^{132} (spin 0)
- Allow e^- capture: Sm^{152*} ($J = 1$) + ν
- Spin of Sm^* in same direction as e^-
- $Sm^{152*} \rightarrow Sm + \gamma$ (Sm has $J = 0$)
 $\Rightarrow \gamma$ has helicity of Sm^* in forward direction
- Select forward γ : Use Sm target.
 Forward γ has enough energy to interact. Backward doesn't
 "Resonant scattering"

$$\gamma + Sm^{152} \rightarrow Sm^{152*} \rightarrow \gamma + Sm^{152}$$

- Measure polarization by passing γ through magnetized iron; electron with spin opposite photon can be absorbed
- If γ beam in same direction as B , transmission is greater for left-handed than for right-handed γ 's

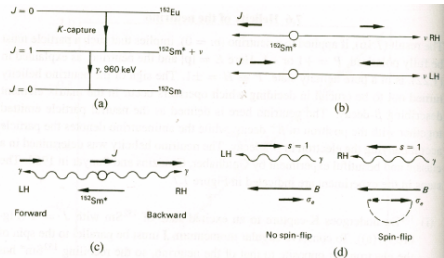


Fig. 7.7. Principal steps in the experiment to determine the neutrino helicity, as described in the text.

- From Perkins, Introduction to High Energy Physics
- See also Goldhaber and Cahn for discussion of this experiment

The ν is left-handed!

Incorporating Parity Violation into Fermi's 4-Point Interaction

- For EM, $J = \bar{\psi}\gamma_\mu\psi$ where γ_μ is a vector operator
- For WI, will generalize to $J = \bar{\psi}\mathcal{O}\psi$
- What possible Lorentz forms can \mathcal{O} have?
 - ▶ S,P Spin 0: ℓ and $\bar{\ell}$ have same helicity
 - ▶ V,A Spin 1: ℓ and $\bar{\ell}$ have opposite helicity
 - ▶ T Spin 2: ℓ and $\bar{\ell}$ have opposite helicity
- Experiments have shown that only V and A currents exist
- For leptons:

$$J_{lept} = \psi_e \gamma_\mu \left(\alpha + \beta \frac{\sigma \cdot p}{E} \right) \psi_\nu$$

if $\beta = -\alpha$, LH ν .

- Experimentally, for leptons:

$$\begin{aligned} J_{lept} &= \psi_e \gamma_\mu \frac{1}{2} \left(1 - \frac{\sigma \cdot p}{E} \right) \psi_\nu \\ &= \psi_e \gamma_\mu \frac{1}{2} (1 - \gamma_5) \psi_\nu \end{aligned}$$

We'll talk more about the quarks Thursday

Helicity and Chirality

- For massless fermions, operator to project states of particular helicity are:

$$P_R = \frac{1}{2} \left(1 + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E} \right)$$
$$P_L = \frac{1}{2} \left(1 - \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E} \right)$$

- For massive fermions, need 4-component spinor and 4-component operator

$$P_{R,L} = \frac{1}{2} (1 \pm \gamma^5)$$

- Because direction of spin wrt momentum changes under boosts, this operator cannot represent helicity per se
- Instead, projects out state of polarization $P = \pm v/c$
 - In spite of this, everyone writes

$$\frac{1}{2} (1 - \gamma^5) u \equiv u_L$$

$\frac{1}{2} (1 \pm \gamma^5)$ are called the chiral projection operators

- You will prove the relevant properties of $P_{R,L}$ on next week's homework

Classification of Weak Decays

- Leptonic: only leptons in final state. Eg:

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

$$\pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

- Semileptonic: Both leptons and hadrons in final state. Eg:

$$n \rightarrow p e^- \bar{\nu}_e$$

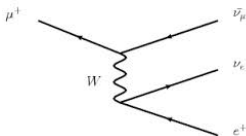
$$K^0 \rightarrow \pi^0 e^+ \nu_e$$

- Hadronic: Only hadrons in final state. Eg:

$$K^0 \rightarrow \pi^+ \pi^-$$

$$\Lambda \rightarrow p \pi^-$$

Muon Decay



- From dimensional analysis: $\Gamma \propto G_F^2 m_\mu^5$
 - Implicitly assumes couplings to e and μ are same

$$G_F^e = G_F^\mu$$

- Full calculation gives

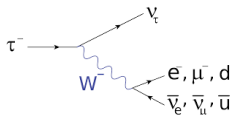
$$\Gamma_\mu \equiv \frac{1}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^2}$$

where

$$\frac{G_F}{\sqrt{2}} = \frac{g_{wk}^2}{8M_W^2}$$

- Full calculation can be found in many books, including Griffiths

Tau Decay



- $m_\tau = 1.777 \text{ GeV}$
- Several possible decays:

$$\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$$

$$\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$$

$$\tau^- \rightarrow d\bar{u} \nu_\tau$$

In last case, the $d\bar{u}$ turns into hadrons with 100% probability

- All diagrams look like μ -decay
- If $G_F^\mu = G_F^e = G_F$, predict:

$$\begin{aligned} \Gamma_{\tau^- \rightarrow e^-} &= \Gamma_{\tau^- \rightarrow \mu^-} \\ &= (m_\tau/m_\mu)^2 \Gamma(\mu) \end{aligned}$$

(difference in available phase space)

- Using the measured τ -lifetime and BR, check consistency of G_F

$$G_F^\tau/G_F^\mu = 1.0023 \pm 0.0033$$

$$G_F^e/G_F^\mu = 1.000 \pm 0.004$$

Lepton universality for G_F

- For quark decays, need a factor of 3 for color. Predict

$$BR(\tau \rightarrow \text{hadrons}) = \frac{3}{3+1+1} = 60\%$$

- Experimental result:

$$BR(\tau \rightarrow \text{hadrons}) = (64.76 \pm 0.06)\%$$

Difference from 60% understood (QCD corrections)

More on G_F for quarks Thurs