Problem Set 11 solutions

Question 1: Feynman diagrams with W bosons (24 points)

Learning objectives

In this question you will:

- Apply knowlege of W Feynman diagrams to various physical processes

Draw a Feynman diagram involving a W boson for each of the followingbprocesses:

1a.

$\tau^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\tau$

1b.

$K^0 \rightarrow \pi^- + e^+ + \nu_e$

1c.

$D^+ \rightarrow \overline{K^0} + \mu^+ + \nu_\mu$ (Note: A D^+ meson is a $c\bar{d}$ combination)

1d.

$\tau^+ \rightarrow \nu_\tau + \pi^+$

1e.

$\Lambda \rightarrow p + e^- + \bar{\nu}_e$

1f.

$\nu_e + e^- \rightarrow \nu_e + e^-$

Question 2: What is X? (20 points)

Learning objectives

In this question you will:

- Use conservation laws to determine what particles are needed to make a process possible via the weak interaction

For each of the weak interations listed below replace the unknown X with the appropriate particle:

2a.

$\pi^+ \rightarrow \pi^0 + e^+ + X$

$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$

2b.

$X \rightarrow e^+ \nu_e \bar{\nu}_\mu$

$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$

2c.

$K^+ \rightarrow X e^+ \nu_e$

$K^+ \to \pi^0 e^+ \nu_e$

2d

$X + p \to n + e^+$

$\nu_u + p \to n + e^+$

2e.

$D^0 \to K^- + \pi^0 + \nu_e + X$

$D^0 \to K^- + \pi^0 + \nu_e + e^+$

Question 3 Helicity suppression in two body weak decays (30 points)

Learning objectives

In this question you will:

- Apply the expression for leptonic weak decays to the case of heavy mesons and demonstrate that the decay rates to different lepton species vary by many orders of magnitude

Consider the rare leptonic decay $B^+ \to \ell^+ \nu_\ell$ (where the B^+ is a $u\bar{b}$ meson

3a.

Explain in words, with an accompanying diagram, why this decay is rare.

This decay occurs through an annihilation of the quark (u) and antiquark ($\bar{b}$) that make up the B^+. the Feynman diagram is

There are three reasons why this decay is rare:

- The decay rate depends on the wave function at the origin squared since it is an annihilation
- The rate is proportional to $|V_{ub}|^2$ and $|V_{ub}| = 3.8 \times 10^{-3}$
- The decay is helicity suppressed (see formula from 3b)

3b.

We saw in class that the charged pion has a partial width for its decay to a muon

$$\Gamma(\pi^+ \to \mu^+ \nu) = \frac{G^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

where f_π is a constant that is related to the π wave function at the origin, has units of mass. Using the analog of this expression, calculate the relative rates for the B^+ to decay to the 3 lepton species e, μ and τ.

The ratio of the rates is given by

$$\Gamma(B^+ \to \tau\nu_\tau) : \Gamma(B^+ \to \mu\nu_\mu) : \Gamma(B^+ \to e\nu_e) = m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right) : m_\mu^2 \left(1 - \frac{m_\mu^2}{m_B^2}\right) : m_e^2 \left(1\right.$$

using $m_e = 0.5$ MeV, $m_\mu = 105$ MeV, $m_\tau = 1777$ MeV and $M_{B^+} = 5279$ MeV we find:

$$\Gamma(B^+ \to \tau\nu_\tau) : \Gamma(B^+ \to \mu\nu_\mu) : \Gamma(B^+ \to e\nu_e) = 0.25 : 1.1 \times 10^4 : 2.8 \times 10^6$$

3c.

From the particle data group web page, state whether this decay has been observed for the cases of $\ell = e$, μ and τ and if the decay has been observed, give the observed branching ratio and its uncertainy and the partial width for the decay.

The decay $B^+ \to \tau^+\nu_\tau$ has been observed with a branching ratio of $1.09 \pm 0.24) \times 10^{-4}$ There is evidence for the decay $B^+ \to \mu^+\nu_\mu$ with a branching ratio between 2.90×10^{-7} and 1.07×10^{-6} at 90% confidence The decay $B^+ \to e + \nu_e$ has not been observed. The 90% limit is a branching ratio $< 9.8 \times 10^{-7}$

Question 4 Cabbibo Allowed and Cabbibo Suppressed Decays (26 points)

Learning objectives

In this question you will:

- Apply knowlege of W Feynman diagrams and of the CKM matrix to calculate relative decay rates

Estimate the relative rates for the following four decay modes of the $D^0(c\overline{u})$ meson: $D^0 \to K^-\pi^+$, $D^0 \to \pi^-\pi^+$, $D^0 \to K^+\pi^-$, $D^0 \to K^+K^-$. As part of your answer, draw Feynman diagrams involving W bosons for each decay.

The relevant Feynman diagrams for these decay are:

The ratios of the decay rates go as the square of the matrix elements. Each diagram has two CKM matrix elements in it. To understand when you use the complex conjugate of the matrix element, look at Thomson Fig 14.5 and 14.6 (although since we here are taking the matrix elements squared, the complex conjugation won't change the answer). From the diagrams above:

$$
\begin{array}{llll}
D^0 \to K^-\pi^+ & D^0 \to \pi^-\pi^+ & D^0 \to K^+\pi^- & D^0 \to K^+K^- \\
\left|V_{ud}^* V_{cs}\right|^2 : & \left|V_{ud}^* V_{cd}\right|^2 : & |V_{us}^* V_{cd}|^2 : & |V_{us}^* V_{cs}|^2 \\
|0.974 \times 0.973|^2 : & |0.974 \times 0.225|^2 : & |0.225 \times 0.225|^2 : & |0.225 \times 0.973|^2 \\
0.89 : & 0.048 : & 0.0026 : & 0.048
\end{array}
$$