Physics 129: Particle Physics
Lecture 21: Weak Interactions (II): Quark Charged Current Interactions

Nov 5, 2020

- Suggested Reading:
  - Thomson Chapter 11
  - Griffiths 10.1-10.5
- Reminder: Quiz #3 next week

Our Weak Interaction Roadmap

- Unlike strong and EM, weak interactions don't conserve parity
  - Vertex selects left-handed state for of particles (and right handed state for anti-particles)
    - Subject of the last lecture; will review today
- $W^{\pm}$ coupling to leptons respect flavor familes (e, μ, τ) but coupling to quarks do not
  - Coupling not diagonal in quark flavor: Need to change basis
    - Main topic for today
  - Introduction of this change in basis gives new phenomenology, including mixing and CP violation
    - Will be discussed next week
- $W^{\pm}$ has charge, so it couples to photon
  - Cannot write down a weak theory independent of QED
  - Unified electroweak theory includes Z^0 as well as $W^{\pm}$ and γ
    - Topic for the week of Nov 17
- Need mechanism to give $W^{\pm}$ and Z^0 mass
  - This is the Higgs mechanism
    - Discuss this after Thanksgiving

Reminder: Charged Current Weak Interactions with Parity Violation

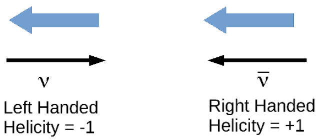
- Wu et al showed that polarized Co^{60} decays had angular distribution

$$\begin{aligned} I(\theta) &= 1 + \alpha \left(\frac{\sigma \cdot p}{E} \right) \\ &= 1 + \alpha \frac{v}{c} \cos\theta \end{aligned}$$

  with later experiments verifying that $\alpha = -1$

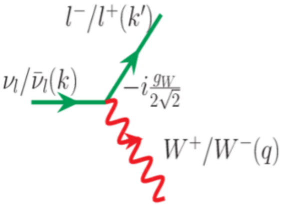
  - This violates parity conservation

- Parity violation is maximal
  - ν are always left handed and $\overline{\nu}$ are always right handed



Putting parity violation into the Feynman rules

- $W^{\pm}$ is charged: fermion vertex must conserve charge
- Lepton number must also be conserved
- Examples:
  - $W^+ \to e^+ \nu_e$
  - $W^- \to \mu^- \overline{\nu}_\mu$



- Coupling factor at vertex:

$$\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^\mu\left(1-\gamma^5\right)$$

- $\sqrt{2}$ an historical artifact
- $\frac{1}{2}\left(1-\gamma^5\right)$ makes the interaction left-handed

  You will prove this on next week's homework

Helicity for particles with mass

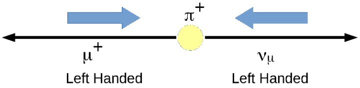
- The factor $\frac{1}{2}\left(1-\gamma^5\right)$ forces massless particle fermions to always have their spin anti-aligned with their direction of motion and massless antiparticle fermions to always have their spins aligned with the direction of motion
- For massive fermions, it's a bit more complicated
  - Dirac spinors have 4-components
  - The components with the "wrong" alignment have factors of v/c relative to those of "right" alignment
  - So, unless particles ultra-relativistic both polarizations exist, although not wit the same rate
    - This is why Wu saw an intensity

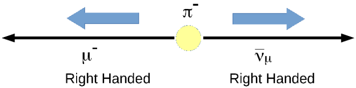
$$I(\theta) = 1 - \frac{v}{c}\cos\theta$$

rather than $1 - \cos\theta$

- This is an extremely important fact
  - Let's see how it works for pion decay

Helicity and Pion Decay

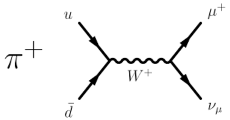




- Spin 0 pion decays to two spin $\frac{1}{2}$ fermions
  - Total spin must be 0: antisymmetric spin sate
- Neutrino is left handed and anti-neutrino is right handed
  - This forces charged lepton to be in "wrong" alignment
  - Only possible due to components proportional to v/c
  - More massive leptons have lower v and will be favored in the decay
  - This is called "helicity suppression"

Next two pages go through this in greater detail

Pion Decay (I) $\pi^-(q) \to \mu^-(p) + \overline{\nu}_\mu(k)$



$\tau_{\pi^+} = 2.6 \times 10^{-8}$ s

- $u\bar{d}$ annihilation into virtual W^+
- Depends on π^+ wave function at origin
  - Need phenomenological parameter that characterizes unknown wave function
- Write matrix element

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} J_\pi^\mu \overline{u}(p)\gamma_\mu (1-\gamma_5) v(k)$$

where $J_\pi^\mu = f(q^2)q^\mu$ since q^μ is the only available 4-vector

- But $q^2 = m_\pi^2$ so $J_\pi = f_\pi q^\mu$. f_π has units of mass (matrix element must be dimensionless)
- After spinor calculation, result for decay width:

$$\Gamma = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- This came from:

$$|\mathcal{M}|^2 \sim G_F^2 m_\mu^2 \left(m_\pi^2 - m_\mu^2\right) f_\pi^2$$

$$\text{Phase Space} \sim \frac{|p|}{8\pi m_\pi^2}$$

- We'll examine what this means on the next page

Pion Decay (II)

- From previous page

$$\Gamma = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

- Result for electron same with $m_\mu \to m_e$
- Thus

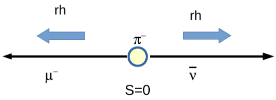
$$\frac{\Gamma_e}{\Gamma_\mu} = \frac{m_e^2}{m_\mu^2} \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2$$

- Since $m_e = 0.51$ MeV, $m_\mu = 105.65$ MeV and $m_{pi+} = 139.57$ MeV

$$\frac{\Gamma_e}{\Gamma_\mu} \sim 1.2 \times 10^{-4}$$

This agrees with measurements

- As promised, result demonstrates helicity suppression

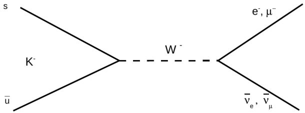


- Spin 0 pion, right-handed antineutrino forces μ^- to be right-handed
- But μ^- wants to be left-handed
  - rh component $\sim (v/c)^2 \sim m_\mu$
- The less relativistic the decay product is, the larger the decay rate

Charged Kaon Decays

- $K^{\pm}$ mass larger than $\pi^{\pm}$
- More options for decay

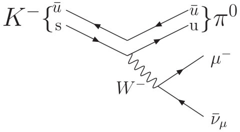
Leptonic



Same calculation as for $\pi^{\pm}$
Helicity suppression make decay rate to muons larger that to electrons

$$BR(K^- \to \mu^- \overline{\nu}_\mu) = (63.56 \pm 0.11) \times 10^{-2}$$
$$BR(K^- \to e^- \overline{\nu}_e) = (1.582 \pm 0.007) \times 10^{-5}$$

Semi-Leptonic

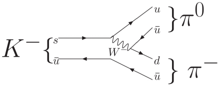


3-body decay: No helicity suppression

$$BR(K^- \to \pi^0 \mu^- \overline{\nu}_\mu) = (3.352 \pm 0.033)\ \%$$
$$BR(K^- \to \pi^0 e^- \overline{\nu}_e) = (5.07 \pm 0.04)\ \%$$

More phase space for decay to e

- Hadronic (several diagrams possible, including:)



$$BR(K^- \to \pi^- \pi^0) = (20.67 \pm 0.08)\ \%$$
$$BR(K^- \to \pi^- \pi^0 \pi^0) = (1.760 \pm 0.023)\ \%$$
$$BR(K^- \to \pi^- \pi^- \pi^+) = (5.583 \pm 0.024)\ \%$$

Some Observations

- Leptonic decays of μ and τ demonstrate that G_F the same for all lepton species
  - See discussion of τ decay in Tuesday's lecture
- Leptonic decay of charged pion and kaon tell us nothing about G_F since f_π and f_K (which depend on wf at origin) are unknown
- If we want to ask whether G_F is the same for hadronic currents as leptonic ones, we need to look at semileptonic decays
  - Analog of β-decay
- But, well have to make sure that we are not affected by stong interaction corrections!

(V-A) and the Hadronic Current

- At low q^2 we measure WI using hadrons and not quarks
- Need to worry about whether binding of quarks in hadron affects the coupling

$$1 - \gamma^5 \rightarrow C_V - C_A \gamma^5$$

- Experimentally, for the neutron

$$C_V = 1.00 \pm 0.003; \quad C_A = 1.26 \pm 0.02$$

  - Vector coupling unaffected: protected by charge conservation (CVC)
  - Axial vector coupling modified (PCAC)
- Experimental implication: for precision tests of hadronic weak interactions, study decays that can only occur through C_V term
  - This means decays between states of the same parity
  - Best option is "superallowed" β-decay with $0^+ \rightarrow 0^+$ transition
  - In addition to no axial vector component, such transitions cannot occur via γ decay

Is G_F Really Universal?

- Muon decay rate is

$$\Gamma = \frac{G_F^2 {m_\mu}^5}{192\pi^3}$$

  in approximation where m_e ignored
- Same formula holds for nuclear β-decay
- A good choice of decay: $O^{14} \to N^{14*}\ e^+ \nu_e\ (0^+ \to 0^+)$
- Correcting for available phase space we find

$$\begin{aligned} G_\mu &= 1.166 \times 10^{-5} \\ G_\beta &= 1.136 \times 10^{-5} \end{aligned}$$

Close but not the same!

- What's going on?

Extend Our Study to More Hadron Decays

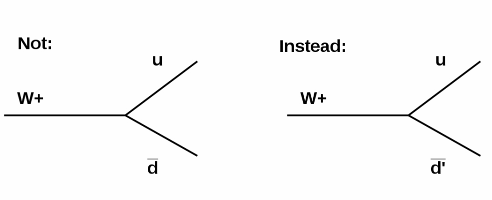
- Compare the following:

	Decay	Quark Level Decay
0^{14}:	$p \to n e^+ \nu_e$	$u \to d e^+ \nu_e$
	$\pi^- \to \pi^0 e^- \overline{\nu}_e$	$d \to u e^- \overline{\nu}_e$
	$K^- \to \pi^0 e^- \overline{\nu}_e$	$s \to u e^- \overline{\nu}_e$
	$\mu^+ \to \overline{\nu}_\mu e^+ \nu_e$	

- After correcting for phase space factors, G_F obtained from p and π^- agree with each other, but are slightly less than obtained from μ.
- G_F obtained from K^- decay seems to be much smaller
- Either G_F is not universal, or something else is going on!

An Explanation: Choice of weak eigenstates

- Suppose strong and weak eigenstates of quarks not the same
- Weak coupling:



- Here d' is an admixture of down-type quarks
- Normalization of w.f. for quarks means if $d' = \alpha d + \beta s$, then $\sqrt{\alpha^2 + \beta^2} = 1$
- Can force this normalization by writing α and β in terms of an angle

$$d' = d\cos\theta_C + s\sin\theta_c$$

or

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_C \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

The Cabbibo Angle

- Using

$$d' = d\cos\theta_C + s\sin\theta_c$$

we predict

$$\begin{aligned} p \;\&\; \pi \text{ decay} &\quad\propto\quad G_F^2 \cos^2\theta_C \\ K \text{ decay} &\quad\propto\quad G_F^2 \sin^2\theta_C \\ \mu \text{ decay} &\quad\propto\quad G_F^2 \end{aligned}$$

- Using experimental measurements, find

$$\begin{aligned} \cos\theta_c &= 0.97420 \pm 0.00021 \\ \sin\theta_c &= 0.2243 \pm 0.0005 \end{aligned}$$

- However, in addition to the d' there is an orthoginal down-type combination

$$s' = s\cos\theta_c - d\sin\theta_c$$

Does it iteract weakly?

A New Quark (Discussion from the Early 1970's)

- It's odd to have one charge 2/3 quark and two charge -1/3 quarks
- Suppose there is a heavy 4^{th} quark
- We could then have two families of quarks. In strong basis:

$$\begin{pmatrix} u \\ d \end{pmatrix}, \quad \begin{pmatrix} c \\ s \end{pmatrix}$$

  Call this new quark "charm"

- Then, the weak basis is

$$\begin{pmatrix} u \\ d' = d\cos\theta_C + s\sin\theta_c \end{pmatrix}, \quad \begin{pmatrix} c \\ s' = s\cos\theta_c - d\sin\theta_c \end{pmatrix}$$

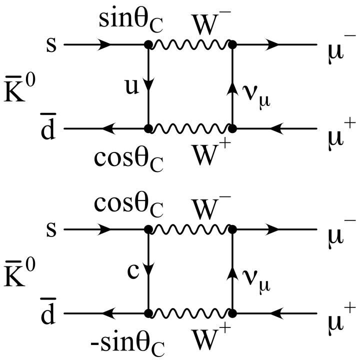
- There is a good argument for this charm quark in addition to G_F
  - This is the reason people were so excited when the J/ψ was seen in 1974

The GIM Mechanism (I)

- Glashow, Iliopoulous, Maiani (GIM) proposed existence of this 4^{th} quark (charm)
- Charm couples to the s' in same way u couples to the d'
- Reason for introducing charm: to explain why flavor changing neutral currents (FCNC) are highly suppressed
- Two examples of FCNC suppression:
  1. $BR(K_L^0 \to \mu^+\mu^-) = 6.84 \times 10^{-9}$
  2. $BR(K^+ \to \pi^+\nu\nu)/BR(K^+ \to \pi^0\mu\nu) < 10^{-7}$
- Why are these decay rates so small?
- It turns out that there is also a Z that couples to $f\overline{f}$ pairs, but it does not change flavor (same as γ)
- If only vector boson was the $W^{\pm}$, would require two bosons to be exchanged
  - Need second order charged weak interactions, but even this would give a bigger rate than seen unless there is a cancellation

The GIM Mechanism (II)

- Consider the "box" diagram



  - $\mathcal{M}$ term with u quark $\propto \cos\theta_C \sin\theta_C$
  - $\mathcal{M}$ term c quark $\propto -\cos\theta_C \sin\theta_C$
  - Same final state, so we add $\mathcal{M}$'s
  - Terms cancel in limit where we ignore quark masses

This Cancellation is Not a Accident!

- Matrix relating strong basis to weak basis is unitary

$$d'_i = \sum_j U_{ij} d_j$$

- Therefore is we sum over down-type quark pairs

$$\begin{aligned} \sum_i \overline{d}'_i d'_i &= \sum_{ijk} \overline{d}_j U^\dagger_{ji} U_{ik} d_k \\ &= \sum_j \overline{d}_j d_j \end{aligned}$$

- If an interaction is diagonal in the weak basis, it stays diagonal in the strong basis
- Independent of basis, there are no $d \longleftrightarrow s$ transitions

No flavor changing neutral current weak interactions
(up to terms that depend on the quark masses)

Some Questions and Answers about the GIM Mechanism

- Why is mixing in the down sector?
  - This is convention.
  - Charged current interactions always involve an up-type and a down-type quark
  - Can always define basis to move all mixing into either up or down sector
- Why is there no Cabbibo angle in the lepton sector?
  - Actually, there is!
  - Before people observed neutrino oscillations, they thought ν's were massless.
  - If all ν were massless, or had same mass, then free to redefine flavor basis to remove the mixing

    We now need to define mixing angles for neutrinos as well as quarks

More Than Two Generations

- Generalize to N families of quark ($N = 3$ as far as we know)
- U is a unitary $N \times N$ matrix and d'_i is an N-column vector

$$d'_i = \sum_{j=1}^{N} U_{ij} d_j$$

  U is called the CKM matrix

- How many independent parameters do we need to describe U?
  - $N \times N$ matrix: N^2 elements
  - But each quark has an unphysical phase: can remove $2N - 1$ phases (leaving one for the overall phase of U)
  - So, U has $N^2 - (2N - 1)$ independent elements
- However, an orthogonal $N \times N$ matrix has $\frac{1}{2}N(N-1)$ real parameters
  - So U has $\frac{1}{2}N(N-1)$ real parameters
  - $N^2 - (2N-1) - \frac{1}{2}N(N-1)$ imaginary phases ($= \frac{1}{2}(N-1)(N-2)$)
- $N = 2$ 1 real parameter, 0 imaginary
- $N = 3$ 3 real parameters, 1 imaginary
- Three generations requires an imaginary phase: CP Violation inherent

The CKM Matrix

- Write hadronic current

$$J^{\mu} = -\frac{g}{\sqrt{2}} \left(\overline{u}\ \overline{c}\ \overline{t} \right) \gamma_{\mu} \frac{(1-\gamma_5)}{2} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- V_{CKM} gives mixing between strong (mass) and (charged) weak basis
- Often write as

$$V_{CKM} = \begin{pmatrix} V_{ud} & Vus & V_{ub} \\ V_{cd} & Vcs & V_{cb} \\ V_{td} & Vts & V_{tb} \end{pmatrix}$$

- Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Here λ is the $\approx \sin\theta_C$.

Best Fit for CKM Matrix from PDG

- From previous page

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Impose Unitary and use all experimental measurements

$$\lambda = 0.22453 \pm 0.00044 \qquad A = 0.836 \pm 0.015$$
$$\rho = 0.122^{+0.018}_{-0.17} \qquad \eta = 0.355^{+0.12}_{-0.11}$$

- Result for the magnitudes of the elements is:

$$\begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 000032 \end{pmatrix}$$

- We'll talk about this more next week