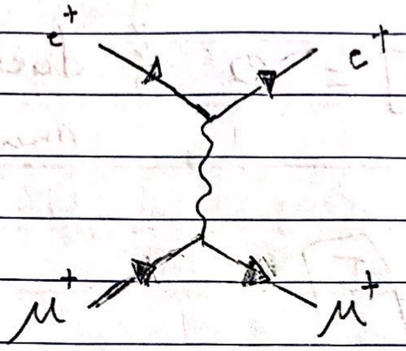
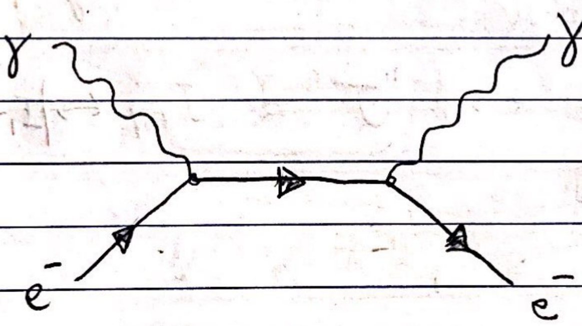
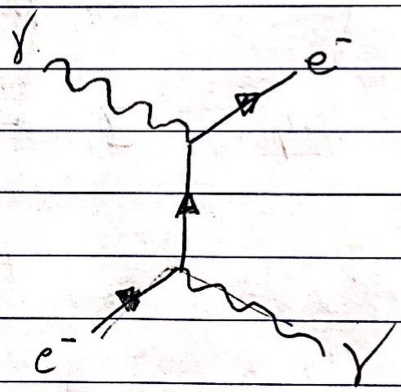
HW 8

1) a) $e^+\mu^+ \to e^+\mu^+$ lowest scattering order

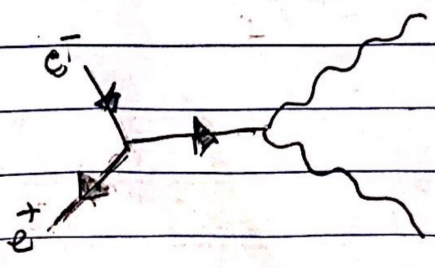


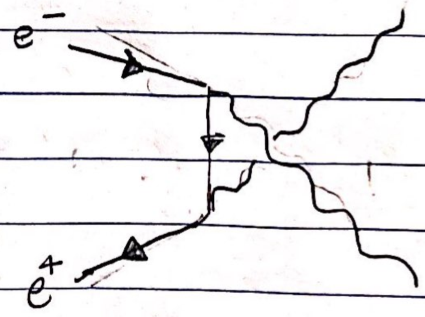
1b) $\gamma e^- \to \gamma e^-$



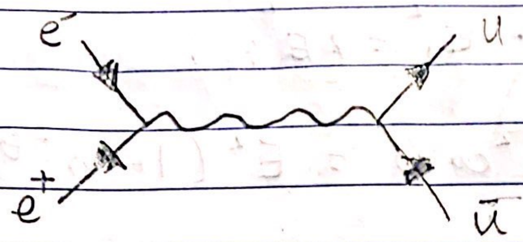


1c) $e^+e^- \to \gamma\gamma$





1d) $e^+e^- \rightarrow \mu\bar{\mu}$



2) $e^+e^- \rightarrow \mu^+\mu^-$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos\theta)$$

$$\langle|\mathcal{M}|^2\rangle = e^4(1+\cos\theta) \xrightarrow{\text{show}} \langle|\mathcal{M}|^2\rangle = 2e^4\left(\frac{t^2+u^2}{s^2}\right)$$

$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$

$$t = (p_3-p_1)^2 = (p_4-p_2)^2$$

$$u = (p_3-p_2)^2 = (p_4-p_1)^2$$

→ in the limit that $\sqrt{s} \gg m_\mu$, the p^μ can be written as:

$$p_1 = \begin{pmatrix} E \\ 0 \\ 0 \\ E \end{pmatrix} \qquad p_2 = \begin{pmatrix} E \\ 0 \\ 0 \\ -E \end{pmatrix}$$

→

$$p_3 = \begin{pmatrix} E \\ E\sin\theta \\ 0 \\ E\cos\theta \end{pmatrix} \qquad p_4 = \begin{pmatrix} E \\ -E\sin\theta \\ 0 \\ -E\cos\theta \end{pmatrix}$$

2) cont)

$$p_1 \cdot p_2 = E^2 + 0 + 0 + E^2 = 2E^2$$

$$p_1 \cdot p_3 = E^2 + 0 + 0 + E^2 \cos\theta = E^2(1+\cos\theta)$$

$$p_1 \cdot p_4 = E^2 + 0 + 0 + (-E^2\cos^2\theta) = E^2(1-\cos\theta)$$

rewrite $(1+\cos^2\theta)$ using these: (to get in terms of stu)

$$(p_1 \cdot p_3)^2 = E^4(1 + 2\cos\theta + \cos^2\theta)$$

$$(p_1 \cdot p_4)^2 = E^4(1 - 2\cos\theta + \cos^2\theta)$$

$$\therefore (p_1 \cdot p_3)^2 + (p_1 \cdot p_4)^2 = \underline{2E^4}(\underline{1+\cos^2\theta})$$

$$(p_1 \cdot p_2)^2 = \underline{4E^4}$$

$$\therefore \langle |\mathcal{M}|^2 \rangle = e^4(1+\cos^2\theta) = 2e^4 \frac{[(p_1 \cdot p_3)^2 + (p_1 \cdot p_4)^2]}{(p_1 \cdot p_2)^2}$$

$\sqrt{s} \gg m_\mu$ limit again:

$$s = (p_1+p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2$$
$$= m_1^2 + m_2^2 + 2p_1 \cdot p_2 \approx \underline{2p_1 \cdot p_2}$$

$$t = (p_1 - p_3)^2 \approx -2\underline{p_1 \cdot p_3}$$

$$u = (p_1 - p_4)^2 \approx -2\underline{p_1 \cdot p_4}$$

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$$\therefore \langle |\mathcal{M}|^2 \rangle = e^4(1+\cos^2\theta) = 2e^4 \frac{(p_1\cdot p_3)^2 + (p_1\cdot p_4)^2}{(p_1\cdot p_2)^2}$$

$$= 2e^4 \left[\frac{(t/2)^2 + (u/2)^2}{(s/2)^2} \right]$$

$$\therefore \boxed{\langle |\mathcal{M}|^2 \rangle = 2e^4 \left[\frac{t^2 + u^2}{s^2} \right]} \checkmark$$

3)

a) $\dfrac{d\sigma}{d\Omega^*} = \dfrac{1}{64\pi^2 s} \dfrac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2$, $p_f^* = p_i^* = E$.

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \langle |\mathcal{M}|^2 \rangle$$

$$= \frac{1}{64\pi^2 s}\left(2e^4 \frac{t^2+u^2}{s^2} \right), \quad \alpha = \frac{e^2}{4\pi}$$

$$= \frac{2e^4}{64\pi^2 s}\left(\frac{(p_1\cdot p_3)^2 + (p_1\cdot p_4)^2}{(p_1\cdot p_2)^2} \right)$$

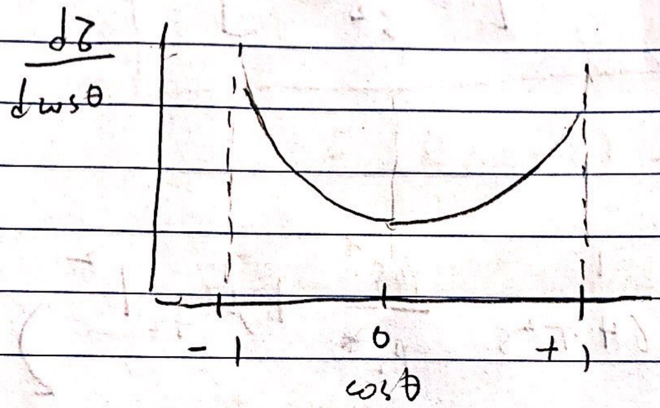
$$= \frac{2e^4}{64\pi^2 s}\left[\frac{2\cancel{E^4}(1+\cos^2\theta)}{4\cancel{E^4}} \right] = \frac{1e^4}{64\pi^2 s}(1+\cos^2\theta)$$

$$= \boxed{\left(\frac{\alpha^2}{4s}\right)(1+\cos^2\theta) = \frac{d\sigma}{d\theta}}$$

3b) ϕ - Independent cross section:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\phi\, d(\cos\theta)} \longrightarrow \frac{d\sigma}{d(\cos\theta)} = \int \frac{d\sigma}{d\Omega}\, d\phi = \frac{d\sigma}{d\Omega}(2\pi)$$

$$\therefore \frac{d\sigma}{d(\cos\theta)} = (2\pi)\left(\frac{\alpha^2}{4s}\right)(1+\cos^2\theta)$$



$\}$ $1+x^2$ curve

3c) $$\sigma = \int \frac{d\sigma}{d\cos\theta}\, d\cos\theta$$

$$= 2\pi \frac{\alpha^2}{4s} \int_{-1}^{+1} (1+\cos^2\theta)\, d\cos\theta$$

$$= \frac{\pi\alpha^2}{2s}\left[\cos\theta + \frac{\cos^3\theta}{3}\right]_{-1}^{1}$$

$$= \frac{\pi\alpha^2}{2s}\left(1+\frac{1}{3}+1+\frac{1}{3}\right) = \frac{\pi\alpha^2}{2s}\left(\frac{8}{3}\right)$$

$$\therefore \boxed{\sigma = \frac{4\pi\alpha^2}{3s}}$$