PSET 9

1) a) $x = \dfrac{Q^2}{2 P\cdot q}$, $y \equiv \dfrac{P\cdot q}{P\cdot k}$ $\quad q \equiv k - k'$

$Q^2 = -q^2$

$|\vec{p}_e^{\,*}| = |\vec{p}_q^{\,*}| = E^*$ $\longleftrightarrow$ $\vec{p}_q^{\,*} + \vec{p}_e^{\,*} = 0.$

$M = m_p$

$\theta^* =$ angle with z-axis.

Show that $y = \sin^2\left(\frac{\theta^*}{2}\right)$

$$y = \frac{P\cdot q}{P\cdot k} \stackrel{?}{=} \sin^2\left(\frac{\theta^*}{2}\right).$$

$$\frac{P\cdot q}{P\cdot k} = \frac{P\cdot(k-k')}{ME} = \frac{M(E-E')}{ME}$$

$$\therefore = \underbrace{1 - E'/E}\ = y$$

$$1 - \frac{E'}{E} \stackrel{?}{=} \sin^2\left(\frac{\theta^*}{2}\right)$$

$\hookrightarrow$ $$\cancel{E' = \frac{E}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta^*}{2}\right)}}$$

$$\cancel{1 - \frac{1}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta^*}{2}\right)} = \frac{\frac{2E}{M}\sin^2\left(\frac{\theta^*}{2}\right)}{1 + \frac{2E}{M}\sin^2\left(\frac{\theta^*}{2}\right)}}$$

$ep \to e + X$

$E^2 = m^2 + p$

$E \approx p$

Invariants:

$$P \cdot q = P \cdot (k - k') = M(E - E') \equiv M\nu$$

$$Q^2 = -q^2 = -(k-k')^2 = 2EE'(1-\cos\theta)$$

$$y = \frac{P \cdot q}{P \cdot k} = 1 - E'/E = \frac{(E - E^*)}{E} \equiv \nu/E$$

$$\vec{p}_q^{\,*} + \vec{p}_e^{\,*} = 0 \quad \Big| \quad |\vec{p}_e^{\,*}| = |\vec{p}_q^{\,*}| \equiv E^*$$

$$Q^2 = 4EE^* \sin^2\left(\frac{\theta^*}{2}\right) = -q^2$$

$$x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2(M\nu)}$$

$$\nu = \frac{Q^2}{2Mx}, \quad \nu = Ey$$

$$Ey = \frac{Q^2}{2Mx}$$

$$y = \frac{Q^2}{2ME}\frac{1}{x} = \frac{4\cancel{E}E^* \sin^2\left(\frac{\theta^*}{2}\right)}{2M\cancel{E}x}$$

$E^* = |\vec{p}_e^{\,*}| = |\vec{p}_q|$

$$y = \frac{2E^*}{M}\sin^2\left(\frac{\theta^*}{2}\right).$$

$$\boxed{y \propto \sin^2\left(\frac{\theta^*}{2}\right)}$$

?

1b)

$$\frac{d\sigma^{eq}}{d\Omega} = e_q^2 \frac{\alpha^2}{8p^{*2}\sin^4(\theta^*/2)}\left[1+\cos^4\left(\frac{\theta^*}{2}\right)\right]$$

$y = \sin^2\left(\frac{\theta^*}{2}\right) \longrightarrow$ { using this for the following problems as its given

$$d\Omega = 2\pi\, d(\cos\theta^*)$$

$$\frac{d\sigma}{\cancel{d\Omega}}\left(\frac{\cancel{d\Omega}}{dy}\right) = \frac{d\sigma}{dy} \quad \Bigg| \quad \frac{d\Omega}{dy} = \frac{2\pi\, d(\cos\theta^*)}{dy}$$

$$\frac{dy}{d(\cos\theta^*)} = \frac{d}{d(\cos\theta^*)}\left(\sin^2\left(\frac{\theta^*}{2}\right)\right) = \frac{d}{d(\cos\theta^*)}\left[\frac{1}{2} - \frac{1-\cos\theta^*}{2}\right.$$

$$\underline{\frac{d\Omega}{dy} = 2\pi\left(-\frac{2}{1}\right)} \Longleftarrow \underline{\frac{dy}{d(\cos\theta^*)} = -\frac{1}{2}}$$

$$\frac{d\sigma^{eq}}{d\Omega}\left(\frac{d\Omega}{dy}\right) = \boxed{\frac{d\sigma^{eq}}{dy} = -e_q^2\frac{\pi\alpha^2}{2p^{*2}\sin^4\left(\frac{\theta^*}{2}\right)}\left[1+\cos^4\left(\frac{\theta^*}{2}\right)\right]}$$

$$= (-4\pi)\frac{d\sigma^{eq}}{d\Omega}$$

$$= -e_q^2\frac{\pi\alpha^2}{2p^{*2}y^2}\left[1+(1-y)^2\right]$$

$$1+(1-y)^2 = y^2-2y+2$$

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1c)

$$\frac{d^2\sigma^{ep}}{dx\,dy} \quad \text{using} \quad f_i(x)$$

$$\frac{d\sigma^{ep}}{dy} = \boxed{\frac{-e_q^2\pi\alpha^2}{2p^{*2}y^2}} \left[1+(1-y)^2\right] = \frac{-e_q^2\pi\alpha^2}{p^{*2}y^2}\left[(1-y)+\frac{y^2}{2}\right]$$

differentiate

$$\frac{d^2\sigma}{dy} = \frac{\pi\alpha^2}{p^{*2}y^2}\left[(1-y)+\frac{y^2}{2}\right]\cdot e_i^2 \underbrace{f_i(x)\,\underline{\underline{\delta x}}}$$

↳ for specific quark flavor ←

overall:

$$\boxed{\frac{d^2\sigma^{ep}}{dx\,dy} = \frac{\pi\alpha^2}{p^{*2}y^2}\left[(1-y)+\frac{y^2}{2}\right]\left(\sum_i e_i^2 f_i(x)\right)}$$

1d) $F_2(x) \equiv \sum_i e_i^2\, x\, f_i(x) = x\sum_i e_i^2 f_i(x)$

$$\hookrightarrow \frac{F_2(x)}{x} = \sum_i e_i^2 f_i(x)$$

$$\therefore \boxed{\frac{d^2\sigma^{ep}}{dx\,dy} = \frac{\pi\alpha^2}{p^{*2}y^2}\left[(1-y)+\frac{y^2}{2}\right]\frac{F_2(x)}{x}}$$

$$F_2(x) = 2x\,F_1(x)$$

$(x_1 - x_2)(p_1 + p_2) = x_1 p_1 + x_1 p_2 - x_2 p_1 - x_2 p_2$

$f_i(x)$

$i = j$ b/c quark/antiquark must be same sp.

2a)

$$\frac{d\sigma(pp \to \mu^+\mu^- X)}{d\hat{s}} = \sum_{ij} f_i^{(q)}(x_1) f_j^{(\bar{q})}(x_2) \frac{d\sigma(q_i \bar{q}_j \to \mu^+\mu^- X)}{d\hat{s}}$$

Show that $\hat{s} = x_1 x_2 s$

$s = (\text{CoM energy})^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2$

$p_3 = x_1 p_1$

$p_4 = x_2 p_2$

$m \ll p$

$x_1, x_2 \equiv$ fraction of protons' original momentum

$pp: \; s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 \approx 2p_1 p_2$

$\mu^+\mu^-: \; \hat{s} = (p_3 + p_4)^2 \simeq 2p_3 p_4 = 2(x_1 p_1)(x_2 p_2)$

$= x_1 x_2 (2p_1 p_2)$

$\hat{s} = x_1 x_2 (2p_1 p_2)$

$\therefore \boxed{\hat{s} = x_1 x_2 s}$

2b) prove $p_{\parallel} = (x_1 - x_2)\frac{\sqrt{s}}{2} \equiv$ momentum along beamline

$p_{\parallel} = x_1 p_1 - x_2 p_2$

p_{total} in CoM $= 0 \to |p_1| = |p_2| \equiv p$.

$\hookrightarrow E = p$

$p_{\parallel} = (x_1 - x_2) p = (x_1 - x_2) E$

$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 p_2 = 4p^2 = 4E^2 \longrightarrow$

$s = 4E^2 \rightarrow E = \frac{\sqrt{s}}{2}$

$$\boxed{p_{T1} = (x_1 - x_2)\,E = (x_1 - x_2)\frac{\sqrt{s}}{2}}$$