Physics 129: Particle Physics
Lecture 7: Symmetries and Conservation Laws (Part I)

Sept 17, 2020

- Suggested Reading:
  - Thomson Sections 1.1, 3.1, 9.1-9.2
  - Griffiths Chapter 4
  - Perkins Sections 3.1-3.10

Our Goal:

Determine the Lagrangian $\mathcal{L}$ that describes the fundamental particles in the Standard Model and the interactions between these particles:

- Deduce the form of $\mathcal{L}$ through experimental measurements of
  - Particle decays
  - Scattering cross sections
  - Mass spectra
- First step: make observations that constrain the form of the Lagrangian (determine what possible terms are allowed)
  - Learn what the symmetries and conservation laws obeyed by each type of interation
- Next step: use properties of each particle species to determine which are pointlike (fundamental) and which are composed of smaller constituents
  - Will find that leptons are pointlike but hadrons have substructure
- Final step: postulate form for $\mathcal{L}$; test postulate using detailed measurements

  This is an ongoing activity as we continue to look for new physics beyond the Standard Model

Particle Decays: Fermi's Golden Rule (FGR)

Reminder from Lecture 3:

- The *transition rate* W_{ka} is the transition probability per unit time for going from state $|a\rangle$ to a state with energy in the range δ around E_k
- Fermi's Golden Rule tells us how to calculate W_{ka}:

$$\frac{d}{dt}\left(P_{a\to k}\right) \equiv W_{ka} = 2\pi\lambda^2 \left|H'_{ka}\right|^2 \mathcal{D}(E_k)$$

- Decays of fundamental particles will obey this rule
  - $\mathcal{D}(E_k)$ depends only on the kinematics (masses and momenta of decay products)
  - "Matrix element" $\lambda\,|H'_{ka}|$ contains all the information about $\mathcal{L}$
  - λ explicitly pulled out here to remind us that there is a perturbative parameter that characterizes the strength of the interaction
  - Usually, the λ is incorporated into the definition of H'

$\Rightarrow$ Measuring decay rates provides information on the strength of the interaction

Decay rates, lifetimes and particle widths

- Decay rate measured in 1/sec. Moving to natural units:

$$\frac{1}{\text{sec}} \to \frac{\hbar}{\text{sec}} \to \text{MeV}$$

- Called "partial width": $\Gamma_{ka} \equiv W_{ka}$ measured in units of energy
- Lifetime of a particle is related to its decay width summed over all possible decay channels:

$$\Gamma_{tot} = \sum_{1}^{n} \Gamma_i$$

$$\tau = \frac{1}{\Gamma_{tot}}$$

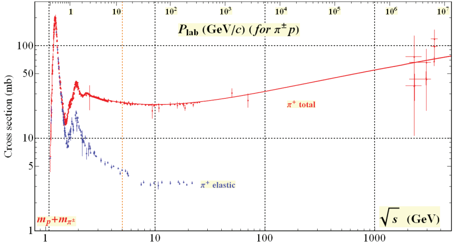
- If a particle is unstable, it's mass cannot be describes as a δ-function in energy
- Uncertainty principle

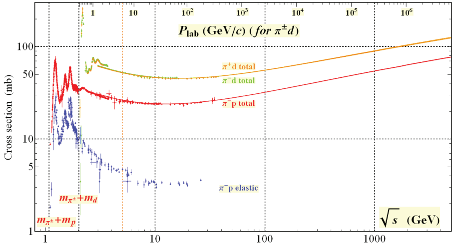
$$\Delta E \Delta t \geq \hbar$$

- Finite lifetime means mass distribution has finite width

  Using FGR, after correcting for density of states, can determine strength of interaction that causes the particle to decay from its lifetime/width

Using Scattering to Measure Particle Widths





- Large bumps: "resonances"
- Eg: near 1236 MeV
  - This is called Δ^{++} resonance
- From the plot: width ≈ 120 MeV $\Rightarrow$ short lifetime
- Using $\Delta E \Delta t \sim \hbar$:

$$\begin{aligned}\Delta t &\sim \frac{\hbar}{\Delta E} \\ &\sim \frac{6.58 \times 10^{-22}\ \text{MeV s}}{120\ \text{MeV}} \\ &\sim 5 \times 10^{-24}\ \text{s}\end{aligned}$$

- Large decay width means short lifetime which means large value for $\lambda H'_{int}$

Decays via the Strong Interaction

- Widths of 10 to 100's of MeV typical of strong decays
- Indicate that strength of interaction is big
- Take a look at in the PDG:
  - Most hadrons aside from the lightest ones decay strongly
  - There are exceptions and these exxeptions will tell a story

From PDG particle table

$\boxed{\rho(770)}$ $I^G(J^{PC}) = 1^+(1^{--})$

See the note in $\rho(770)$ Particle Listings.

Mass $m = 775.26 \pm 0.25$ MeV

$\boxed{\text{Full width } \Gamma = 149.1 \pm 0.8 \text{ MeV}}$

$\boxed{\psi(3770)}$ $I^G(J^{PC}) = 0^-(1^{--})$

Mass $m = 3773.7 \pm 0.4$ MeV (S = 1.4)

$\boxed{\text{Full width } \Gamma = 27.2 \pm 1.0 \text{ MeV}}$

Electromagnetic and Weak Decays

$\boxed{\pi^0}$ $\quad I^G(J^{PC}) = 1^-(0^{-+})$

Mass $m = 134.9768 \pm 0.0005$ MeV $\quad$ (S = 1.1)
$m_{\pi^\pm} - m_{\pi^0} = 4.5936 \pm 0.0005$ MeV
Mean life $\tau = (8.52 \pm 0.18) \times 10^{-17}$ s $\quad$ (S = 1.2)
$c\tau = 25.5$ nm

$\boxed{\pi^\pm}$ $\quad I^G(J^P) = 1^-(0^-)$

Mass $m = 139.57039 \pm 0.00018$ MeV $\quad$ (S = 1.8)
Mean life $\tau = (2.6033 \pm 0.0005) \times 10^{-8}$ s $\quad$ (S = 1.2)
$c\tau = 7.8045$ m

- π are the lightest hadrons
  - Cannot decay via strong interaction since no channel with available where decay products feel strong interaction
- π^0 can decay to 2 photons via electromagnetic interaction

$$\pi^0 \to \gamma\gamma$$

  - Technique to measure decay width complicated; won't discuss here
- π^+ can decays via weak interaction

$$\begin{aligned} \pi^+ &\to \mu^+\nu_\mu \\ \pi^- &\to \mu^-\overline{\nu}_\mu \end{aligned}$$

But why doesn't it decay electromagnetically (eg $\to \mu\gamma$?)

  - e, μ and τ number individually conserved (aside from small effect from ν oscillations)

What we just learned

- Strong, electromagnetic and weak interactions have very different strengths
- If strong decay possible, it will dominate the decay rate
- If strong decay not possible but electromagnetic is, electromagnetic will dominate
- If electromagnetic also not possible, decay must be weak
- If a particle decays weakly, we should ask why it can't decay strongly or electromagnetically
  - In case of $\pi^{\pm}$ decay, this led us to postulate conservation of lepton number
  - We can then check this postulate in other interactions to see if it is correct

Can learn about conservation laws using particle decays

Extending the argument

- We developed our strategy using decays
- But same idea can be used for scattering cross sections, angular distributions and other observables
- If a process can occur via strong interaction, the strong interaction will dominate
  - Even if EM or weak process present, difficult to observe over strong background
  - Processes that cannot proceed via strong interaction allow us to study EM and weak processes
  - Or, alternately, if EM and weak processes populate phase space differently from the strong process, can isolate these smaller processes

Different interactions, different conserved quantities

- For π decay, reason strong interaction not possible was obvious
- In other cases, must work harder to understand why
- We will see today and next week that symmetries and conservation laws satisfied by the different interactions are not the same
  - Example: Strong and EM interactions conserve quark flavor, but weak interaction does not
- Understanding these symmetries and conservation laws tells us about $\mathcal{L}$

Symmetries and Conservation Laws

- Symmetry of H: Operator R leaves H unchanged

$$R^{-1}H(t)R = H(t)$$

- Relationship between symmetries and conservation laws:

$$i\frac{dQ}{dt} = i\frac{\partial Q}{\partial t} + [Q, H]$$

If operator has no explicit time dependence

$$[Q, H] = 0 \implies < Q > \text{ is conserved}$$

- Conserved quantum $\#$'s are associated with operators that commute with H (Noether's Theorm)
- Most common examples:
  - space-time invariance (translations) $\iff$ energy-momentum conservation
  - space-time invariance (rotations) $\iff$ angular momentum conservation

In general, we write these invariance principles in terms of infinitesmal transformation

Nature of H_{int} determines the symmetries we observe

Symmetries of Interest in Particle Physics

- Continuous Space-Time Transformations
  - Translations
  - Rotations
  - Extension of Poincare group to include fermionic anticommuting spinors (SUSY)
- Discrete Transformations
  - Space Time Inversion (Parity≡P)
  - Particle-Antiparticle Interchange (Charge Conjugation≡C)
  - Time Reversal (≡T)
  - Combinations of these: CP, CPT
- Continuous Transformations of Internal Symmetries
  - Isospin
  - $SU(3)_{\text{flavor}}$
  - $SU(3)_{\text{color}}$
  - Weak Isospin

Continuous Space Time Transformations

- Translations
  - Infinitesmal: $\mathcal{D} = 1 + \delta r \frac{\partial}{\partial r}$
  - Finite: $\mathcal{D} = e^{ip\Delta r}$
- Rotations
  - Infinitesmal: $\mathcal{R} = 1 + \delta\phi \frac{\partial}{\partial \phi}$
  - Finite: $\mathcal{R} = e^{iJ_z\Delta\phi}$

- Symmetries under continuous transformations lead to additive conservation laws

All interactions are invariant under these global space-time transformations

Intrinsic Spin

- From QM, know particles have intrinsic spin
  - Spin $\frac{1}{2}$: electrons, protons, neutrons
  - Spin 1: photon
- Simple extension of the algebra used for orbital angular momentum
  - For spin-$\frac{1}{2}$ particles

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

  - Transformation: $\psi \rightarrow e^{\imath\vec{\sigma}\cdot\hat{n}\theta/2}\psi$

$$\begin{aligned} \psi' &\rightarrow \psi + \delta\psi \\ \delta\psi &= i\theta\hat{n}\cdot\left(\frac{\vec{\sigma}}{2}\psi\right) \end{aligned}$$

  - θ defines the magnitude of the rotation angle in spin space
  - $\hat{n}$ is the axis of rotation
  - The Pauli matrices $\vec{\sigma}$ are a representation of SU(2)
    - 2×2:' "fundamental representation" of SU(2)

Determining spin of other particles

- Experimentally determined from particle's decays and interactions
  - Eg ν is fermion since β-decay $n \to pe^- \overline{\nu}$
- Measure rates or angular distributions to further determine value of spin
  - Eg: he π^+:

    $$pp \to \pi^+ d$$

    Principle of Detailed Balance: $|\mathcal{M}_{if}|^2 = |\mathcal{M}_{fi}|^2$

    $$\begin{aligned}\sigma(pp \to \pi^+ d) &= (2s_\pi + 1)(2s_d + 1){p_\pi}^2 \\ \sigma(\pi^+ d \to pp) &= \frac{1}{2}(2s_p + 1)^2 {p_\pi}^2\end{aligned}$$

    ($\frac{1}{2}$ due to identical particles in final state)
  - Spin of the π^+ is 0:
- We'll see more examples of this on HW and in class

  Don't forget for identical particles, we need to symmetrize (bosons) or antisymmetrize (fermions) the wave function!

Discrete Transformations: P, C, T

- These symmetries depend on characteristics of the Lagrangian
- Because EM interaction symmetric under P, C and T, we are familiar with them from Quantum Mechanics
- Strong interaction also symmetric under C, P and T separately
- Weak interaction is not:
  - P violation as large as it can be (totally left handed ν)
  - Small violation of simulataneous application of C and P ($\approx 10^{-3}$ effect)
  - All field theories invariant under simultaneous application of C, P and T (CPT theorem)

$\Rightarrow$ Must test whether each symmetry is respected by each interaction

- Symmetries under discrete transformations lead to multiplicative conservation laws

Parity

- Parity operator defined as spatial inversion

$$\begin{aligned} (x, y, z) &\longrightarrow (-x, -y, -z) \\ P(\psi(\vec{r})) &= \psi(-\vec{r}) \end{aligned}$$

- Repetition of the operations gives $P^2 = 1$
  - P is a unitary operator with eigenvalues ± 1

- If system is an eigenstate of P, its eigenvalue is called the parity of the system

Reminder: Parity and orbital angular momentum

- Something familiar from atomic physics and quantum mechanics:

$$
\begin{aligned}
\psi(r,\theta,\phi) &= \chi(r) Y_{\ell m}(\theta,\phi) \\
&= \chi(r)\sqrt{\frac{2\ell+1)(\ell-m)}{4\pi(\ell+m)!}} P_{\ell m}(\cos\theta) e^{\imath m\phi}
\end{aligned}
$$

- Spatial inversion:

$$\vec{r} \to -\vec{r} \text{ is equiv to } \theta \to \pi-\theta,\ \phi \to \phi+\pi.$$

Thus:

$$
\begin{aligned}
e^{\imath m\phi} &\to e^{\imath m(\phi+\pi)} \to (-1)^m \\
P_{\ell m}(\cos\theta) &\to (-1)^{\ell+m} P_{\ell m}(\cos\theta) \\
Y_{\ell m}(\theta,\phi) &\to (-1)^{\ell} Y_{\ell m}(\theta,\phi)
\end{aligned}
$$

- Spherical harmonics have parity $(-1)^{\ell}$

More on the Parity Operator

- Define $U_P \equiv P$ such that $U_P \psi(\vec{r}) = \psi(-\vec{r})$
- $U_P^\dagger = U_P = U_P^{-1}$
- How do various operators transform under P?

$$\begin{aligned} U_P\, \vec{r}\, U_P^{-1} &= -\vec{r} \\ U_P\, \vec{p}\, U_P^{-1} &= -\vec{p} \\ U_P\, \vec{L}\, U_P^{-1} &= +\vec{L} \\ U_P\, \vec{S}\, U_P^{-1} &= +\vec{S} \end{aligned}$$

Notes:

1. Parity is a multiplicative quantum number

$$P(\psi = \phi_a \phi_b) = P(\phi_a) P(\phi_b)$$

2. Spin must be an axial vector since L is an axial vector
3. $\vec{r}$ and $\vec{p}$ are called vectors and $\vec{L}$ and $\vec{S}$ are called axial vectors
4. $\vec{r} \cdot \vec{p}$ is called a scalar and $\vec{r} \cdot \S$ is called a pseudoscalar
5. Vectors and pseudoscalars are odd under P, axial vectors and pseudoscalars are even

Parity and Elementary Particles

- If parity is a good symmetry of H_{int}, all elementary particles must be eigenstates of P with eigenvalues ± 1.
- To determine if parity is a good symmetry, see if it's possible to define eigenstates for each elementary particle
(independent of reaction)
Note: It is not necessarily true that definition be *unique* as long as we can define it in a consistent one
- Experimental Facts:
  - Both Strong and EM interactions conserve parity
  - Weak interactions do not

We'll talk more about this in a few weeks

Elementary Particles Have Intrinsic Parity

- The Photon
  - Electric current is a vector not an axial vector so $P(\gamma) = -1$
- Spin-$\frac{1}{2}$ Particles
  - Dirac Eq and definition of vector current require particle and anti-particle to have opposite parity
  - Since they are always pair produced, it is a matter of convention as to which is $+$ and which is $-$
  - We'll talk about this more in a few weeks
- Pions
  - Pions are bosons with spin 0 and three charge states

    π^+, π^0, π^-
  - Since bosons, they can be produced singly:

    P can be measured by studying reactions
  - See next two pages for details

Parity of the Charged Pion

- Study $\pi^- d \to nn$
  - π capture from s-wave (mesonic x-ray spectrum and rate)
  - Spin(d)=1 and Spin(π)=0 and $L = 0$ so $J = 1$ for initial state
  - What are the possibilities for the nn state?

$$\begin{array}{cc} L = 0 & S = 1 \\ L = 1 & S = 0, 1, 2 \\ L = 2 & S = 1 \end{array}$$

  - Fermi statistics: nn w.f. must be anti-symmetric

    Symmetry of wf: $(-1)^\ell (-1)^{s+1}$
  - Only $L = 1, S = 1$ state is possible
  - Thus nn are in a ${}^3P_1$ state with parity $(-1)^\ell = -1$
  - To determine P of deuteron: p and n have $P = 1$. Also, we know $L = 0$ so deuteron has $P = 1$

$\Rightarrow \pi^-$ has $P = -1$ (pseudoscalar)

Parity of the Neutral Pion

- Main decay mode $\pi^0 \to \gamma\gamma$
  - But to measure P in this mode, must measure γ polarization
- Instead use $\pi^0 \to (e^+e^-)(e^+e^-)$ (BR$\sim 10^{-4}$)
  - Look at polarization planes of e^+e^- pairs: Two possible forms

$$\begin{aligned} \psi \propto (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2) &= \cos\phi \;\; \text{scalar} \\ \psi \propto (\vec{\epsilon}_1 \times \vec{\epsilon}_2) \cdot \vec{k} &= \sin\phi \;\; \text{pseudoscalar} \end{aligned}$$

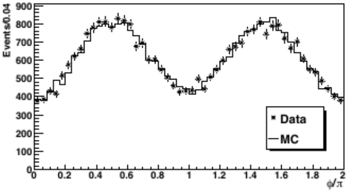


FIG. 4: Distribution of the angle ϕ, in units of π, between the planes of the two e^+e^- pairs. The solid histogram shows the Monte Carlo expectation for negative parity.

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$\Rightarrow \pi^0$ has $P = -1$ (pseudoscalar)