Physics 129: Particle Physics
Lecture 22: Weak decays of hadrons (continued), Mixing in the K^0 and B Systems

Nov 10, 2020

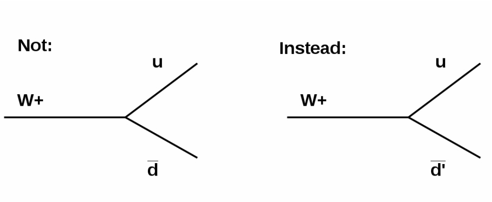
- Suggested Reading:
  - Thomson Chapter 14.2-14.6
  - Griffiths 10.5
- Homework schedule for the rest of the semester:
  - HW # 11: Released later today, due Wed Nov 18
  - HW # 12: Released Nov 18, due Wed Dec 2
  - HW # 13: Released Nov 25, due Sat Dec 11
- Final project details to be posted next week, due by Dec 15

Our Weak Interaction Roadmap

- Unlike strong and EM, weak interactions don't conserve parity
  - Vertex selects left-handed state for of particles (and right handed state for anti-particles)
    - Subject of last Tuesday's lecture
- $W^{\pm}$ coupling to leptons respect flavor familes (e, μ, τ) but coupling to quarks do not
  - Coupling not diagonal in quark flavor: Need to change basis
    - Subject of last Thursday's lecture
  - Introduction of this change in basis gives new phenomenology, including mixing and CP violation
    - This week's lectures
- $W^{\pm}$ has charge, so it couples to photon
  - Cannot write down a weak theory independent of QED
  - Unified electroweak theory includes Z^0 as well as $W^{\pm}$ and γ
    - Topic for the week of Nov 17
- Need mechanism to give $W^{\pm}$ and Z^0 mass
  - This is the Higgs mechanism
    - Discuss this after Thanksgiving

Review: Choice of weak eigenstates

- Suppose strong and weak eigenstates of quarks not the same
- Weak coupling:



- Here d' is an admixture of down-type quarks
- But wf must remain properly normalized
  - That means transformation $d \leftrightarrow d'$ must be unitary

The Cabbibo Angle

- If we had only 2 quark generations, would need only 1 number to relate the bases
  - Expressing it as an angle θ_C ensures proper normalization
- For two generations:

$$\begin{aligned} d' &= d\cos\theta_C + s\sin\theta_c \\ s' &= s\cos\theta_C - d\sin\theta_c \end{aligned}$$

- With this formulation:

$$\begin{aligned} p\ \&\ \pi \text{ decay} &\propto G_F^2\cos^2\theta_C \\ K \text{ decay} &\propto G_F^2\sin^2\theta_C \\ \mu \text{ decay} &\propto G_F^2 \end{aligned}$$

- Using experimental measurements, find

$$\begin{aligned} \cos\theta_c &= 0.97420 \pm 0.00021 \\ \sin\theta_c &= 0.2243 \pm 0.0005 \end{aligned}$$

Review: The GIM Mechanism (I)

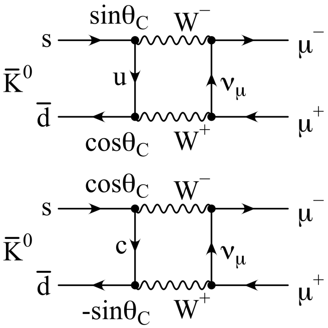
- Flavor changing neutral currents (FCNC) are highly suppressed
- Two examples of FCNC suppression:
    1. $BR(K_L^0 \to \mu^+\mu^-) = 6.84 \times 10^{-9}$
    2. $BR(K^+ \to \pi^+\nu\nu)/BR(K^+ \to \pi^0\mu\nu) < 10^{-7}$
- Why are these decay rates so small?
    - Z that couples to $f\overline{f}$ pairs, but it does not change flavor (same as γ)
    - Two $W^\pm$ exchange can produce FCNC
    - Need second order charged weak interactions, but even this would give a bigger rate than seen unless there is a cancellation

Review: The GIM Mechanism (II)

- Reminder:

$$\begin{aligned} d' &= d\cos\theta_C + s\sin\theta_c \\ s' &= s\cos\theta_C - d\sin\theta_c \end{aligned}$$

- Consider the "box" diagram



- $\mathcal{M}$ term with u quark $\propto \cos\theta_C \sin\theta_C$
- $\mathcal{M}$ term c quark $\propto -\cos\theta_C \sin\theta_C$
- Same final state, so we add $\mathcal{M}$'s
- Terms cancel in limit where we ignore quark masses

Review: More Than Two Generations

- Generalize to N families of quark ($N = 3$ as far as we know)
- U is a unitary $N \times N$ matrix and d'_i is an N-column vector

$$d'_i = \sum_{j=1}^{N} U_{ij} d_j$$

  U is called the CKM matrix

- How many independent parameters do we need to describe U?
  - $N \times N$ matrix: N^2 elements
  - But each quark has an unphysical phase: can remove $2N - 1$ phases (leaving one for the overall phase of U)
  - So, U has $N^2 - (2N - 1)$ independent elements
- However, an orthogonal $N \times N$ matrix has $\frac{1}{2}N(N-1)$ real parameters
  - So U has $\frac{1}{2}N(N-1)$ real parameters
  - $N^2 - (2N-1) - \frac{1}{2}N(N-1)$ imaginary phases $(= \frac{1}{2}(N-1)(N-2))$
- $N = 2$ 1 real parameter, 0 imaginary
- $N = 3$ 3 real parameters, 1 imaginary
- Three generations requires an imaginary phase: CP Violation inherent

The CKM Matrix

- Write hadronic current

$$J^{\mu} = -\frac{g}{\sqrt{2}} \left(\overline{u}\ \overline{c}\ \overline{t}\right) \gamma_{\mu} \frac{(1-\gamma_5)}{2} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- V_{CKM} gives mixing between strong (mass) and (charged) weak basis
- Often write as

$$V_{CKM} = \begin{pmatrix} V_{ud} & Vus & V_{ub} \\ V_{cd} & Vcs & V_{cb} \\ V_{td} & Vts & V_{tb} \end{pmatrix}$$

- Wolfenstein parameterization:

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Here λ is the $\approx \sin\theta_C$.

Best Fit for CKM Matrix from PDG

- From previous page

$$V_{CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

- Impose Unitary and use all experimental measurements

$$\lambda = 0.22453 \pm 0.00044 \qquad A = 0.836 \pm 0.015$$
$$\rho = 0.122^{+0.018}_{-0.17} \qquad \eta = 0.355^{+0.12}_{-0.11}$$

- Result for the magnitudes of the elements is:

$$\begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359 \pm 0.00011 & 0.04214 \pm 0.00076 \\ 0.00896 \pm 00024 & 0.04133 \pm 0.00074 & 0.999105 \pm 000032 \end{pmatrix}$$

Weak hadron decays and the CKM matrix

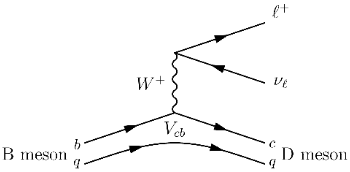
- Since hadronic current is

$$J^{\mu} = -\frac{g}{\sqrt{2}} \left(\overline{u}\ \overline{c}\ \overline{t}\right) \gamma_{\mu} \frac{(1-\gamma_5)}{2} V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

  We get a V_{CKM} factor for each $Wq\overline{q}$ vertex

  - We need to keep track of these factors when we calculate decay rates

- Example: Semileptonic decay of B meson



NB: overline on b-quark difficult to see!

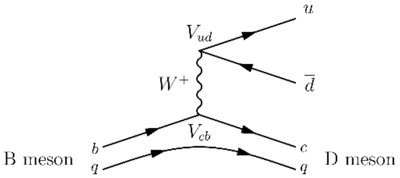
  - B mesons: $q\overline{b}$
  - $\overline{B}$ mesons: $b\overline{q}$
  - Pseudo-scalar mesons decay weakly through $W^{\pm}$ emission

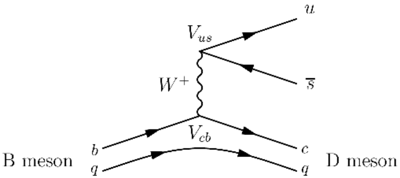
  Factor of V_{cb} in matrix element, $|V_{cb}|^2$ in decay rate

  - From previous page: $V_{cb} = 0.04214 \pm 0.00076$
  - This gives a factor $\approx 1.76 \times 10^{-3}$ in decay rate
  - Explains why weakly decaying B mesons have relatively long lifetimes

$$\tau(B^+) = 1.6 \times 10^{-12}\ \mathrm{s}$$

How does this work for hadronic decays?





NB: overline on b-quark difficult to see!

- Now have a CKM element at each vertex
- Also need a factor for probability that our $u\overline{d}$ turns into a π^+ (or a ρ^+)
- Note also that diagrams where W decays between the two quarks are possible
- And in some cases annihilation diagrams also possible
- But can estimate relative rates for decays
  - Eg, expect

$$\frac{BR(B \to DK^+)}{BR(B \to DK^+)} \approx \left|\frac{V_{us}}{V_{ud}}\right|^2$$

Implications of the CKM picture

- We have already seen in GIM mechanism that 2^{nd} order weak interactions with two W's exchanged can be important
- Phenomenology of these interactions is rich
- First system where it was explored: Neutral kaons
- Kaons played essential role in understanding particle physics
- Have already seen two examples:
  - $K^+ \to 2\pi, 3\pi$: Parity violation
  - $K^0 \to \mu^+\mu+$: Very low BR due to GIM mechanism
- Today and Thursday, explore special role of neutral K system
  - Mixing
  - CP Violation
- Will also include some discussion of neutral B mesons, which exhibit similar phenomena

Reminder: Strange Particle Phenomenology

- Stange pseudoscalar mesons: 2 isodoublets

$$\begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \begin{pmatrix} \overline{K}^0 \\ K^- \end{pmatrix}$$

where $K^+ = \overline{s}u$, $K^- = \overline{u}s$, $K^0 = \overline{s}d$, $\overline{K}^0 = \overline{d}s$

- Because strangeness conserved in SI, K^0 and $\overline{K}^0$ are distinct particles
- Strange particles are pair produced via SI

$$\begin{aligned} \pi^- p &\rightarrow \Lambda K^0 \\ \pi^+ p &\rightarrow p K^+ \overline{K}^0 \end{aligned}$$

- First reaction has much lower threshold than second
  - Can produce a pure K^0 beam
- Today, neutal K beams produced using high intensity proton beams hitting targets
  - Background a big issue for K^0 experiments, most notably from neutrons

Neutral Kaon Decays

- $m(K^0) = 497$ MeV. Not many decay modes open
  - Fully leptonic decays highly suppressed (GIM)
  - $K^0 \to \pi^- \ell^+ \nu_\ell$ (and charge conjugate) occurs
  - Since parity not conserved in weak interactions, both 2π and 3π decays possible
- Since both K^0 and $\overline{K}^0$ can decay to same states, they can *mix* through virtual decays

$$K^0 \longleftrightarrow \left\langle \begin{array}{c} \pi\pi \\ \pi\pi\pi \end{array} \right\rangle \longleftrightarrow \overline{K}^0$$

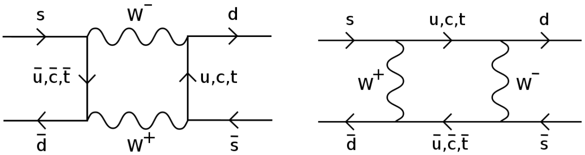
- These are 2^{nd} order weak onteractions with $\Delta S = 2$
- If we start with a pure K^0 state at $t = 0$, it will at some later time have a combination of K^0 and $\overline{K}^0$

$$|K(t)\rangle = \alpha(t)\left|K^0\right\rangle + \beta(t)\left|\overline{K}^0\right\rangle$$

with $\sqrt{\alpha^2 + \beta^2} = 1$

Neutral Kaon Mixing (I)

- Can describe this 2^{nd} order weak interactions in quark terms:



- If there were no weak interactions, K^0 and $\overline{K}^0$ would be degenerate in mass
- Weak Interactions break the degeneracy
- Because physical observables (eg mass, lifetime) are eigenstates of complete Hamiltonian (SI+WI), must select correct linear combination of K^0 and $\overline{K}^0$ define the states that propagate and decay

  The mass and lifetime eigenstates are not the flavor eigenstates!

Neutral Kaon Mixing (I)

- We can *almost* guess the correct basis to use
  - We know weak interactions don't conserve P since ν are LH and $\overline{\nu}$ are RH
  - Parity would turn a LH ν into a RH ν
  - But Charge Conjugation turns a ν into a $\overline{\nu}$
  - Hence, CP turns a a LH ν into a RH $\overline{\nu}$
- Weak Interaction Lagrangian appears to be CP invariant
- In fact, CP is violated in CKM matrix ($\sim 10^{-3}$ effect)
- But the CP basis is close to the correct one and that's what we'll use today
  - We'll add the small CP violating piece on Thurs

Neutral Kaon Mixing (II)

- Neutral Kaons transform under CP (not unique definition)

$$
\begin{aligned}
CP\left|K^0\right\rangle &= \left|\overline{K}^0\right\rangle \\
CP\left|\overline{K}^0\right\rangle &= \left|K^0\right\rangle
\end{aligned}
$$

- Therefore, we can write

$$
\begin{aligned}
\left|K_1\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle + \left|\overline{K}^0\right\rangle\right) \qquad CP\left|K_1\right\rangle = \left|K_1\right\rangle \\
\left|K_2\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|K^0\right\rangle - \left|\overline{K}^0\right\rangle\right) \qquad CP\left|K_2\right\rangle = -\left|K_2\right\rangle
\end{aligned}
$$

- $\left|K_1\right\rangle$ and $\left|K_2\right\rangle$ are CP eigenstates and *almost* the physical basis

CP of Possible Hadronic Decays

$\pi^0\pi^0$:

- Spin 0 to 2 spin 0 particles: $\ell = 0$

$$\begin{aligned} P(\pi^0\pi^0) &\rightarrow \pi^0\pi^0 \\ C\pi^0 &\rightarrow \pi^0 \\ CP(\pi^0\pi^0) &\rightarrow +1(\pi^0\pi^0) \end{aligned}$$

$\pi^+\pi^-$:

- Spin 0 to 2 spin 0 particles: $\ell = 0$

$$\begin{aligned} P\left(\pi^+(\vec{p})\ \pi^-(-\vec{p})\right) &\rightarrow \pi^+(-\vec{p})\pi^-(\vec{p}) \\ C\pi^\pm &\rightarrow \pi^\mp \\ CP\left(\pi^+\pi^-\right) &\rightarrow +1(\pi^+\pi^-) \end{aligned}$$

$\pi^0\pi^0\pi^0$:

- Any two π^0 combo must have even ℓ (Bose stats)
- $J = 0$ so ℓ of 3^{rd} π^0 also even wrt other two
- But π has intrinsic parity $P = -1$

$$\begin{aligned} P(\pi^0\pi^0\pi^0) &\rightarrow (-1)^3\pi^0\pi^0\pi^0 \\ C\pi^0 &\rightarrow \pi^0 \\ CP(\pi^0\pi^0\pi^0) &\rightarrow -1(\pi^0\pi^0\pi^0) \end{aligned}$$

$\pi^+\pi^-\pi^0$:

- Small Q suggests $\ell = 0$. If so, same argument as above
- Both CP states allowed but $CP(\pi^+\pi^-\pi^0) = -(\pi^+\pi^-\pi^0)$ state highly dominant

2π states have $CP = +1$ and 3π states have $CP = -1$

Hadronic Decays of the $|K_1\rangle$ and $|K_2\rangle$

- Associating the CP states with the decays:

$$|K_1\rangle \rightarrow 2\pi$$
$$|K_2\rangle \rightarrow 3\pi$$

- However, very little phase space for 3π decay: Lifetime of $|K_2\rangle$ much longer than of $|K_1\rangle$
- Physical states called "K-long" and "K-short":

$$\tau(K_S) = 0.9 \times 10^{-10} \text{ sec}$$
$$\tau(K_L) = 0.5 \times 10^{-7} \text{ sec}$$

- We'll use distinction that $|K_1\rangle$, $|K_2\rangle$ are the CP eigenstates and $|K_S\rangle$, $|K_L\rangle$ are true mass eigenstates (including CP violation)

A More Formal Treatment of Mixing

- Write our state ψ as linear combination of K^0 and $\overline{K}^0$:

$$\psi = \alpha \left| K^0 \right\rangle + \beta \left| \overline{K}^0 \right\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- Schrodinger eq tells us

$$i \frac{d\psi}{dt} = H\psi$$

where H is Hermitian matrix: "generalized mass matrix"

- In matrix form:

$$H = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M^*{}_{12} - \frac{i}{2}\Gamma^*{}_{12} & M - \frac{i}{2}\Gamma \end{pmatrix}$$

- Diagonal elements equal from CPT
- If CP is a good symmetry, M_{12} and Γ_{12} are real
- Find eigenstates by diagonalizing the matrix

$$\begin{array}{ll} M = (m_1 + m_2)/2 & \Delta m \equiv M_{12} = (m_1 - m_2)/2 \\ \Gamma \equiv \Gamma_{12} = (\Gamma_1 + \Gamma_2)/2 & \Delta\Gamma = (\Gamma_1 - \Gamma_2)/2 \end{array}$$

Time Dependence (I)

- Write wave functions (ignoring for now CP violation)

$$
\begin{aligned}
|K_1(t)\rangle &= e^{-im_1 t - \Gamma_1 t/2}\,|K_1\rangle \\
|K_2(t)\rangle &= e^{-im_2 t - \Gamma_2 t/2}\,|K_2\rangle
\end{aligned}
$$

- Writing this in terms of strong eigenstates

$$
\begin{aligned}
\left|K^0\right\rangle_{\text{at } t=0} &\Rightarrow \frac{1}{\sqrt{2}}\left[e^{-im_1 t - \Gamma_1 t/2}\,|K_1\rangle + e^{-im_2 t - \Gamma_2 t/2}\,|K_2\rangle\right] \\
\left|\overline{K}^0\right\rangle_{\text{at } t=0} &\Rightarrow \frac{1}{\sqrt{2}}\left[e^{-im_1 t - \Gamma_1 t/2}\,|K_1\rangle - e^{-im_2 t - \Gamma_2 t/2}\,|K_2\rangle\right]
\end{aligned}
$$

- If a state ψ that is purely $\left|K^0\right\rangle$ is produced at $t = 0$, at a later time it will be a combination of $\left|K^0\right\rangle$ and $\left|\overline{K}^0\right\rangle$:

$$
\begin{aligned}
\left\langle K^0\right|\,|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\langle K_1| + \langle K_2|\right)|\psi(t)\rangle &= \frac{1}{2}\left[e^{-im_1 t - \Gamma_1 t/2} + e^{-im_2 t - \Gamma_2 t/2}\right] \\
\left\langle \overline{K}^0\right|\,|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(\langle K_1| - \langle K_2|\right)|\psi(t)\rangle &= \frac{1}{2}\left[e^{-im_1 t - \Gamma_1 t/2} - e^{-im_2 t - \Gamma_2 t/2}\right]
\end{aligned}
$$

Time Dependence (II)

- Square to get probability:

$$
\begin{aligned}
\left|\langle K^0|\,|\psi(t)\rangle\right|^2 &= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m t)\right] \\
\left|\left\langle \overline{K}^0\right|\,|\psi(t)\rangle\right|^2 &= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m t)\right]
\end{aligned}
$$

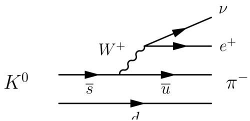
- The $\left|K^0\right\rangle$ and $\left|\overline{K}^0\right\rangle$ oscillate with frequency Δm and at the same time they decay

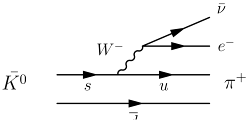
Observing the Oscillation

- Oscillation provides a way to measure ΔM
- Also demonstrates that this QM phenomenon is happening
- How do we see it?
  1. Start with pure K^0 beam (low energy)
     Look at time dependence of hyperon yield in interactions ($\overline{K}^0 p \to \Lambda\pi$)

     $$\Delta m \tau_1 = 0.477 \pm 0.2$$

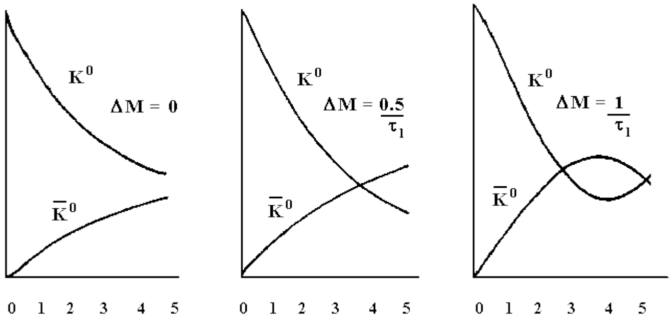
  2. Look for decays that *tag* the flavor: semileptonic
     Observe time dependence in ℓ^+ vs ℓ^- rate





- Note: Phenomenology of K mixing depends on two things
  - Large lifetime difference: time to mix before decaying
  - Small mass difference: short oscillation frequency
- In B system things look somewhat different (we'll discuss later)

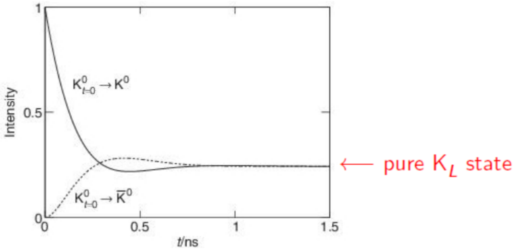
What we expect to see



- Begin with K^0 state at $t = 0$

$$
\begin{aligned}
\left|\left\langle K^0\right| |\psi\rangle\right|^2 &= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m t)\right] \\
\left|\left\langle \overline{K}^0\right| |\psi\rangle\right|^2 &= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1+\Gamma_2)t/2}\cos(\Delta m t)\right]
\end{aligned}
$$

For the measured ΔM



- Start with a K_0 beam
- After many K_S lifetimes, have a pure K_L beam
  - In absence of CP violation, equal parts K_0 and $\overline{K}_0$

Observation of $K_0 - \overline{K}_0$ Oscill using semileptonic decays

- Use lepton flavor to distinguish K_0 and $\overline{K}_0$

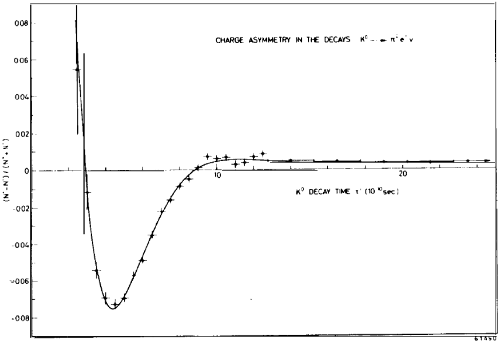
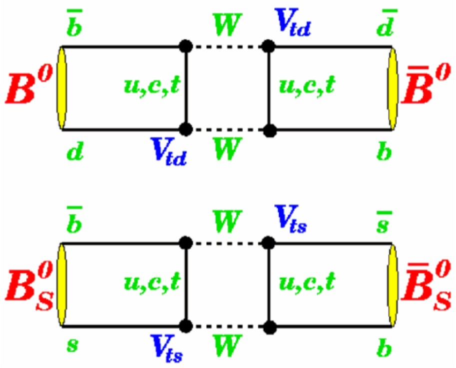


Fig. 3. Time dependence of the charge asymmetry of semileptonic decays.

- Plot shows asymmetry $\frac{N(\ell^+)-N(\ell^-)}{N(\ell^+)+N(\ell^-)}$
- Removes (trivial) lifetime dependence
- We'll come back to the non-zero value at large times Thursday (CP violation)

How About the B system?



- Again, second order in weak interactions
- Different CKM matrix elements for B^0 and B_s
  - Larger ΔM for B_s than B_d
- Many possible final states for the decay
  - Difference in lifetime of the B_L and B_S states small
- NB: D^0-$\overline{D^0}$ mixing also exists, but *very small* since mass differences in down sector smaller

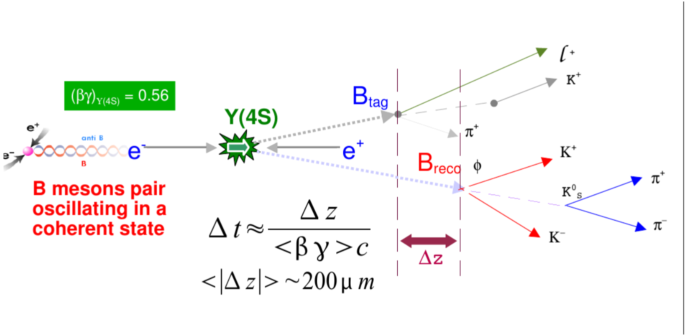
Observing B^0-$\overline{B^0}$ Mixing

- B hadrons produced in Strong or EN interactions: pair produced
  - Flavor conserved in SI and EM
- If one B-hadron identified as B or $\overline{B}$, know the that the other has opposite flavor
- Can "tag" flavor of one B
  - $b \to W^- c \to \ell^- \nu c$, $\overline{b} \to W^+ \overline{c} \to \ell^+ \overline{\nu c}$
  - $b \to W^- c \to W^- D^{+(*)}$, $\overline{b} \to W^+ \overline{c} \to W^+ D^{-(*)}$
  - Fully or partially reconstructed B^+ or B^-
- Study time evolution of other B using same processes to determine flavor (b or $\overline{b}$)
- For B^0 most incisive studies from $\Upsilon(4s)$
- B_s not produced on $\Upsilon(4s)$: hadron colliders for B_s mixing

$e^+e^- \to \Upsilon(4s)$: How do the $B\overline{B}$ pairs behave?

- B and $\overline{B}$ come from $\Upsilon(4s)$ in a coherent $L = 1$ state
  - $\Upsilon(4s)$ is $J^{PC} = 1^{--}$
  - B mesons are scalars
  - Thus, $L = 1$
- $\Upsilon(4s)$ decays strongly so B and $\overline{B}$ produced as flavor eigenstates
  - After production, each meson oscillates in time, but *in phase* so that at any time there is only one B and one $\overline{B}$ until one particle decays
    - Coherent oscillations
  - Once one B decays, the other contines to oscillate, but coherence is broken
  - Possible to have events with two B or two $\overline{B}$ decays
- This common evolution will become important for CP studies
  - Time integrate asymmetries vanish for cases where CP violation comes from mixing diagrams
  - More on this later

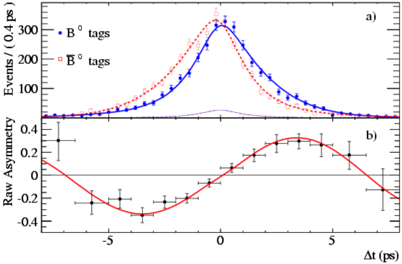
Asymmetric B-Factories



- e^+ amd e^- beams with different energies
  - $\Upsilon(4s)$ boosted along beamline
  - B mesons travel finite distance before decaying
  - Typical distance between decay of the two B mesons: $\sim 200\ \mu$m
- Two B-factories built:
  - SLAC
  - KEK

Example of B Mixing (B^0 and B_s)

- ΔM for $B^0 = 0.510 \pm 0.003 \pm 0.002$ ps^{-1}



- ΔM for $B_s = 17.761 \pm 0.021 \pm 0.007$ ps^{-1}