Physics 129: Particle Physics

Lecture 15: Feynman Diagrams and QED

Oct 15, 2020

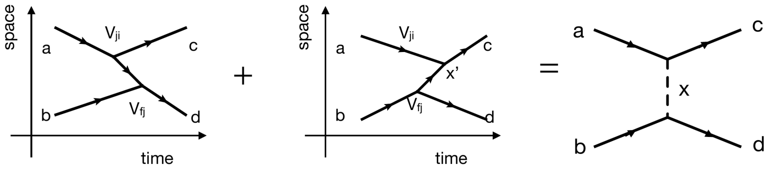
- Suggested Reading:
  - Thomson 5.3-5.4, 6.1-6.4
  - Griffiths 7.4-7.6

  Reminder: Quiz #2 next week. Focus: Lectures 7-13 and HW 4-6 (LIPS but no Dirac Eq for this quiz)

Introduction

- Today's lecture focuses on calculation of cross sections and decay rates using Feynman Diagrams
- A full treatment of this topic would take a semester
- We won't emphasize details of the numerical calcuation
  - Griffiths and Thomson go through calculations in gory detail
- What you should concentrate on is what the Feynman diagrams can do to help us predict the answers
  - How many powers of the coupling constant?
  - What do the propagators tell us about the dependence on s or t?
  - What do spin considerations tell us about which amplitudes will contribute?

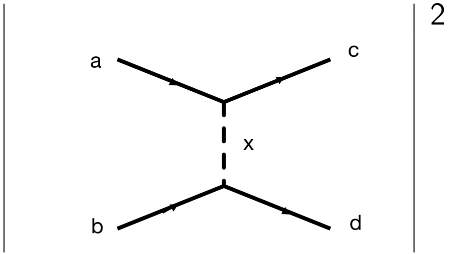
Review: Feynman Diagrams replace time orderd PT ME



- In QM would need to consider two cases
  - a emits particle that is absorbed by b
  - b emits particle that is absorbed by a
- But order of these events not independent of reference frame!
- Turns out that sum of both possibilities is Lorentz invariant (see Thomson 5.3)
- Draw both as a single "Feynman diagram"
- Important:
  - The Feynman diagram is a short hand description of a matrix element and NOT a cross section or decay rate
  - We'll have to square it to calculate transition rates

To emphasize the previous point

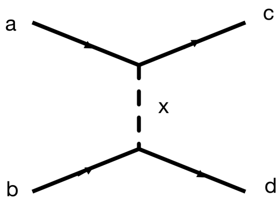
- Writing things in the familiar language of QM PT:



$$\equiv \quad \left| \sum_{n \neq i} \frac{\langle f| V |n\rangle \langle n| V |i\rangle}{E_i - E_f - i\epsilon} \right|^2$$

- Diagram above is 2^{nd} order in PT theory
  - Each vertex in diagram has an interaction strength. This is the perturbative parameter in our expansion
- In QFT, all interaction occur via exchange so no 1^{st} order diagrams
  - When we discuss weak interactions, we'll see that the pre-SM description had a 4-point "Fermi" interaction which was 1^{st} order PT and we'll see that this leads to problems

Drawing Feynman Diagrams



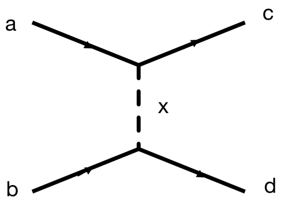
- Two conventions for drawing Feynman Diagrams
  - Time goes left-to-right
  - Time goes up to down

  We'll mainly use the first convention

- Incoming and outgoing particles are physical states we can observe
  - With left-to-right convention, incoming particles on lhs; outgoing on rhs
  - Due to Stuckelberg and Feyname, antiparticles will have arrows reversed

- Everything between incoming and outgoing is how the interaction happens
  - The exchanged particle (labeled X) is "virtual."
  - Direction of arrow is arbitrary for these virtual particles

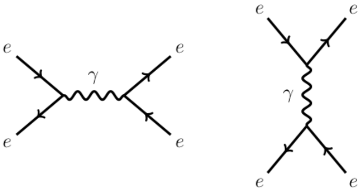
Pieces of the Matrix Element Calculation



- Incoming and outgoing particles have standard wf's
- At each vertex
  - need a constant g that tells us the strength of the interaction
  - Energy and momentum conserved at all vertices
  - The exchanged particle (labeled X) is "virtual". For this particle $E^2 - p^2 \neq m^2$ ("off-shell"')
- For each vertex, there is "propagator" term $\propto \frac{1}{q^2 - m_X^2}$. This term has the same origin as the denominator on page 4
  - For exchange of photon (spin 1), need an additioanl $ig^{\mu\nu}$ for the photon propagator
- Remember, we'll have to square matrix element to get decay rate or scattering cross section

Adding Matrix Elements together

- To calculate a cross section or decay rate, must consider all possible Feynman diagrams
  - If two diagrams have the same initial state and the same final state, must add ME before squaring
  - As is alwaus the case in QM, these diagrams can interfere
- Example: $e^+e^- \to e^+e^-$:



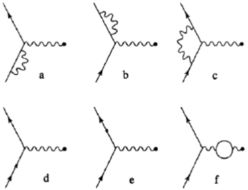
$\rightarrow$ time

- LH diagram: e^+ and e^- annihilate to photon that then splits to e^+e^-
  - s-channel: propagator term: $\frac{1}{s}$
- FH diagram: e^+ and e^- exchange photon and scatter elastically
  - t-channel: propagator term: $\frac{1}{t}$

- For $e^+e^- \to \mu^+\mu^-$ only have the LH diagram
  - That's why we'll use $e^+e^- \to \mu^+\mu^-$ as our example process today

Higher Order Terms

- Perturbative expansion can be continued to include higher orders in PT
- Can determine order of PT by number of vertices *in matrix element squared*
  - Since we add ME before squaring, inteference between terms with different number of vertices possible
  - We'll discuss this in more detail in a couple of weeks
- In QFT, HO corrections can result in infinities in our calculations
  - Systematic prescription to remove infinities called "renormalization"
  - We won't do renormalization calculations here but will talk about some results of such calculations
- One example: Lamb shift: a difference in energy between $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ levels of hydrogen not predicted by Dirac eq alone



The various higher-order graphs that contribute to the Lamb shift: (a) and (b) the electron self-energy graphs; (c) the vertex correction; (d) and (e) the electron mass counterterm; (f) the photon self-energy correction.

What we need to calculate decay rates and cross sections

- Go back to FGR for decays and equivalent of FGR for scattering
- Divide into matrix element and density of states
- Matrix element calculations from Feynman Diagrams
- LIPS factors include delta fns to enforce conservation of energy
- I won't go through derivations of LIPS factors. Quoting results from Thomson:
  - Two body decays of particle a in rest frame:

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2} \int \left| M_{if} \right|^2 d\Omega$$

    where p^* is magnitude of momentum of one of the decay products
  - $a + b \to c + d$ in center of mass:

$$\sigma = \frac{1}{64\pi^2 s} \frac{p_f^*}{p_i^*} \int \left| M_{if} \right|^2 d\Omega$$

- We'll use these results today and later on this semester

Quantum Electrodynamics (QED)

- QED was first QFT and remains the best tested and best understood
  - Developed based on insights from classical E&M
  - Used as a model for how to construct more complicated theories of strong and weak interactions
- Begin with spinor description of fermions from Dirac Eq and description of EM energy density ($\propto E^2 + B^2$) from classical mechanics
- Extend concept of gauge invariance from classical E&M by postulating this is a local rather than a global symmetry
  - Imposing local gauge invariance defines the interaction between the charged fermions and the photon
  - In case of QED this gives back the familar Columb interaction
  - Terminology: the photon is the example of a "gauge boson":
    - Spin 1
    - Mediates interaction between the fermions
- Same general approach works for all QFT: If we define the energy density associated with the gauge bson and impose local gauge invariance, the interaction between the gauge boson and the fermions is defined

The photon in Classical E& M

- Can write Maxwell's equations in terms of a Lorentz invariant 4-vector potential

$$^{\mu} = \left(V, \vec{A}\right)$$

where V and $\vec{A}$ are the familar potential and vector potential

- Can show that Maxwell's eq become (in natural units)

$$\partial^{\nu}\partial_{\nu}A^{\mu} = 4\pi J^{\mu}$$

and that the energy in the EM field can be written

$$E = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where

$$F^{\mu\nu} \equiv \partial_{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

- $F^{\mu\nu}$ is called the energy-momentum tensor
- We also know from classical E&M that the direction of motion of a photon is given by $\vec{E} \times \vec{B}$ so that photons have two possible polarization states

Photon Polarization

- Photons have spin-1 but only have two polarization states
- This is typical for massless particles
  - Only the two transverse polarizations exist for real (not virtual) massless bosons
  - We'll see when we get to the weak interactions where the W and Z have mass, longitudinal polarizations possible
- In QED we will write our photon wave functions as plane wave solutions :

$$A_\mu(x) = \epsilon_\mu^{(\lambda)} e^{-i(\vec{p}\cdot\vec{x}-Et)}$$

  where $\lambda = 1, 2$ is the polarization
- The two orhogonal polarization states are

$$\epsilon_\mu^{(1)} = (0,1,0,0) \quad \epsilon_\mu^{(2)} = (0,0,1,0)$$

- Using this nomenclature, the Coulomb potential energy is

$$\hat{V}_\mu = q\gamma^0\gamma^\mu A_\mu$$

  where the γ^0 is present to make sure the timelike contribgution is $q\gamma^9\gamma^\mu A_0 = q\phi$ as expected for classical E&M

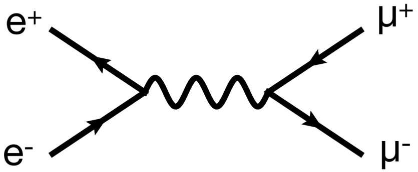
Feynman Rules for QED

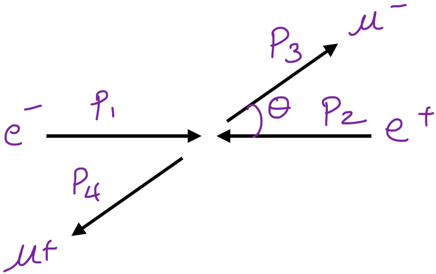
- **External** lines
  - **Spin 1/2**
    - incoming particle $u(p)$ 
    - outgoing particle $\overline{u}(p)$ 
    - incoming antiparticle $\overline{v}(p)$ 
    - outgoing antiparticle $v(p)$ 
  - **Spin 1**
    - incoming photon $\varepsilon^{\mu}(p)$ 
    - outgoing photon $\varepsilon^{\mu}(p)^{*}$ 
- **Internal lines** (propagators)
  - Spin 1: photon $-\frac{ig_{\mu\nu}}{q^2}$ 
  - Spin 1/2 fermion $-\frac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$ 

The Roadmap

- Calculating Feynman diagrams is largely a cookbook process:
  - Draw the Feynman Diagrams
  - Write dow the matrix element
  - Add spin and consider helicity combinations
  - Write incoming and outgoing currents
  - Calculate matrix element
  - Calculate differential cross section
  - Calculate total cross section
- Although we won't be doing too many calculations in 129 it's important to understand how these calculations are done
- You will not be asked to full calculate cross sections and decay rates on homework or exams
  - But you will be expected to argue about how the answer depends on important factors in the calculations

Example: $e^+e^- \to \mu^+\mu^-$

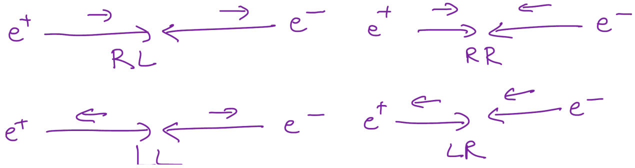




$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\,\frac{-ig_{\mu\nu}}{q^2}\,[\overline{u}(p_4)ie\gamma^\nu v(p_3)]$$

Spin in e^+e^- Annihilation

- In general, electron and positron are not polarized
  - Equal populations of positive and negative helicity
  - There are four possible combinations of spin in the intial state



- Similarly, there are 4 possible helicity combinations in the final state
- This leads to 16 possible combinations (eg $RL \to LR$)
  - All but 4 will end up vanishing
- Physical cross section should average over inital helicity states and sum over final helicity states

The possilble helicity combinations

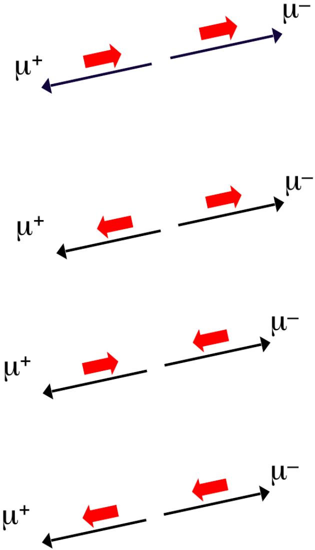
- Look separately at intial and final currents (a current is attached to a vertex)
- Each of these combinations have different spinors
- Eg for RL final state current you get

$$u_{\uparrow}(p)3)\gamma^{\nu}v_{\downarrow}(p_4) = 2E(0, -cos\theta, i, \sin\theta)$$

- Result of LR is

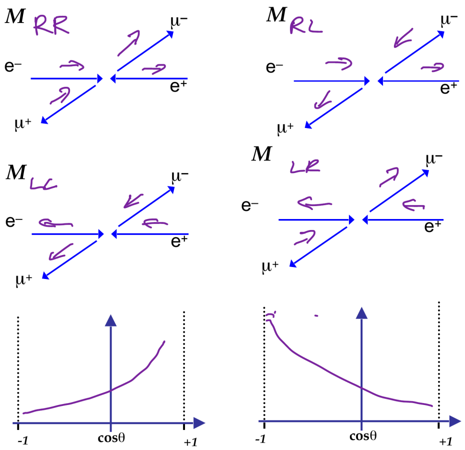
$$u_{\uparrow}(p)3)\gamma^{\nu}v_{\downarrow}(p_4) = 2E(0, -cos\theta, -i, \sin\theta)$$

- The LL and RR results give zero
- This is not an accident
  - Result of the $\overline{\psi}\gamma^{\mu}\psi$ form of the EM current



Possible Helicity States

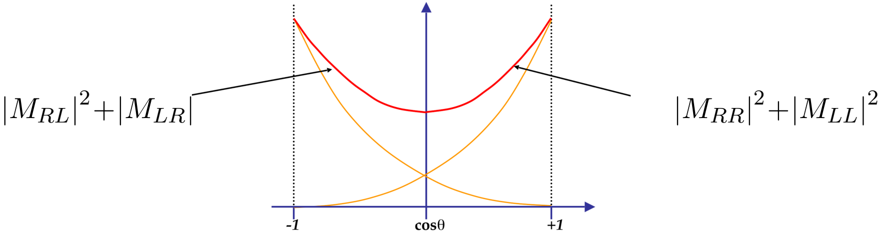
- Calculate non-zero matrix elements
- Different spin states are non identical, so we square each ME separately
- For unpolarized initial state all 4 possibilities equally likely: average over them
- $|M_{RR}|^2 = |M_{LL}|^2 = (1+\cos\theta)^2$
- $|M_{RL}|^2 = |M_{LL}|^2 = (1-\cos\theta)^2$



Differential Cross Sectton

- Calculate cross section by averaging over intial spins and summing over final spins

$$
\begin{aligned}
\frac{d\sigma}{d\cos\theta} &= \frac{1}{4}\frac{1}{64\pi^2 s}\left(|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2+|M_{LL}|^2\right)\\
&= \frac{(4\pi\alpha)^2}{256\pi^2 s}\left(2\left(1+\cos\theta\right)^2+2\left(1-\cos\theta\right)^2\right)\\
&= \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right)
\end{aligned}
$$



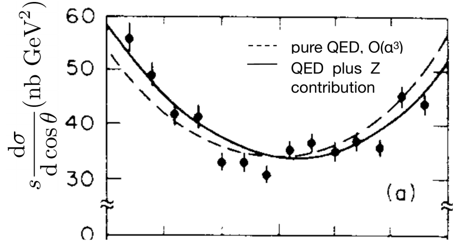
Example from Experiment

- Our result from previous page was:

$$\frac{d\sigma}{d\cos\theta} = \frac{\alpha^2}{4s}\left(1+\cos^2\theta\right)$$

- Measured, eg, at SLAC in at $\sqrt{s} = 29$ GeV:

Mark II Expt. M.E. Levi et al
Phys. Rev. Lett. 51 (1983) 1941



- Slight asymmetry due to contributions from weak process where Z rather than γ in propagator
  - We'll talk more about this in a few weeks
  - But you will get a chance to fit similar data on this week's HW

Total Cross Section

- Total cross section obtained by integrating over angles

$$
\begin{aligned}
\sigma &= \int \frac{d\sigma}{d\Omega} d\Omega \\
&= \frac{\alpha^2}{4s} \int_{-1}^{1} \left(1 + \cos^2\theta\right) d\cos\theta d\phi \\
&= 2\pi \frac{\alpha^2}{4s} \int_{-1}^{1} \left(1 + \cos^2\theta\right) d\cos\theta \\
&= \frac{4\pi\alpha^2}{3s}
\end{aligned}
$$

Lorentz Invariant Form of Cross section

- We have done our calculation in center of mass frame and result for $d\sigma/d\Omega$ written in terms of $\cos\theta$
- But we should be able to rewrite our soln in Lorentz invariant form
- You will prove on this week's HW that starting with

$$\left\langle |M_{fi}|^2 \right\rangle = e^4 \left(1 + \cos^2\theta\right)$$

the Lorentz invariant form is

$$\left\langle |M_{fi}|^2 \right\rangle = 2e^4 \frac{t^2 + u^2}{s^2}$$

- This result is valid in any frame