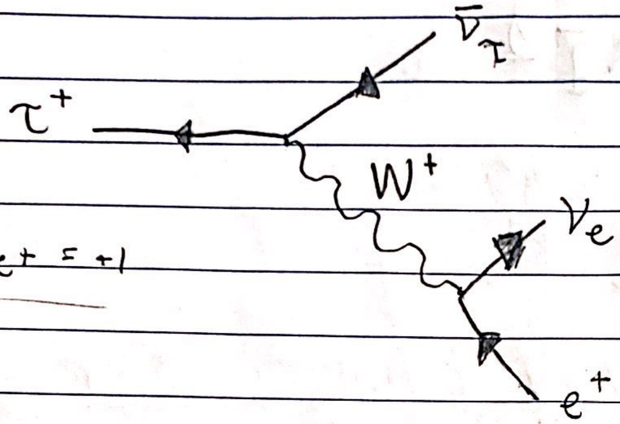
PSET 11 129

Draw a Feynman Diagram (W boson) for each.

1a) $\tau^+ \to e^+ + \nu_e + \bar{\nu}_\tau$

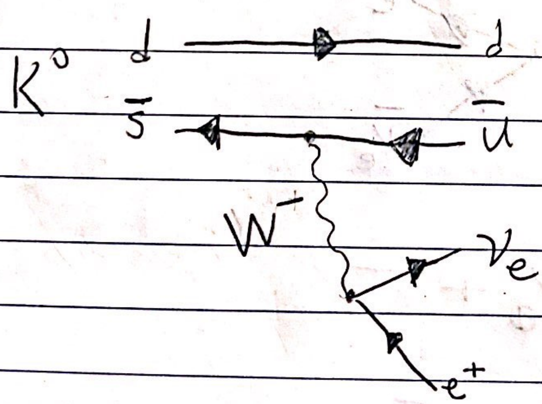
$q_{\bar{\nu}_\tau} = 0$



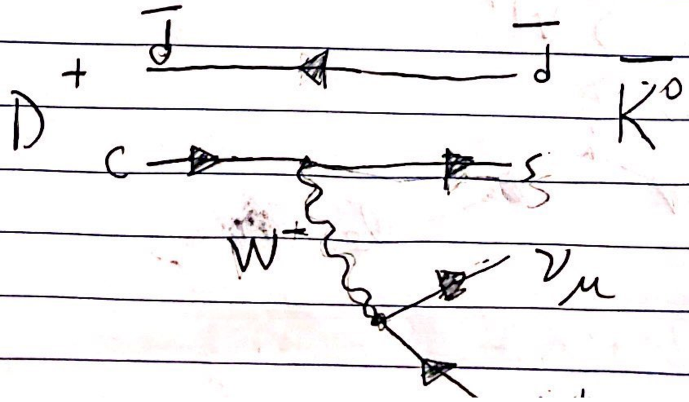

$q_{\tau^+} = +1$

1b) $K^0 \to \pi^- + e^+ + \nu_e$

$q_{\bar{s}} = +2/3$, $q_{\bar{u}} = -2/3$.



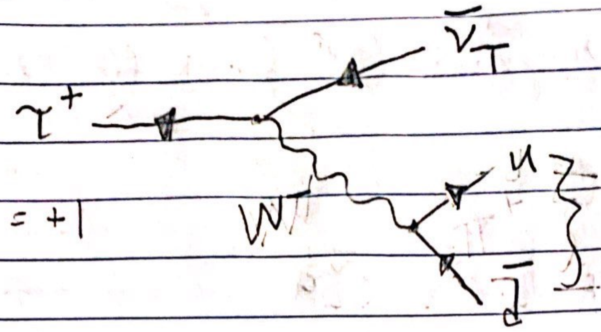

1c) $D^+ \to \bar{K}^0 + \mu^+ + \nu_\mu$ ($D^+$ meson is $c\bar{d}$ combination)




$q_c = 2/3$

$q_s = -1/3$

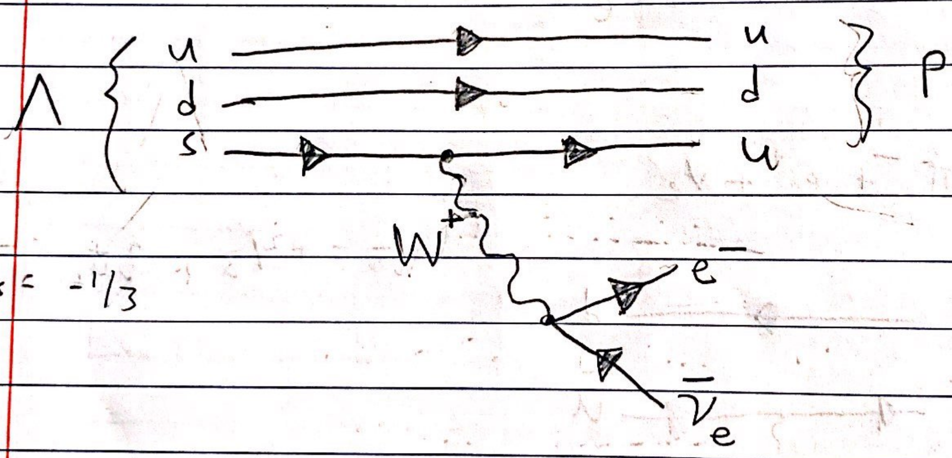
1d) $\tau^+ \rightarrow \bar{\nu}_\tau + \pi^+$




$q_{\tau^+} = +1$

$q_{\bar{\nu}_\tau} = 0$

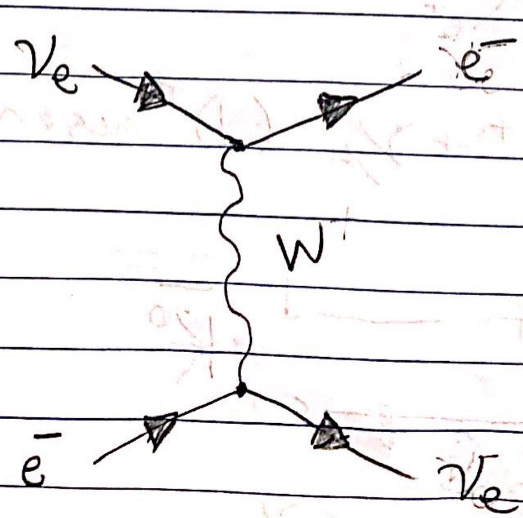
1e) $\underbrace{\Lambda}_{uds} \rightarrow \underbrace{p}_{uud} + e^- + \bar{\nu}_e$




$q_s = -1/3$

$q_u = +2/3$

1f) $\nu_e + e^- \rightarrow \nu_e + e^-$



Find "X" for these weak interactions.

2a) $\pi^+ \rightarrow \pi^0 + e^+ + X$

$\boxed{\pi^+ \rightarrow \pi^0 + e^+ + \nu_e}$ } $\beta^+$ decay

2b) $X \rightarrow e^+ \nu_e \bar{\nu}_\mu$

from W

indicates $\mu^+$, antimuon

$\therefore \boxed{\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu}$

2c) $K^+ \rightarrow X\, e^+ \nu_e$

$u\bar{s} \rightarrow$ quark change

from W

possible candidate: $\pi^0$: for charge conservation

$\therefore \boxed{K^+ \rightarrow \pi^0 e^+ \nu_e}$

2d) $X + p \rightarrow n + e^-$

$\boxed{\bar{\nu}_e + p \rightarrow n + e^-}$ } Inverse $\beta$ decay

2e) $D^0 \rightarrow K^- + \pi^0 + \nu_e + X$

$D^0 = c\bar{u}$, $K^- = \bar{u}s$

W

$\therefore X = e^+$

$\boxed{D^0 \rightarrow K^- + \pi^0 + \nu_e + e^+}$

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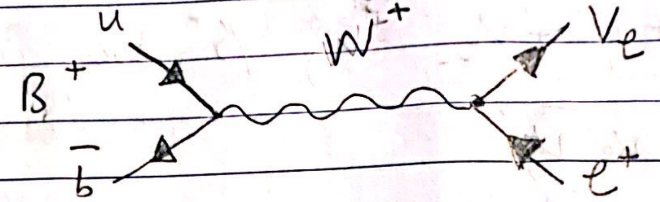
3) Consider $B^+ \to e^+ \nu_e$ $(B^+ : u\bar{b})$.

a) 


$q_u = 2/3$
$q_{\bar{b}} = +1/3$.

b) Based on the CKM matrix,

Thomson pg 370:

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.225 & 0.973 & 0.041 \\ 0.009 & 0.04 & 0.999 \end{pmatrix}$$

$$|V_{ub}| = 0.004$$

this interaction is the one we're concerned with, and its strength is small relative to the other quark interactions (i.e. other matrix elements)

∴ This decay via weak-interaction is considered rare.

(units of $m_\pi$)

3b) $$\Gamma(\pi^+ \to \mu^+ \nu) = \frac{G^2}{8\pi} \cdot f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$

Relative decay rates of

↳ $B^+$ to $e, \mu, \tau$.

$(m_{B^+} = 5279.29$ MeV$)$

$\Gamma(B^+ \to \ell^+ \nu_\ell)$ where $\ell = e, \mu, \tau$.

↳ $m_e = 0.511$ MeV

$m_\mu = 105.658$ MeV

$m_\tau = 1776.84$ MeV

$$\frac{\Gamma(B^+ \to e^+ \nu_e)}{\Gamma(B^+ \to e^+ \nu_e)} = \underline{1}$$

$$\frac{\Gamma(B^+ \to \mu^+ \nu_\mu)}{\Gamma(B^+ \to e^+ \nu_e)} = \overbrace{\cancel{\left(\frac{G^2 f_B^2 m_B 8\pi}{G^2 f_B^2 m_B 8\pi}\right)}}^{1} \frac{m_\mu^2}{m_e^2} \frac{(1 - m_\mu^2/m_B^2)^2}{(1 - m_e^2/m_B^2)^2}$$

$$= \underline{\underline{4.272 \cdot 10^4}}$$

$$\frac{\Gamma(B^+ \to \tau^+ \nu_\tau)}{\Gamma(B^+ \to e^+ \nu_e)} = (1) \frac{m_\tau^2}{m_e^2} \frac{(1 - m_\tau^2/m_B^2)^2}{(1 - m_e^2/m_B^2)^2}$$

$$= \underline{\underline{9.507 \cdot 10^6}}$$

$$\therefore \frac{\Gamma(B^+ \to e^+ \nu_e)}{\Gamma(B^+ \to e^+ \nu_e)} : \frac{\Gamma(B^+ \to \mu^+ \nu_\mu)}{\Gamma(B^+ \to e^+ \nu_e)} : \frac{\Gamma(B^+ \to \tau^+ \nu_\tau)}{\Gamma(B^+ \to e^+ \nu_e)}$$

$$= \boxed{1 : 4.272 \cdot 10^4 : 9.507 \cdot 10^6}$$

$(\Gamma_i / \Gamma)$

3c) from PDG

$\Gamma_{29} = \Gamma(B^+ \to e^+ \nu_e) < 9.8 \cdot 10^{-7}$ CL = 90%

$\Gamma_{30} = \Gamma(B^+ \to \mu^+ \nu_\mu) < 1 \cdot 10^{-6}$ CL = 90%

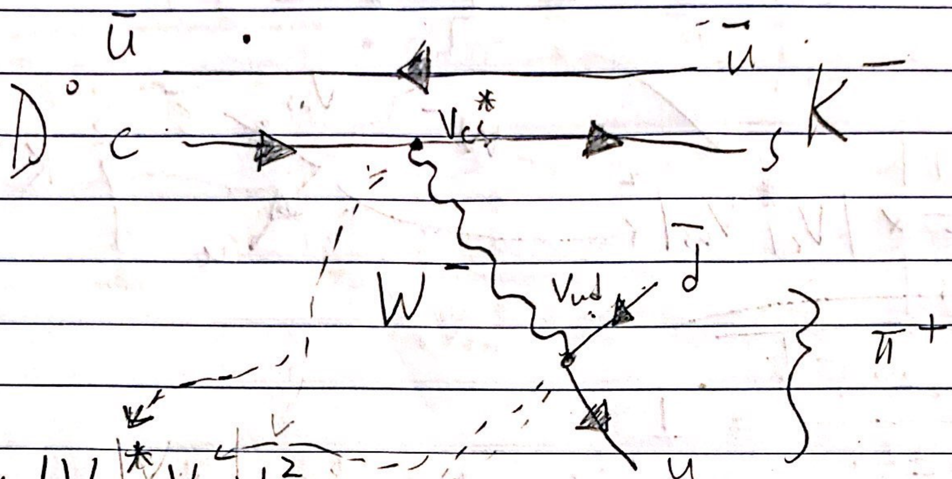
$\Gamma_{31} = \Gamma(B^+ \to \tau^+ \nu_\tau) = (1.14 \pm 0.27) \cdot 10^{-4}$ S = 1.3

All of the decay modes have been observed, but all are much less than 1% (cementing in their "rare" quality). Technically, because $\Gamma_{29}$ and $\Gamma_{30}$ only have established upper limits, the ratio from 3b) is still obtainable. At the very least, the result from 3b) gets the ordering of their magnitudes correct (but not the order of magnitude; confusing)

4) $D^0 : c\bar{u}$ Estimate Relative Rates + Draw Feynman Diagrams

i) $D^0 \to K^- \pi^+$

$(c\bar{u})$ $(\bar{u}s)$ $(u\bar{d})$



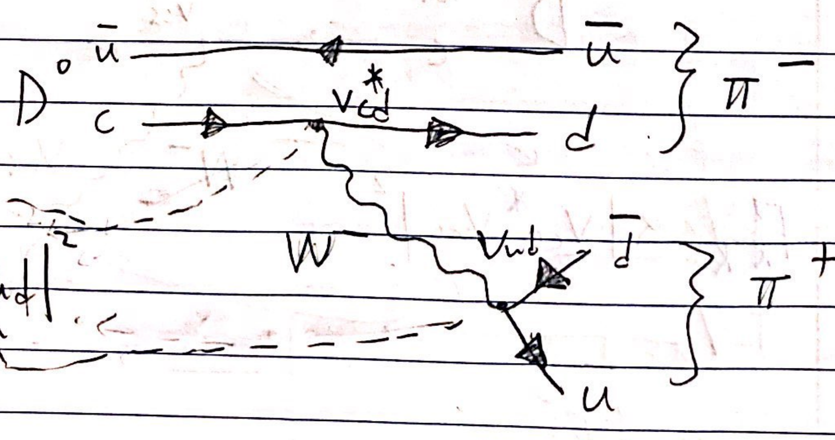

$\Gamma_{K^-\pi^+} \propto |V_{cs}^* V_{ud}|^2$

Fermi's Golden Rule: $\Gamma = 2\pi \sum_f |\langle f|H_i|i\rangle|^2 \rho(f)$

$\Gamma_{K^-\pi^+} \propto |V_{cs}^* V_{ud}|^2$ ✓

ii) $D^0 \to \pi^- \pi^+$

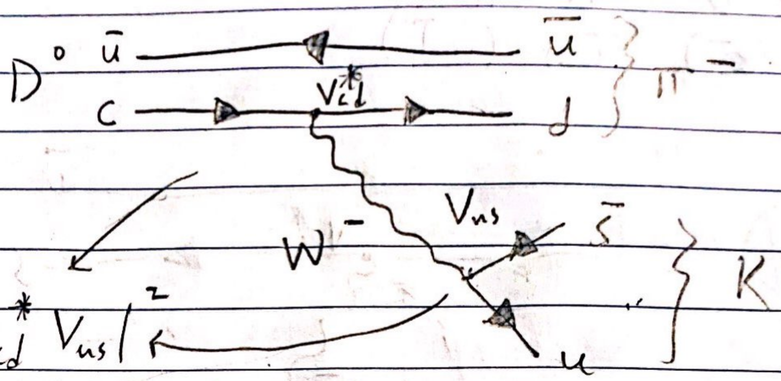
$(c\bar{u})$ $(\bar{u}d)$ $(u\bar{d})$




$\Gamma_{\pi^+\pi^-} \propto |V_{cd}^* V_{ud}|^2$

iii) $D^0 \rightarrow K^+ \pi^-$

$(c\bar{u}) \quad (u\bar{s}) \quad (\bar{u}d)$




$|V_{cd}|$

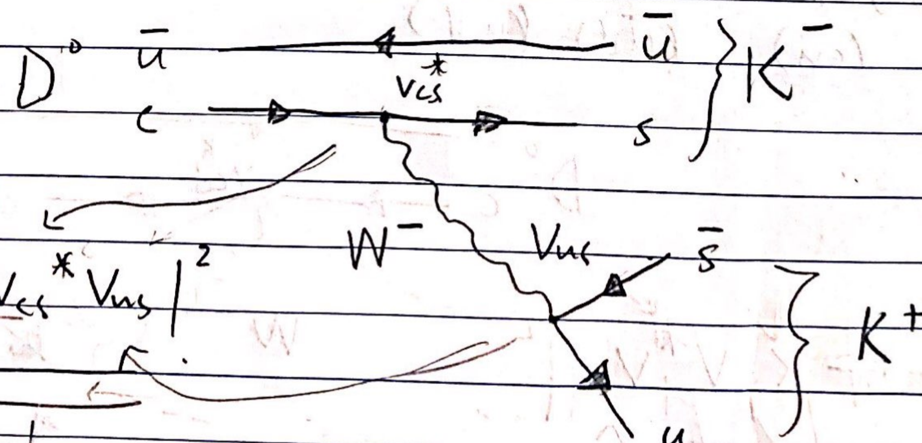
$\Gamma_{\pi^-K^+} \propto |V_{cd}^* V_{us}|^2$

$|V_{us}|$

$= \frac{0.2251}{0.225} = 1$

iv) $D^0 \rightarrow K^+ K^-$

$(c\bar{u}) \quad (u\bar{s}) \quad (\bar{u}s)$




$\Gamma_{K^-K^+} \propto |V_{cs}^* V_{us}|^2$

$|V_{us}|$

$= \frac{0.973}{0.225}$

Then, base the ratios off one of the interactions for a relative ratio estimation

For this problem then:

$$\boxed{\frac{\Gamma(D^0 \to K^-\pi^+)}{\Gamma(D^0 \to K^-\pi^+)} : \frac{\Gamma(D^0 \to \pi^-\pi^+)}{\Gamma(D^0 \to K^-\pi^+)} : \frac{\Gamma(D^0 \to \pi^-K^+)}{\Gamma(D^0 \to K^-\pi^+)} : \frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to K^-\pi^+)}}$$

$$1 : \frac{|V_{cd}^* V_{ud}|^2}{|V_{cs}^* V_{ud}|^2} : \frac{|V_{cd}^* V_{us}|^2}{|V_{cs}^* V_{ud}|^2} : \frac{|V_{cs}^* V_{us}|^2}{|V_{cs}^* V_{ud}|^2}$$

Based on CKM matrix values, then:

$|V_{cd}| = 0.225$, $|V_{ud}| = 0.974$, $|V_{cs}| = 0.973$,

$|V_{us}| = 0.225$, $|V$

$\therefore$ The ratio above resolves to:

$$\boxed{1 : 0.053 : 0.00285 : 0.053}$$