

Problem Set 12 problems

Question 1: Feynman diagrams with γ , W and Z bosons (30 points)

Learning objectives

In this question you will:

- Apply knowledge of Feynman diagrams involving electroweak bosons to various physical processes

Draw the lowest order Feynman diagram or diagrams for each of the following processes

1a.

$$e^+e^- \rightarrow \nu_\tau \bar{\nu}_\tau$$

Write your answer here

1b.

$$\mu^+\mu^- \rightarrow u\bar{u} \text{ where } u \text{ is an up quark}$$

Write your answer here

1c.

$$\bar{\nu}_\mu + e^- \rightarrow \bar{\nu}_\mu + e^-$$

Write your answer here

1d.

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

Write your answer here

1e.

$$p\bar{p} \rightarrow W^+W^- \text{ (consider only the valence quarks in the proton and anti-proton)}$$

Write your answer here

Question 2: Mixing in the K and B systems (30 points)

Learning objectives

In this question you will:

- Review the formulae for flavor mixing and see how the mixing phenomenology depends on $\Delta\Gamma$ and Δm

The phenomenon of K^0 - \bar{K}^0 mixing was discussed in Lecture 22 and in Thomson Section 14.5. If a K^0 is produced via the strong interactions at $t = 0$ the probabilities of the state being observed as a K^0 or as a \bar{K}^0 at a later time t are given by the expressions

$$\begin{aligned} P(K_{t=0}^0 \rightarrow K^0) &= \frac{1}{4} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-(\Gamma_1 + \Gamma_2)t/2} \cos(\Delta m t)] \\ P(K_{t=0}^0 \rightarrow \bar{K}^0) &= \frac{1}{4} [e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1 + \Gamma_2)t/2} \cos(\Delta m t)] \end{aligned}$$

where Γ_1 and Γ_2 are the inverse lifetimes of the K_S and K_L states and Δm is the mass difference between the states. Note: for this problem we will ignore CP violation so that $|K_S\rangle \equiv |K_1\rangle$ and $|K_L\rangle \equiv |K_2\rangle$

2a.

If a beam of 1 million K^0 is produced, using the measured values of the K_S and K_L lifetimes plot the predicted number of K^0 and of \bar{K}^0 that would be present in the beam as a function of time for $0 < t < \frac{10}{\Gamma_1}$ if the value of Δm were 0, 0.5Γ or Γ .

```
In [8]: import numpy as np
import matplotlib.pyplot as plt
```

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In [9]: num_K0 = 10**6

#from slides:
tau_s = 0.9*10**(-10) #sec
tau_p = 0.5*10**(-7) #sec

gamma1 = 1/tau_s
gamma2 = 1/tau_p

gamma = (gamma1 + gamma2)/2
deltam = np.array([0, 0.5, 1])*gamma

times = np.linspace(0, 10/gamma1, 1000)

def KtoK(t, g1, g2, dm, bar = False):
    a = np.exp(-1*g1*t)
    b = np.exp(-1*g2*t)
    c = 2*np.exp(-1*(g1 + g2)*t/2)*np.cos(dm*t)

    if bar == True:
        c *= -1

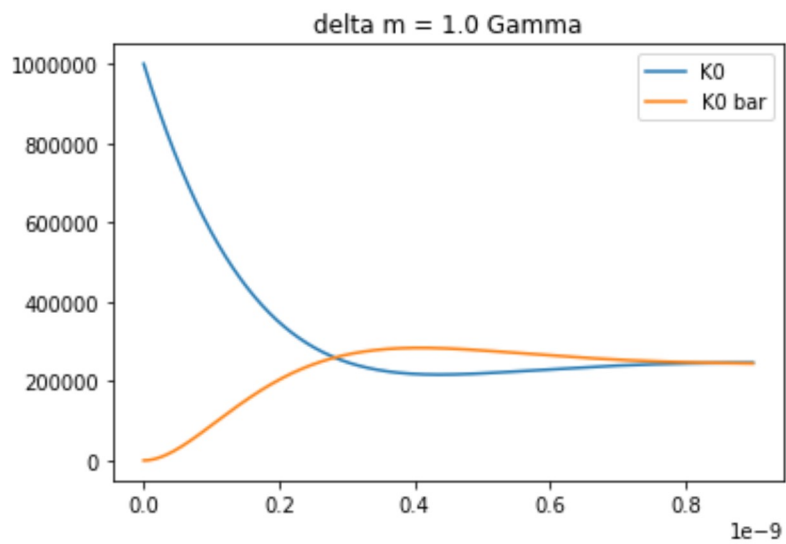
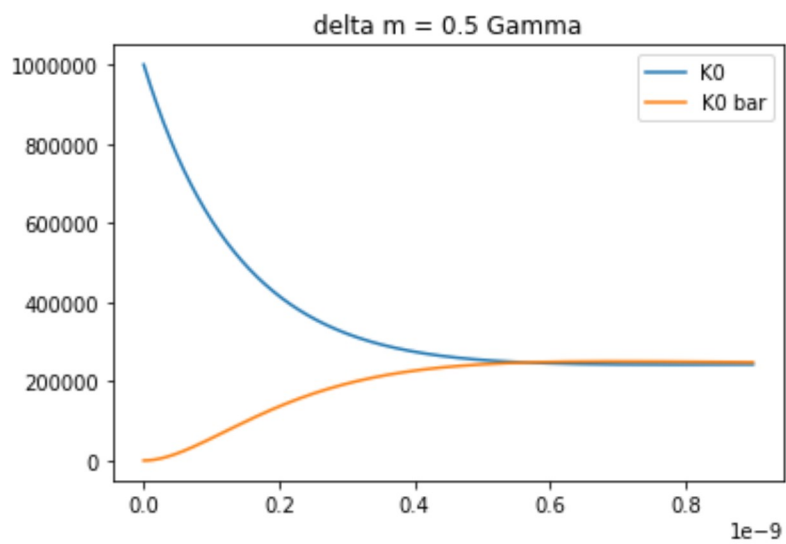
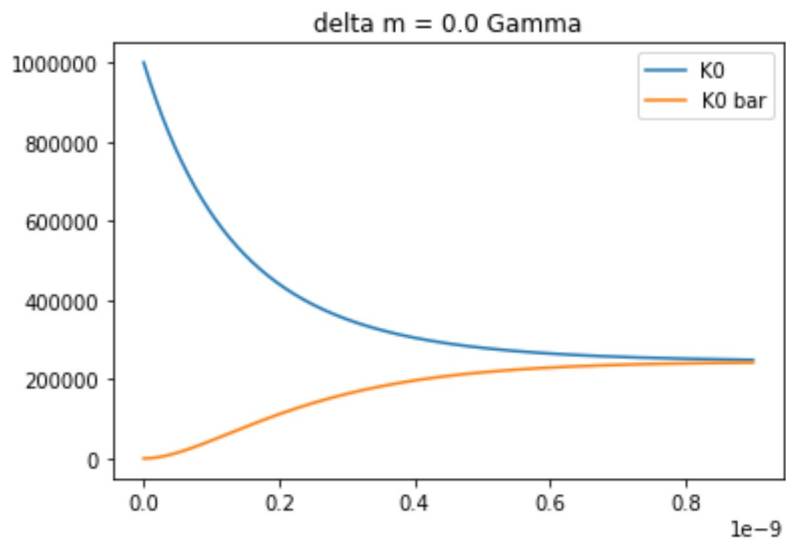
    return (1/4)*(a + b + c)

for i in range(len(deltam)):
    K0s = KtoK(times, gamma1, gamma2, deltam[i])*num_K0
    K0bars = KtoK(times, gamma1, gamma2, deltam[i], bar = True)*num_K0

    titlestr = str(i*0.5) + " Gamma"

    plt.figure()
    plt.title("delta m = " + titlestr)
    plt.plot(times, K0s, label = "K0")
    plt.plot(times, K0bars, label = "K0 bar")
    plt.legend()

plt.show()
```



2b.

Remake the plot above but plotting the predicted number of K_1 and K_2 in the beam instead of K_0 and \overline{K}_0

```
In [10]: # by definition:

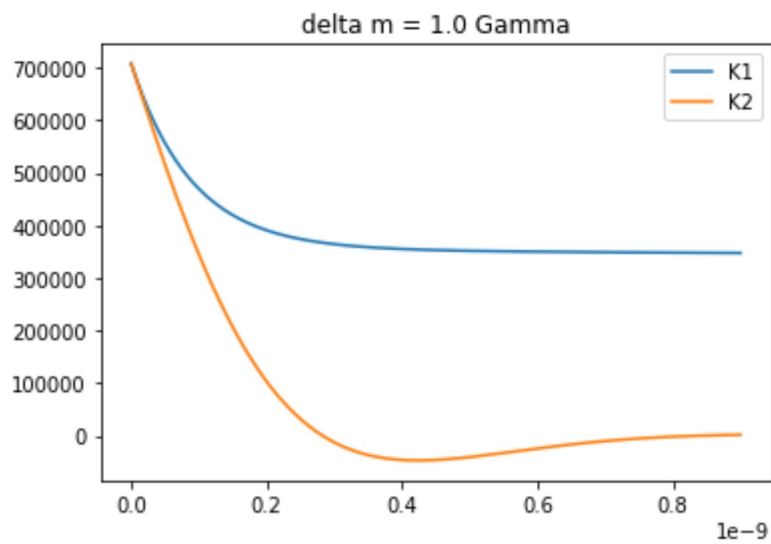
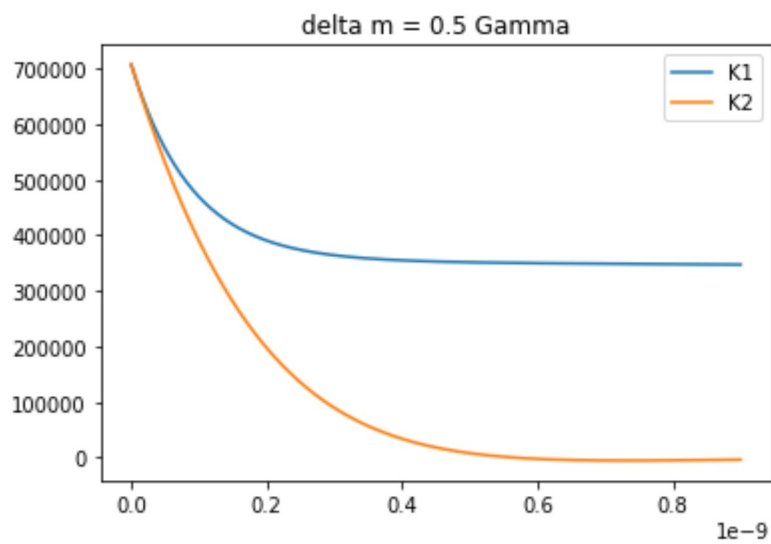
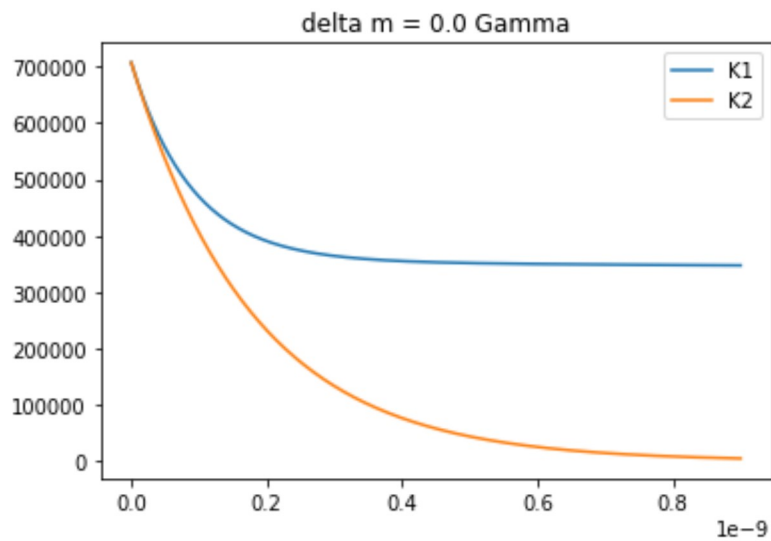
for i in range(len(deltam)):
    K0s = KtoK(times, gamma1, gamma2, deltam[i])*num_K0
    K0bars = KtoK(times, gamma1, gamma2, deltam[i], bar = True)*num_K0

    K1s = (1/np.sqrt(2))*(K0s + K0bars)
    K2s = (1/np.sqrt(2))*(K0s - K0bars)

    titlestr = str(i*0.5) + " Gamma"

    plt.figure()
    plt.title("delta m = " + titlestr)
    plt.plot(times, K1s, label = "K1")
    plt.plot(times, K2s, label = "K2")
    plt.legend()

plt.show()
```



2c.

The measured value of the mass difference is

$$\Delta m = 3.383 \times 10^{-15} \text{ GeV}$$

remake the plots above for this value

```
In [20]: given_dm = 3.383*(10**(-15)) # GeV

### convert to freq
h = 4.135*(10**(-15)) #eV*s
hGeV = h*(10**(-9)) # GeV*s

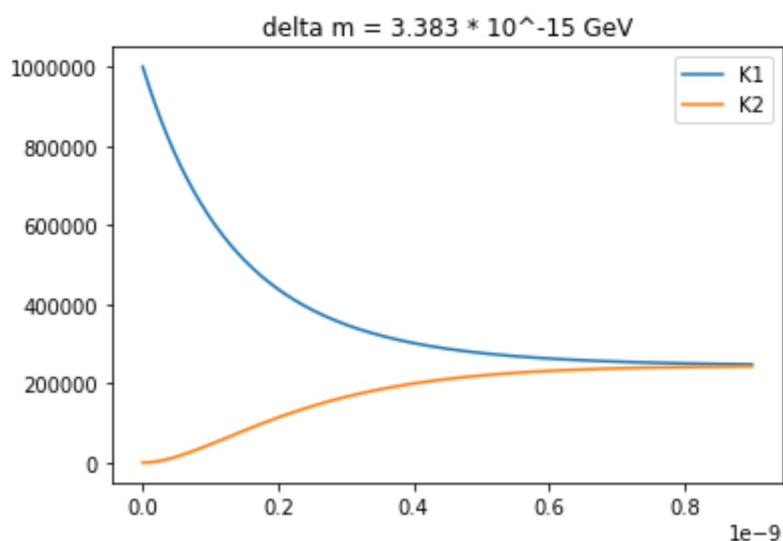
converted_dm = given_dm/hGeV #s

K1s = KtoK(times, gamma1, gamma2, converted_dm)*num_K0
K2s = KtoK(times, gamma1, gamma2, converted_dm, bar = True)*num_K0

titlestr = "3.383 * 10^-15 GeV"

plt.figure()
plt.title("delta m = " + titlestr)
plt.plot(times, K1s, label = "K1")
plt.plot(times, K2s, label = "K2")
plt.legend()

plt.show()
```



2d

The same expressions used for K^0 mixing can be applied to the B^0 and to the B_s . Use the PDG to find the values of Γ_1 , Γ_2 and Δm for the B^0 and the B_s . Using these numbers repeat 2c for these two neutral mesons.

```
In [26]: %matplotlib notebook
```

```
In [28]: # convention: 1 = B0, 2 = B0s
m_B0 = 5279.61 # MeV
m_B0s = 5366.79 # MeV

tau_B0 = 1.520*10**(-12) #s
tau_B0s = 1.510*10**(-12) #s

gammaB0 = 1/tau_B0
gammaB0s = 1/tau_B0s

deltam_B = (m_B0 - m_B0s)/2

deltam_B0s = 1.1688*10**(-8) #MeV
deltam_B0 = 0.32 #MeV

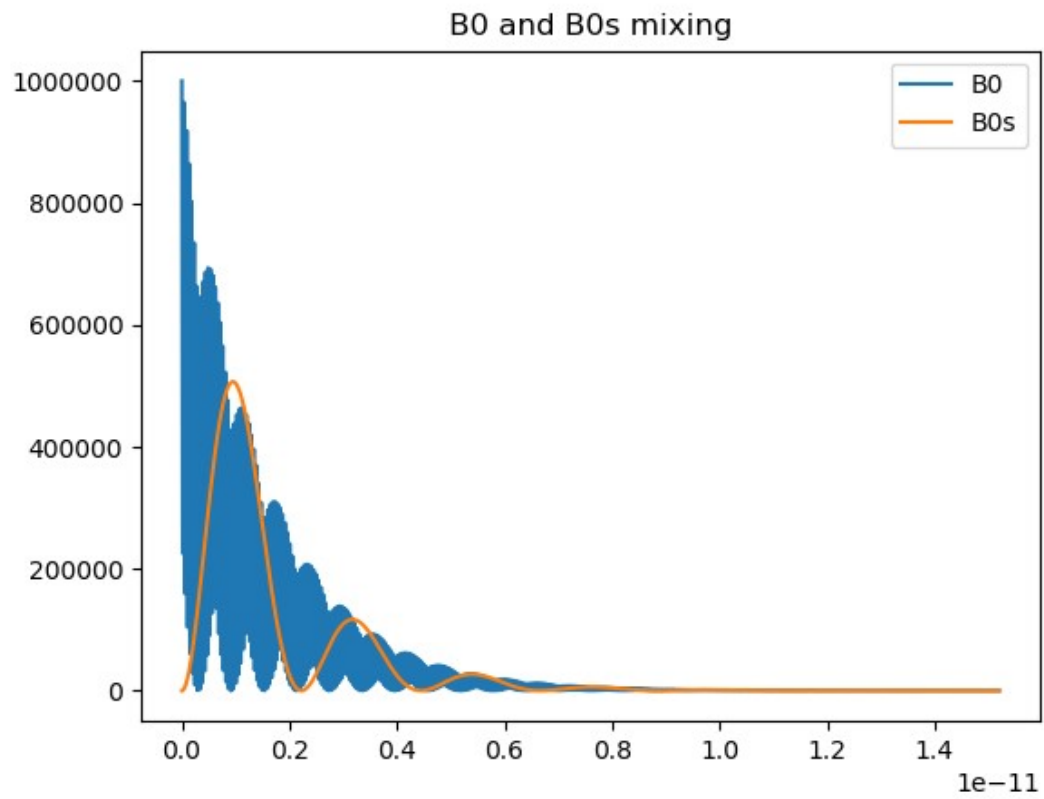
hMeV = h*(10**(-6)) #MeV*s
deltam_B0s /= hMeV #s
deltam_B0 /= hMeV #s

new_times = np.linspace(0, 10/gammaB0, 1000)

B0 = KtoK(new_times, gammaB0, gammaB0s, deltam_B0)*num_K0
B0s = KtoK(new_times, gammaB0, gammaB0s, deltam_B0s, bar = True)*num_K0

plt.figure()
plt.title("B0 and B0s mixing")
plt.plot(new_times, B0, label = 'B0')
plt.plot(new_times, B0s, label = 'B0s')
plt.legend()

plt.show()
```



In []: