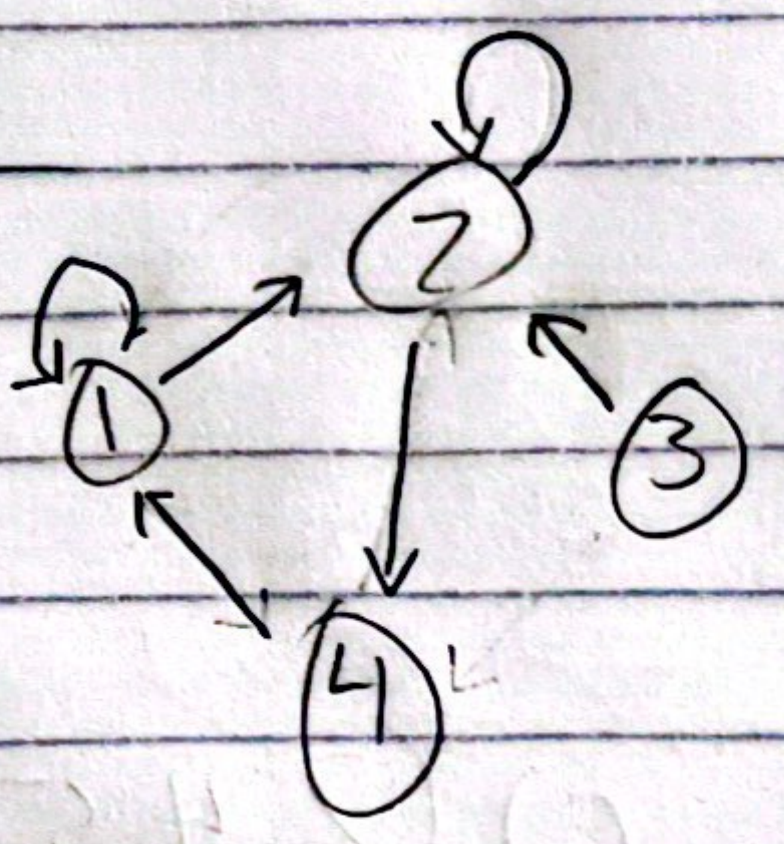
Will Gasser

6.1.1

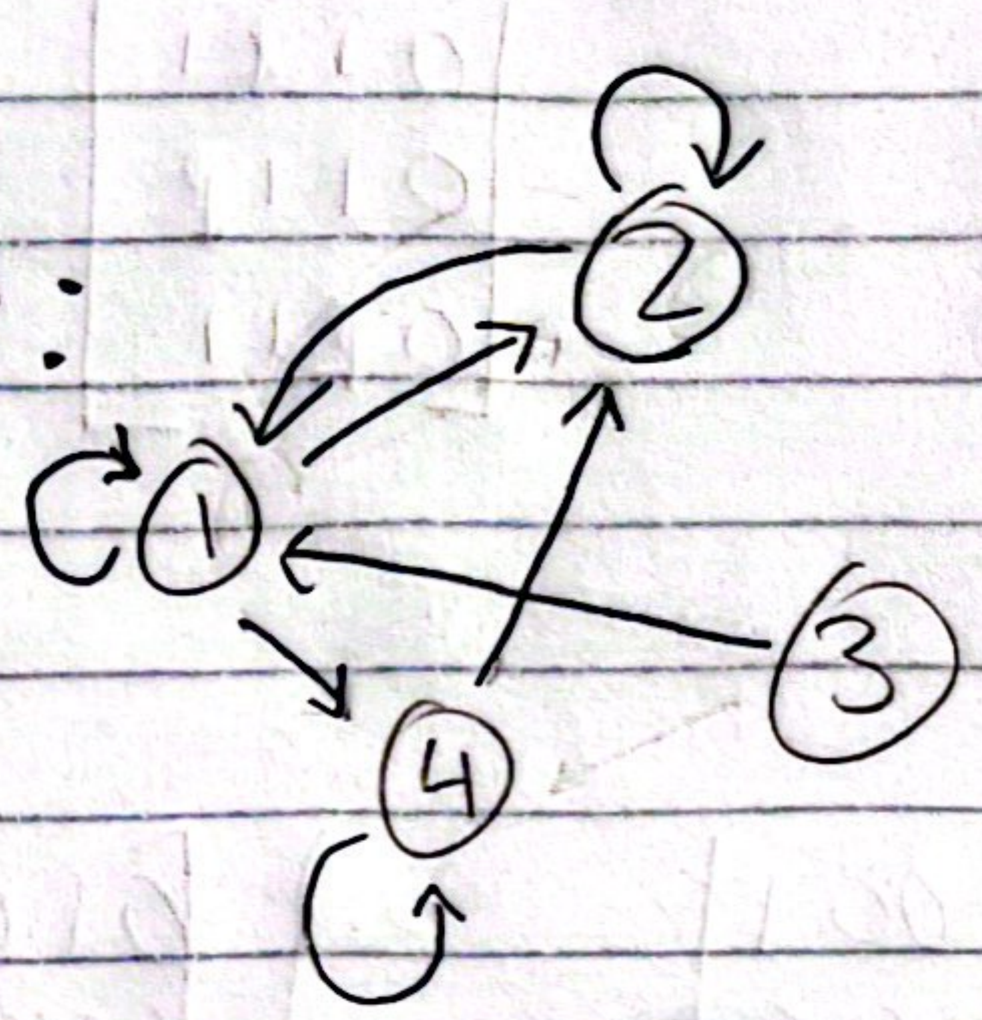
a.) NO
b.) Yes
c.) NO
d.) Yes

6.1.2

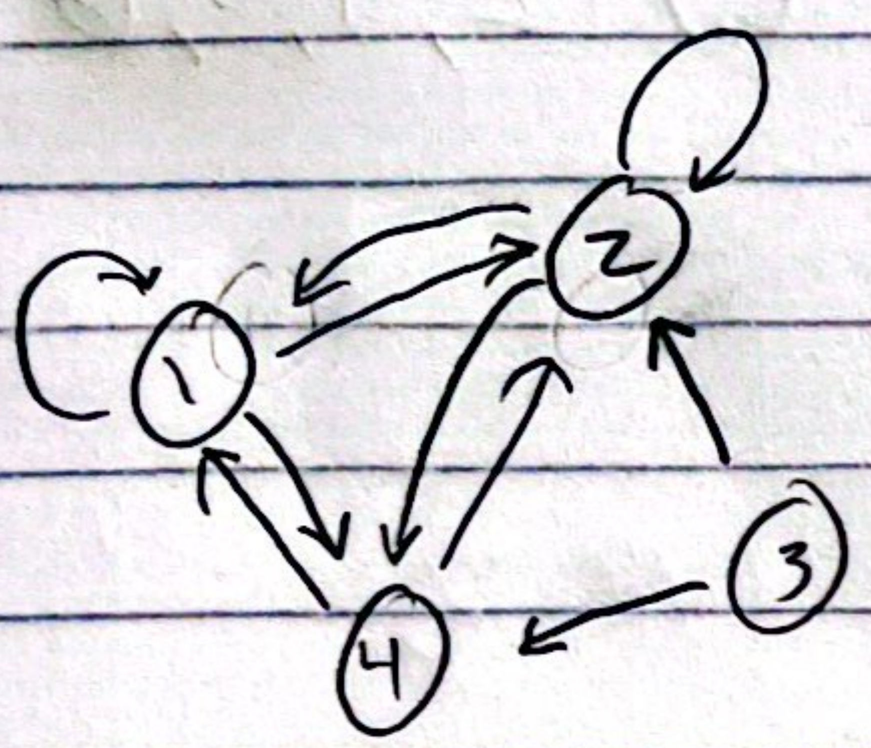
G^2:



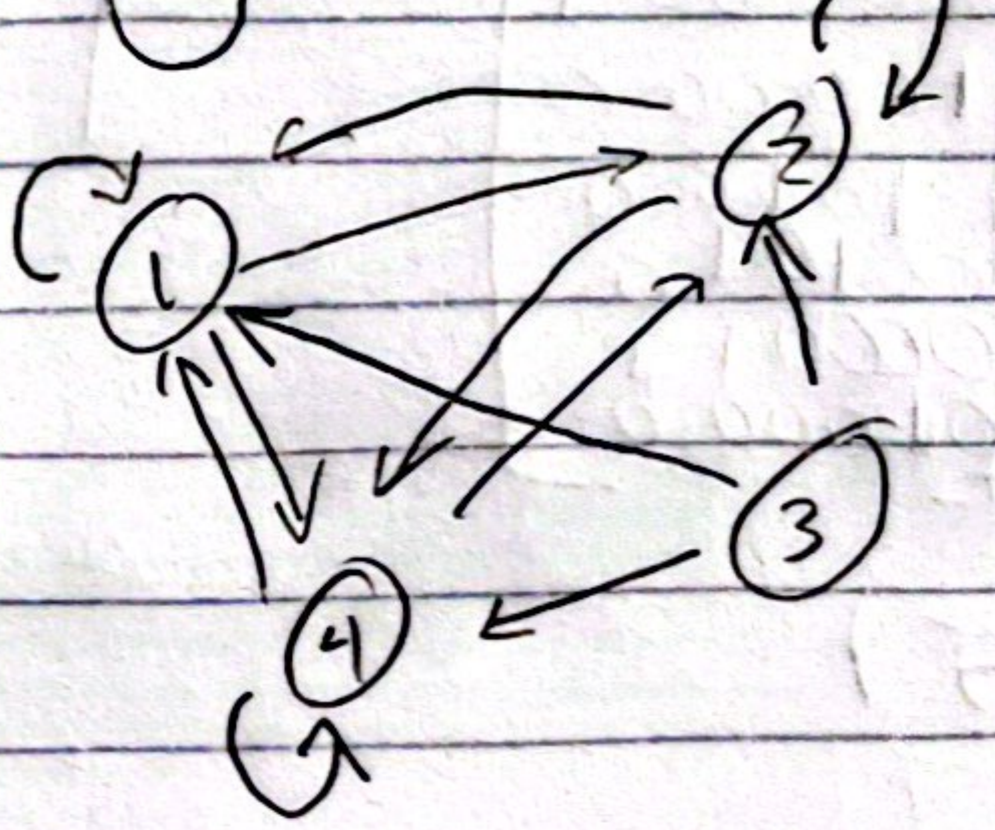
G^3:



G^4:

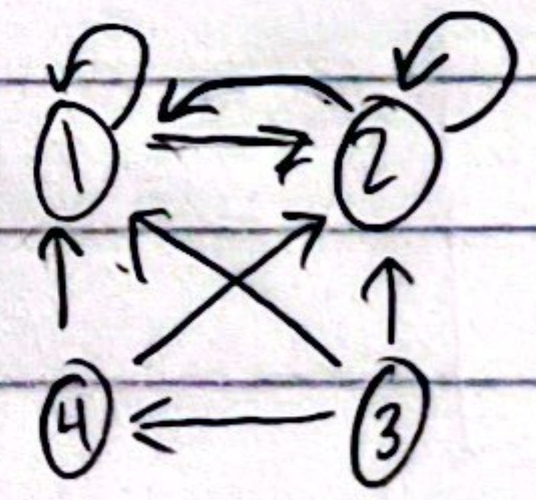


G^+:

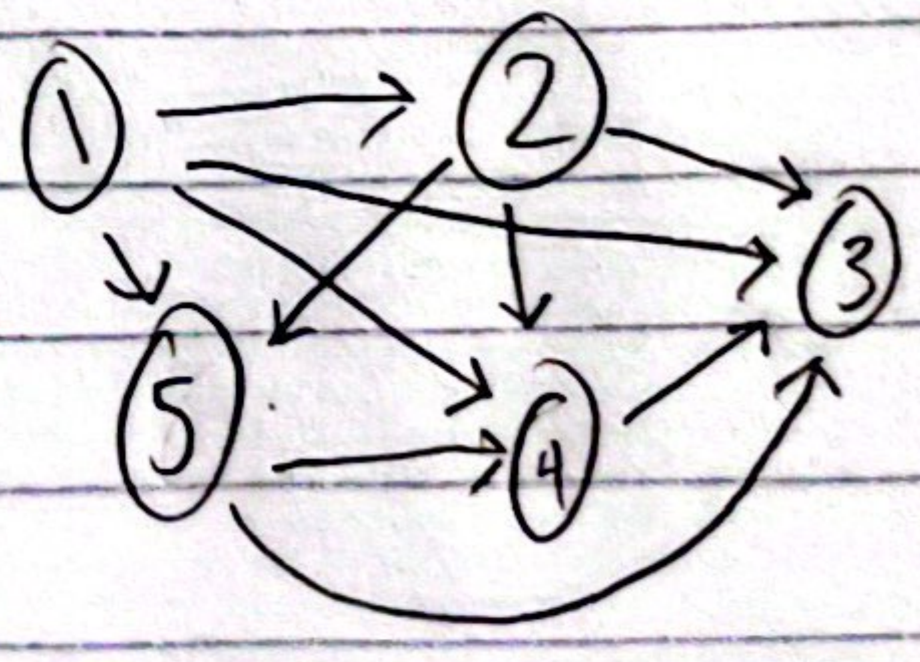


6.1.3

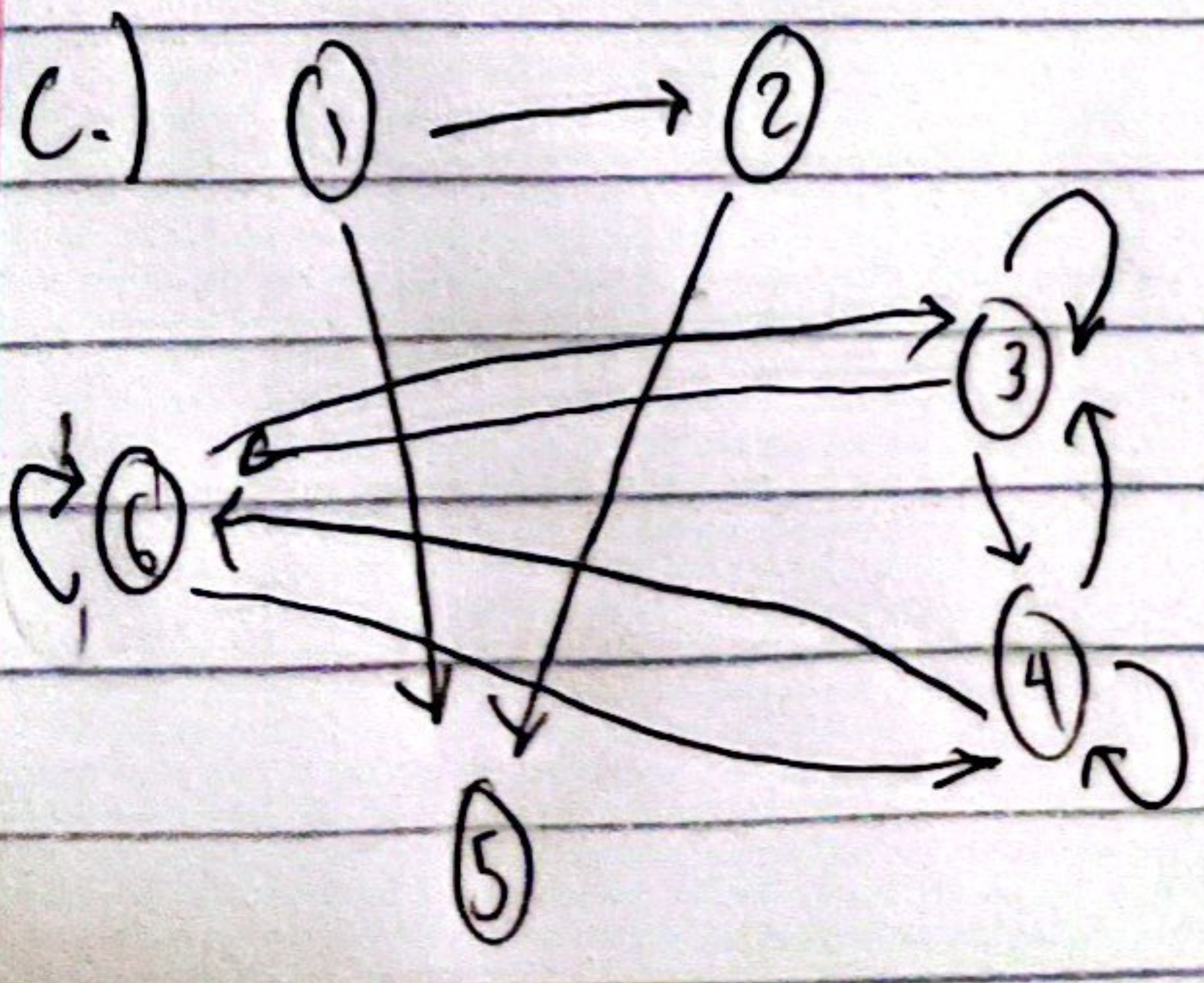
a.)



b.)



c.)



6.2.1

a.)

$$G \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&1&0&0 \end{bmatrix} \quad G^2 \begin{bmatrix} 0&0&1&0 \\ 0&0&0&1 \\ 0&1&0&0 \\ 0&0&1&0 \end{bmatrix} \quad G^3 \begin{bmatrix} 0&0&0&1 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{bmatrix}$$

$$G^4 \begin{bmatrix} 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \\ 0&1&0&0 \end{bmatrix} \quad G^+ \begin{bmatrix} 0&1&1&1 \\ 0&1&1&1 \\ 0&1&1&1 \\ 0&1&1&1 \end{bmatrix}$$

6.2.2

a.)

$$\begin{bmatrix} 0&0&0&0&0&1 \\ 1&0&0&0&0&0 \\ 0&1&0&1&0&0 \\ 0&0&1&0&1&0 \\ 0&0&0&0&1&1 \\ 0&1&0&0&0&0 \end{bmatrix}$$

b.)

$$\begin{bmatrix} 0&1&0&0&0&0 \\ 0&0&0&0&0&1 \\ 1&0&1&0&1&0 \\ 0&1&0&1&1&1 \\ 0&1&0&0&1&1 \\ 1&0&0&0&0&0 \end{bmatrix}$$

c.) (2, 4, 5, 6)

d.) (1, 4, 5)

6.2.3

$$A^4 = \begin{bmatrix} 0&0&1&0&0 \\ 1&0&0&0&0 \\ 0&1&0&0&0 \\ 1&1&0&1&0 \\ 1&1&1&0&1 \end{bmatrix}$$

a. (2, 3, 4)
b. 4
c. NO
d. Yes
e. NO
f. Yes

6.3.1

a.) (J, I, A, F)

b.) (J, H, D, G)

c.) (A,D) Yes (J,F) NO (B,E) NO (G,F) Yes
(D,B) Yes (C,F) NO (A,I) Yes (C,E) NO

6.4.1

a. The relationship is not of equivalence. It is reflexive and symmetric, but not transitive. Sharing a nonbiological parent does not hold.

b. This is an equivalence relation. It is a reflexive and symmetric relationship that is also transitive. If the biological parent is shared they must be related.

The partition would be the group of children from the same mother.

c. This is not reflexive because someone cannot be their own spouse and related to themselves.

d. It is reflexive because 2x and x are integers. Symmetric because $x+y=y+x$. It is finally transitive. If x and y are integers and $x+2$ is an integer, 2 is an integer.

The partitions are all even and odd integers.

6.4.2 $\{7, 2, 13, 44, 56, 34, 99, 31, 4, 17$

remainders: $\{3 | 2 | 1 | 0 | 0 | 2 | 3 | 3 | 0 | 1$

$$S = \{\{7, 99, 31\}, \{2, 34\}, \{13, 17\}, \{44, 56, 4\}\}$$

6.4.5

a.) $x - x = 0$ $\quad x - x = 3(0)$ ✓ $\quad xRx$

Reflexive ✓

$y - x = -(x - y) \rightarrow y - x = -(3m) \rightarrow y - x = 3(n)$

$n = -m$

yRx ✓

Symmetric ✓

$x - z = x - y + y - z \rightarrow x - z = (x - y) + (y - z) \rightarrow 3(m + n)$

$x - z = 3m + 3n$ $\quad xRz$ Transitive ✓

$\boxed{\text{This is an equivalence relation}}$.

b.) $x + x = 2x$ $\quad x + x = 3\left(\frac{2x}{3}\right)$ $\frac{2x}{3}$ ✓ reflexive ✓ xRx

$y + x = x + y$ ✓ Symmetric yRx

$= 3m$

$x + y = 1 + 2 = 3(1)\ xRy$ $\quad y + z = 2 + 4 = 6\ yRz$

$x + z = 1 + 4 = 5$

No this is not an equivalence relation.

not transitive