Title

Mioto Takahashi

CSDS MS

September 26, 2022

1 Lotka-Volterra 4th Equilibrium

1.1 Perquisites

Taking the video lectures into account, we have the continuous dynamics of 2 competing species in the form of logistic growth models.

$$
\begin{aligned}
&\text{Lotka-Volterra} = \begin{cases} \dot{N}_1 & = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha_2 N_2}{K_1} \right) \\ \dot{N}_2 & = r_2 N_2 \left(\frac{K_2 - N_2 - \alpha_1 N_1}{K_2} \right) \end{cases} \\
&\text{where} \begin{cases} a_{12} \equiv \alpha_2 \frac{r_1}{K_1} \\ a_{12} \equiv \alpha_1 \frac{r_2}{K_2} \end{cases}
\end{aligned} \tag{1}
$$

with the following variables,

- i: species ID
- r_i: population growth rate for species i
- N_i: population of species i
- K_i: population maximum capacity
- a_{ij}: interaction between species i and species j (predation and reproduction)

Nullclines for $\dot{N}_1 = 0$ and $\dot{N}_2 = 0$ are,

For $\dot{N}_1 = 0$,

- $N_1^* = 0$ (species 1 is extinct)
- $N_1^{**} = K_1 - \alpha_2 N_2$

For $\dot{N}_2 = 0$,

- $N_2^* = 0$ (species 2 is extinct)
- $N_2^{**} = K_2 - \alpha_1 N_1$

with $*$ and $**$ signifying 2 solutions. The same rule applies to the rest of the paper.

The uncoupled fixed points are the following.

$$N_1^{**} = \frac{K_1 - \alpha_2 K_2}{1 - \alpha_2 \alpha_1} \tag{2}$$

$$N_2^{**} = \frac{K_2 - \alpha_1 K_1}{1 - \alpha_1 \alpha_2} \tag{3}$$