Modeling Complex Systems (CS/CSYS 302), Fall 2022 Assignment #1

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1 Lotka-Volterra 4th Phase Plane

1.1 Perquisites

Taking the video lectures into account, we have the continuous dynamics of 2 competing species in the form of logistic growth models.

$$\text{Lotka-Volterra} = \begin{cases} \dot{N}_1 & = r_1 N_1 \left(\frac{K_1 - N_1 - \alpha_2 N_2}{K_1}\right) \\ \dot{N}_2 & = r_2 N_2 \left(\frac{K_2 - N_2 - \alpha_1 N_1}{K_2}\right) \end{cases}$$
$$\text{where} \begin{cases} a_{12} \equiv \alpha_2 \frac{r_1}{K_1} \\ a_{12} \equiv \alpha_1 \frac{r_2}{K_2} \end{cases} \tag{1}$$

with the following variables,

- i: species ID
- r_i: population growth rate for species i
- N_i: population of species i
- K_i: population maximum capacity
- a_{ij}: interaction between species i and species j (predation and reproduction)

Nullclines for $\dot{N}_1 = 0$ and $\dot{N}_2 = 0$ are,

For $\dot{N}_1 = 0$,

- $N_1^* = 0$ (species 1 is extinct)
- $N_1^{**} = K_1 - \alpha_2 N_2$

For $\dot{N}_2 = 0$,

- $N_2^* = 0$ (species 2 is extinct)
- $N_2^{**} = K_2 - \alpha_1 N_1$

with $*$ and $**$ signifying 2 solutions. The same rule applies to the rest of the paper.

The uncoupled fixed points are the following.

$$N_1^{**} = \frac{K_1 - \alpha_2 K_2}{1 - \alpha_2 \alpha_1} \tag{2}$$

$$N_2^{**} = \frac{K_2 - \alpha_1 K_1}{1 - \alpha_1 \alpha_2} \tag{3}$$

1.2 Determining the Vector Space

The 4th phase space, succeeding the 3 phase spaces discussed in the lecture, has the following constraints.

$$\begin{aligned} K_1 &> \frac{K_2}{\alpha_1} \\ K_2 &> \frac{K_1}{\alpha_2} \end{aligned} \tag{4}$$

This system in a phase space should resemble Figure 1.

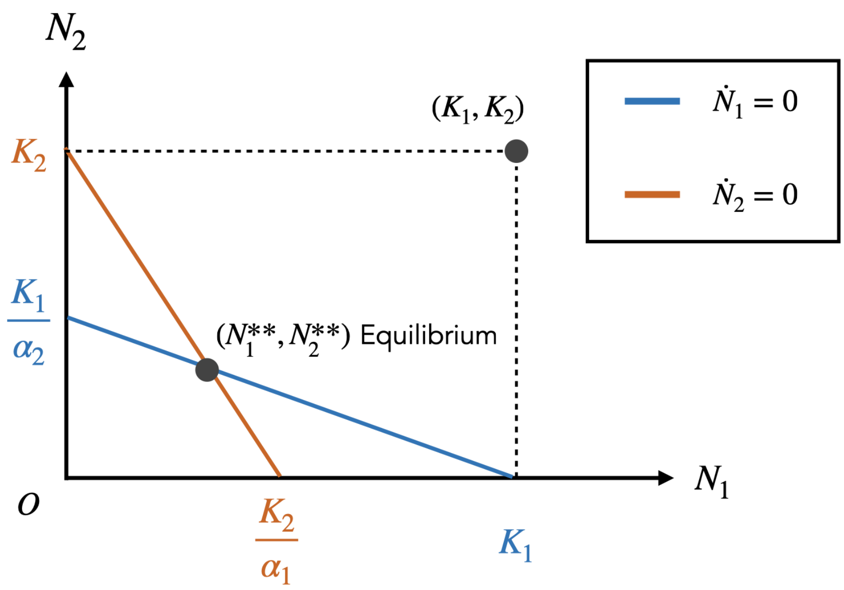

Figure 1: Nullclines for N_1 and N_2 and the equilibrium of the Lotka-Volterra system when Equation 4 is met.

In determining the vector space, or rather the vectors' orientation on arbitrary points in the phase plane, we need a sample of a point on the phase plane. For simplicity, we take (K_1, K_2).

Now we determine the orientation of the vector at (K_1, K_2).