Math 146B (010) Winter 2021 Homework Set 1 Due Wed Jan.13

1. Determine the linear dependence of the following functions:

$$f_1(t) = 2t - 3, \qquad f_2(t) = t^2 + 1, \qquad f_3(t) = 2t^2 - t, \qquad f_4(t) = t^2 + t + 1.$$

   If they are linear dependent, find a linear relation between them.

2. Verify the given functions are the solutions of the differential equation. Compute their Wronskian and show that they form a fundamental set of solutions.

$$y''' + 2y'' - y' - 2y = 0; \qquad \text{given } y_1 = e^t, \ y_2 = e^{-t}, \ y_3 = e^{-2t}.$$

3. Verify the given functions are the solutions of the differential equation. Compute their Wronskian and show that they form a fundamental set of solutions. ($y = y(x)$, where y is the unkown function and x is the variable.)

$$x^3y''' + x^2y'' - 2xy' + 2y = 0; \qquad \text{given } y_1 = x, \ y_2 = x^2, \ y_3 = \frac{1}{x}.$$

4. Find the general solution of the given differential equations.

   (a) $y''' - y'' - y' + y = 0$;

   (b) $y''' - 3y'' + 3y' - y = 0$.

5. Solve the initial value problem

$$y''' + y' = 0; \qquad y(0) = 0, \ y'(0) = 1, \ y''(0) = 2.$$