fee_sigma_analysis_second_interval

October 2, 2024

[53]:
```python
import warnings
warnings.filterwarnings('ignore')
# Hide input cells in Jupyter Notebook
from IPython.display import display, HTML

display(HTML("<style>.jp-InputArea { display: None !important; }</style>"))
```

```
<IPython.core.display.HTML object>
```

[13]:
```python
import numpy as np
import pandas as pd

file_path = '/home/shiftpub/AMM-Python/results/dummy_results/
 ↪bitcoin_simulation_result_second.csv'
results = pd.read_csv(file_path)
unique_fee_rate = results['fee_rate'].unique()
```

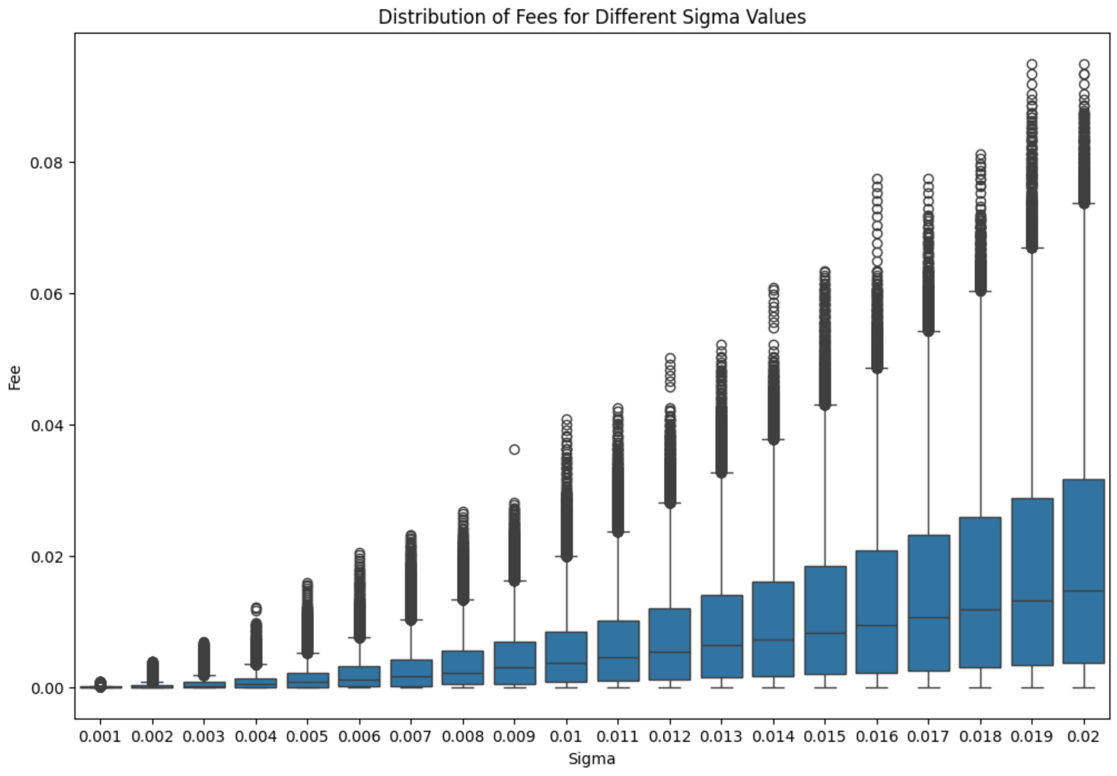
Parameter	Value
Initial Token A	1,000,000 units (1 million)
Initial Token B	1,000,000 units (1 million)
Fee Rate	`np.round(np.arange(min_fee_rate, max_fee_rate, min_fee_rate), 4)` Example: [0.0005, 0.001, 0.0015, …, 0.02] (total 40 values)
Sigma (Volatility)	`np.round(np.arange(0.001, 0.021, 0.001), 3)` Example: [0.001, 0.002, …, 0.02] (total 20 values)
Time Interval (dt)	`1/3600` seconds (1 second interval)
Simulation Length	50,000 steps
Market Spread	0.01
Drift Rate	0.0001
Start Price (Token A, B)	50,000
Iterations per Combination	300 (for each combination of `sigma` and `fee rate`)

0.0.1 Plan:

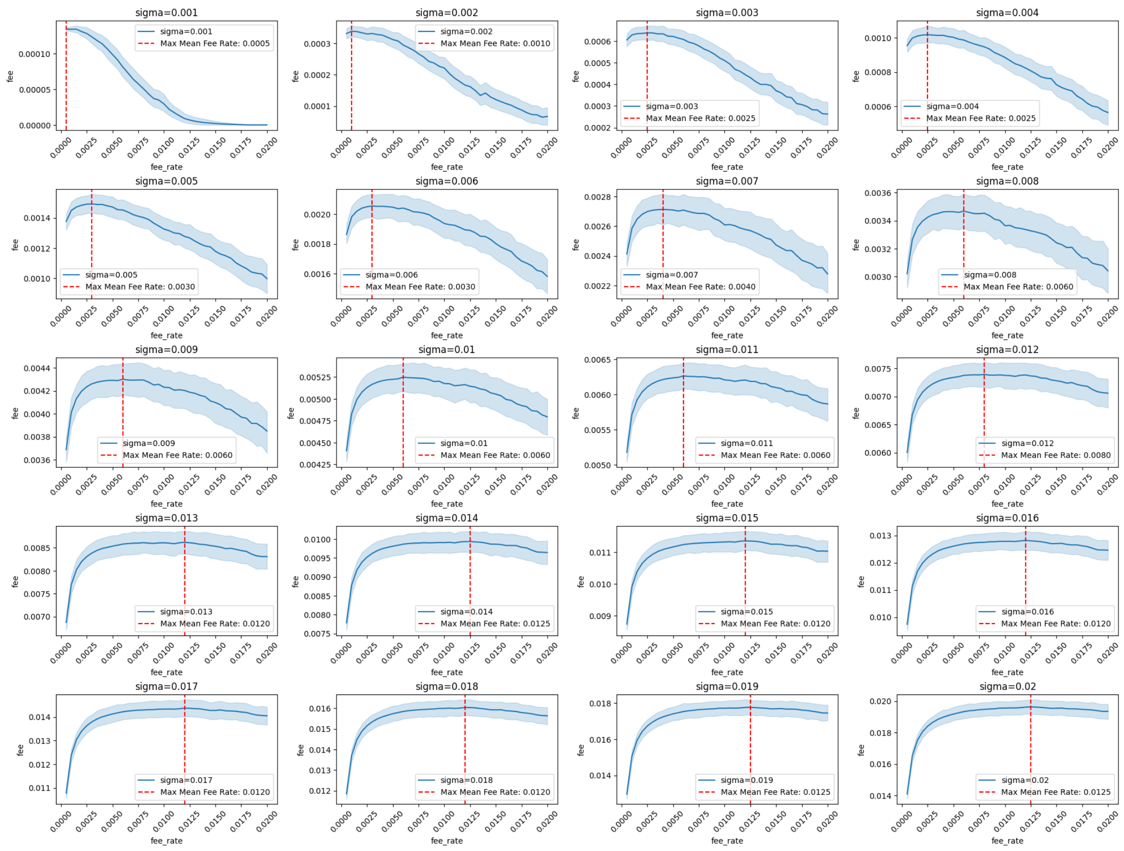
```
I. Demonstrate the overall relationship between sigmas and fee collected in AMM.
```

```
II. Explore the Max Mean fee rate for each sigma.

III. Examine the pattern between sigma and Max Mean fee rate.
```

[7]:
```python
import matplotlib.pyplot as plt
import seaborn as sns

# Set up the plot
plt.figure(figsize=(12, 8))

# Draw the boxplot
sns.boxplot(x='sigma', y='fee', data=results)

# Set the labels and title
plt.xlabel('Sigma')
plt.ylabel('Fee')
plt.title('Distribution of Fees for Different Sigma Values')

# Show the plot
plt.show()
```



```
[8]: import matplotlib.pyplot as plt
     import seaborn as sns

     unique_sigma = results['sigma'].unique()

     # Define the number of rows and columns for the subplots grid
     num_rows = 5
     num_cols = 4

     # Create subplots
     fig, axes = plt.subplots(num_rows, num_cols, figsize=(20, 15))
     axes = axes.flatten()  # Flatten the axes for easy iteration

     # Loop through each sigma value and create a subplot
     for i, sigma in enumerate(unique_sigma):
         df = results[results['sigma'] == sigma].copy()  # Ensure a copy to avoid␣
      ↪warnings

         # Calculate the fee_rate with the highest mean fee
         max_fee_rate = df.groupby('fee_rate')['fee'].mean().idxmax()

         # Plot the fee vs fee_rate
         sns.lineplot(x='fee_rate', y='fee', data=df, ax=axes[i],␣
      ↪label=f'sigma={sigma}')

         # Add a red dashed line to indicate the fee_rate with the highest mean fee
         axes[i].axvline(x=max_fee_rate, color='red', linestyle='--', label=f'Max␣
      ↪Mean Fee Rate: {max_fee_rate:.4f}')

         # Set labels and title
         axes[i].set_xlabel('fee_rate')
         axes[i].set_ylabel('fee')

         # make x tick labels more readable rotate 45 degrees
         axes[i].tick_params(axis='x', rotation=45)
         axes[i].set_title(f'sigma={sigma}')
         axes[i].legend()

     # Remove any unused subplots
     for j in range(i + 1, len(axes)):
         fig.delaxes(axes[j])

     plt.tight_layout()
     plt.show()
```



0.0.2 Statistics Analysis

1. **Confidence Metric Calculation**:
   - Perform **pairwise ANOVA** between the **max mean fee rate** and all other fee rates.
   - The **confidence metric** is calculated as:

$$\text{Confidence Metric} = \frac{1}{\sum \text{p-values from pairwise comparisons}}$$

   - This metric quantifies how confidently the max mean fee rate differs from other fee rates.
2. **ANOVA p-value Heatmap**:
   - Use a **heatmap** to visualize the p-values from ANOVA for all fee rates for each sigma.
   - Each subplot in the grid represents the pairwise p-values for a different sigma value, providing insight into how fee rates compare statistically.
3. **Pairwise T-Test Heatmap**:
   - Perform **pairwise t-tests** between all fee rates for each sigma.
   - Generate a **heatmap** for each sigma to display the p-values from the t-tests, indicating the significance of differences between fee rates.

I. Confidence Metric

[50]:
```python
from statsmodels.formula.api import ols
import statsmodels.api as sm
```

```
from statsmodels.stats.multicomp import pairwise_tukeyhsd

confidence_scores = []

def calculate_confidence(tukey_summary, max_fee_rate):
    confidence_sum = 0
    for result in tukey_summary.summary().data[1:]:
        fee_rate_1, fee_rate_2, p_value = result[0], result[1], result[4]
        if max_fee_rate in [fee_rate_1, fee_rate_2]:
            confidence_sum += p_value

    # Higher confidence means lower p-values
    return 1 / confidence_sum if confidence_sum > 0 else 0

for sigma in unique_sigma:
    df_sigma = results[results['sigma'] == sigma].copy()
    max_fee_rate = df_sigma.groupby('fee_rate')['fee'].mean().idxmax()
    # Perform ANOVA
    model = ols('fee ~ C(fee_rate)', data=df_sigma).fit()
    anova_table = sm.stats.anova_lm(model, typ=2)
    # Perform Tukey's HSD for pairwise comparison
    tukey = pairwise_tukeyhsd(endog=df_sigma['fee'],␣
  ↪groups=df_sigma['fee_rate'], alpha=0.05)
    confidence_scores.append(calculate_confidence(tukey, max_fee_rate))


# Plot the confidence scores
plt.figure(figsize=(12, 8))
plt.plot(unique_sigma, confidence_scores, marker='o')
plt.xlabel('Sigma')
plt.ylabel('Confidence Metric')
plt.title('Confidence Metric for Different Sigma Values')
plt.show()
```

```
/home/shiftpub/miniconda3/envs/amm-env/lib/python3.9/site-
packages/scipy/integrate/_quadpack_py.py:1272: IntegrationWarning: The integral
is probably divergent, or slowly convergent.
  quad_r = quad(f, low, high, args=args, full_output=self.full_output,
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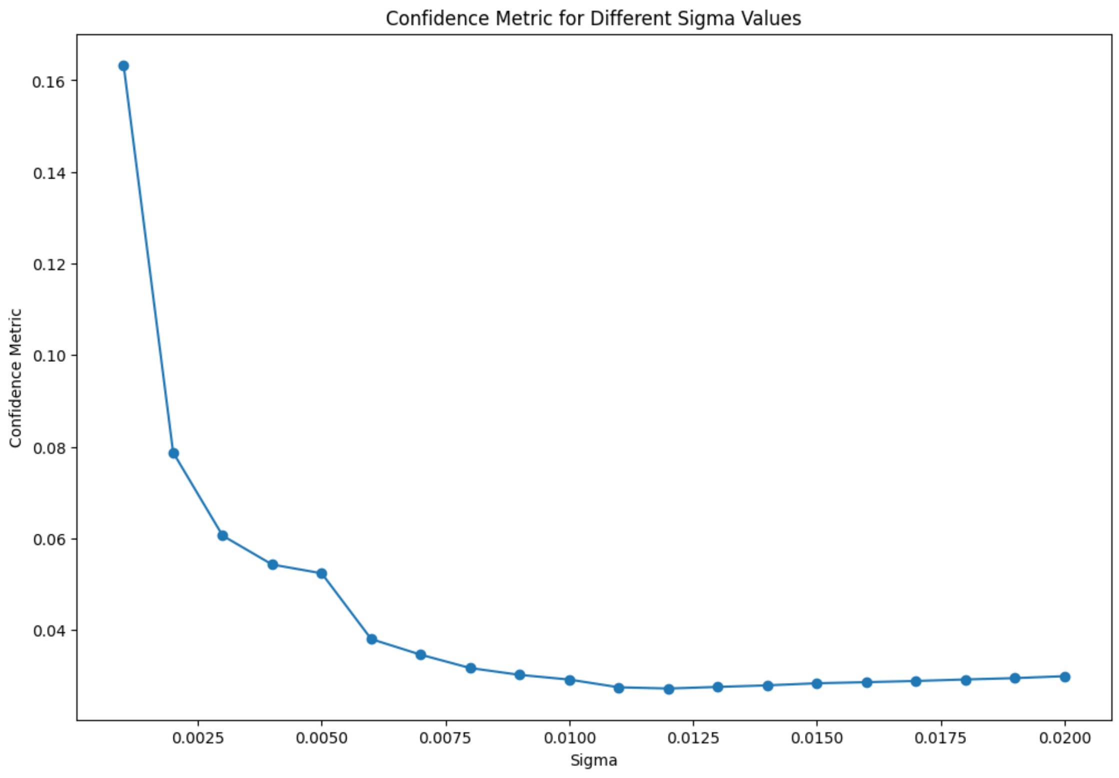

II. ANOVA p-value Heatmap

[46]:
```python
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from statsmodels.formula.api import ols
from statsmodels.stats.multicomp import pairwise_tukeyhsd

# Unique sigma values
unique_sigma = results['sigma'].unique()

# Define the number of rows and columns for the subplots grid
num_rows = 5
num_cols = 4

# Create subplots
fig, axes = plt.subplots(num_rows, num_cols, figsize=(20, 15))
```

```
axes = axes.flatten()  # Flatten the axes for easy iteration

# Loop through each sigma value and create a heatmap subplot for each
for i, sigma in enumerate(unique_sigma):
    df = results[results['sigma'] == sigma].copy()  # Filter the dataframe for␣
 ↪the current sigma

    # Perform ANOVA
    model = ols('fee ~ C(fee_rate)', data=df).fit()

    # Perform Tukey's HSD for pairwise comparison
    tukey = pairwise_tukeyhsd(endog=df['fee'], groups=df['fee_rate'], alpha=0.
 ↪05)

    # Extract unique fee rates
    fee_rates = sorted(df['fee_rate'].unique())

    # Create an empty matrix to store p-values
    p_value_matrix = np.ones((len(fee_rates), len(fee_rates)))  # Initialize␣
 ↪with ones

    # Fill the matrix with p-values from Tukey's HSD
    for row in tukey.summary().data[1:]:
        group1, group2, meandiff, p_adj, lower, upper, reject = row
        idx1 = fee_rates.index(float(group1))
        idx2 = fee_rates.index(float(group2))
        p_value_matrix[idx1, idx2] = p_adj
        p_value_matrix[idx2, idx1] = p_adj  # Fill symmetrically

    # Mask the lower triangle (including the diagonal)
    mask = np.triu(np.ones_like(p_value_matrix, dtype=bool))

    # Plot the heatmap in the current subplot
    sns.heatmap(p_value_matrix, xticklabels=fee_rates, yticklabels=fee_rates,␣
 ↪cmap='coolwarm', annot=False, mask=mask, ax=axes[i])

    # Set title for each subplot
    axes[i].set_title(f'sigma={sigma}')

# Remove any unused subplots
for j in range(i + 1, len(axes)):
    fig.delaxes(axes[j])

# Adjust layout
plt.tight_layout()
plt.show()
```

```
/home/shiftpub/miniconda3/envs/amm-env/lib/python3.9/site-
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  quad_r = quad(f, low, high, args=args, full_output=self.full_output,
```
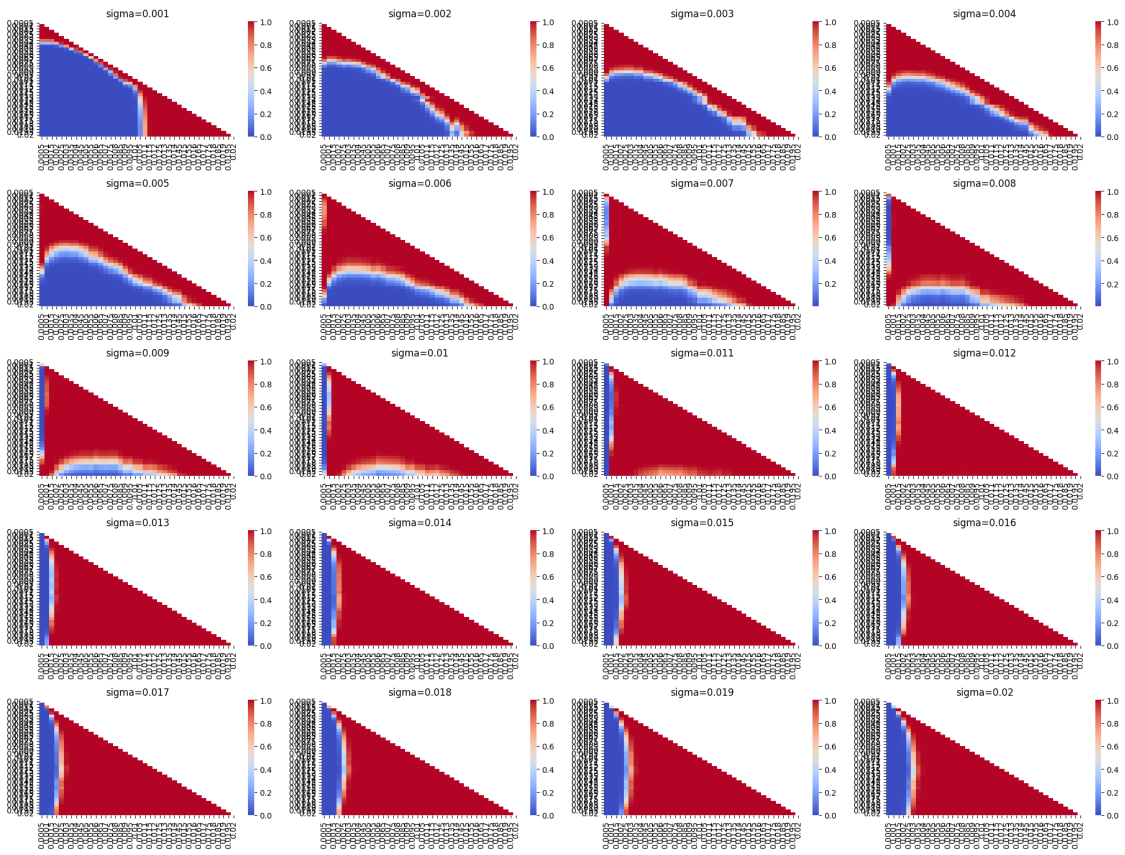
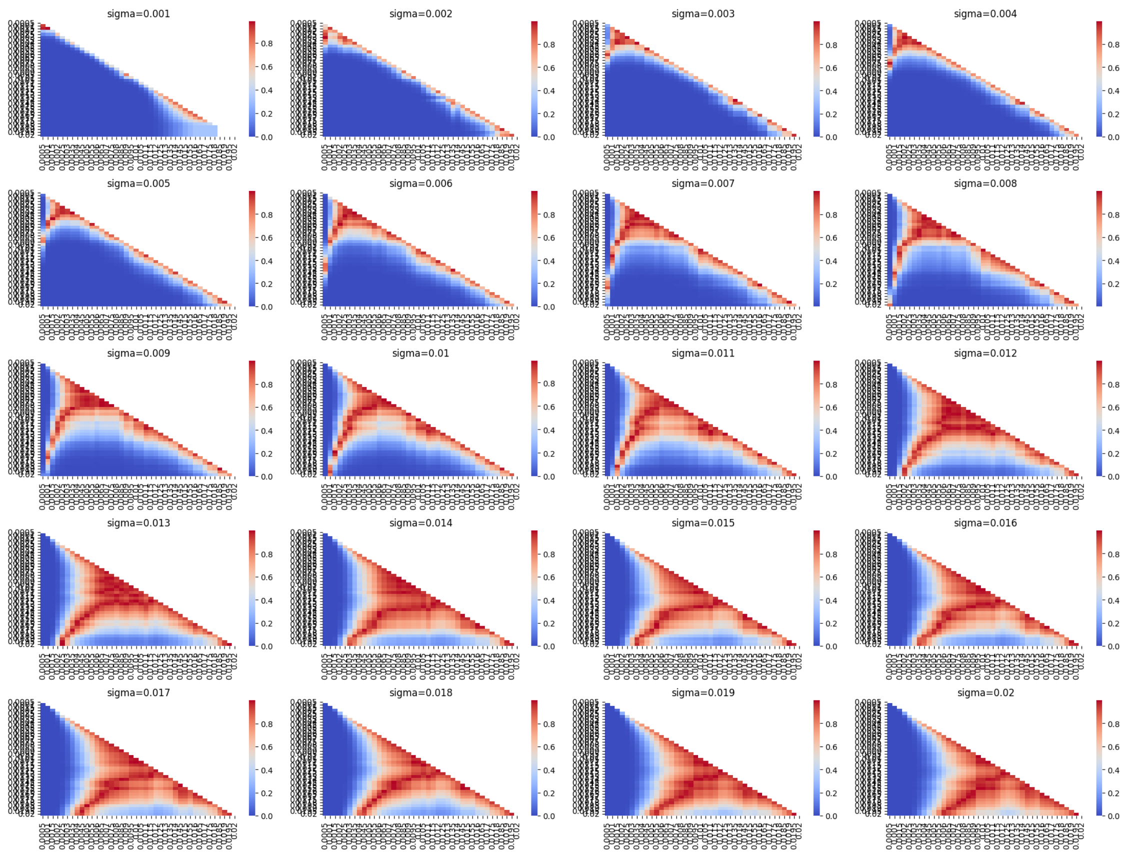



III. Pairwise T-Test Heatmap

[51]:
```
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
from scipy.stats import ttest_ind

# Unique sigma values
unique_sigma = results['sigma'].unique()

# Define the number of rows and columns for the subplots grid
num_rows = 5
num_cols = 4

# Create subplots
fig, axes = plt.subplots(num_rows, num_cols, figsize=(20, 15))
axes = axes.flatten()  # Flatten the axes for easy iteration

# Loop through each sigma value and create a heatmap subplot for each
for i, sigma in enumerate(unique_sigma):
    df = results[results['sigma'] == sigma].copy()  # Filter the dataframe for␣
 ↪the current sigma

    # Extract unique fee rates
    fee_rates = sorted(df['fee_rate'].unique())

    # Create an empty matrix to store p-values
    p_value_matrix = np.ones((len(fee_rates), len(fee_rates)))  # Initialize␣
 ↪with ones

    # Perform pairwise t-tests for each pair of fee rates
    for idx1, fee_rate_1 in enumerate(fee_rates):
        for idx2, fee_rate_2 in enumerate(fee_rates):
            if idx1 != idx2:
                # Extract the fees for the two fee rates
                fees_1 = df[df['fee_rate'] == fee_rate_1]['fee']
                fees_2 = df[df['fee_rate'] == fee_rate_2]['fee']

                # Perform t-test
                t_stat, p_value = ttest_ind(fees_1, fees_2, equal_var=False)  #␣
 ↪Assume unequal variances

                # Store the p-value in the matrix
                p_value_matrix[idx1, idx2] = p_value

    # Mask the lower triangle (including the diagonal)
    mask = np.triu(np.ones_like(p_value_matrix, dtype=bool))

    # Plot the heatmap in the current subplot
    sns.heatmap(p_value_matrix, xticklabels=fee_rates, yticklabels=fee_rates,␣
 ↪cmap='coolwarm', annot=False, mask=mask, ax=axes[i])
```

```
    # Set title for each subplot
    axes[i].set_title(f'sigma={sigma}')

# Remove any unused subplots
for j in range(i + 1, len(axes)):
    fig.delaxes(axes[j])

# Adjust layout
plt.tight_layout()
plt.show()
```

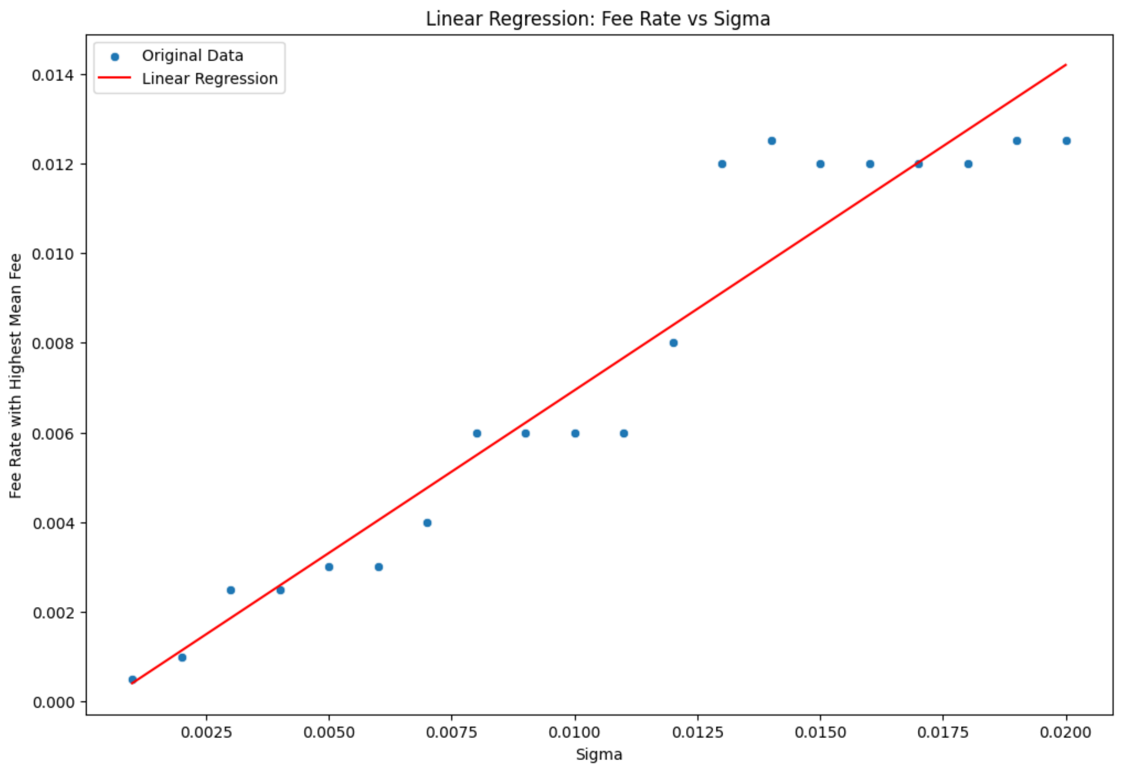


0.0.3 Linear Regression : Max Mean fee rate VS. Sigma

[12]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score
```

```
# List to store sigma and corresponding fee_rate with highest mean fee
sigma_values = []
max_fee_rates = []

# Loop through each sigma
for sigma in unique_sigma:
    df = results[results['sigma'] == sigma].copy()  # Filter the dataframe for␣
 ↪current sigma

    # Calculate the fee_rate with the highest mean fee for this sigma
    max_fee_rate = df.groupby('fee_rate')['fee'].mean().idxmax()

    # Store the sigma and the corresponding max fee_rate
    sigma_values.append(sigma)
    max_fee_rates.append(max_fee_rate)

# Prepare the data (assuming you already have sigma_values and max_fee_rates␣
 ↪lists)
plot_data = pd.DataFrame({'sigma': sigma_values, 'max_fee_rate': max_fee_rates})

# Features and target
X = plot_data[['sigma']]  # Independent variable
y = plot_data['max_fee_rate']  # Dependent variable

# Initialize and fit the linear model
linear_model = LinearRegression()
linear_model.fit(X, y)

# Make predictions
y_pred_linear = linear_model.predict(X)

# Plot the original data
plt.figure(figsize=(12, 8))
sns.scatterplot(x='sigma', y='max_fee_rate', data=plot_data, label='Original␣
 ↪Data')

# Plot Linear Regression line
plt.plot(plot_data['sigma'], y_pred_linear, label='Linear Regression',␣
 ↪color='red')

# Set labels and title
plt.xlabel('Sigma')
plt.ylabel('Fee Rate with Highest Mean Fee')
plt.title('Linear Regression: Fee Rate vs Sigma')
plt.legend()
```

```python
# Show the plot
plt.show()

# Print performance metrics
print("Linear Regression R^2:", r2_score(y, y_pred_linear))
print("Linear Regression Coefficients:", linear_model.coef_)
print("Linear Regression Intercept:", linear_model.intercept_)
```



```
Linear Regression R^2: 0.9244377778351344
Linear Regression Coefficients: [0.72556391]
Linear Regression Intercept: -0.00031842105263157894
```

[ ]: