Homework #1, Jinwoo Shin

Problem 1.1

Assume that risk free interest 3.5% is annual basis.

1.1.(1)

Answer: We can say there is an arbitrage opportunity. Steps are as following and the arbitrage profit is 66.75

We make Portfolio X(t) s.t.

In t=0,

(1) long p/s in forward on 1 gold with delivery in 1year
(2) Short p/s in spot of 1 gold
(3) invest $5,050 for risk-free rate in 1year

Then X(0) = 0 since spot price of gold is $5,050

And in t=1,

- invest $5,050 for risk-free rate in year becomes [5,050×(1.035) = 5226.75]
- Since Forward price is 5,200, We can get 1gold by $5,200.
- And We buy back 1 gold which is short sold in t=0

Then X(1) = 5,266.75-5,200 = 66.75

Since our portfolio X(t) is X(0)=0 but X(1) >0, there is an arbitrage opportunity. ■

1.1.(2)

Answer: We can say there is an arbitrage opportunity. Steps are as following and the arbitrage profit is 33.25

We make Portfolio X(t) s.t.

In t=0,

(1) short p/s in forward on 1 gold with delivery in 1year
(2) borrow $5,050 for risk-free rate in 1year
(3) long p/s in spot of 1 gold

Then X(0) = 0 since spot price of gold is \$5,050

And in t=1,

- Since Forward price is \$5,300, We can get \$5,300 with delivery for 1 gold(which we bought at t=0)
- And our debt becomes [5,050×(1.035) = 5226.75]

If We repay all debt in t=1, X(1) = 5300-5,266.75 = 33.25

Since our portfolio X(t) is X(0)=0 but X(1) >0, there is an arbitrage opportunity. ■

Problem 1.2

1.2.(1) **Answer: 3.2764%**

Let r(1) is annual compounding rate which is same with semi-annual compounding rate 3.25.

Then by definition, (1+r(1)) = $\left(1 + (0.0325/2)\right)^2 \approx$ 1.032764

r(1) = 1.032764-1 = 0.032764

∴ r(1) = 3.2764% ■

1.2.(2) **Answer: 3.2369%**

Let r(2) is Quarterly compounding rate which is same with semi-annual compounding rate 3.25.

Then by definition, $\left(1 + (\, r\,(\,2\,)/\,4\,)\right)^4 \;=\; \left(1 + (0.0325/2)\right)^2$

Then (1+r(2) / 4) =$\left(1 + (0.0325/2)\right)^{\frac{1}{2}}$

r(2) = ($\left(1 + (0.0325/2)\right)^{\frac{1}{2}} - 1$) × 4 ≈ 0.032369

∴ r(2) = 3.2369% ■

1.2.(3) **Answer: 3.2239%**

Let r(3) is continuous compounding rate which is same with semi-annual compounding rate 3.25.

Then by definition, exp(r(3)) = $\left(1 + (0.0325/2)\right)^2$

r(3) = $\ln\left(1 + (0.0325/2)^2\right)$ = 0.032239

∴ r(3) = 3.2239% ■

1.3. **Answer** : **10,130.9472**

Let r is daily basis rate which is same with APR for excellent Credit.

Then by definition, (1+0.1715) = $(1 + (\mathrm{r}/365))^{365}$

So 1+(r/365) = $(1 + 0.1715)^{(1/365)}$ ----- (1)

We need to calculate total balance X after 30days including interest,

So X = \$10,000×$(1 + (\mathrm{r}/365))^{30}$

Then by (1),

X = \$10,000×$\left(1 + (\mathrm{r}/365)\right)^{30}$= \$10,000×$(1 + 0.1715)^{(30/365)}$ ≅ 10,130.9472 ■

1.4. (1) Answer : **102.8314**

bond b1 is bond with annual coupon 5.0% paid twice a year with maturity 2y. So cash flows and each cash flow's discount factor and present value are as follows :

Time	**Cash flows**(ⓐ)	**Discount Factor**(ⓑ)	**Present Value**(ⓒ=ⓐ×ⓑ)
t= 0.5	2.5	exp(-0.03×0.5)	2.5×exp(-0.015)
t= 1	2.5	exp(-0.03)	2.5×exp(-0.03)
t= 1.5	2.5	exp(-(0.035×1.5))	2.5×exp(-0.0525)
t = 2	102.5	exp(-(0.035×2))	102.5×exp(-0.07)

∴ Price of b1 :

2.5×exp(-0.015) + 2.5×exp(-0.03) + 2.5×exp(-0.0525) + 102.5×exp(-0.07) ≈**102.8314** ■

1.4.(2) **Answer : 111.8428**

bond b2 is bond with annual coupon 6.0% paid twice a year with maturity 10y. So cash flows and each cash flow's discount factor and present value are as follows :

Time	Cash flows (ⓐ)	Discount Factor (ⓑ)	Present Value (ⓒ=ⓐ×ⓑ)
t= 0.5	3	exp(-0.03×0.5)	3 × exp(-0.015)
t= 1	3	exp(-0.03)	3 × exp(-0.03)
t= 1.5	3	exp(-(0.035×1.5))	3 × exp(-0.0525)
t = 2	3	exp(-(0.035×2.0))	3 × exp(-0.07)
t = 2.5	3	exp(-(0.0425×2.5))	3 × exp(-0.10625)
t = 3	3	exp(-(0.0425×3))	3 × exp(-0.1275)
t = 3.5	3	exp(-(0.0425×3.5))	3 × exp(-0.14875)
t = 4	3	exp(-(0.0425×4))	3 × exp(-0.17)
t = 4.5	3	exp(-(0.0425×4.5))	3 × exp(-0.19125)
t = 5	3	exp(-(0.0425×5))	3 × exp(-0.2125)
t = 5.5	3	exp(-(0.045×5.5))	3 × exp(-0.2475)
t = 6	3	exp(-(0.045×6))	3 × exp(-0.27)
t = 6.5	3	exp(-(0.045×6.5))	3 × exp(-0.2925)
t = 7	3	exp(-(0.045×7))	3 × exp(-0.315)
t = 7.5	3	exp(-(0.045×7.5))	3 × exp(-0.3375)
t = 8	3	exp(-(0.045×8))	3 × exp(-0.36)
t = 8.5	3	exp(-(0.045×8.5))	3 × exp(-0.3825)
t = 9	3	exp(-(0.045×9))	3 × exp(-0.405)
t = 9.5	3	exp(-(0.045×9.5))	3 × exp(-0.4275)
t = 10	103	exp(-(0.045×10))	3 × exp(-0.45)

∴ Price of b2 : sum of PV(ⓒ in above sheet) ≈ **111.8428** ■

1.4.(3) **Answer** : bond yield of **B1** =**3.4908%, B2 = 4.4650 %**

Let y1 is the yield of B1.

Then By definition, $\sum_{i=1}^{4}(2.5 \times exp(-y1 \times 0.5i)) + 100 \times exp(-y1 \times 2)$ = 102.8314

Using Excel's What-if Analysis ➔ Goal Seek, y1 ≈**3.4908%** ■

(Spread Sheet For 1.4.(3-1))

			0.034908	
	Time	Cash Flow	yield	PV
	0.5	2.5	0.034908	2.456743
	1	2.5	0.034908	2.414235
	1.5	2.5	0.034908	2.372463
	2	102.5	0.034908	95.58792
			SUM	102.8314

Let y2 is the yield of B2.

Then By definition, $\sum_{i=1}^{20}(3\ \times\ exp(-y2 \times 0.5i))+\ 100 \times exp(-y2 \times 10)$ = 111.8428

Using Excel's What-if Analysis ➔ Goal Seek, y2 ≈ **4.4650 %** ■

(Spread Sheet is in the Next Page.)

(Spread Sheet For 1.4.(3-2))

			0.04465		
	Time	Cash Flow	yield	PV	
	0.5	3	0.04465	2.933767	
	1	3	0.04465	2.868997	
	1.5	3	0.04465	2.805656	
	2	3	0.04465	2.743714	
	2.5	3	0.04465	2.683139	
	3	3	0.04465	2.623902	
	3.5	3	0.04465	2.565972	
	4	3	0.04465	2.509322	
	4.5	3	0.04465	2.453922	
	5	3	0.04465	2.399745	
	5.5	3	0.04465	2.346765	
	6	3	0.04465	2.294954	
	6.5	3	0.04465	2.244287	
	7	3	0.04465	2.194738	
	7.5	3	0.04465	2.146284	
	8	3	0.04465	2.098899	
	8.5	3	0.04465	2.05256	
	9	3	0.04465	2.007244	
	9.5	3	0.04465	1.962929	
	10	103	0.04465	65.90601	
			SUM	111.8428	

1.5.

1.5.(1) **Answer : 0.0.25M USD Gain**

By Long P/S of forward for Euro, US company pays 1.15M USD for 1.0M Euro.

If the company buy 1.0M Euro at T=6M by spot rate, US company should pay 1.175M USD for 1.0M Euro.

∴ Company's profit for forward : 1.175M-1.15M = 0.025M USD ■

1.5.(2) **Answer : 8.8904%**

Assume that r_{usd} and r_{EUR} are continuous rate.

Then

t = 0 : 1.1 USD = 1.0EUR

t= 0.5M : 1.1 USD × $\exp(0.5r_{usd})$ = 1.0EUR × $\exp(0.5r_{EUR})$

∴ 1.0 EUR = 1.1 USD × $\exp(0.5(r_{usd}-r_{EUR}))$ ---- (1)

By forward FX rate, 1.0 EUR = 1.15 USD ---- (2)

By (1) & (2), 1.15 USD = 1.1 USD × $\exp(0.5(r_{usd}-r_{EUR}))$

∴ $\exp(0.5(r_{usd}-r_{EUR})) = 1.15/1.1$

➔ $r_{usd}-r_{EUR} = 2 \times \ln(1.15/1.1) = 0.088904$

∴ $r_{usd}-r_{EUR} = 8.8904\%$ ■