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## Problem 1.1 – Forward Pricing & Arbitrage (Gold)

**Methodology.**
I computed the no-arbitrage forward price using the cost-of-carry model. With spot price S0, risk-free rate r, and maturity T=1 year. I then compared the market forward price to F^*. If the market forward is below F^*, a reverse cash-and-carry arbitrage exists (short spot, invest proceeds, go long forward). If the market forward is above F^*, a cash-and-carry arbitrage exists (borrow, buy spot, short forward). The arbitrage profit equals the difference between the theoretical forward price and the market forward price, per ounce.

**Results.**

- When the market forward is below the theoretical forward price, an arbitrage opportunity exists via reverse cash-and-carry, generating a positive risk-free profit.

- When the market forward is above the theoretical forward price, an arbitrage opportunity exists via cash-and-carry, again generating a positive risk-free profit.

**Excel Implementation.**
I computed F^* and the profit using Excel formulas and verified the arbitrage direction based on the comparison of market forward vs. theoretical forward.
*(See Excel screenshot.)*

| | | | | |
|---|---|---|---|---|
| Problem 1.1 | | | | |
| Case i | | | Case ii | |
| Spot Price S0 | 5050 | | Spot Price S0 | 5050 |
| Risk-free rate r | 3.5% | | Risk-free rate r | 3.5% |
| Time T (years) | 1 | | Time T (years) | 1 |
| Market forward Fmkt | 5200 | | Market forward Fmkt | 5300 |
| Investment | 5226.75 | | Investment | 5226.75 |
| Profit per oz | 26.75 | | Profit per oz | 73.25 |

| | | | | |
|---|---|---|---|---|
| Problem 1.1 | | | | |
| Case i | | | Case ii | |
| Spot Price S0 | 5050 | | Spot Price S0 | =C4 |
| Risk-free rate r | 0.035 | | Risk-free rate r | =C5 |
| Time T (years) | 1 | | Time T (years) | =C6 |
| Market forward Fmkt | 5200 | | Market forward Fmkt | 5300 |
| Investment | =C4*(1+C5)^C6 | | Investment | =F4*(1+F5)^F6 |
| Profit per oz | =C8-C7 | | Profit per oz | =F7-F8 |

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# Problem 1.2 – Equivalent Interest Rates

**Methodology.**
I first converted the quoted semi-annual rate into an **effective annual rate (EAR)**.

Then I converted the EAR into equivalent rates under:

- **Annual compounding:** equal to the EAR
- **Quarterly compounding**
- **Continuous compounding**

**Results.**
The equivalent annual, quarterly, and continuously compounded rates were computed consistently from the same EAR, ensuring all rates imply the same one-year growth.

**Excel Implementation.**
I used Excel to compute the EAR and then transformed it into the requested compounding conventions.
*(See Excel screenshot.)*

| | |
|---|---|
| Problem 1.2 | |
| Quoted rate (semi-annual) | 3.25% |
| Compounding per year (m) | 2 |
| Annual Rate | 3.276% |
| Annual Compounding | 3.276% |
| Quarterly Compounding | 3.237% |
| Continous Compounding | 3.224% |

| | |
|---|---|
| Problem 1.2 | |
| Quoted rate (semi-annual) | =0.0325 |
| Compounding per year (m) | =2 |
| Annual Rate | =(1+C13/C14)^C14-1 |
| Annual Compounding | =C15 |
| Quarterly Compounding | =4*((1+C15)^(1/4)-1) |
| Continous Compounding | =LN(1+C15) |

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## Problem 1.3 – Credit Card Balance with Daily Compounding

**Methodology.**
Credit card APRs are compounded daily. The balance after 30 days was computed as:

where PV=$10,000 and APR corresponds to the "Excellent Credit" rate from the table.

**Results.**
Using the Excellent Credit APR, the total balance after 30 days increases due to daily compounding, and the interest accrued equals the difference between the ending balance and the initial balance.

**Excel Implementation.**
I applied the daily compounding formula directly in Excel and computed both the ending balance and interest accrued.
*(See Excel screenshot.)*

| | |
|---|---|
| Problem 1.3 | |
| Balance (PV) | 10000 |
| APR (Excellent Credit) | 17.15% |
| Compounding per year | 365 |
| Days | 30 |
| Balance after 30 days | 10141.923 |
| Interst Accrued | 141.92 |

| | |
|---|---|
| Problem 1.3 | |
| Balance (PV) | 10000 |
| APR (Excellent Credit) | 0.1715 |
| Compounding per year | 365 |
| Days | 30 |
| Balance after 30 days | =C21*(1+C22/C23)^C24 |
| Interst Accrued | =C25-C21 |

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## Problem 1.4 – Bond Pricing Using Zero Rates & Yield to Maturity

**Methodology.**
Each bond was priced by discounting its cash flows using the appropriate **zero rate** for the cash flow's maturity:

For B1 (2Y, 5% coupon, semiannual) and B2 (10Y, 6% coupon, semiannual), I constructed cash-flow schedules and applied the corresponding zero rates by maturity bucket.
Yields to maturity (YTM) were then computed by solving for the discount rate

that equates the present value of cash flows to the bond price using Excel's `RATE()` function (annualized from semiannual).

**Results.**

- **B1 price** was above par, implying a **YTM below the coupon rate**, which is consistent with fixed-income theory.

- **B2 price** and YTM reflected higher long-term zero rates, producing a higher YTM than B1.

**Excel Implementation.**
I built a cash-flow table, discounted each payment using the appropriate zero rate, summed present values to obtain prices, and computed YTM using RATE(). (See Excel s*creenshot.)*

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Problem 1.4 | | | | | | | | |
| Part 1 | | | | | | | | |
| Bond 1 (B1) | | | | | | | | |
| | | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor | Present Value |
| 1 | 3.00% | | 1 | 0.5 | 2.5 | 3.00% | 0.985 | 2.463 |
| 2 | 3.50% | | 2 | 1 | 2.5 | 3.00% | 0.971 | 2.427 |
| 5 | 4.25% | | 3 | 1.5 | 2.5 | 3.50% | 0.950 | 2.374 |
| 10 | 4.50% | | 4 | 2 | 102.5 | 3.50% | 0.934 | 95.685 |
| | | | | | | | | |
| Face (FV) | 100 | | Bond Price B1 | 102.950 | | | | |
| Coupon (annual) | 5.00% | | | | | | | |
| Maturity (years) | 2 | | | | | | | |
| Payments per year | 2 | | | | | | | |
| | | | | | | | | |
| | | | | | | | | |
| Part 2 | | | | | | | | |
| Bond 2 (B2) | | | | | | | | |
| | | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor | Present Value |
| 1 | 3.00% | | 1 | 0.5 | 3 | 3.00% | 0.985 | 2.956 |
| 2 | 3.50% | | 2 | 1 | 3 | 3.00% | 0.971 | 2.913 |
| 5 | 4.25% | | 3 | 1.5 | 3 | 3.50% | 0.950 | 2.849 |
| 10 | 4.50% | | 4 | 2 | 3 | 3.50% | 0.934 | 2.801 |
| | | | 5 | 2.5 | 3 | 4.25% | 0.901 | 2.704 |
| | | | 6 | 3 | 3 | 4.25% | 0.883 | 2.648 |
| Face (FV) | 100 | | 7 | 3.5 | 3 | 4.25% | 0.864 | 2.593 |
| Coupon (annual) | 0.06 | | 8 | 4 | 3 | 4.25% | 0.847 | 2.540 |
| Maturity (years) | 10 | | 9 | 4.5 | 3 | 4.25% | 0.829 | 2.488 |
| Payments per year | 2 | | 10 | 5 | 3 | 4.25% | 0.812 | 2.436 |
| Total periods | 20 | | 11 | 5.5 | 3 | 4.50% | 0.785 | 2.355 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Bond 2 (B2) | | | | | | | | |
| | | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor | Present Value |
| 1 | 3.00% | | 1 | 0.5 | 3 | 3.00% | 0.985 | 2.956 |
| 2 | 3.50% | | 2 | 1 | 3 | 3.00% | 0.971 | 2.913 |
| 5 | 4.25% | | 3 | 1.5 | 3 | 3.50% | 0.950 | 2.849 |
| 10 | 4.50% | | 4 | 2 | 3 | 3.50% | 0.934 | 2.801 |
| | | | 5 | 2.5 | 3 | 4.25% | 0.901 | 2.704 |
| | | | 6 | 3 | 3 | 4.25% | 0.883 | 2.648 |
| Face (FV) | 100 | | 7 | 3.5 | 3 | 4.25% | 0.864 | 2.593 |
| Coupon (annual) | 0.06 | | 8 | 4 | 3 | 4.25% | 0.847 | 2.540 |
| Maturity (years) | 10 | | 9 | 4.5 | 3 | 4.25% | 0.829 | 2.488 |
| Payments per year | 2 | | 10 | 5 | 3 | 4.25% | 0.812 | 2.436 |
| Total periods | 20 | | 11 | 5.5 | 3 | 4.50% | 0.785 | 2.355 |
| | | | 12 | 6 | 3 | 4.50% | 0.768 | 2.304 |
| | | | 13 | 6.5 | 3 | 4.50% | 0.751 | 2.254 |
| | | | 14 | 7 | 3 | 4.50% | 0.735 | 2.204 |
| | | | 15 | 7.5 | 3 | 4.50% | 0.719 | 2.156 |
| | | | 16 | 8 | 3 | 4.50% | 0.703 | 2.110 |
| | | | 17 | 8.5 | 3 | 4.50% | 0.688 | 2.064 |
| | | | 18 | 9 | 3 | 4.50% | 0.673 | 2.019 |
| | | | 19 | 9.5 | 3 | 4.50% | 0.658 | 1.975 |
| | | | 20 | 10 | 103 | 4.50% | 0.644 | 66.325 |
| | | | | | | | | |
| | | | Bond Price B2 | 112.691 | | | | |
| | | | | | | | | |
| Part 3 | | | | | | | | |
| Yield to Maturity | | | | | | | | |
| | | | | | | | | |
| Bond | Price | YTM | | | | | | |
| B1(2Y, 5%) | 102.950 | 3.46% | | | | | | |
| B2(10Y, 6%) | 112.691 | 4.42% | | | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Problem 1.4 | | | | | | | |
| Part 1 | | | | | | | |
| Bond 1 (B1) | | | | | | | |
| | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor |
| 1 | 0.03 | | 1 | =E33/2 | =100*0.05/2 | =XLOOKUP(F33,$B$3 | =1/(1+H33)^F33 |
| 2 | 0.035 | | 2 | =E34/2 | =100*0.05/2 | =XLOOKUP(F34,$B$3 | =1/(1+H34)^F34 |
| 5 | 0.0425 | | 3 | =E35/2 | =100*0.05/2 | =XLOOKUP(F35,$B$3 | =1/(1+H35)^F35 |
| 10 | 0.045 | | 4 | =E36/2 | =G35+100 | =XLOOKUP(F36,$B$3 | =1/(1+H36)^F36 |
| | | | | | | | |
| Face (FV) | 100 | | Bond Price B1 | =SUM(J33:J36) | | | |
| Coupon (annual) | 0.05 | | | | | | |
| Maturity (years) | 2 | | | | | | |
| Payments per year | 2 | | | | | | |
| | | | | | | | |
| | | | | | | | |
| Part 2 | | | | | | | |
| Bond 2 (B2) | | | | | | | |
| | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor |
| 1 | 0.03 | | 1 | =E48/2 | =100*0.06/2 | =XLOOKUP(F48,$B$3 | =1/(1+H48)^F48 |
| 2 | 0.035 | | 2 | =E49/2 | =100*0.06/2 | =XLOOKUP(F49,$B$3 | =1/(1+H49)^F49 |
| 5 | 0.0425 | | 3 | =E50/2 | =100*0.06/2 | =XLOOKUP(F50,$B$3 | =1/(1+H50)^F50 |
| 10 | 0.045 | | 4 | =E51/2 | =100*0.06/2 | =XLOOKUP(F51,$B$3 | =1/(1+H51)^F51 |
| | | | 5 | =E52/2 | =100*0.06/2 | =XLOOKUP(F52,$B$3 | =1/(1+H52)^F52 |
| | | | 6 | =E53/2 | =100*0.06/2 | =XLOOKUP(F53,$B$3 | =1/(1+H53)^F53 |
| Face (FV) | 100 | | 7 | =E54/2 | =100*0.06/2 | =XLOOKUP(F54,$B$3 | =1/(1+H54)^F54 |
| Coupon (annual) | =0.06 | | 8 | =E55/2 | =100*0.06/2 | =XLOOKUP(F55,$B$3 | =1/(1+H55)^F55 |
| Maturity (years) | 10 | | 9 | =E56/2 | =100*0.06/2 | =XLOOKUP(F56,$B$3 | =1/(1+H56)^F56 |
| Payments per year | 2 | | 10 | =E57/2 | =100*0.06/2 | =XLOOKUP(F57,$B$3 | =1/(1+H57)^F57 |
| Total periods | 20 | | 11 | =E58/2 | =100*0.06/2 | =XLOOKUP(F58,$B$3 | =1/(1+H58)^F58 |
| | | | 12 | =E59/2 | =100*0.06/2 | =XLOOKUP(F59,$B$3 | =1/(1+H59)^F59 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Bond 2 (B2) | | | | | | | |
| Max T | R(T) | | Period | Time (Years) | Cashflow | Zero Rate | Discount Factor |
| 1 | 0.03 | | 1 | =E48/2 | =100*0.06/2 | =XLOOKUP(F48,$B$3 | =1/(1+H48)^F48 |
| 2 | 0.035 | | 2 | =E49/2 | =100*0.06/2 | =XLOOKUP(F49,$B$3 | =1/(1+H49)^F49 |
| 5 | 0.0425 | | 3 | =E50/2 | =100*0.06/2 | =XLOOKUP(F50,$B$3 | =1/(1+H50)^F50 |
| 10 | 0.045 | | 4 | =E51/2 | =100*0.06/2 | =XLOOKUP(F51,$B$3 | =1/(1+H51)^F51 |
| | | | 5 | =E52/2 | =100*0.06/2 | =XLOOKUP(F52,$B$3 | =1/(1+H52)^F52 |
| | | | 6 | =E53/2 | =100*0.06/2 | =XLOOKUP(F53,$B$3 | =1/(1+H53)^F53 |
| Face (FV) | 100 | | 7 | =E54/2 | =100*0.06/2 | =XLOOKUP(F54,$B$3 | =1/(1+H54)^F54 |
| Coupon (annual) | =0.06 | | 8 | =E55/2 | =100*0.06/2 | =XLOOKUP(F55,$B$3 | =1/(1+H55)^F55 |
| Maturity (years) | 10 | | 9 | =E56/2 | =100*0.06/2 | =XLOOKUP(F56,$B$3 | =1/(1+H56)^F56 |
| Payments per year | 2 | | 10 | =E57/2 | =100*0.06/2 | =XLOOKUP(F57,$B$3 | =1/(1+H57)^F57 |
| Total periods | 20 | | 11 | =E58/2 | =100*0.06/2 | =XLOOKUP(F58,$B$3 | =1/(1+H58)^F58 |
| | | | 12 | =E59/2 | =100*0.06/2 | =XLOOKUP(F59,$B$3 | =1/(1+H59)^F59 |
| | | | 13 | =E60/2 | =100*0.06/2 | =XLOOKUP(F60,$B$3 | =1/(1+H60)^F60 |
| | | | 14 | =E61/2 | =100*0.06/2 | =XLOOKUP(F61,$B$3 | =1/(1+H61)^F61 |
| | | | 15 | =E62/2 | =100*0.06/2 | =XLOOKUP(F62,$B$3 | =1/(1+H62)^F62 |
| | | | 16 | =E63/2 | =100*0.06/2 | =XLOOKUP(F63,$B$3 | =1/(1+H63)^F63 |
| | | | 17 | =E64/2 | =100*0.06/2 | =XLOOKUP(F64,$B$3 | =1/(1+H64)^F64 |
| | | | 18 | =E65/2 | =100*0.06/2 | =XLOOKUP(F65,$B$3 | =1/(1+H65)^F65 |
| | | | 19 | =E66/2 | =100*0.06/2 | =XLOOKUP(F66,$B$3 | =1/(1+H66)^F66 |
| | | | 20 | =E67/2 | =G66+100 | =XLOOKUP(F67,$B$3 | =1/(1+H67)^F67 |
| | | | Bond Price B2 | =SUM(J48:J67) | | | |
| Part 3 | | | | | | | |
| Yield to Maturity | | | | | | | |
| Bond | Price | YTM | | | | | |
| B1(2Y, 5%) | =F38 | =RATE(4,2.5,-C75,100 | | | | | |
| B2(10Y, 6%) | =F69 | =RATE(20,3,-C76,100 | | | | | |

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## Problem 1.5 – FX Forward Hedge & Interest Rate Differential

### (i) Hedging Gain/Loss.

The company hedges a €1,000,000 payment with a long EUR forward. The payoff at maturity is:

Since the spot at maturity exceeded the forward rate, the hedge generated a positive gain, reducing the USD cost of the euro payment relative to paying spot at maturity.

### (ii) Implied Interest Rate Differential.

Using covered interest parity. I solved for the implied annualized differential over the 6-month maturity (with a simple-compounding approximation, consistent with course convention).

### Excel Implementation.

I computed the hedge payoff and the implied interest rate differential directly in Excel using the forward/spot ratio.

| | |
|---|---|
| Problem 1.5 | |
| | |
| EUR National | 1000000 |
| Forward Rate F | 1.15 |
| Spot Maturity X(t) | 1.175 |
| Gain/Loss(USD) | 25000 |
| | |
| Spot X0 | 1.1 |
| Forward F0 | 1.15 |
| T (years) | 0.5 |
| rUSD - rEUR | 9.09% |
| Continous Compounding | 8.89% |

| | |
|---|---|
| Problem 1.5 | |
| | |
| EUR National | 1000000 |
| Forward Rate F | 1.15 |
| Spot Maturity X(t) | 1.175 |
| Gain/Loss(USD) | =(C84-C83)*C82 |
| | |
| Spot X0 | 1.1 |
| Forward F0 | 1.15 |
| T (years) | 0.5 |
| rUSD - rEUR | =((C88/C87)-1)/C89 |
| Continous Compounding | =LN(C88/C87)/C89 |