FE620 HW1

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##Problem 1.1

```r
S0 <- 5050
r  <- 0.035
T  <- 1

F_star <- S0 * exp(r * T)

F1 <- 5200
F2 <- 5300

check_arbitrage <- function(F, F_star) {
  if (F > F_star) {
    cat("Since the market forward price is greater than the fair forward 
price,\n")
    cat("an arbitrage opportunity exists.\n")
    cat("The arbitrage profit is:\n")
    cat(round(F - F_star, 2), "\n")
  } else if (F < F_star) {
    cat("Since the market forward price is lower than the fair forward 
price,\n")
    cat("an arbitrage opportunity exists.\n")
    cat("The arbitrage profit is:\n")
    cat(round(F_star - F, 2), "\n")
  } else {
    cat("No arbitrage opportunity exists.\n")
  }
}

cat("The fair forward price using continuous compounding is:\n")
```

```
## The fair forward price using continuous compounding is:
```

```r
cat("F* =", round(F_star, 2), "\n\n")
```

```
## F* = 5229.88
```

```r
cat("i):\n")
```

```
## i):
```

```r
check_arbitrage(F1, F_star)
```

```
## Since the market forward price is lower than the fair forward price,
## an arbitrage opportunity exists.
```

```
## The arbitrage profit is:
## 29.88

cat("\n")

cat("ii):\n")

## ii):

check_arbitrage(F2, F_star)

## Since the market forward price is greater than the fair forward price,
## an arbitrage opportunity exists.
## The arbitrage profit is:
## 70.12
```

##Problem 1.2

```
r2 <- 0.0325

r1 <- (1 + r2 / 2)^2 - 1
r4 <- 4 * ((1 + r2 / 2)^(1/2) - 1)
rc <- 2 * log(1 + r2 / 2)

cat("i) Annual compounding rate:\n")

## i) Annual compounding rate:

cat(round(r1 * 100, 4), "%\n\n")

## 3.2764 %

cat("ii) Quarterly compounding rate:\n")

## ii) Quarterly compounding rate:

cat(round(r4 * 100, 4), "%\n\n")

## 3.2369 %

cat("iii) Continuous compounding rate:\n")

## iii) Continuous compounding rate:

cat(round(rc * 100, 4), "%\n")

## 3.2239 %
```

##Problem 1.3 right

```
balance <- 10000
APR <- 0.1715
days <- 30
```

```r
r365 <- 365 * ((1 + APR)^(1/365) - 1)

balance_30 <- balance * (1 + r365 / 365)^days

cat("Since interest is compounded daily, we first compute the equivalent daily rate.\n\n")

## Since interest is compounded daily, we first compute the equivalent daily rate.

cat("The total balance after 30 days is:\n")

## The total balance after 30 days is:

cat(round(balance_30, 2), "\n")

## 10130.95
```

##Problem 1.4

```r
options(digits = 16)

zero_rate <- function(t) {
  if (t <= 1) return(0.03)
  if (t <= 2) return(0.035)
  if (t <= 5) return(0.0425)
  return(0.045)
}

price_bond <- function(coupon, maturity, freq = 2, face = 100) {
  times <- seq(1/freq, maturity, by = 1/freq)
  cashflows <- rep(face * coupon / 100 / freq, length(times))
  cashflows[length(cashflows)] <- cashflows[length(cashflows)] + face

  discounts <- sapply(times, function(t) exp(-zero_rate(t) * t))
  sum(cashflows * discounts)
}

B1 <- price_bond(5, 2)
B2 <- price_bond(6, 10)


yield_bond <- function(price, coupon, maturity, freq = 2) {
  f <- function(y) {
    times <- seq(1/freq, maturity, by = 1/freq)
    cashflows <- rep(coupon / freq, length(times))
    cashflows[length(cashflows)] <- cashflows[length(cashflows)] + 100
    sum(cashflows / (1 + y / freq)^(freq * times)) - price
  }
  uniroot(f, c(0, 0.2))$root
```

```
}

y1 <- yield_bond(B1, 5, 2)
y2 <- yield_bond(B2, 6, 10)


cat("Using the given zero rate curve:\n\n")
```

```
## Using the given zero rate curve:
```

```
cat("1) The price of the 2-year bond is:\n")
```

```
## 1) The price of the 2-year bond is:
```

```
cat(round(B1, 3), "\n\n")
```

```
## 102.831
```

```
cat("2) The price of the 10-year bond is:\n")
```

```
## 2) The price of the 10-year bond is:
```

```
cat(round(B2, 3), "\n\n")
```

```
## 111.843
```

```
cat("3) The yield of the 2-year bond is:\n")
```

```
## 3) The yield of the 2-year bond is:
```

```
cat(round(y1 * 100, 3), "%\n\n")
```

```
## 3.521 %
```

```
cat("The yield of the 10-year bond is:\n")
```

```
## The yield of the 10-year bond is:
```

```
cat(round(y2 * 100, 3), "%\n")
```

```
## 4.516 %
```

##Problem 1.5

```
X0 <- 1.100
Xfwd <- 1.150
XT <- 1.175
T <- 0.5

gain <- 1e6 * (XT - Xfwd)
rate_diff <- (1 / T) * log(Xfwd / X0)

cat("At maturity, the company pays USD 1.15m for EUR 1.0m.\n\n")
```

```
## At maturity, the company pays USD 1.15m for EUR 1.0m.
cat("The realized gain from hedging is:\n")
## The realized gain from hedging is:
cat(round(gain, 2), "USD\n\n")
## 25000 USD
cat("The implied interest rate differential is:\n")
## The implied interest rate differential is:
cat(round(rate_diff * 100, 3), "%\n")
## 8.890000000000001 %
```