

Homework #1

Problem 1.1

Assume that the spot price of gold is \$5,050 per ounce. Suppose that the market quotes a forward contract on gold with delivery in 1 year at \$5,200.00. The risk-free interest rate is $r = 3.5\%$.

- i) Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage. What is the profit we can expect to make?
 - ii) Consider now the case when the forward price is \$5,300.00. Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage.

	A	B	C
1	Problem 1.1		
2	Maturity of 1 year		
3	spot price	5,050.00	
4	interest rate	3.50	
5	interest	176.75	=B3 * (B4/100)
6	calculated forward price	5,226.75	=B3 + B5

Problem 1.2

Assume that an interest rate is quoted as 3.25% with semi-annual compounding. What is the rate when expressed with: i) annual compounding. ii) quarterly compounding? iii) continuous compounding.

A	B	C	D	E	F	G	H	I	J	K	L	M	N
32 Problem 1.2													
33 semi-annual compounding rate is 3.25%													
34 annual compounding rate	1.03276406	sanity check											
35													
36 case i			case ii			case iii							
37 m1 = compounding rate1	2.00000		m1	2.00000		m1	2		formulaB40	=B\$39*((1 + \$B\$38/\$B\$37)^(\$B\$37/\$B\$39) - 1)			
38 r1 = interest rate with m1	0.03250		r1	0.03250		r1	0.0325		formulaE40	=E\$39*((1 + E\$38/E\$37)^(E\$37/E\$39) - 1)			
39 m2 = compounding rate2	1.00000		m2	4.00000					formulaH40	=H\$37 * LN(1 + H\$38/H\$37)			
40 r2 = new interest rate	0.0327641		r2	0.03237		r2	0.032239						
41													

Problem 1.3

Credit card companies quote the APR on the outstanding balance. APR means Annual Percentage Rate and is the interest rate with annualized compounding. Typical APRs are shown in the Table below. However, credit card interest rate is compounded daily, for 365 days a year. Suppose the balance on a credit card is \$10,000. Compute the total balance including interest after 30 days, if the customer has Excellent Credit?

A	B	C	D
43 Problem 1.3			
44 Amount =	10,000.00		
45 Rate =	0.1715	over one year	
46 Time =	0.08219	30 days of 365 days	
47 Rate * Time =	0.014096		
48 Balance after 30 days	10,141.957	=B44 * EXP(B47)	
49			

Problem 1.4

We are given the zero rates $R(T)$ for several maturities in Table 2. These zero rates apply for all maturities in the ranges shown. 1) Price a bond B1 with annual coupon 5.0% paid twice a year with maturity 2Y. 2) Price a bond B2 with annual coupon 6.0% paid twice a year, with maturity 10Y. 3) Compute the bond yields of the two bonds in points 1) and 2).

	A	B	C	D	E	F	G	H
50	Problem 1.4							
51	(1)							
52	principal	100.00	cpnRateYear	0.05	cpnYear	5	cpn6m	2.5
53		2.46277985			=H\$52*EXP(-0.03*0.5)			
54		2.42611383	1yr		=H\$52*EXP(-0.03*1)			
55		2.37213580			=H\$52*EXP(-0.035*1.5)			
56		7.26102949	sum of just coupons		=SUM(A53:A55)			
57		95.57036654	2yr		=(\$B\$52+\$H\$52)*EXP(-0.035*2)			
58		102.83139603			=SUM(A56:A57)			
59	(2)							
60	principal	100.00	cpnRateYear	0.06	cpnYear	6	cpn6m	3
61		2.95533582			=H\$60*EXP(-0.03*0.5)			
62		2.91133660	1yr		=H\$60*EXP(-0.03*1)			
63		2.84656296			=H\$60*EXP(-0.035*1.5)			
64		2.79718146	2yr		=H\$60*EXP(-0.035*2)			
65		2.69759946			=H\$60*EXP(-0.0425*2.5)			
66		2.64088025	3yr		=H\$60*EXP(-0.0425*3)			
67		2.58535360			=H\$60*EXP(-0.0425*3.5)			
68		2.53099445	4yr		=H\$60*EXP(-0.0425*4)			
69		2.47777824			=H\$60*EXP(-0.0425*4.5)			
70		2.42568095	5yr		=H\$60*EXP(-0.0425*5)			
71		2.34225066			=H\$60*EXP(-0.045*5.5)			
72		2.29013848	6yr		=H\$60*EXP(-0.045*6)			
73		2.23918574			=H\$60*EXP(-0.045*6.5)			
74		2.18936662	7yr		=H\$60*EXP(-0.045*7)			
75		2.14065592			=H\$60*EXP(-0.045*7.5)			
76		2.09302898	8yr		=H\$60*EXP(-0.045*8)			
77		2.04646167			=H\$60*EXP(-0.045*8.5)			
78		2.00093043	9yr		=H\$60*EXP(-0.045*9)			
79		1.95641221			=H\$60*EXP(-0.045*9.5)			
80		46.16713451	sum of just coupons		=SUM(A61:A79)			
81		65.67569962	10yr		=(\$B\$60+\$H\$60)*EXP(-0.045*10)			
82		111.842834			=SUM(A80:A81)			
83	(c)							
84	Yield for (1)				"=2.5e^(-y0.5) + 2.5e^(-y1.0) + 2.5e^(-y1.5) + 102.5e^(-y2.0)"			
85		0.03521442			=YIELD("02/06/2026","02/06/2028",D52,A58,100,2)		approximately	
86								
87	Yield for (2)				=3e^(-y0.5) + 3e^(-y1.0) + 3e^(-y1.5) + 3e^(-y2.0) + 3e^(-y2.5) + 3e^(-y3.0) +			
88					3e^(-y3.5) + 3e^(-y4.0) + 3e^(-y4.5) + 3e^(-y5.0) + 3e^(-y5.5) + 3e^(-y6.0) +			
89					3e^(-y6.5) + 3e^(-y7.0) + 3e^(-y7.5) + 3e^(-y8.0) + 3e^(-y8.5) + 3e^(-y9.5) + 103e^(-y10.0)			
90		0.045152036			=YIELD("02/06/2026","02/06/2036",D60,A82,100,2)		approximately	

Problem 1.5

A US company is due to make a payment of 1.0m Euros in 6 months. They plan to hedge this payment by taking a long position in a forward contract for 1.0m Euros with maturity 6 months, at a forward exchange rate 1.150. The current EUR/USD rate is $X_0 = 1.100$, and the actual exchange rate realized at maturity is $X(6M) = 1.175$. i) What is the gain or loss of the company at maturity? ii) What is the interest rate differential $r_{USD} - r_{EUR}$ for maturity 6M implied by the quoted forward FX rate?

A	B	C	D	E	F	G	H	I	J	K	L
93 Problem 1.5											
94 case (i)											
95 To avoid exposure to a weaker dollar, the company chose to go long a 6m forward contract to purchase EUR at a forward rate of 1.15 USD.											
96 At the 6m maturity, the cost to purchase a EUR increased to 1.175 USD in the spot market											
97 To hedge their USD position by purchasing the EUR with 1.15 USD as defined in the forward contract, the company made a good decision since the dollar weakend by 0.025											
98											
99 forward contract amount	1,000,000.00 EUR										
100 spot rate at T=0	1.100 EUR/USD										
101 spot rate at T=6m	1.175 EUR/USD										
102 forward rate	1.150 EUR/USD										
103 spot rate at T=6m - forward rate	0.025 EUR/USD	=B101-B102									
104 Scenario A from my email											
105 if the company did NOT go long the contract											
106 if bought 1m EUR at T=0	1,100,000.00 USD	=B99*B100									
107 instead of buy 1m EUR at T=6m	1,150,000.00 USD	=B99*B102									
108 then the gain (or savings) is	50,000.00	=B107-B106									
109 Scenario B from my email											
110 however, if the company committed to the contract											
111 since buy 1m EUR at T=6m	1,150,000.00										
112 instead of spot rate at T=6m	1,175,000.00 USD	=B99*B101									
113 then the gain (or savings) is	25,000.00 USD	=B112-B111									
114											
115 (ii)											
116 $R(F) = Re^((R_{USD} - R_{EUR}) * T)$											
117 $1.150 = 1.100 * exp(R_{USD} - R_{EUR})^{*0.5})$											
118 $1.04545 = exp(R_{USD} - R_{EUR})^{*0.5})$											
119 $\ln(1.04545) = \ln(exp(R_{USD} - R_{EUR})^{*0.5}))$											
120 $0.044452 = (R_{USD} - R_{EUR})^{*0.5}$											
121 $0.0889 = R_{USD} - R_{EUR}$	assuming the company DID go long the contract										

Please see photo of email I sent on the next page

Ara Meyer - ameyer3

To: Hao Fu

Fri 2/6/2026 10:28 PM

Hi,

I have a question regarding Problem 1.5 - I will rewrite it here:

Problem 1.5 A US company is due to make a payment of 1.0m Euros in 6 months. They plan to hedge this payment by taking a long position in a forward contract for 1.0m Euros with maturity 6 months, at a forward exchange rate 1.150. The current EUR/USD rate is $X_0 = 1.100$, and the actual exchange rate realized at maturity is $X(6M) = 1.175$. i) What is the gain or loss of the company at maturity?

Are we expected to judge the decision made by the company to hedge at all or just assume that the forward contract is a given? The word "plan" makes this a bit ambiguous to me and leads me to believe that we need to compare the decision to hedge vs not to hedge since the gain/loss would be different in the two scenarios.

Scenario A. If the company purchased the EUR at $T=0$ instead of going long the forward, the company would pay less for the EUR at $T=0$ than purchasing the EUR at $T=6m$ which is at a higher rate. The savings would be \$50,000 USD.

Scenario B. If the company "committed" to going long a forward contract, then they are purchasing the EUR at the forward rate which is lower than the spot price at 6m. The savings would be \$25,000.

This is also open to interpretation of the possibility of this being a loss since the company only saved \$25,000 instead of \$50,000.

Please advise