Assignment #1 FE 620

Problem 1.1

i) There is an arbitrage because the quoted forward price is below the no-arbitrage level implied by the spot price and interest rate, so by shorting gold today, investing the proceeds at the risk-free rate, and locking in a cheap long forward, you can secure a risk-free profit of about $30 per ounce at maturity.

FE 620

i) $5050 \times e^{(0.035)(1)}$

$\rightarrow 5050 \times 1.03563 = 5229.88$

Investment of $5,050 becomes $5,229.88

Guaranteed Profit $\rightarrow$ 5,229.88 − 5,200 = 29.88 //

ii) In this case there is no arbitrage opportunity in this case because the quoted forward price of $5,300 is above the no-arbitrage forward price $5,229.88 implied by the spot price and the risk-free rate, so any potential strategy would fail to lock in a risk-free profit once financing costs are taken into account.

Problem 1.2

i) $\left(1+\frac{0.0325}{2}\right)^2 \Rightarrow (1+0.01625)^2 = \underbrace{1.03276}_{\text{EQUIVALENT ANNUAL RATE}}$

$\Rightarrow 1.03276 - 1 = 0.03276 = \underbrace{3.28\%}_{\text{ANNUAL compounded RATE}}$

ii) $\left(1+\frac{0.0325}{2}\right)^2 = 1.03276$

$\left(1+\frac{R}{m}\right)^m \Rightarrow \left(1+\frac{R}{4}\right)^4 = (1.03276)^{1/4} = 1.00809$

each QUARTER $1 Grows by About 0.809%.

Hence $4 \times 0.00809 = 0.03236 \Rightarrow \underbrace{3.24\%}_{\text{QUARTERLY compounded RATE}}$

iii) Effective 1yr Growth

$\left(1+\frac{0.0325}{2}\right)^2 = (1.01625)^2 = 1.03276$

$\Rightarrow e^{R_c \cdot 1} = 1.03276$

$\Rightarrow R_c = \ln(1.03276) = 0.03223 \Rightarrow \underbrace{3.22\%}_{\text{continuously compounded rate}}$

Problem 1.3

Total balance after 30 days = $10,141.92, meaning the interest over the 30 days is about $141.92

$R_d = \frac{0.1715}{365} = \underbrace{0.00046986}_{\text{daily rate}}$

compound for 30 days: $10{,}000\left(1+\frac{0.1715}{365}\right)^{30}$

$\Rightarrow 10{,}000\,(1.00046986)^{30} = 10{,}141.92$

Problem 1.4

1) SemiAnnual coupon $\rightarrow 100 \times \frac{0.05}{2} = 2.50$

2 Yr Cash Flow:

$T = 0.5 \rightarrow 2.5$
$\;\;= 1 \rightarrow 2.5$
$\;\;= 1.5 \rightarrow 2.5$
$\;\;= 2 \rightarrow 102.5$

$\Rightarrow D(\tau) = e^{-R(\tau)\tau}$

$B_1 = 2.5e^{-0.03(0.5)} + 2.5e^{-0.03(1.0)} + 2.5e^{-0.035(1.5)} + 102.5e^{-0.035(2.0)}$

$\rightarrow 2.4628 + 2.4261 + 2.3722 + 95.5705 = 102.83$ per \$100 FV //

2) SA Coupon $\rightarrow 100 \times \frac{0.06}{2} = 3$

Final Payment at 10 Yr = 103 (per \$100 FV)

$B_2 = 3e^{-0.03(0.5)} + 3e^{-0.03(1.0)} + 3e^{-0.035(1.5)} + 3e^{-0.035(2.0)} + \sum^{5} 3e^{-0.0425} + \sum^{10} 3e^{-0.045} + 103e^{-0.045(10)}$

$\Rightarrow 2.955 + 2.912 + 2.846 + 2.796 + 16.03 + 20.14 + 65.67$

$\Rightarrow B_2 = 113.35$ //

3) I rewrote the PV of coupons as a geometric series using $q = e^{-0.5y}$, then solved for $y$by trial-and-error until the PV matched the bond price.

3)

$$\text{Price} = \sum_i \text{CASH FLOW}_i \, e^{-yT_i}$$

Price $B_1 = 102.83$

CF → $T = 0.5, 1, 1.5 : 2.5$

$= 2.0 : 102.5$

$$\Rightarrow 102.93 = 2.5e^{-0.5y} + 2.5e^{-1.0y} + 2.5e^{-1.5y} + 102.5e^{-2.0y}$$

$\Rightarrow$ Yield $B_1 = 3.45\%$ //

Price $B_2 = 113.35$

SA coupon $= 3$

LAST PAYMENT AT 10YR $= 103$

$$\Rightarrow 113.35 = \sum_{i=1}^{20} 3e^{-y(0.5i)} + 100e^{-10y}$$

$$q^i = e^{-0.5y}$$

$$q^{20} = e^{-10y}$$

$$\rightarrow 113.35 = 3\sum_{i=1}^{20} q^i + 100q^{20}$$

$$\sum_{i=1}^{20} q^i = q\,\frac{1-q^{20}}{1-q}$$

$$\rightarrow 113.35 = 3\left(q\,\frac{1-q^{20}}{1-q}\right) + 100q^{20}$$

Yield $B_2 = 4.29\%$ //

Problem 1.5

i) PAY OFF $= (X_T - K) \times 1,000,000$ EUROS

$X_T = 1.175$

$K = 1.150$

$\Rightarrow (1.175 - 1.150) \times 1,000,000 = 0.025 \times 1,000,000 = \$25,000$ //

ii) $F_0 = X_0 \, e^{(R_{USD} - R_{EUR})T}$

$F_0 = 1.150$

$X_0 = 1.100$

$T = 0.5$ YEARS (6 MONTHS)

$\Rightarrow \dfrac{F_0}{X_0} = e^{(R_{USD} - R_{EUR})T}$

$\Rightarrow \ln\left(\dfrac{1.150}{1.100}\right) = (R_{USD} - R_{EUR}) \times 0.5$

$\Rightarrow R_{USD} - R_{EUR} = \dfrac{1}{0.5} \ln\left(\dfrac{1.150}{1.100}\right) = \dfrac{1}{0.5} \ln(1.04545)$

$\Rightarrow R_{USD} - R_{EUR} = 2 \times 0.04445 = 0.0889$

$\Rightarrow R_{USD} - R_{EUR} = 8.89\%$ //