FE621 Computational Finance
Homework 1 Report

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Overview

This report follows the assignment stub and covers: (i) data gathering from Yahoo Finance using `yfinance`, (ii) implied volatility and option diagnostics under Black–Scholes, (iii) AMM expected fee revenue via numerical integration, and (iv) a bonus double-integral trapezoidal-rule exercise. All results come from the fully-commented notebook `FE621_HW1_full_pipeline.ipynb`. DATA1 was downloaded at `[DATA1_TIMESTAMP_YYYY-MM-DD HH:MM:SS TZ]` and DATA2 was downloaded at `[DATA2_TIMESTAMP_YYYY-MM-DD HH:MM:SS TZ]`.

Part 1 (20 points): Data gathering component

1. Downloader function and data cleaning

A Python function connects to Yahoo Finance using `yfinance` to download:

- equity/ETF/index data (TSLA, SPY, $^VIX)and$
- `option chains (calls and puts) for TSLA and SPY for the next three monthly expirations (third Friday).`

The options are consolidated and cleaned by dropping duplicates (contract symbol), filtering for valid bid/ask with nonzero volume, and keeping consistent column labels. When bid and ask exist and volume is nonzero, the mid price is used for implied volatility:

$$\text{mid} = \frac{\text{bid} + \text{ask}}{2}.$$

2. Downloaded symbols, expirations, and recorded quantities

The assignment asks for the underlying prices at download time, a short-term rate, and time-to-maturity. Table 1 reports the recorded spot values S_0 *at the exact moment of download* for DATA1 and DATA2 (including the option-chain "as-of" timestamp), along with the short-term rate r pulled for the same calendar day. All time-to-maturity values τ are computed from the recorded download timestamp to each expiration date, expressed in years.

We use the next three monthly expirations (third Friday):

2026-02-20, 2026-03-20, 2026-04-17.

Listed options have many maturities (weeklies, monthlies, quarterlies, LEAPS) because different participants need exposure over different horizons. More listed dates also helps concentrate liquidity around commonly traded expirations and supports hedging of calendar and event risk.

Table 1: Recorded spot values at download time, short-term rate used in downstream calculations, and option snapshot timestamps (DATA1 and DATA2).

Reference date	Ref close (ET)	TSLA spot	SPY spot	ˆVIX spot	r (annual, dec)	Options
2026-02-12	2026-02-12T16:00:00-05:00	416.839996	681.219971	20.740000	0.036400	2026-02-

3. Description of downloaded symbols

TSLA is the common stock of Tesla, Inc. SPY is an ETF that tracks the S&P 500 index, allowing diversified index exposure to trade intraday like a stock. VIX is the CBOE Volatility Index, which summarizes the market-implied 30-day volatility for the S&P 500 derived from option prices.

4. Notes on DATA1/DATA2 and practical limitation

Yahoo Finance provides option-chain snapshots at runtime (not free historical chains). Therefore, for each day (DATA1 and DATA2) the underlying price S_0 and the option-chain snapshot timestamp are recorded at download time, and these recorded values (along with the same-day short-term rate r) are used consistently throughout Parts 2–4.

Part 2 (50 points): Analysis of the data

5. Black–Scholes pricing (no toolbox for pricing)

Let S_0 be spot, K strike, r the short-term annual rate, and $\tau = T - t$ time-to-maturity in years. Under Black–Scholes with volatility σ,

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)\tau}{\sigma\sqrt{\tau}}, \qquad d_2 = d_1 - \sigma\sqrt{\tau}.$$

The European call and put prices are:

$$C = S_0\Phi(d_1) - Ke^{-r\tau}\Phi(d_2), \qquad P = Ke^{-r\tau}\Phi(-d_2) - S_0\Phi(-d_1),$$

where $\Phi(\cdot)$ is the standard normal CDF.

6. Bisection method for implied volatility (DATA1)

For each DATA1 option contract, define $f(\sigma) = \mathrm{BS}(\sigma) - \mathrm{mid}$. Bisection brackets the root and iterates until tolerance 10^{-6} is reached. Mid prices are used only when bid and ask exist and volume is nonzero.

ATM implied volatility uses the strike closest to S_0. Average implied volatility uses moneyness S_0/K and averages contracts with $S_0/K \in [0.90, 1.10]$.

7. Newton method for implied volatility and speed comparison

Newton's method updates $\sigma_{n+1} = \sigma_n - f(\sigma_n)/f'(\sigma_n)$. For implied volatility, $f'(\sigma)$ is the Black–Scholes Vega. Table 3 summarizes average timing results.

Table 2: Implied volatility results (DATA1). "AvgIV(0.9–1.1)" averages vols for moneyness $S_0/K \in [0.9, 1.1]$.

Underlying	Expiry	Type	Spot	K_ATM	IV_ATM	AvgIV(0.9-1.1)	N	Bisc iters	Bisc
SPY	2026-02-20	call	681.219971	681.000000	0.167283	0.164889	20	22.700000	5.581
SPY	2026-02-20	put	681.219971	681.000000	0.156156	0.153400	20	23.000000	5.701
SPY	2026-03-20	call	681.219971	681.000000	0.169897	0.168875	20	23.000000	5.726
SPY	2026-03-20	put	681.219971	681.000000	0.174406	0.173920	20	23.000000	5.713
SPY	2026-04-17	call	681.219971	681.000000	0.163162	0.162911	20	23.000000	6.317
SPY	2026-04-17	put	681.219971	681.000000	0.174995	0.174625	20	22.900000	6.090
TSLA	2026-02-20	call	416.839996	417.500000	0.372070	0.383489	20	22.600000	6.146
TSLA	2026-02-20	put	416.839996	417.500000	0.348630	0.354525	20	23.000000	5.873
TSLA	2026-03-20	call	416.839996	415.000000	0.433581	0.435468	17	22.750000	5.712
TSLA	2026-03-20	put	416.839996	415.000000	0.421037	0.422655	17	23.000000	5.538
TSLA	2026-04-17	call	416.839996	415.000000	0.447076	0.447706	17	23.000000	5.742
TSLA	2026-04-17	put	416.839996	415.000000	0.436231	0.437700	17	23.000000	5.927

Table 3: Average performance comparison between bisection and Newton across all contracts.

Underlying	AvgMs_bisect	AvgMs_newton	AvgIters_bisect	AvgIters_newton
SPY	5.855165	1.165527	22.933333	3.158333
TSLA	5.823557	1.290614	22.891667	3.566667

8. Interpretation: TSLA vs SPY, maturity, and relation to ^{V}IX

TSLA implied volatilities are much higher (about 35%–45%) than SPY (about 15%–18%), which matches TSLA being a single-name equity with more idiosyncratic risk than a diversified index ETF. The recorded VIX level is about 20.74%, above the SPY implied vols in this snapshot. This can happen because VIX is a standardized, option-implied volatility measure targeting an approximately 30-day horizon and can differ from option-implied vols at specific strikes and maturities.

Across maturities the term structure is not perfectly monotone in this sample, but longer maturities generally reflect more uncertainty. Across strikes, skew/smile patterns appear (Figures 1–6). Differences between calls and puts can reflect skew (downside protection demand), dividends (SPY), and the fact that listed equity options are American-style.

9. Put–Call parity check

For European options without dividends, put–call parity is:

$$C - P = S_0 - Ke^{-r\tau}.$$

We compare observed mids to parity-implied values. Table 4 shows near-ATM examples for the nearest maturity, and Table 5 summarizes deviations.

Deviations are expected due to bid/ask frictions, American exercise features (parity becomes an inequality), and dividends for SPY (parity should include the present value of expected dividends).

10. Implied volatility smile/skew plots

We plot implied volatility versus strike for the nearest maturity, and then overlay the three maturities.

Table 4: Put–call parity comparison (sample near ATM, nearest maturity). Errors are parity-implied minus observed mid.

Underlying	Expiry	K	Call mid	Put mid	Put from parity (via call)	Call from parity (v
SPY	2026-02-20	681.000000	7.115000	5.905000	6.351939	6.
SPY	2026-02-20	682.000000	6.490000	6.285000	6.726141	6.
SPY	2026-02-20	680.000000	7.770000	5.540000	6.007736	7.
SPY	2026-02-20	683.000000	5.890000	6.690000	7.125344	5.
SPY	2026-02-20	679.000000	8.435000	5.205000	5.673534	7.
SPY	2026-02-20	684.000000	5.310000	7.115000	7.544546	4.
SPY	2026-02-20	678.000000	9.115000	4.885000	5.354331	8.
SPY	2026-02-20	685.000000	4.755000	7.530000	7.988749	4.
SPY	2026-02-20	677.000000	9.820000	4.580000	5.060129	9.
SPY	2026-02-20	686.000000	4.225000	8.005000	8.457951	3.
TSLA	2026-02-20	417.500000	9.000000	8.750000	9.327052	8.
TSLA	2026-02-20	415.000000	10.375000	7.650000	8.204045	9.
TSLA	2026-02-20	420.000000	7.700000	9.950000	10.525058	7.
TSLA	2026-02-20	412.500000	11.875000	6.650000	7.206039	11.
TSLA	2026-02-20	422.500000	6.550000	11.325000	11.873064	6.
TSLA	2026-02-20	410.000000	13.500000	5.750000	6.333033	12.
TSLA	2026-02-20	425.000000	5.550000	12.800000	13.371070	4.
TSLA	2026-02-20	407.500000	15.250000	5.000000	5.585026	14.
TSLA	2026-02-20	427.500000	4.650000	14.400000	14.969077	4.
TSLA	2026-02-20	405.000000	17.050000	4.300000	4.887020	16.

Table 5: Summary parity deviations (mean absolute error and maximum absolute error).

Underlying	MAE_put	MAE_call	MaxAbs_put	MaxAbs_call
SPY	0.724479	0.724479	1.493357	1.493357
TSLA	0.617123	0.617123	0.932539	0.932539

11. Greeks (analytic vs finite differences)

We compute $\Delta = \partial C/\partial S$, $\Gamma = \partial^2 C/\partial S^2$, and Vega $= \partial C/\partial \sigma$ analytically under Black–Scholes and approximate them with finite differences. Table 6 shows near-ATM examples for the nearest maturity, and Table 7 summarizes absolute differences.

The differences are very small, which suggests the finite-difference step sizes used in the notebook are numerically stable for these contracts.

12. DATA2 pricing using IV from DATA1

For DATA2, we recompute Black–Scholes prices using the DATA2 spot S_0, implied volatility from DATA1, and the DATA2 short-term rate. Table 8 summarizes errors versus market mids, and Table 9 shows near-ATM examples.

Pricing errors are expected because the implied volatility surface changes from day to day, and this comparison uses DATA1 implied volatilities to price DATA2 contracts.

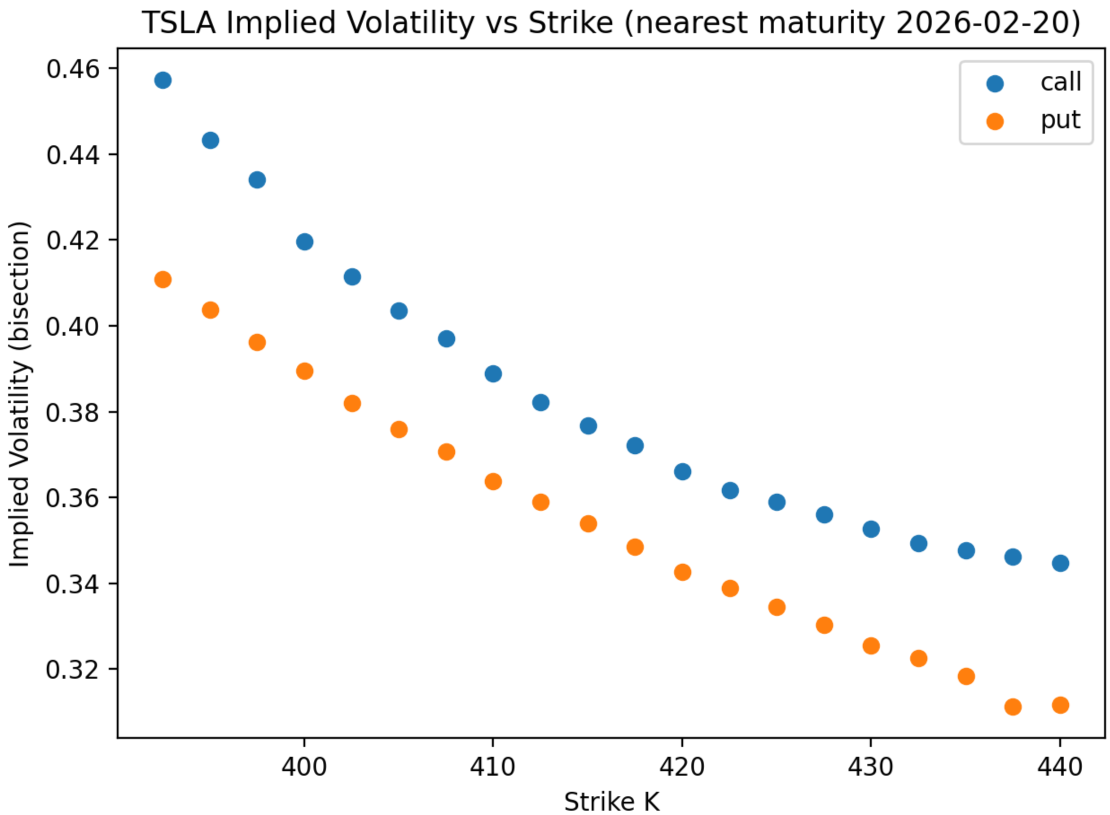

Figure 1: TSLA implied volatility vs strike (nearest maturity).

Part 3 (30 points): Numerical integration — AMM arbitrage fee revenue

(a) Derive swap amounts

Let reserves at time t be (x_t, y_t) with $k = x_t y_t$, pool price $P_t = y_t/x_t$, and fee rate $\gamma \in (0, 1)$.
Case 1: $S_{t+1} > \dfrac{P_t}{1-\gamma}$ (BTC cheaper in the pool). Arbitragers swap USDC→BTC. Let post-trade reserves be (x', y') where

$$x' = x_t - \Delta x, \qquad y' = y_t + (1-\gamma)\Delta y,$$

and $(x')(y') = k$. The boundary condition is

$$\frac{y'}{x'} = (1-\gamma)S_{t+1}.$$

Solving gives

$$x' = \sqrt{\frac{k}{(1-\gamma)S_{t+1}}}, \qquad y' = \sqrt{k(1-\gamma)S_{t+1}}.$$

Thus,

$$\Delta x = x_t - x', \qquad \Delta y = \frac{y' - y_t}{1-\gamma},$$

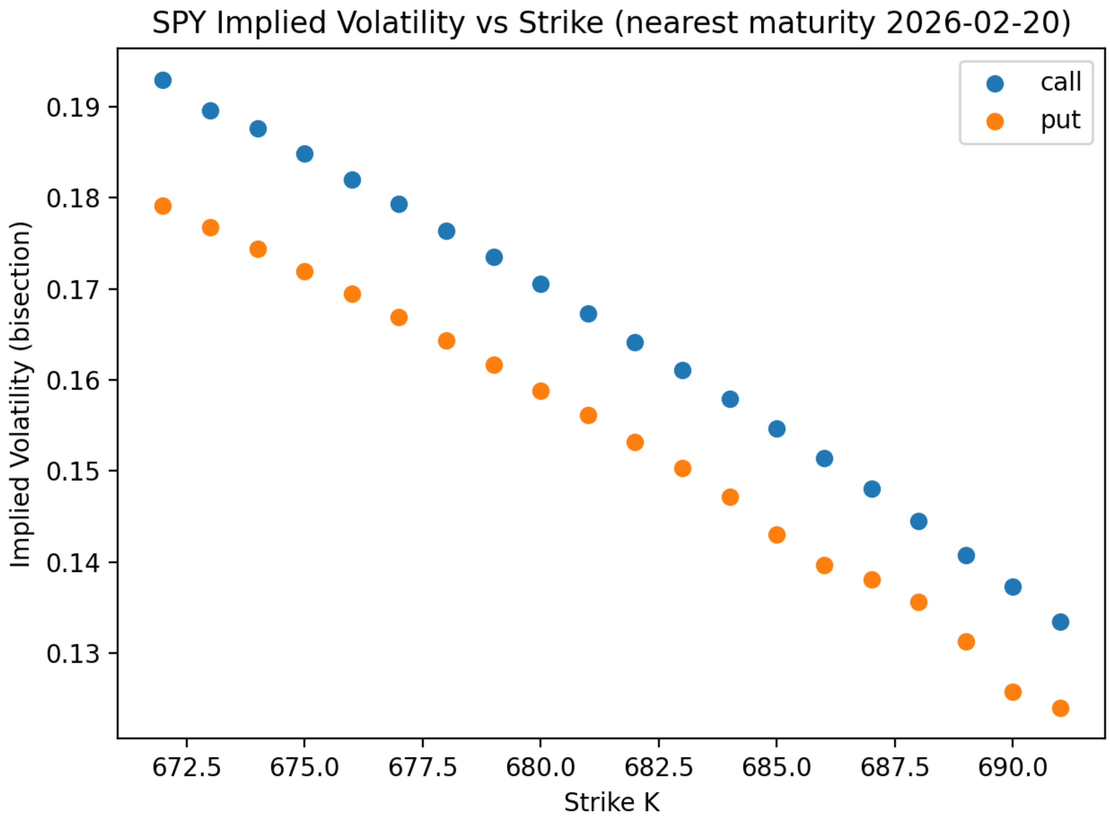

Figure 2: SPY implied volatility vs strike (nearest maturity).

and fee revenue in USDC is $R(S_{t+1}) = \gamma \Delta y$.

Case 2: $S_{t+1} < P_t(1-\gamma)$ (BTC cheaper outside). Arbitragers swap BTC→USDC. Let

$$x' = x_t + (1-\gamma)\Delta x, \qquad y' = y_t - \Delta y,$$

with $x'y' = k$ and boundary condition

$$\frac{y'}{x'} = \frac{S_{t+1}}{1-\gamma}.$$

Solving gives

$$x' = \sqrt{\frac{k(1-\gamma)}{S_{t+1}}}, \qquad y' = \sqrt{\frac{kS_{t+1}}{1-\gamma}},$$

so

$$\Delta x = \frac{x' - x_t}{1-\gamma}, \qquad \Delta y = y_t - y',$$

and fee revenue (converted to USDC) is

$$R(S_{t+1}) = \gamma \Delta x\, S_{t+1}.$$

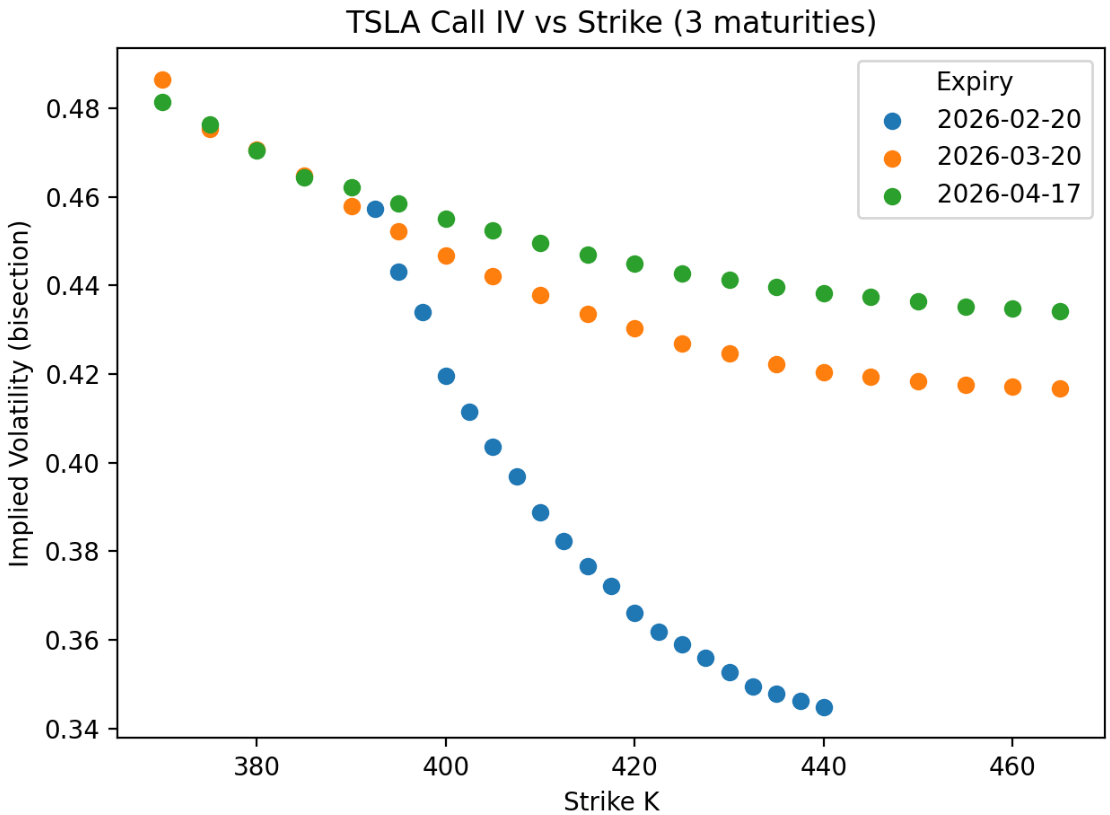

Figure 3: TSLA calls: implied volatility vs strike for three maturities.

(b) Expected fee revenue via trapezoidal rule

With $x_t = y_t = 1000$, $P_t = 1$, $S_t = 1$, and $\Delta t = 1/365$, assume one-step GBM:

$$S_{t+1} = S_t \exp\Big(-\tfrac{1}{2}\sigma^2 \Delta t + \sigma\sqrt{\Delta t}\, Z\Big), \quad Z \sim N(0,1).$$

We compute the expected one-step fee revenue $E[R(S_{t+1})]$ numerically using the trapezoidal rule over the regions where arbitrage trades occur.

(c) Optimal fee rate under different volatilities

Table 10 reports $E[R]$ for $\sigma \in \{0.2, 0.6, 1.0\}$ and $\gamma \in \{0.001, 0.003, 0.01\}$ and selects $\gamma^*(\sigma)$. Figure 7 plots $\sigma \mapsto \gamma^*(\sigma)$ over a grid.

Part 4 (Bonus 10 points): Double integral trapezoidal rule

1. Analytical integrals

For $f_1(x, y) = xy$,

$$\int_0^1 \int_0^3 xy \, dy \, dx = \frac{9}{4} = 2.25.$$

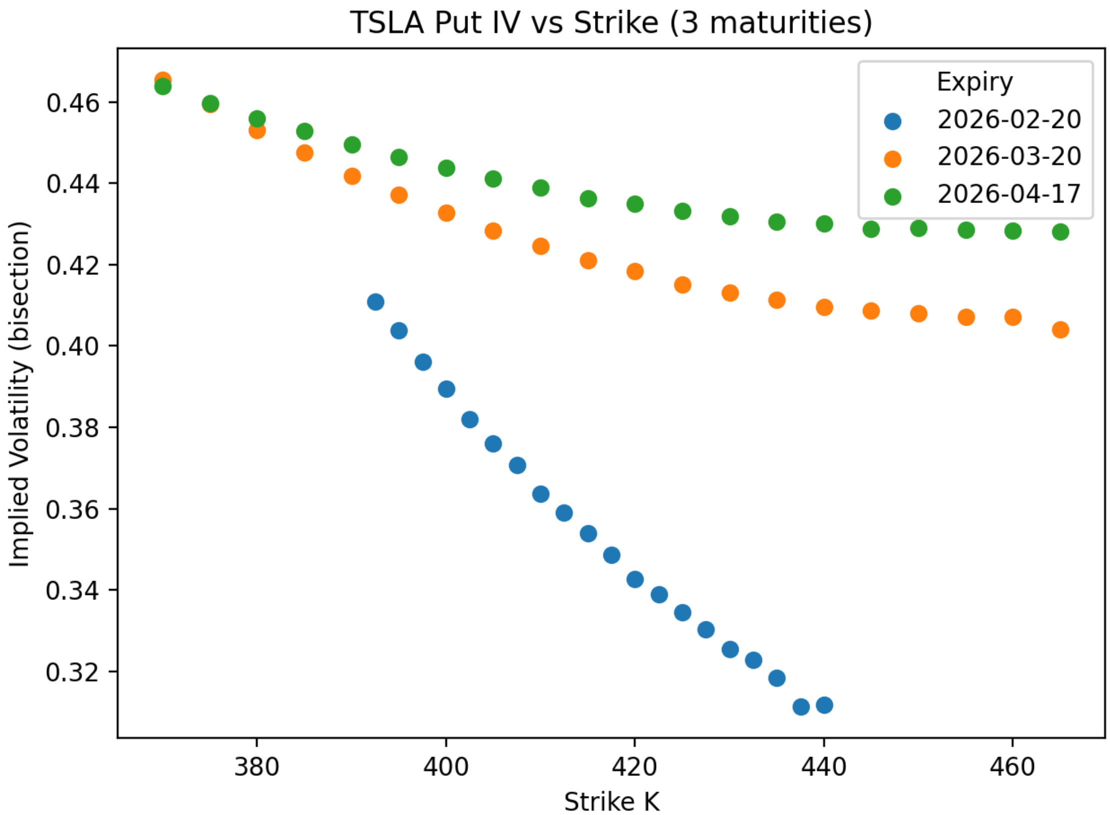

Figure 4: TSLA puts: implied volatility vs strike for three maturities.

For $f_2(x, y) = e^{x+y}$,

$$\int_0^1 \int_0^3 e^{x+y}\, dy\, dx = (e^3 - 1)(e - 1) = e^4 - e^3 - e + 1 \approx 32.794331.$$

2. Numerical trapezoidal approximations and errors

Using four $(\Delta x, \Delta y)$ pairs, we apply the composite trapezoidal rule to approximate both integrals and compute absolute errors (Table 11).

For $f_1(x, y) = xy$, the trapezoidal rule is exact up to roundoff on this grid. For the exponential function, the error decreases as the grid is refined, consistent with trapezoidal-rule convergence.

References

1. Clewlow, L. and Strickland, C. *Implementing Derivative Models.* Wiley, 1996.
2. Mariani, M. C. and Florescu, I. *Quantitative Finance.* Wiley, 2020.
3. Rouah, F. D. *The Heston Model and Its Extensions in Matlab and C.* Wiley, 2013.
4. Angeris, G. et al. "An analysis of Uniswap markets." arXiv:1911.03380, 2019.

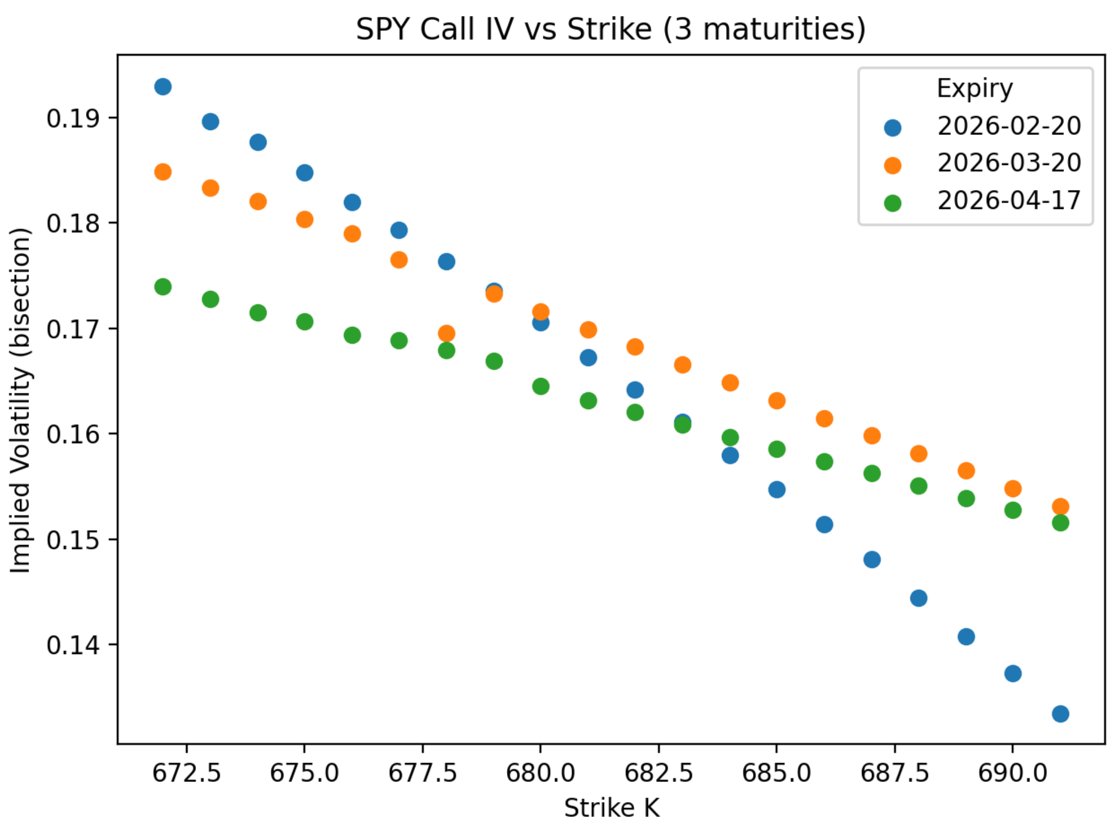

Figure 5: SPY calls: implied volatility vs strike for three maturities.

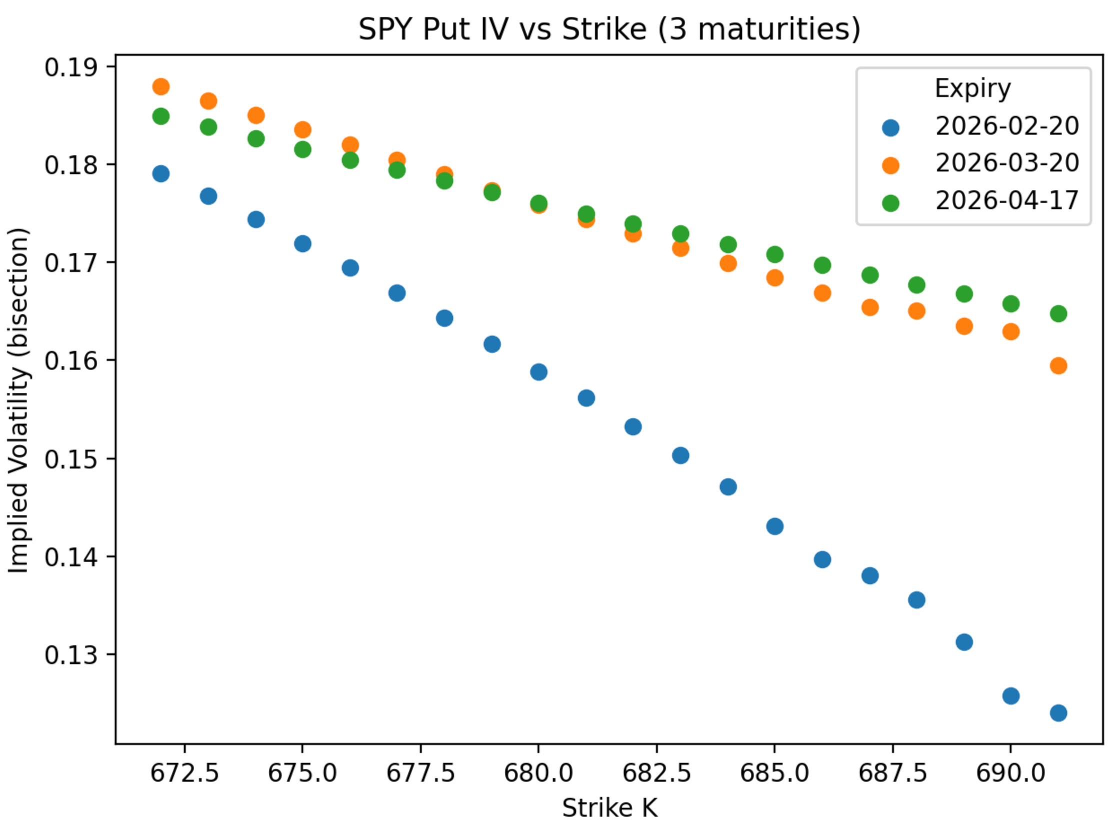

Figure 6: SPY puts: implied volatility vs strike for three maturities.

Table 6: Greeks comparison (sample near ATM, nearest maturity).

Underlying	Expiry	K	σ_{IV}	delta_analytic	delta_fd	gamma_analytic	g
SPY	2026-02-20	6.810000e+02	1.672832e-01	5.229814e-01	5.229812e-01	2.360752e-02	2.36
SPY	2026-02-20	6.820000e+02	1.641992e-01	4.991611e-01	4.991611e-01	2.409084e-02	2.40
SPY	2026-02-20	6.800000e+02	1.705483e-01	5.458620e-01	5.458617e-01	2.304062e-02	2.30
SPY	2026-02-20	6.830000e+02	1.611374e-01	4.744774e-01	4.744775e-01	2.449841e-02	2.44
SPY	2026-02-20	6.790000e+02	1.735262e-01	5.678720e-01	5.678716e-01	2.246532e-02	2.24
SPY	2026-02-20	6.840000e+02	1.579420e-01	4.489311e-01	4.489314e-01	2.483984e-02	2.48
SPY	2026-02-20	6.780000e+02	1.763681e-01	5.890366e-01	5.890361e-01	2.186776e-02	2.18
SPY	2026-02-20	6.850000e+02	1.547030e-01	4.225709e-01	4.225714e-01	2.508656e-02	2.50
SPY	2026-02-20	6.770000e+02	1.793555e-01	6.092423e-01	6.092417e-01	2.122298e-02	2.12
SPY	2026-02-20	6.860000e+02	1.513831e-01	3.954309e-01	3.954316e-01	2.522758e-02	2.52
TSLA	2026-02-20	4.175000e+02	3.720704e-01	5.053073e-01	5.053051e-01	1.737315e-02	1.73
TSLA	2026-02-20	4.150000e+02	3.767094e-01	5.483581e-01	5.483527e-01	1.703450e-02	1.70
TSLA	2026-02-20	4.200000e+02	3.660921e-01	4.611549e-01	4.611563e-01	1.757464e-02	1.75
TSLA	2026-02-20	4.125000e+02	3.822086e-01	5.899272e-01	5.899189e-01	1.648227e-02	1.64
TSLA	2026-02-20	4.225000e+02	3.617230e-01	4.167654e-01	4.167705e-01	1.748130e-02	1.74
TSLA	2026-02-20	4.100000e+02	3.888324e-01	6.293225e-01	6.293119e-01	1.574432e-02	1.57
TSLA	2026-02-20	4.250000e+02	3.590146e-01	3.732859e-01	3.732946e-01	1.709041e-02	1.70
TSLA	2026-02-20	4.075000e+02	3.969637e-01	6.658790e-01	6.658667e-01	1.485624e-02	1.48
TSLA	2026-02-20	4.275000e+02	3.559831e-01	3.308186e-01	3.308307e-01	1.650138e-02	1.65
TSLA	2026-02-20	4.050000e+02	4.036322e-01	7.003625e-01	7.003488e-01	1.395095e-02	1.39

Table 7: Summary of absolute differences between analytic Greeks and finite-difference approximations.

Underlying	MeanAbsDeltaDiff	MeanAbsGammaDiff	MeanAbsVegaDiff	MaxAbsDeltaDiff	MaxAbsGam
SPY	2.846737e-07	2.426658e-08	5.528957e-07	1.907353e-06	8.5504
TSLA	5.020567e-06	1.425633e-07	2.003553e-07	2.113434e-05	7.1904

Table 8: DATA2 pricing error summary by underlying, expiry, and option type.

Underlying	Expiry	Type	N	MAE	MeanError	RMSE
SPY	2026-02-20	call	20	0.245008	-0.245008	0.245973
SPY	2026-02-20	put	20	0.525754	-0.525754	0.526286
SPY	2026-03-20	call	20	0.030681	-0.030681	0.032715
SPY	2026-03-20	put	20	0.348944	-0.348944	0.349136
SPY	2026-04-17	call	20	0.029665	0.029665	0.031808
SPY	2026-04-17	put	20	0.293493	-0.293493	0.293684
TSLA	2026-02-20	call	20	0.228318	-0.228318	0.252496
TSLA	2026-02-20	put	20	0.715342	-0.715342	0.725989
TSLA	2026-03-20	call	20	0.089677	0.004434	0.103107
TSLA	2026-03-20	put	20	0.523618	-0.523618	0.534033
TSLA	2026-04-17	call	20	0.082230	0.069153	0.106669
TSLA	2026-04-17	put	20	0.463561	-0.463561	0.470686

Table 9: DATA2 pricing (sample near ATM, nearest maturity). Error is BS price minus observed mid.

Underlying	Expiry	Type	K	Market mid	BS price (using IV from DATA1)	Error (BS - m
SPY	2026-02-20	call	682.000000	6.490000	6.220749	-0.2692
SPY	2026-02-20	call	681.000000	7.115000	6.846434	-0.2685
SPY	2026-02-20	call	683.000000	5.890000	5.621355	-0.2686
SPY	2026-02-20	call	680.000000	7.770000	7.502946	-0.2670
SPY	2026-02-20	call	684.000000	5.310000	5.043812	-0.2661
SPY	2026-02-20	call	679.000000	8.435000	8.171412	-0.2635
SPY	2026-02-20	put	682.000000	6.285000	5.732000	-0.5530
SPY	2026-02-20	put	681.000000	5.905000	5.353622	-0.5513
SPY	2026-02-20	put	683.000000	6.690000	6.136919	-0.5530
SPY	2026-02-20	put	680.000000	5.540000	4.992236	-0.5477
SPY	2026-02-20	put	684.000000	7.115000	6.563937	-0.5510
SPY	2026-02-20	put	679.000000	5.205000	4.661637	-0.5433
TSLA	2026-02-20	call	417.500000	9.000000	8.683443	-0.3165
TSLA	2026-02-20	call	415.000000	10.375000	10.080444	-0.2945
TSLA	2026-02-20	call	420.000000	7.700000	7.370471	-0.3295
TSLA	2026-02-20	call	412.500000	11.875000	11.607367	-0.2676
TSLA	2026-02-20	call	422.500000	6.550000	6.212397	-0.3376
TSLA	2026-02-20	call	410.000000	13.500000	13.262734	-0.2372
TSLA	2026-02-20	put	417.500000	8.750000	7.931853	-0.8181
TSLA	2026-02-20	put	415.000000	7.650000	6.854043	-0.7959
TSLA	2026-02-20	put	420.000000	9.950000	9.117908	-0.8320
TSLA	2026-02-20	put	412.500000	6.650000	5.883544	-0.7664
TSLA	2026-02-20	put	422.500000	11.325000	10.483277	-0.8417
TSLA	2026-02-20	put	410.000000	5.750000	5.019185	-0.7308

Table 10: AMM expected one-step fee revenue $E[R]$ and optimal $\gamma^*(\sigma)$ among the three candidates.

σ	γ	E[R]	$\gamma^*(\sigma)$
0.200000	0.001	0.003685	
0.200000	0.003	0.008522	
0.200000	0.01	0.009430	
0.200000	BEST	0.009430	0.010000
0.600000	0.001	0.011923	
0.600000	0.003	0.032983	
0.600000	0.01	0.081082	
0.600000	BEST	0.081082	0.010000
1.000000	0.001	0.020061	
1.000000	0.003	0.057384	
1.000000	0.01	0.160690	
1.000000	BEST	0.160690	0.010000

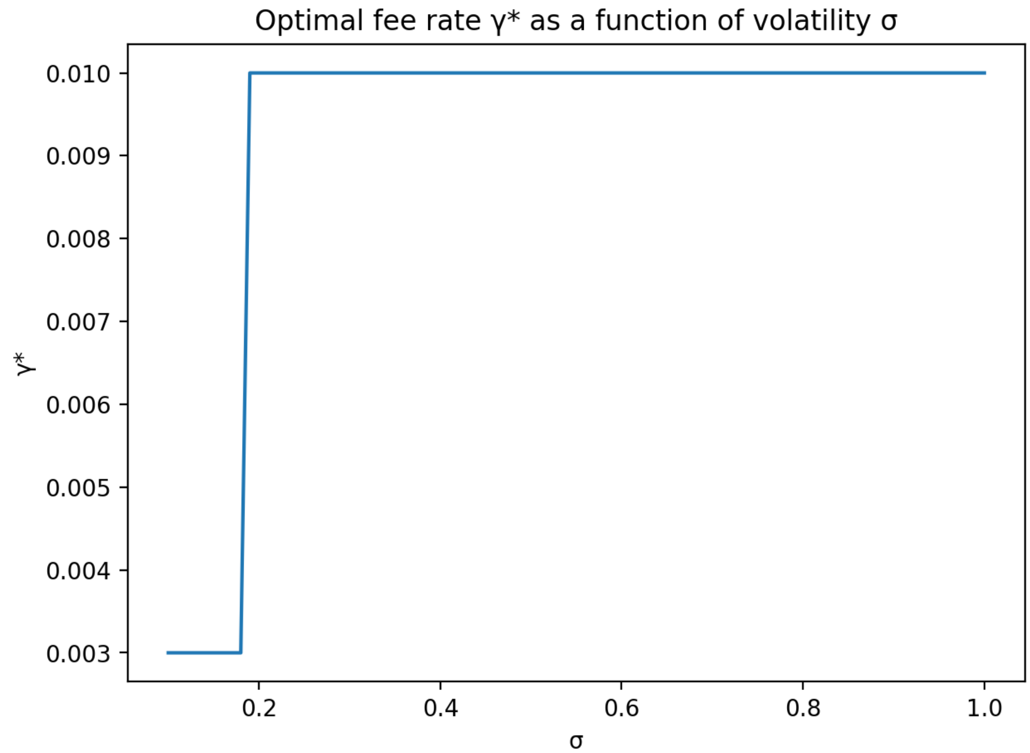

Figure 7: Optimal fee rate $\gamma^*(\sigma)$ on a volatility grid.

Table 11: Double-integral trapezoidal approximations and errors for four grid choices.

Δx	Δy	I1 approx	I1 true	I1 error	I2 approx	I2 true	
2.000000e-01	5.000000e-01	2.250000e+00	2.250000e+00	4.440892e-16	3.299242e+01	3.279433e+01	1
1.000000e-01	3.000000e-01	2.250000e+00	2.250000e+00	-4.440892e-16	3.286264e+01	3.279433e+01	6
5.000000e-02	2.000000e-01	2.250000e+00	2.250000e+00	4.440892e-16	3.282336e+01	3.279433e+01	2
2.500000e-02	1.000000e-01	2.250000e+00	2.250000e+00	-1.332268e-15	3.280159e+01	3.279433e+01	7