Homework #1: Interest rates, bonds and FX

FE-620

Due 8 February 2026

Problem 1.1

Assume that the spot price of gold is \$5,050 per ounce. Suppose that the market quotes a forward contract on gold with delivery in 1 year at \$5,200.00 The risk-free interest rate is r = 3.5%.

i) Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage. What is the profit we can expect to make?

ii) Consider now the case when the forward price is \$5,300.00. Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage.

i. Forward Price at \$5,200.00
Theoretical Price Equation: F = S * e^(r*t)
Calculation: 5,050 * e^(0.035 * 1) = \$5,229.85
Since the Market Price (\$5,200) is less than the Theoretical Price (\$5,229.85), an arbitrage opportunity exists.
Arbitrage Steps: Short gold at spot, invest proceeds at 3.5%, and enter long forward at \$5,200.
Profit = 5226.75 − 5200 = 26.75 per ounce

ii. Forward Price at \$5,300.00
Since the Market Price (\$5,300) is greater than the Theoretical Price (\$5,229.85), an arbitrage opportunity exists.
Arbitrage Steps: Borrow at 3.5%, buy gold at spot, and enter short forward at \$5,300.
Profit = 5300 − 5226.75 = 73.25 per ounce

Problem 1.2

Assume that an interest rate is quoted as 3.25% with semi-annual compounding. What is the rate when expressed with:

i) annual compounding.

Effective annual rate:

$EAR = (1 + 0.0325 / 2)^2 - 1$

$EAR = (1.01625)^2 - 1$

$EAR \approx$ **3.276%**

ii) quarterly compounding?

$(1 + rq)^4 = (1.01625)^2$

$rq = (1.01625)^{(1/2)} - 1$

$rq \approx 0.00809$

Quarterly nominal rate:

$4 \times 0.809\% \approx$ **3.236%**

iii) continuous compounding.

$e^{rc} = (1.01625)^2$

$rc = \ln(1.01625^2)$

$rc \approx 0.03224 =$ **3.224%**

Problem 1.3
Credit card companies quote the APR on the outstanding balance. APR means Annual Percentage Rate and is the interest rate with annualized compounding. Typical APRs are shown in the Table below. However, credit card interest rate is compounded *daily*, for 365 days a year.

Suppose the balance on a credit card is $10,000. Compute the total balance including interest after 30 days, if the customer has Excellent Credit?

Given
APR = 17.15%
Daily rate = 0.1715 / 365
Balance = 10,000
Time = 30 days

Formula:
Balance = 10,000 × (1 + 0.1715 / 365)^30

Calculation:
Daily rate ≈ 0.0004699
Balance ≈ 10,000 × (1.0004699)^30
Balance ≈ **10,141.10**
Interest after 30 days ≈ **141.10**

Table 1: Current Credit Card Interest Rates. As of Sep 8 2025, from https://wallethub.com/edu/cc/current-credit-card-interest-rates/128285.

Category	Interest Rate
Excellent Credit	17.15%
Good Credit	23.33%
Fair Credit	26.55%
Student Credit Cards	19.04%

Problem 1.4

We are given the zero rates $R(T)$ for several maturities in Table 2. These zero rates apply for all maturities in the ranges shown.

1) Price a bond B_1 with annual coupon 5.0% paid twice a year with maturity 2Y.

Coupon = 5% annually → 2.5% per half-year
Maturity = 2 years → 4 payments
Face value = 100
Cash flows:
t = 0.5, 1.0, 1.5, 2.0
Coupons = 2.5 each
Final = 102.5
Discount rates:
0–1Y: 3.00%, 1–2Y: 3.50%
Discount factors:
DF(0.5) = 1 / (1 + 0.03/2)^1
DF(1.0) = 1 / (1 + 0.03/2)^2
DF(1.5) = 1 / (1 + 0.035/2)^3
DF(2.0) = 1 / (1 + 0.035/2)^4
Bond price B1 ≈ **102.03**

2) Price a bond B_2 with annual coupon 6.0% paid twice a year, with maturity 10Y.

Coupon = 6% annually → 3% per half-year
Maturity = 10 years → 20 payments
Discount each cash flow:
0–1Y: 3.00%, 1–2Y: 3.50%, 2–5Y: 4.25%, 5–10Y: 4.50%
Sum of discounted coupons + discounted principal:
Bond price B2 ≈ **109.4**

3) Compute the bond yields of the two bonds in points 1) and 2).

Price = Σ CFt / (1 + y/2)^t
Results: Yield B1 ≈ **3.45%,** Yield B2 ≈ **4.45%**

Table 2: Data for Problem 1.4.

T	$R(T)$
[0,1Y]	3.00%
(1Y,2Y]	3.50%
(2Y,5Y]	4.25%
(5Y,10Y]	4.50%

Problem 1.5

A US company is due to make a payment of 1.0m Euros in 6 months. They plan to hedge this payment by taking a long position in a forward contract for 1.0m Euros with maturity 6 months, at a forward exchange rate 1.150. The current EUR/USD rate is $X_0 = 1.100$, and the actual exchange rate realized at maturity is $X(6M) = 1.175$.

i) What is the gain or loss of the company at maturity?

Net result in USD = 1,000,000 * (1.175 − 1.150)

Net result = **25,000** USD

ii) What is the interest rate differential $r_{USD} - r_{EUR}$ for maturity $6M$ implied by the quoted forward FX rate?

Covered interest parity:

F = X0 × (1 + rUSD) / (1 + rEUR)

1.150 / 1.100 = (1 + rUSD) / (1 + rEUR)

1.04545 ≈ (1 + rUSD − rEUR)

So:

rUSD − rEUR ≈ **4.545%** for 6 months