

Homework #1

- 1.1) Spot price $S_0 = \$5,050$
Risk-free rate $r = 3.5\%$

$$\text{forward price } (F_0) = 5050 * 1.035 = 5226.75$$

(i) Forward price = \$5,200 is less than F_0
Therefore arbitrage exists

(ii) Forward price = \$5,300 is more than F_0
Therefore arbitrage does not exist

$$\text{Profit} = 5300 - 5226.75 = \$73.25$$

- 1.2) Given:
Interest rate = 3.25% with semi-annual compounding
- i) Annual interest rate
 $(1 + 0.0325/2)^2 = 1.032764$
 $r = 3.2764\%$
- ii) Quarterly compounding:
 $(1 + r/4)^4 = 1.0328$
 $r = 3.28\%$
- iii) Continuous compounding:
 $r = \ln(1.0328) = 3.227\%$

- 1.3) Balance = \$10,000
APR = 17.15%
Daily compounding (365 days)
Time = 30 days

Balance after 30 days:
 $10000 * (1 + 0.1715/365)^{30} = \$10,141.00$

1.4) Bond B1 Details:

Face Value = \$100

Coupon Rate = 5% annually, paid semi-annually

Coupon per period = $5\% \times 100 / 2 = \$2.50$

Maturity = 2 years

Zero Rates from Table:

$R(T) = 3.00\%$ for maturities up to 1 year

$R(T) = 3.50\%$ for maturities between 1 and 2 years

Present Value Calculations:

$$PV(0.5) = 2.50 / (1.03)^{0.5} = 2.463$$

$$PV(1.0) = 2.50 / (1.03)^1 = 2.427$$

$$PV(1.5) = 2.50 / (1.035)^{1.5} = 2.375$$

$$PV(2.0) = 102.50 / (1.035)^2 = 93.445$$

Bond Price:

$$2.463 + 2.427 + 2.375 + 93.445 = \$100.71$$

Yield Calculations:

$$2.50e^{(-y*0.5)} + 2.50 e^{(-y*01)} + 2.50 e^{(-y*1.5)} + 102.50 e^{(-y*2)} = 100.71$$

Bond Price = \$100.71

Yield = 4.64%

Bond B2:

Bond B2 Details:

Face Value = \$100

Coupon Rate = 6% annually, paid semi-annually

Coupon per period = $6\% \times 100 / 2 = \$3.00$

Maturity = 10 years (20 semi-annual periods)

Cash Flows:

Periods 1-19: \$3.00 each period

Period 20: \$103.00 (coupon + principal)

Present Value Calculations (using zero rates):

$$PV(0.5) = 3.00 / (1.03)^{0.5}$$

$$PV(1.0) = 3.00 / (1.03)^1$$

$$PV(1.5) = 3 / (1.035)^{1.5}$$

$$\begin{aligned}
PV(2) &= 3 / (1.035)^2 \\
PV(2.5) &= 3 / (1.0425)^2.5 \\
PV(3) &= 3 / (1.0425)^3 \\
PV(3.5) &= 3 / (1.0425)^3.5 \\
PV(4) &= 3 / (1.0425)^4 \\
PV(4.5) &= 3 / (1.0425)^4.5 \\
PV(5) &= 3 / (1.0425)^5 \\
PV(5.5) &= 3 / (1.045)^5.5 \\
PV(6) &= 3 / (1.045)^6 \\
PV(6.5) &= 3 / (1.045)^6.5 \\
PV(7) &= 3 / (1.045)^7 \\
PV(7.5) &= 3 / (1.045)^7.5 \\
PV(8) &= 3 / (1.045)^8 \\
PV(8.5) &= 3 / (1.045)^8.5 \\
PV(9) &= 3 / (1.045)^9 \\
PV(9.5) &= 3 / (1.045)^9.5 \\
PV(10.0) &= 103.00 / (1.045)^10
\end{aligned}$$

Bond Price:

Sum of discounted cash flows $\approx \$108.90$

Yield Calculation

$$\begin{aligned}
&3e^{(-y*0.5)} + 3e^{(-y*01)} + 3e^{(-y*1.5)} + 3e^{(-y*2)} + 3e^{(-y*2.5)} + 3e^{(-y*3)} + 3e^{(-y*3.5)} + 3e^{(-y*4)} + 3e^{(-y*4.5)} + 3e^{(-y*5)} + 3e^{(-y*5.5)} + 3e^{(-y*6)} + 3e^{(-y*6.5)} + \\
&3e^{(-y*7)} + 3e^{(-y*7.5)} + 3e^{(-y*8)} + 3e^{(-y*8.5)} + 3e^{(-y*9)} + 3e^{(-y*9.5)} + 102.50 \\
&e^{(-y*10)} = 108.90
\end{aligned}$$

$$y = 4.47\%$$

1.5) Notional = €1,000,000

Forward rate = 1.150

Spot rate = 1.100

Spot at maturity = 1.175

(i) Gain/Loss:

Hedged payment = \$1,150,000

Unhedged payment = \$1,175,000

Loss (opportunity cost) = \$25,000

(ii) Interest rate differential:

USD – EUR = $\ln(1.150 / 1.100)$

= 4.44% (annualized)