FE-620 Homework #1 Answers

Problem 1.1

Fair forward price (no storage, no income): $F^* = S0 \cdot e^{(rT)} = 5050.00 \cdot e^{(0.0350 \cdot 1)} = 5229.88$.

i) Forward quoted at 5200.00 below fair value arbitrage exists (reverse cash-and-carry).

Steps: (1) Borrow 1 oz of gold and sell it today for $5,050. (2) Invest $5,050 at 3.5% risk-free for 1 year. (3) Enter a long forward to buy 1 oz in 1 year at $5,200. (4) At maturity, use the investment proceeds to buy gold via the forward and return the borrowed gold.

Profit per ounce = $F^* - F = 5229.88 - 5200.00 = 29.88$.

ii) Forward quoted at 5300.00 above fair value arbitrage exists (cash-and-carry).

Steps: (1) Borrow $5,050 at 3.5% and buy 1 oz of gold today. (2) Short the forward at $5,300. (3) At maturity deliver the gold into the forward for $5,300 and repay the loan.

Profit per ounce = $F - F^* = 5300.00 - 5229.88 = 70.12$.

Problem 1.2

Given: 3.25% quoted with semi-annual compounding.

i) Annual compounding rate: $(1+0.0325/2)^2 - 1 = 3.2764\%$.

ii) Quarterly compounding rate R_q: $(1+R_q/4)^4 = (1+0.0325/2)^2 \Rightarrow R_q = 3.2369\%$.

iii) Continuous compounding rate r_c: $\ln(1+\text{effective annual}) = 3.2239\%$.

Problem 1.3

Excellent credit APR = 17.15%, balance = $10,000, daily compounding for 30 days.

Convert APR (annual) to an equivalent daily rate: $r_d = (1+APR)^{(1/365)} - 1 = 0.00043375$.

Balance after 30 days: $10{,}000 \cdot (1+r_d)^{30} = \$10{,}130.95$.

Problem 1.4

Assume face value = 100 and discount factors use continuously-compounded zero rates by bucket:

[0,1Y]: 3.00%, (1,2Y]: 3.50%, (2,5Y]: 4.25%, (5,10Y]: 4.50%.

t (years)	Zero rate R	Cashflow	Discount Factc	PV
0.5	0.03	2.5	0.985329278	2.463323195
1	0.03	2.5	0.970873786	2.427184466
1.5	0.035	2.5	0.949706642	2.374266605
2	0.035	102.5	0.9335107	95.68484679
			Price	102.9496211

1) Bond B1 (5% coupon, semiannual, 2Y) price = 102.949.

t (years)	Zero rate R	Cashflow	Discount Factc	PV
0.5	0.03	3	0.985329278	2.955987834
1	0.03	3	0.970873786	2.912621359
1.5	0.035	3	0.949706642	2.849119926
2	0.035	3	0.9335107	2.800532101
2.5	0.0425	3	0.901176464	2.703529393
3	0.0425	3	0.882616026	2.647848079
3.5	0.0425	3	0.864437855	2.593313566
4	0.0425	3	0.846634078	2.539902234
4.5	0.0425	3	0.829196984	2.487590951
5	0.0425	3	0.81211902	2.436357059
5.5	0.045	3	0.784983273	2.354949819
6	0.045	3	0.767895738	2.303687215
6.5	0.045	3	0.751180165	2.253540496
7	0.045	3	0.734828458	2.204485373
7.5	0.045	3	0.718832694	2.156498083
8	0.045	3	0.703185127	2.109555381
8.5	0.045	3	0.687878176	2.063634529
9	0.045	3	0.672904428	2.018713283
9.5	0.045	3	0.658256628	1.974769884
10	0.045	103	0.643927682	66.32455125
			Price	112.6911878

2) Bond B2 (6% coupon, semiannual, 10Y) price = 112.69

3) Yield to maturity (semiannual compounding):
y(B1) = **3.4608%**, y(B2) = **4.4163%**

Problem 1.5
i) With hedge, USD paid = 1.150m. Without hedge at maturity = 1.175m. Gain (savings) = 0.025m = **$25,000**.
ii) Covered interest parity (continuous): $F = X_0 e^{(r_{USD} - r_{EUR})T} \Rightarrow r_{USD} - r_{EUR} = \ln(F/X_0)/T$.

$$r_{USD} - r_{EUR} = \frac{\ln\left(\frac{1.150}{1.100}\right)}{0.5} = \mathbf{8.8904}\%.$$