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Homework #1

1.2 a) $S_0 = 5050$ \$/oz, $r = 0.035$, $T = 1$ year

First check for arbitrage.

$$F_0^* = S_0 e^{rT}$$

Plugging in $F_0^* = 5050e^{0.035 \cdot 1}$
≈ 5229.88

Fair Forward is 5229.88

The forward is $5200 < 5229.88$
$\Rightarrow$ So Arbitrage exists

In order to realize the arbitrage,
At $t=0$ short 1 oz of gold
Invest the 5,050 at 0.035, buy a bond
Go long 1 forward with delivery price 5200

At $t=1$ for each ounce, the arbitrage profit would be

$$\boxed{\$29.88 \text{ per oz}}$$

1.1 b.) If the market forward is 5300, there is an arbitrage opportunity since 5300 > 5229.88

Borrow $S_0 = 5050$ and buy one ounce of gold
Short one forward with delivery $F_0 = 5300$

At $t=0$, Borrow +5050, Buy gold −5050,
Enter forward

At $t=1$, Deliver gold into the forward and receive +5300 and repay the loan −5229.88

So the arbitrage profit is $\boxed{\$70.12 \text{ per oz}}$

1.2) a.) The growth factor is $\left(1+\frac{r}{m}\right)^m$ where r is the rate and m is the compounding factor

After 1 year, $\left(1+\frac{0.0325}{2}\right)^2 = 1.0328$

So the annual rate is $\boxed{3.28\%}$ ~~per year~~

1.2 (b.) With quarterly compounding,

$$\left(1+\frac{r}{4}\right)^4 = 1.0328$$

$$\cancel{1+}\frac{r}{\cancel{4}} \approx (1.008084-1)4$$

$$r = 0.03236$$

So the quarterly rate is $\boxed{3.236\%}$

c.) For continuously,

$$e^{R_c} = 1.0328$$

$$R_c = \ln(1.0328)$$

$$R_c = 0.03224$$

So the continuously compounding rate is $\boxed{3.224\%}$

1.3 Balance = 10,000 17.15% for excellent credit

Daily compound rate $B_t = B_0\left(1+\frac{APR}{365}\right)^t$

$$B_{30} = 10000\left(1+\frac{0.1715}{365}\right)^{30} = 10,141.90$$

So the Balance would be $\boxed{\$10,141.90}$

1.4(a) Coupon per half-year $C = \frac{0.05}{2} \cdot 100 = 2.5$
Let $F = 100$, face value

Semiannual payment at $t = 0.5, 1.0, 1.5, 2.0$

$R = 3\%$ $t \le 1$, $R = 3.5\%$ for $1 \le t \le 2$,

$$\text{Bond} = 2.5e^{-0.03(0.05)} + 2.5e^{-0.03(1.0)} + 2.5e^{-0.035(1.5)} + 102.5e^{-0.035(2.0)}$$

Assuming continuous compounding,

$$\text{Bond} = 2.5(0.9851119) + 2.5(0.9704455) + 2.5(0.9488) + 102.5(0.9324)$$

$$= \boxed{102.83 \text{ for } 100 \text{ face}}$$

b) $C = \frac{0.06}{2} = 3$, for 10 years there are 20 payout times

$$B_2 = 3(e^{-0.03(0.5)} + e^{-0.03(1.0)}) + 3(e^{-0.035(1.5)} + e^{-0.035(2.0)})$$

$$+ 3 \sum_{t \in \{2.5, 3.0, 3.5, 4, 4.5, 5\}} e^{-0.0425t}$$

$$+ 3 \sum_{t \in \{5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5\}} e^{-0.045t}$$

$$+ 103 \sum e^{-0.045 \cdot (10)}$$

→

1 b.) Adding it up bond 2 = $\boxed{111.84 \text{ per } 100 \text{ face}}$

c.) Getting bond yields: $C = 2.5$

$$B_1 \Rightarrow 102.83 = \sum_{k=1}^{3} \frac{2.5}{(1+\frac{y}{2})^k} + \frac{102.5}{(1+\frac{y}{2})^4}$$

Solving gives 0.03521

$\boxed{\text{so the semiannual-compounded rate for } B_1 \text{ is } 3.521\%}$

$$B_2 \Rightarrow 111.84 = \sum_{k=1}^{19} \frac{3}{(1+\frac{y}{2})^k} + \frac{103}{(1+\frac{y}{2})^{20}}$$

$= 0.04515$

$\boxed{\text{so the semi annual-compounded rate for } B_2 \text{ is } 4.515\%}$

1.5 a.) the forward's value to a long position is

$$\text{Payoff} = (X_T - F_0) \times \text{Notional in EUR}$$

Plugging in $\text{Payoff} = (1.175 - 1.150) \times 1{,}000{,}000$

$= \$25{,}000$

so $\boxed{\$25{,}000 \text{ gain at maturity}}$

1.5 (b) $F_0 = X_0 e^{(r_{USD} - r_{EUR})T}$ for continuous compounding

$r_{USD} - r_{EUR} = \frac{1}{T} \ln\left(\frac{F_0}{X_0}\right)$ → solving for differential

plug ⇒ $r_{USD} - r_{EUR} = \frac{1}{0.5} \ln\left(\frac{1.150}{1.100}\right) =$

$2\ln(1.04545) \approx 2(0.04445 = 0.0889$

so $r_{USD} - r_{EUR} = 0.0889$

$\boxed{\text{or } 8.89\% \text{ per year continually compounding}}$