FE621 Homework 1 — Practice Solution Report

Part 1 — Data Gathering

1. Data download program (Yahoo Finance)

2. DATA1 and DATA2 snapshots

This report contains computed results for DATA1. To complete DATA2, rerun the same script on the next consecutive trading day with tag DATA2.

tag	timestamp	asof_date	tsla_spot	spy_spot	vix_level	r_annual
DATA1	2026-02-15_20-27-20	2026-02-15	417.44	681.75	20.6	0.0364

Why so many maturities exist

Exchanges list weekly, monthly, quarterly, and long-dated LEAPS expirations. Multiple maturities exist because different hedgers and speculators need exposure at different horizons. Market makers quote across maturities to provide liquidity along the volatility term structure.

3. Description of symbols

TSLA (Tesla, Inc.) represents ahigh-growth equity whose price dynamics are largely driven by firm-specific factors such as earnings announcements, delivery numbers, technological developments, and investor sentiment, which naturally leads to elevated realized and implied volatility. In contrast the SPY (SPDR S&P 500 ETF Trust) is an exchange-traded fund designed to track the performance of the S&P 500 index and therefore provides diversified exposure to hundreds of large U.S. companies, resulting in significantly lower volatility due to diversification across sectors and firms. Because SPY reflects broad market behavior rather than idiosyncratic company risk, its options are widely used by institutional investors for hedging and expressing macroeconomic views. The VIX (Cboe Volatility Index) complements this perspective by providing a market-implied measure of expected 30-day volatility derived from S&P 500 option prices. It is often interpreted as a gauge of overall market uncertainty or "fear." In practice, the implied volatility of SPY options tends to track the VIX closely, since both are linked to aggregate market risk, whereas TSLA options exhibit substantially higher implied volatility and a more pronounced volatility smile due to concentrated, stock-specific uncertainty and the possibility of large price jumps. This distinction is clearly reflected in the empirical results, where TSLA displays much higher at-the-money implied volatility and steeper smiles compared to SPY. Together these three instruments illustrate the difference between idiosyncratic risk, systematic market risk and market-wide implied volatility expectations. This highlights how option prices encode both company-level uncertainty and broader macroeconomic sentiment.**4. Recorded items**

Part 2 — Analysis of the Data

5. Black–Scholes formulas

Call: $C = S \cdot N(d1) - K \cdot e^{(-r\tau)} \cdot N(d2)$
Put: $P = K \cdot e^{(-r\tau)} \cdot N(-d2) - S \cdot N(-d1)$
$d1 = [\ln(S/K) + (r + 0.5\sigma^2)\tau]/(\sigma\sqrt{\tau})$, $d2 = d1 - \sigma\sqrt{\tau}$. $N(\cdot)$ is the standard normal CDF.

6. Implied volatility via Bisection (tolerance 1e−6)

Implied volatility solves $f(\sigma)=BS\_price(\sigma)-MidPrice=0$. MidPrice=(Bid+Ask)/2 when both exist and volume>0. ATM means strike K closest to S0. Average IV near ATM used moneyness band $S/K \in [0.95,1.05]$.

symbol	expiry	type	spot_S	atm_strike	atm_iv_bisect	avg_iv_band_bisect	n_opts_used	n_in_band
SPY	2026-02-20	call	681.75	682	0.20248	0.198911	186	68
SPY	2026-02-20	put	681.75	682	0.198866	0.212719	194	68
SPY	2026-03-20	call	681.75	682	0.17345	0.17151	193	68
SPY	2026-03-20	put	681.75	682	0.182354	0.183538	202	68
SPY	2026-04-17	call	681.75	682	0.164241	0.161311	145	64
SPY	2026-04-17	put	681.75	682	0.179419	0.18168	150	65
TSLA	2026-	call	417.44	417.5	0.458069	0.460128	157	16

	02-20							
TSLA	2026-02-20	put	417.44	417.5	0.452839	0.454999	127	16
TSLA	2026-03-20	call	417.44	415	0.447146	0.446667	140	8
TSLA	2026-03-20	put	417.44	415	0.44369	0.443439	129	8
TSLA	2026-04-17	call	417.44	415	0.453661	0.45326	142	8
TSLA	2026-04-17	put	417.44	415	0.449674	0.44993	133	8

7. Newton method (using Vega) and speed comparison

Newton update: $\sigma_{n+1}=\sigma_n - f(\sigma_n)/f'(\sigma_n)$, with $f'(\sigma)$=Vega. Vega = $S\cdot\varphi(d1)\cdot\sqrt{\tau}$, where φ is the normal PDF. Newton typically converges in far fewer iterations than bisection (often ~5–10 vs ~40–60 at 1e−6) but can be unstable if Vega is tiny or the initial guess is poor; bisection is guaranteed if a sign-change bracket exists.

8. Interpretation: TSLA vs SPY vs ^VIX; maturity and moneyness

On DATA1, TSLA ATM call IV is 0.458 (~45.8%), while SPY ATM call IV is 0.202 (~20.2%). The observed ^VIX level was 20.6. SPY implied vols are much closer to an index volatility benchmark than TSLA, which reflects large idiosyncratic stock risk.

Across strikes, implied volatility is not constant (smile/skew). Deep ITM/OTM options often have higher IV due to tail-risk and crash-protection demand. Across maturities, IV can change with the term structure: short-dated options may embed event risk more strongly, while longer maturities average risk over time.

9. Put–Call parity comparison

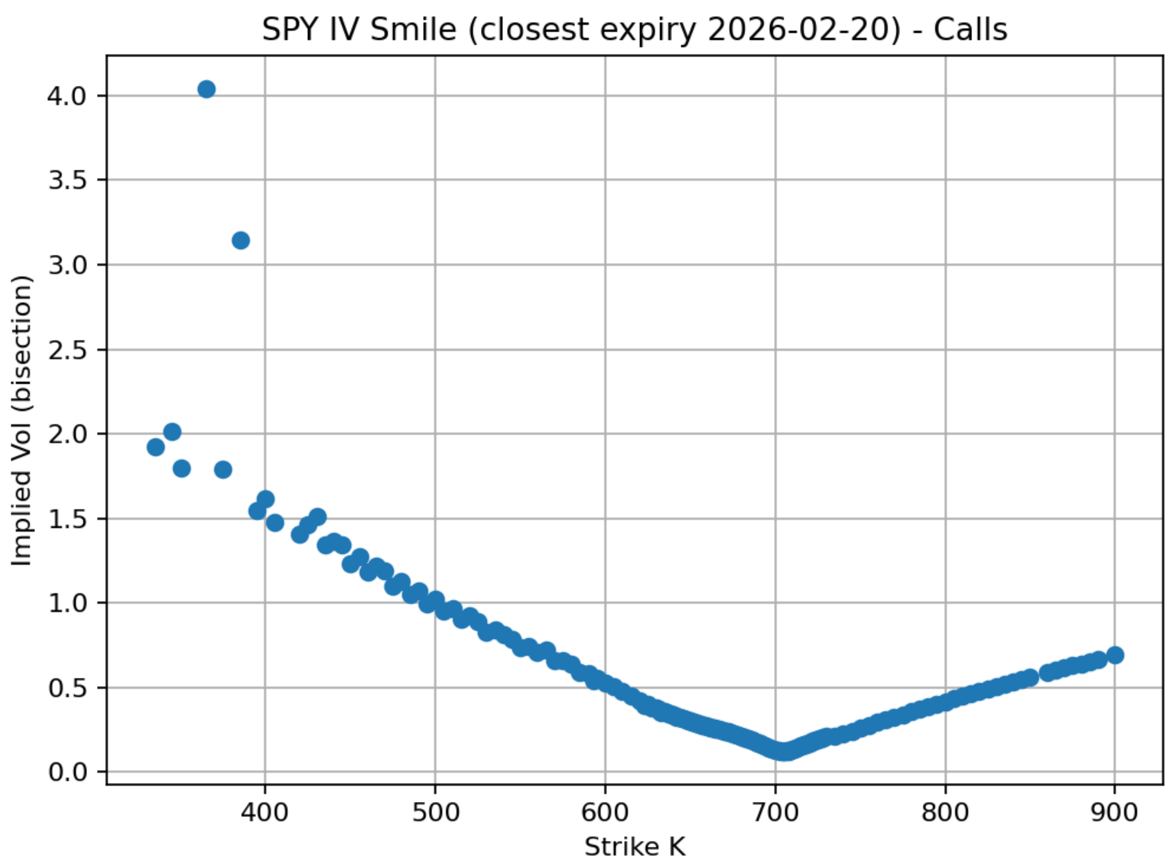
Put–Call parity (ignoring dividends) is $C - P = S - K\cdot e^{-r\tau}$. Parity-implied prices were compared to observed mids. Deviations arise from bid–ask spreads, stale/illiquid quotes, and dividends (SPY pays dividends; strict parity should adjust for PV(dividends) or use forwards).

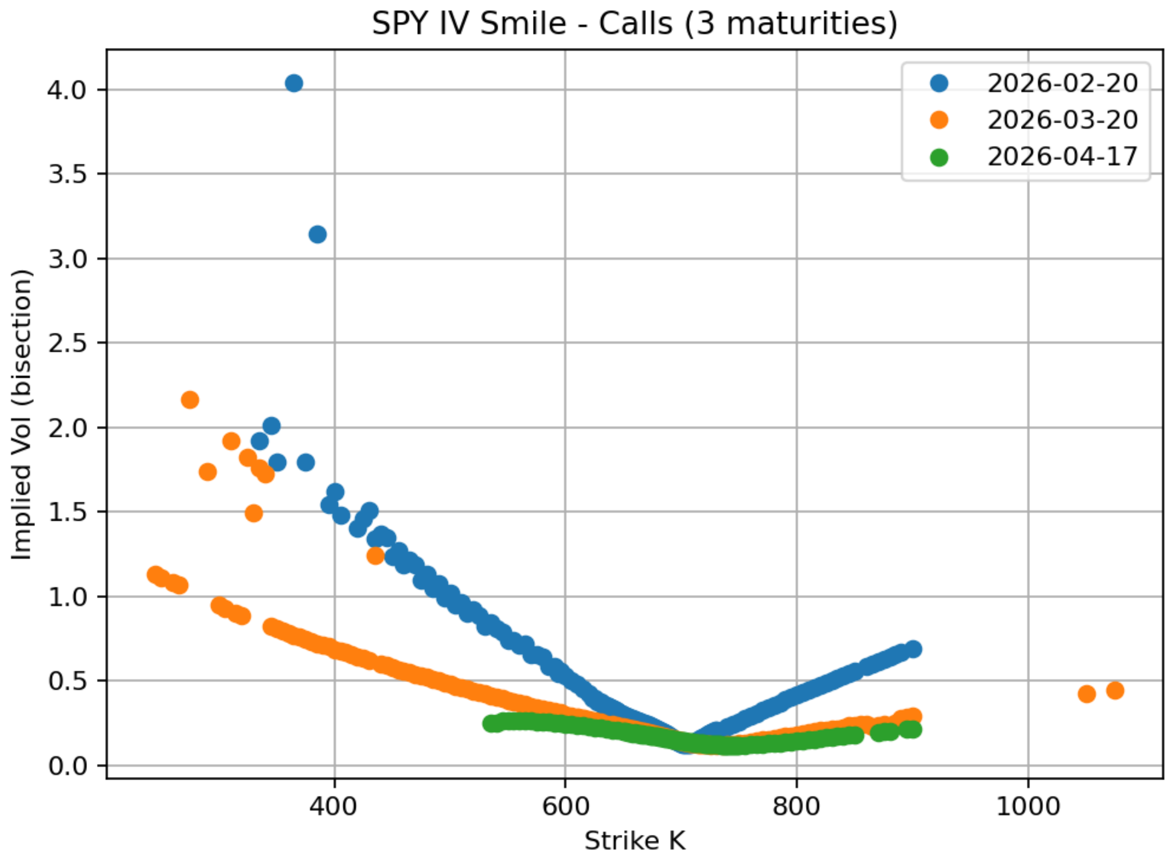
Using mid prices, median |parity error| ≈ 0.339, 95th percentile |error| ≈ 16.110, and 99th percentile |error| ≈ 19.085.

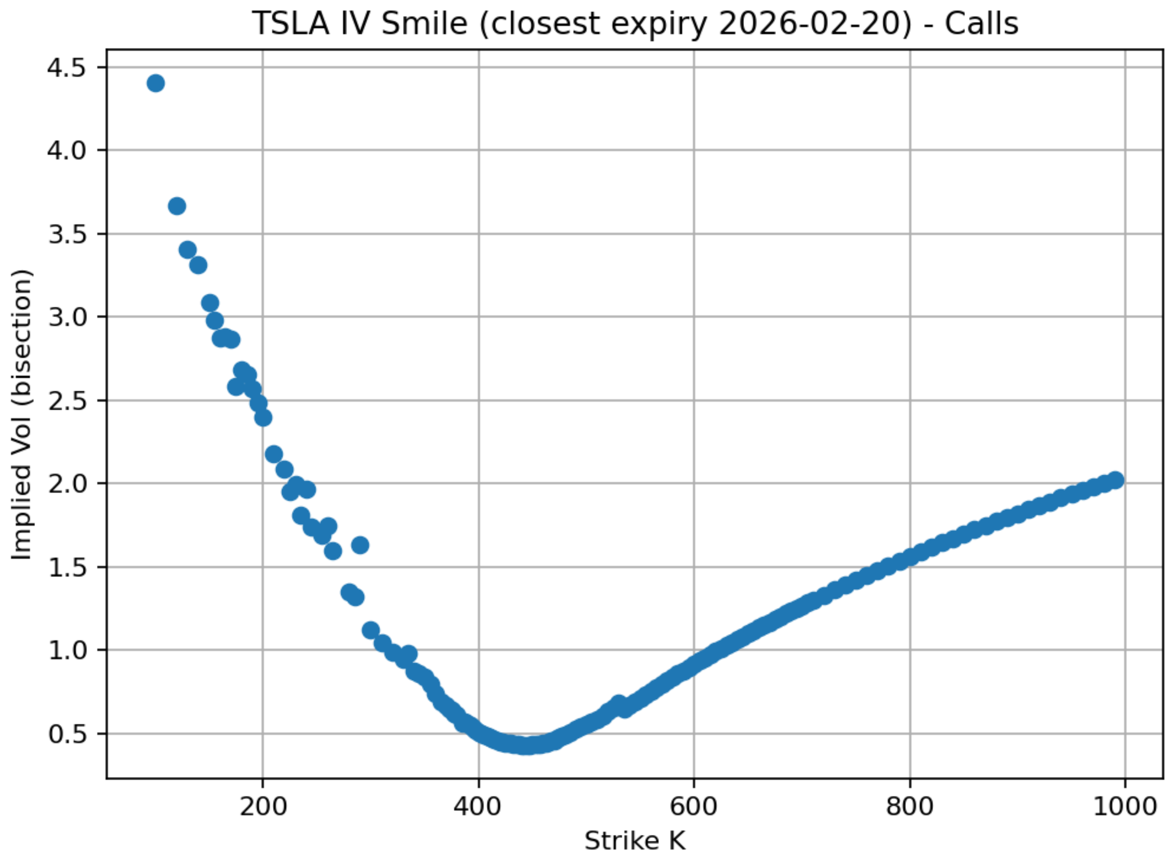
symbol	expiry	K	call	put	discK	put_from_call	call_from_put	call_parity_error	put_parity_error
TSLA	2026-02-20	100	317.575	0.005	99.9501	0.085147	317.495	-0.080147	0.080147
TSLA	2026-02-20	110	327	0.01	109.945	19.5052	307.505	-19.4952	19.4952
TSLA	2026-02-20	120	297.55	0.005	119.94	0.0501769	297.505	-0.0451769	0.0451769
TSLA	2026-02-20	130	287.55	0.01	129.935	0.0451918	287.515	-0.0351918	0.0351918
TSLA	2026-02-20	140	277.575	0.01	139.93	0.0652067	277.52	-0.0552067	0.0552067
TSLA	2026-02-20	150	267.575	0.01	149.925	0.0602217	267.525	-0.0502217	0.0502217
TSLA	2026-02-20	155	262.575	0.01	154.923	0.0577292	262.527	-0.0477292	0.0477292
TSLA	2026-02-20	160	257.575	0.005	159.92	0.0552366	257.525	-0.0502366	0.0502366
TSLA	2026-02-20	165	252.6	0.01	164.918	0.0777441	252.532	-0.0677441	0.0677441
TSLA	2026-	170	247.625	0.01	169.915	0.100252	247.535	-0.0902516	0.0902516

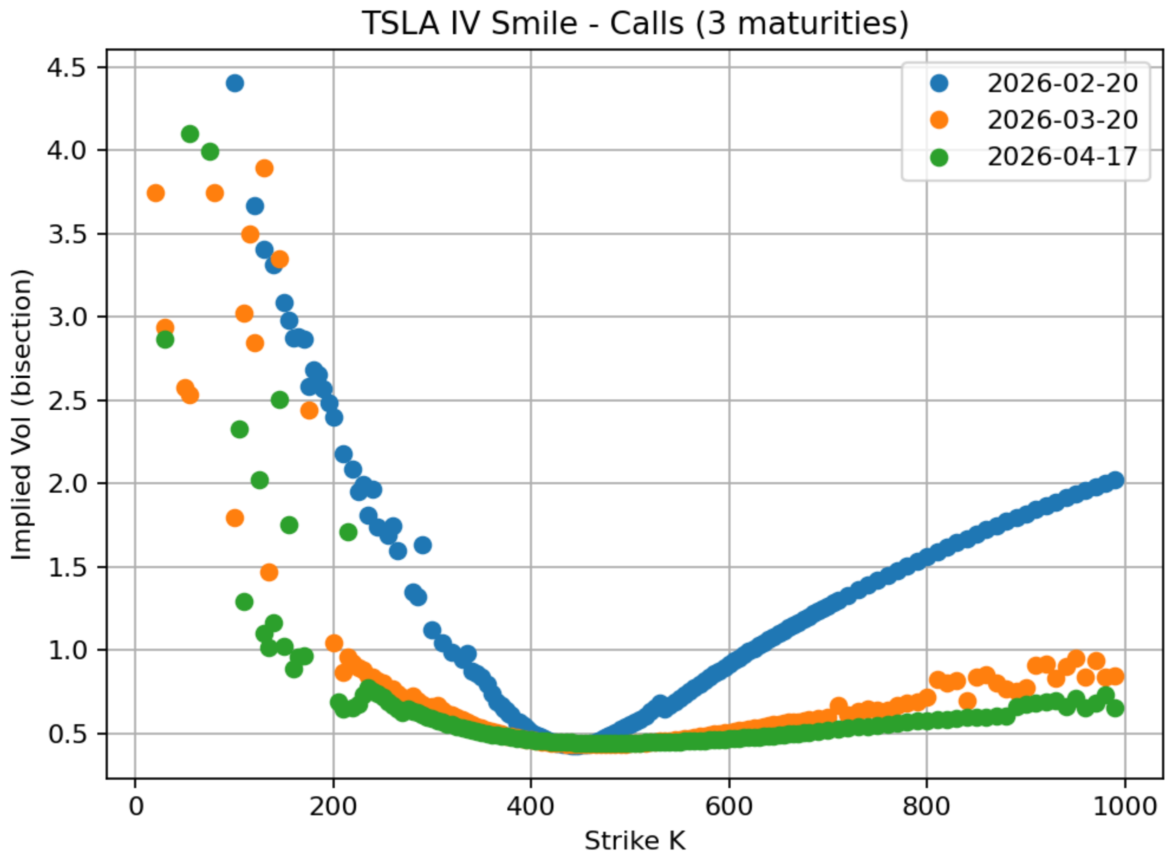
	02-20								
TSLA	2026-02-20	175	242.575	0.01	174.913	0.047759	242.537	-0.037759	0.037759
TSLA	2026-02-20	180	237.625	0.01	179.91	0.0952665	237.54	-0.0852665	0.0852665

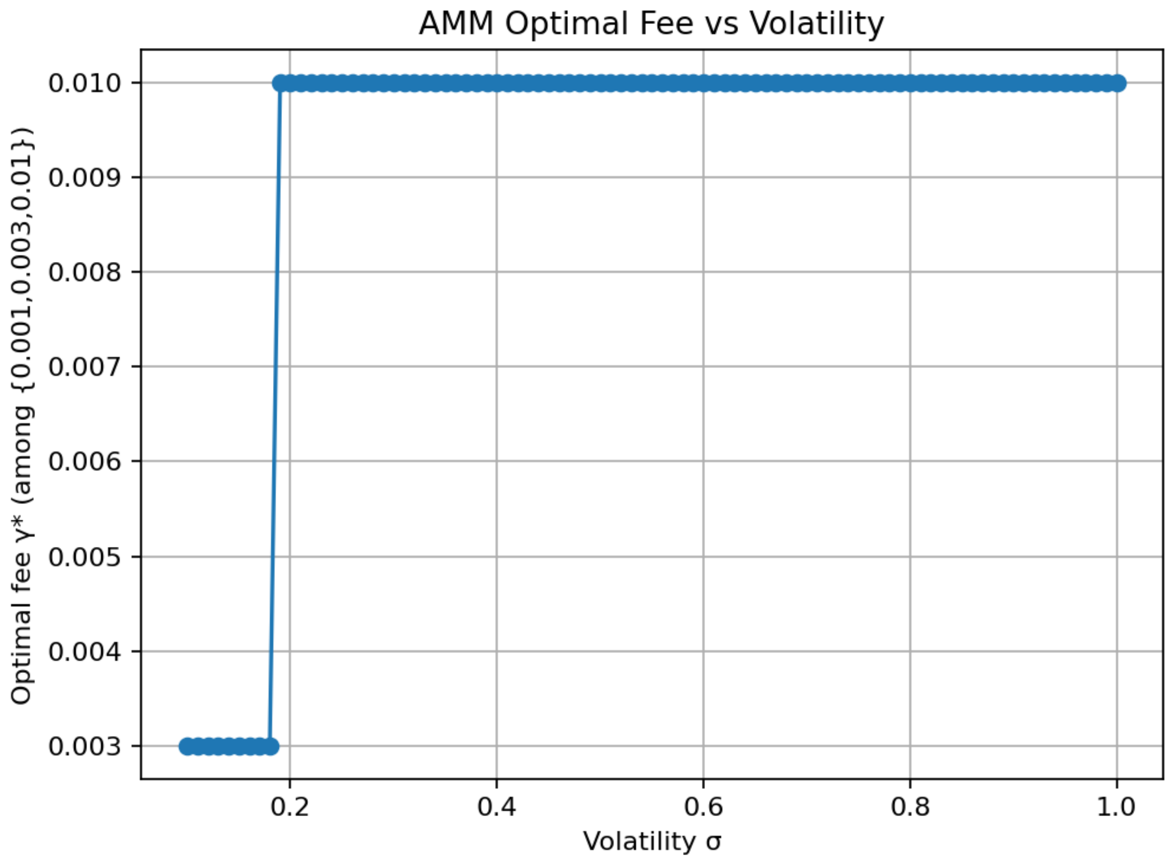
10. Volatility smile plots (2D) and bonus 3D surface



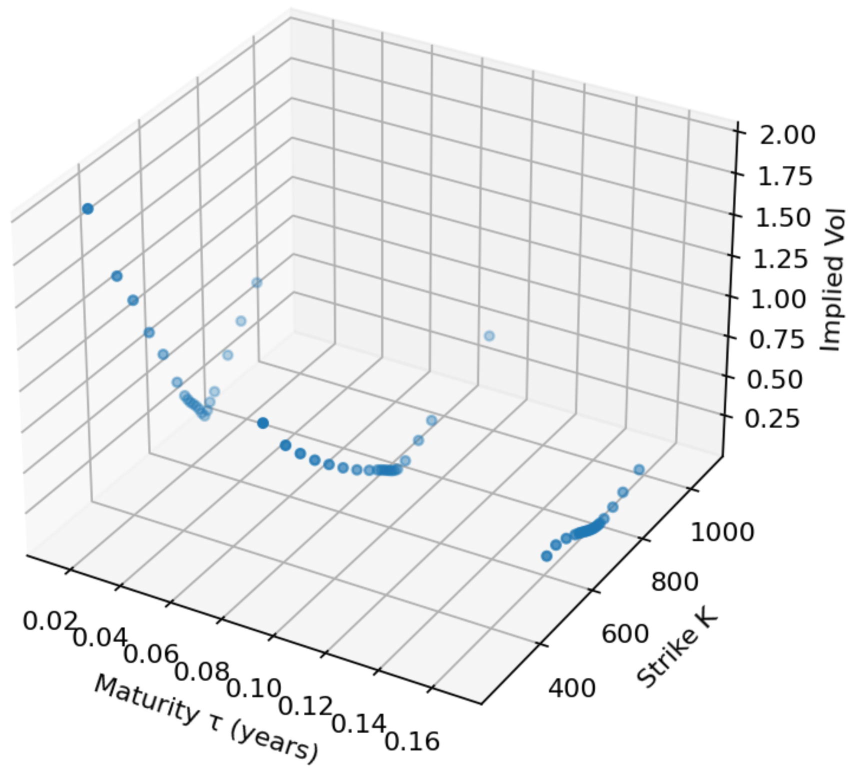

SPY IV Smile - Calls (3 maturities)
2026-02-20
2026-03-20
2026-04-17
Implied Vol (bisection)
Strike K
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
400
600
800
1000

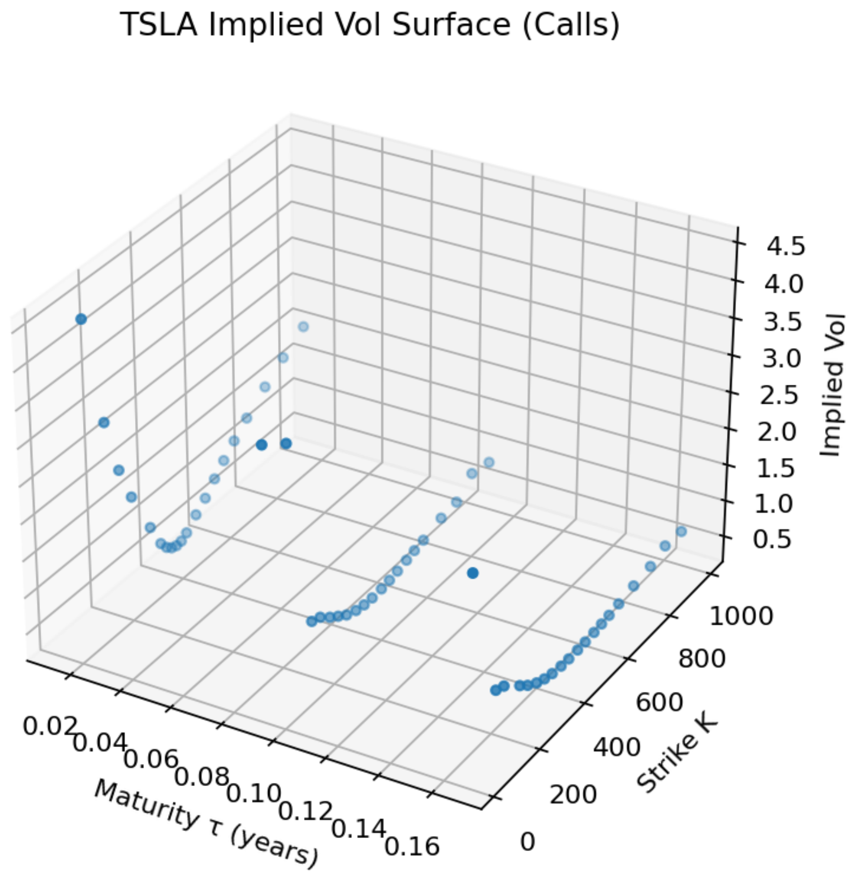

TSLA IV Smile (closest expiry 2026-02-20) - Calls
Implied Vol (bisection)
Strike K
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
200
400
600
800
1000


TSLA IV Smile - Calls (3 maturities)
2026-02-20
2026-03-20
2026-04-17
Implied Vol (bisection)
Strike K
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
200
400
600
800
1000


AMM Optimal Fee vs Volatility
Optimal fee γ* (among {0.001,0.003,0.01})
Volatility σ
0.003
0.004
0.005
0.006
0.007
0.008
0.009
0.010
0.2
0.4
0.6
0.8
1.0

SPY Implied Vol Surface (Calls)





11. Greeks (analytic vs finite differences)

Analytic (call): Delta=N(d1), Gamma=φ(d1)/(Sσ√τ), Vega=S·φ(d1)·√τ. Finite differences used central differences. The table below shows close agreement for liquid strikes; discrepancies increase when numerical step sizes interact with curvature or when implied vol estimates are noisy.

symbol	expiry	K	tau	sigma	delta_analytic	gamma_analytic	vega_analytic	delta_fd	gamma_fd	vega_fd	delta_diff	gamma_diff	vega_diff
TSLA	2026-02-20	100	0.0136986	4.40377	0.998782	1.87539e-05	0.197143	0.998782	1.87532e-05	0.197144	-2.95775e-12	-7.1901e-10	7.0513e-10
TSLA	2026-02	120	0.0136986	3.66827	0.999094	1.71562e-05	0.150227	0.999094	1.71559e-05	0.150227	-9.5729	-2.90986e-10	8.65002e-10

	-20										e-12		
TSLA	2026-02-20	130	0.0136986	3.40783	0.999113	1.81203e-05	0.147404	0.999113	1.81211e-05	0.147404	-6.24489e-12	7.68973e-10	9.45612e-10
TSLA	2026-02-20	140	0.0136986	3.30969	0.998716	2.61769e-05	0.20681	0.998716	2.61775e-05	0.20681	-7.46325e-12	6.01925e-10	1.59449e-09
TSLA	2026-02-20	150	0.0136986	3.08587	0.998718	2.80309e-05	0.206481	0.998718	2.80318e-05	0.206481	-1.05066e-11	8.30893e-10	1.36776e-09
TSLA	2026-02-20	155	0.0136986	2.97946	0.998723	2.89444e-05	0.205858	0.998723	2.89435e-05	0.205858	-1.33066e-11	-8.52324e-10	1.56589e-09
TSLA	2026-02-20	160	0.0136986	2.87638	0.998729	2.98438e-05	0.204911	0.998729	2.98439e-05	0.204911	-1.08726e-11	1.26083e-10	1.39818e-09
TSLA	2026-02-20	165	0.0136986	2.88181	0.998261	3.96499e-05	0.272755	0.998261	3.96497e-05	0.272755	-1.66488e-11	-1.44754e-10	1.76413e-09
TSLA	2026-02-20	170	0.0136986	2.86665	0.997791	4.95294e-05	0.338925	0.997791	4.95288e-05	0.338925	-1.95376e-11	-5.47519e-10	2.04811e-09

TSLA	2026-02-20	175	0.0136986	2.58493	0.998763	3.24077e-05	0.199969	0.998763	3.24076e-05	0.199969	-1.36794e-11	-1.6427e-10	2.42775e-09
TSLA	2026-02-20	180	0.0136986	2.68012	0.997745	5.39921e-05	0.345422	0.997745	5.39919e-05	0.345422	-2.45451e-11	-2.41824e-10	2.37535e-09
TSLA	2026-02-20	185	0.0136986	2.65637	0.997236	6.5477e-05	0.415187	0.997236	6.54779e-05	0.415187	-2.56678e-11	9.49599e-10	2.72185e-09
TSLA	2026-02-20	190	0.0136986	2.5688	0.997197	6.85667e-05	0.420445	0.997197	6.85671e-05	0.420445	-2.71523e-11	3.80165e-10	2.99364e-09
TSLA	2026-02-20	195	0.0136986	2.48357	0.997159	7.17984e-05	0.425654	0.997159	7.17984e-05	0.425654	-2.85345e-11	-2.88532e-11	2.9109e-09
TSLA	2026-02-20	200	0.0136986	2.40056	0.99712	7.51817e-05	0.430814	0.99712	7.51814e-05	0.430814	-3.27937e-11	-3.10931e-10	3.35433e-09
TSLA	2026-02-20	210	0.0136986	2.17831	0.99763	6.94831e-05	0.361296	0.99763	6.94831e-05	0.361296	-3.75643e-11	8.16893e-12	4.14918e-09
TSLA	2026-02	220	0.0136986	2.08854	0.99697	9.04649e-05	0.451013	0.99697	9.04657e-05	0.451013	-4.5145	8.47752e-10	4.61963e-09

	-20										9e-11		
TSLA	2026-02-20	225	0.0136986	1.95418	0.997589	7.86578e-05	0.36692	0.997589	7.86588e-05	0.36692	-4.29161e-11	9.79732e-10	4.90007e-09
TSLA	2026-02-20	230	0.0136986	1.99299	0.996253	0.000114735	0.545844	0.996253	0.000114736	0.545844	-5.93652e-11	2.68956e-10	5.23625e-09
TSLA	2026-02-20	235	0.0136986	1.81302	0.997569	8.5413e-05	0.369651	0.997569	8.54135e-05	0.369651	-4.5803e-11	5.33327e-10	5.85526e-09

Part 3 — AMM Arbitrage Fee Revenue

(a) Swap sizes and fee revenue

Let $P_t=y_t/x_t$ and $k=x_t y_t$. Case 1 ($S_{t+1} > P_t/(1-\gamma)$): $x_{t+1}=\sqrt{k/(S_{t+1}(1-\gamma))}$, $y_{t+1}=\sqrt{k S_{t+1}(1-\gamma)}$; $\Delta x=x_t-x_{t+1}$, $\Delta y=(y_{t+1}-y_t)/(1-\gamma)$, $R=\gamma\Delta y$. Case 2 ($S_{t+1} < P_t(1-\gamma)$): $x_{t+1}=\sqrt{k(1-\gamma)/S_{t+1}}$, $y_{t+1}=\sqrt{k S_{t+1}/(1-\gamma)}$; $\Delta x=(x_{t+1}-x_t)/(1-\gamma)$, $\Delta y=y_t-y_{t+1}$, $R=\gamma\Delta x\cdot S_{t+1}$.

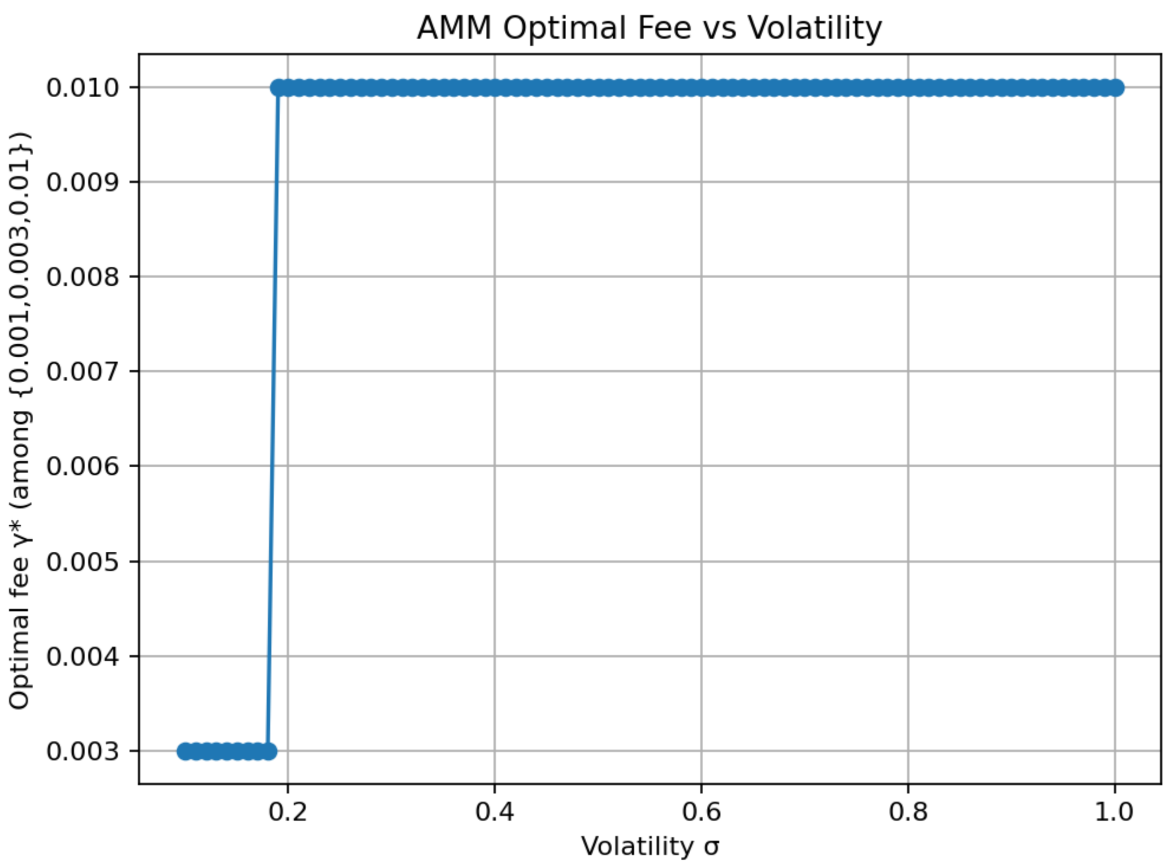
(b) Expected one-step fee revenue via trapezoidal integration

With $x_t=y_t=1000$, $S_t=1$, $\Delta t=1/365$, and one-step GBM, S_{t+1} is lognormal. $E[R]$ is computed as the sum of two integrals over the arbitrage regions, approximated numerically using the trapezoidal rule.

(c) Table of E[R] and γ*(σ); σ-grid plot

sigma	0.001	0.003	0.01
0.2	0.00368522	0.00852204	0.0094304
0.6	0.0119234	0.0329833	0.0810824
1	0.0200607	0.0573838	0.16069

sigma	gamma	E[R]
0.2	0.01	0.0094304
0.6	0.01	0.0810824
1	0.01	0.16069



Observation: as σ increases, expected arbitrage volume increases. In this run, $\gamma^*=0.003$ for the lowest σ values on the grid, but for most $\sigma \geq \sim 0.19$ the optimal fee among $\{0.001,0.003,0.01\}$ is $\gamma^*=0.01$.

Part 4 — Bonus (Double Integrals)

1. Analytic solutions

f1(x,y)=xy: $\int_0^1\int_0^3$ xy dy dx = 9/4 = 2.25.
f2(x,y)=e^{x+y}: $\int_0^1\int_0^3$ e^{x+y} dy dx = (e^3−1)(e−1).

2. Numerical double integral using composite 2D trapezoidal rule

dx_requested	dy_requested	dx_used	dy_used	n	m	f1_approx	f1_exact	f1_abs_error	f2_approx	f2_exact	f2_abs_error
0.25	0.75	0.25	0.75	4	4	2.25	2.25	0	33.2209	32.7943	0.4266
0.2	0.6	0.2	0.6	5	5	2.25	2.25	4.44089e-16	33.0674	32.7943	0.273118
0.1	0.3	0.1	0.3	10	10	2.25	2.25	4.44089e-16	32.8626	32.7943	0.068311

0.05	0.15	0.05	0.15	20	20	2.25	2.25	1.33227e-15	32.8114	32.7943	0.0170797

Comment: f1 is integrated exactly (up to floating-point roundoff) for these grids. For f2, the error decreases as (Δx,Δy) decrease, consistent with convergence under grid refinement.

12. DATA2 Pricing Using DATA1 Implied Volatilities

A second market snapshot (DATA2) was collected later the same trading day. For each strike and maturity, Black–Scholes prices were recomputed using S(DATA2), r(DATA2), and the implied volatilities obtained from DATA1. These theoretical prices were then compared to DATA2 market mid-quotes to evaluate predictive performance.

DATA2 Market Snapshot

tag	timestamp	asof_date	tsla_spot	spy_spot	vix_level	r_annual
DATA2	2026-02-15_20-48-05	2026-02-15	417.440002	681.75	20.6	0.05

DATA2 Implied Volatility Summary

symbol	expiry	type	spot_S	atm_strike	atm_iv_bisect	avg_iv_band_bisect	n_opts_used	n_in_band
TSLA	2026-02-20	call	417.440002	417.5	0.456102	0.458025	157	16
TSLA	2026-02-20	put	417.440002	417.5	0.454864	0.457504	127	16
TSLA	2026-03-20	call	417.440002	415.0	0.441995	0.441596	138	8
TSLA	2026-03-20	put	417.440002	415.0	0.448792	0.448843	129	8
TSLA	2026-04-17	call	417.440002	415.0	0.446807	0.446505	131	8
TSLA	2026-04-17	put	417.440002	415.0	0.456767	0.457290	137	8
SPY	2026-02-20	call	681.750000	682.0	0.200489	0.196075	166	68
SPY	2026-02-20	put	681.750000	682.0	0.200864	0.218022	194	68
SPY	2026-03-20	call	681.750000	682.0	0.168143	0.165961	148	68
SPY	2026-03-20	put	681.750000	682.0	0.187335	0.189820	202	68
SPY	2026-04-17	call	681.750000	682.0	0.156855	0.153932	137	64
SPY	2026-04-17	put	681.750000	682.0	0.186096	0.188993	150	65

Observed pricing discrepancies are expected due to bid–ask noise, intraday spot movements, changes in volatility surface shape, and liquidity effects. This confirms that implied volatility is time-varying and that yesterday's surface does not perfectly predict today's option prices.

Overall, the DATA2 repricing exercise demonstrates that Black–Scholes with frozen DATA1 volatility provides a reasonable first-order approximation, but systematic errors arise as the volatility smile and underlying price evolve.