BRANDON TANAKA CHAPWANYA
PRICING AND HEDGING
HOMEWORK 1

$T = 1$
$S_0 = \$5050$ per ounce
$F_0 = \$5200$
$r = 3{,}5\%$ per annum (assuming continuous compounding)

Fair forward price $\Rightarrow$

$$F_f = S_0 e^{r \cdot T} = 5050 \times e^{0{,}035(1)}$$
$$= \$5\,229{,}88$$

$F_0 < F_f$, hence quoted forward is underpriced, meaning there is arbitrage opportunity.

Strategy! at T_0:

- Short-sell the underlying asset, that is borrow gold and sell it at spot price.
- Invest proceeds \$5050 at the risk free rate, $r = 3{,}5\%$ (continuously compounded) for 1 year
- Enter a long-forward contract to buy 1 ounce of gold in 1 year for \$5200.

at T_1 (1 year)

- Investment grows to: $\$5050 \times e^{0{,}035(1)}$
pay amount $\Rightarrow$ \$5229.88 and receive gold
- return borrowed gold to close the short position
- Arbitrage profit is remaining cash \$29.88
$\$5229{,}88 - 5200 = \underline{\underline{\$29.88}}$.

(ii) $S_0 = 5050$

$F_0 = 5300$

$r = 3{,}5\%$ (continuously compounding)

Fair forward price, $F_f = S_0 e^{r.T}$

$= 5050 \times e^{0{,}035(1)}$

$= \$\ 5229.88$

$F_0 > F_f$, hence quoted forward is overpriced, meaning there is arbitrage opportunity.

Strategy: at T_0

- Borrow money from bank at risk free rate 3,5%.
- Buy underlying asset at spot price \$5050
- Enter a short-forward contract to sell 1 ounce of gold in 1 year for \$5300.

at T_1:

- Deliver 1 ounce of gold via short-forward contract and receive \$5300
- Repay loan, [$\$5050 \times e^{0{,}035(1)} = \$5229{,}88$]
- Remaining cash \$70.12 is arbitrage profit

$\$\ 5300 - \$5229{,}88 = \underline{\$70.12}$.

Q2)

Effective Annual Rate $= \left(1 + \frac{R_2}{2}\right)^2$

Annual compounding, $EAR = 1 + R_1$

Equating,

$$1 + R_1 = \left(1 + \frac{R_2}{2}\right)^2$$

$$1 + R_1 = \left(1 + \frac{0{,}0325}{2}\right)^2$$

$$1 + R_1 = (1{,}01625)^2$$

$$R_1 = 1{,}0327640625 - 1$$

$$= 0{,}032764025$$

$$\approx \underline{3{,}2764\,\%}$$

(ii) $EAR = 1 + R_1 = \left(1 + \frac{R_4}{4}\right)^4$

$$(1{,}0327640625)^{1/4} = 1 + \frac{R_4}{4}$$

$$1{,}00809216 = 1 + \frac{R_4}{4}$$

$$4(0{,}00809216) = R_4$$

$$R_4 = 0{,}03236864$$

Quarterly compounding rate $R_4 \approx \underline{3{,}2369\,\%}$

(iii) Continuous compounding rate R_c,

$$\left(1+\frac{R_m}{m}\right)^m = \left(1+\frac{R_n}{n}\right)^n = e^{R_c}$$

$$R_c = 2\ln(1{,}01625)$$
$$= 2[0{,}0161193990\text{-}9]$$
$$= 0{,}0322387981 8$$
$$R_c \approx \underline{3{,}2239\%}.$$

Q3 Effective annual rate $r = 0{,}1715$
Annual compounding $= 1+r$
Daily compounding $= (1+r_d)^{365}$

$$\therefore \quad (1+r_d)^{365} = 1+r$$
$$r_d = (1+r)^{\frac{1}{365}} - 1$$

$P = \$10\,000$; $n = 30$ days

Balance $\Rightarrow 10\,000 \times (1+0{,}1715)^{\frac{30}{365}}$
$\Rightarrow 10000 \times (1{,}0130946)$
$\Rightarrow \$10\,130{,}95$

$\therefore$ total balance after 30 days, including interest is approximately $\underline{\$\,10\,130{,}95}$.

Problem 1.4 - Bond B1 Pricing (2 Year, 5% Coupon Semi-Annual)

Bond Parameters:

Face Value:	100
Annual Coupon Rate:	0.05
Semi-Annual Coupon:	2.5
Maturity (Years):	2

Zero Rates (Continuously Compounded):

Maturity Range	Zero Rate
[0, 1Y]	3.00%
(1Y, 2Y]	3.50%
(2Y, 5Y]	4.25%
(5Y, 10Y]	4.50%

Cash Flow Valuation:

Time (Years)	Cash Flow	Zero Rate	Discount Factor	Present Value
0.5	$2.50	3.00%	0.9851	$2.4628
1	$2.50	3.00%	0.9704	$2.4261
1.5	$2.50	3.50%	0.9489	$2.3721
2	$102.50	3.50%	0.9324	$95.5704
		Bond Price:		**$102.8314**

Yield Calculation:

Trial Yield (y):	3.49%
Calculated Price:	$102.8314
Difference:	$0.0000

Problem 1.4 - Bond B2 Pricing (10 Year, 6% Coupon Semi-Annual)

Bond Parameters:

Face Value:	100
Annual Coupon Rate:	0.06
Semi-Annual Coupon:	3
Maturity (Years):	10

Zero Rates (see Bond B1 sheet for details)

Cash Flow Valuation:

Time (Years)	Cash Flow	Zero Rate	Discount Factor	Present Value
0.5	$3.00	3.00%	0.9851	$2.9553
1	$3.00	3.00%	0.9704	$2.9113
1.5	$3.00	3.50%	0.9489	$2.8466
2	$3.00	3.50%	0.9324	$2.7972
2.5	$3.00	4.25%	0.8992	$2.6976
3	$3.00	4.25%	0.8803	$2.6409
3.5	$3.00	4.25%	0.8618	$2.5854
4	$3.00	4.25%	0.8437	$2.5310
4.5	$3.00	4.25%	0.8259	$2.4778
5	$3.00	4.25%	0.8086	$2.4257
5.5	$3.00	4.50%	0.7808	$2.3423
6	$3.00	4.50%	0.7634	$2.2901
6.5	$3.00	4.50%	0.7464	$2.2392
7	$3.00	4.50%	0.7298	$2.1894
7.5	$3.00	4.50%	0.7136	$2.1407
8	$3.00	4.50%	0.6977	$2.0930
8.5	$3.00	4.50%	0.6822	$2.0465
9	$3.00	4.50%	0.6670	$2.0009
9.5	$3.00	4.50%	0.6521	$1.9564
10	$103.00	4.50%	0.6376	$65.6757
			Bond Price:	**$111.8428**

Yield Calculation:

Trial Yield (y):	4.46%
Calculated Price:	$111.8428
Difference:	$0.0000

Problem 1.4 - Summary of Results

Bond	Maturity	Coupon Rate	Price	Yield
B1	2 years	5.00%	$102.83	3.49%
B2	10 years	6.00%	$111.84	4.46%

Formulas Used:

Discount Factor:	D(T) = exp(-R*T)
Present Value:	PV = Cash Flow * D(T)
Bond Price:	Sum of all PVs
Yield:	Rate y where Price = Sum(CF * exp(-y*T))

Problem 1.5 - FX Forward Hedging

Given:

Parameter	Symbol	Value	Units
Payment Amount	Notional	1,000,000	EUR
Time to Maturity	T	0.5	years
Spot Rate (today)	X_0	1.100	USD/EUR
Forward Rate	F	1.150	USD/EUR
Spot Rate at Maturity	X(6M)	1.175	USD/EUR

Part i) Gain or Loss at Maturity

Description	Calculation	Amount (USD)
Payment WITH Hedge	F *Notional	$1,150,000.00
Payment WITHOUT Hedge	X(6M)* Notional	$1,175,000.00
Gain (Loss) from Hedge	No Hedge -With Hedge	**$25,000.00**

Interpretation:
A positive value indicates a GAIN from hedging.
The company saved money by locking in the forward rate.

Part ii) Interest Rate Differential (r_USD - r_EUR)

Formula: F = exp[(r_USD - r_EUR) × T] × X_0
Solving for (r_USD - r_EUR):

Step	Calculation	Value
1. F / X_0	F / X_0	1.04545
2. ln(F / X_0)	0.044451763	0.04445
3. r_USD - r_EUR	ln(F / X_0)/T	**8.89%**

Verification:

Forward Rate Check	EXP(r_diff * T) *X_0		1.150
Should Equal Forward Rate:		1.15	1.150
Difference (should be ≈0):		0	0.000000

Economic Interpretation:
- USD interest rates are higher than EUR interest rates by 8.89%
- This causes EUR to appreciate in the forward market (F > X_0)
- The forward premium reflects interest rate parity

SUMMARY OF RESULTS

Question	Answer
i) Gain or Loss at Maturity	**$25,000.00**
ii) Interest Rate Differential	**8.89%**

Key Formulas for FX Forward Hedging

	Formula
Hedging Gain	Gain = Notional × [X(T) - F]
FX Forward Rate	F = exp[(r_d - r_f)T] × X_0
Interest Differential	(r_d - r_f) = (1/T) × ln(F/X_0)
Discount Factor	D(T) = exp(-r × T)

Description
Long position gains when spot > forward
Interest rate parity condition
Implied from forward premium/discount
Continuous compounding