Homework #1: Interest rates, bonds and FX

FE-620

Due 8 February 2026

Problem 1.1

Assume that the spot price of gold is \$5,050 per ounce. Suppose that the market quotes a forward contract on gold with delivery in 1 year at \$5,200.00 The risk-free interest rate is $r = 3.5\%$.

i) Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage. What is the profit we can expect to make?

ii) Consider now the case when the forward price is \$5,300.00. Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage.

$T = 1$ yr $\quad r = 0.035$

$g = 5{,}050$

$G = 5{,}200$

i) Theoretical price

$G = g(1+r)^T = 5{,}050(1+0.035)^1 = 5{,}226.76$

$5{,}226.75 > 5{,}200 \rightarrow$ yes arbitrage opportunity

① Sell gold at \$5,050

② invest \$5,050 for 1 year @ 3.5%

③ Enter forward contract @ 5,200 in 1 year

\$26.75 per oz profit

ii) Yes forward > theoretical

① Borrow 5,050 @ 3.5% to buy gold

② Enter forward to sell gold @ 5,300 in yr

③ After 1 yr, settle contract for \$5,300

$5{,}300 - 5{,}226.75 = \$73.25/oz$

Problem 1.2

Assume that an interest rate is quoted as 3.25% with semi-annual compounding. What is the rate when expressed with:

i) annual compounding.

ii) quarterly compounding?

iii) continuous compounding.

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

$$Ann = \left(1 + \frac{R_m}{m}\right)^m$$

i) $R_{ann} = \left(1 + \frac{0.0325}{2}\right)^2 = (1.01625)^2 = 1.03276$

$$R_{ann} = 3.28\%$$

ii) $R_q = \left(1 + \frac{R_q}{4}\right)^4 = \left(1 + \frac{R_s}{2}\right)^2$

$\left(1 + \frac{R_q}{4}\right) = \left(1 + \frac{R_s}{2}\right)^{2/4} = (1.01625)^{1/2}$

$= 1.008092$

$\hookrightarrow R_q = 4(1.008092 - 1) = 0.032369$

$$R_q = 3.24\%$$

$$e^{R_c} = \left(1 + \frac{R_s}{m}\right)^m$$

iii) $R_c = m \ln\left(1 + \frac{R_m}{m}\right)$

$R_c = 2 \ln\left(1 + \frac{0.0325}{2}\right) = 2 \times \ln(1.01625) = 0.032239$

$$R_c = 3.22\%$$

Problem 1.3

Credit card companies quote the APR on the outstanding balance. APR means Annual Percentage Rate and is the interest rate with annualized compounding. Typical APRs are shown in the Table below. However, credit card interest rate is compounded *daily*, for 365 days a year.

Suppose the balance on a credit card is \$10,000. Compute the total balance including interest after 30 days, if the customer has Excellent Credit?

Initial \$ $M = 10{,}000$

$APR = 0.1715$

$t = 30$ days

$$r_{daily} = \frac{APR}{365} = 0.0004699$$

$$M' = M \times (1 + r_{daily})^t$$

$$= 10{,}000(1 + 0.0004699)^{30}$$

$$= \$10{,}141.93$$

Table 1: Current Credit Card Interest Rates. As of Sep 8 2025, from `https://wallethub.com/edu/cc/current-credit-card-interest-rates/128285`.

Category	Interest Rate
Excellent Credit	17.15%
Good Credit	23.33%
Fair Credit	26.55%
Student Credit Cards	19.04%

Problem 1.4

We are given the zero rates $R(T)$ for several maturities in Table 2. These zero rates apply for all maturities in the ranges shown.

1) Price a bond B_1 with annual coupon 5.0% paid twice a year with maturity 2Y.

2) Price a bond B_2 with annual coupon 6.0% paid twice a year, with maturity 10Y.

3) Compute the bond yields of the two bonds in points 1) and 2).

Duration of Bond

$$D = \sum_{i=1}^{n} t_i \left[\frac{C_i e^{-y t_i}}{B} \right]$$

C_i = cash flow B = price

t_i = time y = yield

$$B = \sum_{i=1}^{n} C_i \cdot D(T_i) = \sum_i C_i e^{-R(T_i) T_i}$$

Coupon $= \frac{0.05 \times 100}{2} = 2.5$

1) $T = 0.5 \rightarrow 2.5 e^{-0.03 \times 0.5} = 2.4628$

$T = 1 \rightarrow 2.5 e^{-0.03 \times 1} = 2.4261$

$T = 1.5 \rightarrow 2.5 e^{-0.035 \times 1.5} = 2.3721$

$T = 2 \rightarrow (100 + 2.5) e^{-0.035 \times 2} = 95.5704$

$\Sigma = 102.8314 = B_1 = \$102.83$

2) $C = \frac{0.06 \times 100}{2} = 3$

$T: 0.5 - 1 = 3e^{-0.03 \times .5} + 3e^{-0.03 \times 1} = 2.955 + 2.911 = 5.866$

$T: 1 - 2 = 3e^{-0.035 \times 1.5} + 3e^{-0.035 \times 2} = 2.846 + 2.797 = 5.643$

$T: 2 - 5 = \sum_{T=2.5}^{5} 3e^{-0.0425 \times T} = 15.366$

$T: 5 - 10 = \sum_{T=5.5}^{10} 3e^{-0.045 \times T} + 100 e^{-0.045 \times 10} = 19.3 + 65.676$

$\Sigma = 111.851 \quad B_2 = \$111.85$

3)

$$B = \sum_{t=1}^{n} \frac{C/2}{(1+y/2)^{2t}} + \frac{100}{(1+y/2)^{2n}}$$

$$B_1 = \sum_{i=1}^{3} \frac{2.5}{(1+y/2)^i} + \frac{102.5}{(1+y/2)^4}$$

102.83

$y = 0.03522$ 3.52%

$$B_2 = \sum_{t=1}^{19} \frac{3}{(1+y/2)^t} + \frac{103}{(1+y/2)^{20}} = 4.52\%$$

111.85

Table 2: Data for Problem 1.4.

T	$R(T)$
[0,1Y]	3.00%
(1Y,2Y]	3.50%
(2Y,5Y]	4.25%
(5Y,10Y]	4.50%

forward vs spot rate

Problem 1.5

A US company is due to make a payment of 1.0m Euros in 6 months. They plan to hedge this payment by taking a long position in a forward contract for 1.0m Euros with maturity 6 months, at a forward exchange rate 1.150. The current EUR/USD rate is $X_0 = 1.100$, and the actual exchange rate realized at maturity is $X(6M) = 1.175$.

i) What is the gain or loss of the company at maturity?

ii) What is the interest rate differential $r_{USD} - r_{EUR}$ for maturity $6M$ implied by the quoted forward FX rate?

$X(0) = \#\ USD \rightarrow EUR = 1.1$

$$X_{fwd} = \frac{D_{EUR}(T)}{D_{usd}(T)} X(0)$$

$$X_{fwd} = e^{-(r_{EUR} - r_{USD})T} X(0)$$

i) @Maturity

Contract $1M \times 1.15 = 1,150,000$

Market $1M \times 1.175 = \$1,175,000$

Gain: $1.175M - 1.15M = +\$25K$

ii) $$X_{fwd} = \frac{1 + r_{USD} \times T}{1 + r_{EUR} \times T} \times X(0) = \frac{1.15}{1.1} = \frac{1 + 0.5 r_{USD}}{1 + 0.5 r_{EUR}} = 1.04545$$

$$r_{USD} - r_{EUR} = \frac{X_{fwd} - X(0)}{X(0)} \times \frac{1}{T} = \frac{1.15 - 1.1}{1.1} \times \frac{1}{1/2}$$

$$\cancel{0.5}(r_{USD} - r_{EUR}) = \frac{1.15 - 1.1}{1.1} = \frac{0.04545}{0.5}$$

$r_{USD} - r_{EUR} = 0.0909$

9.09% per annum