Homework #1: Interest rates, bonds and FX

FE-620 Spring 26

Due 8 February 2026

Problem 1.1
Assume that the spot price of gold is \$5,050 per ounce. Suppose that the market quotes a forward contract on gold with delivery in 1 year at \$5,200.00. The risk-free interest rate is $r = 3.5\%$.

i) Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage. What is the profit we can expect to make?

ii) Consider now the case when the forward price is \$5,300. Is there an arbitrage opportunity? If yes, describe the steps required to realize the arbitrage.

Solution.

The fair forward price is $F_*(T) = S_0 e^{rT} = 5229.88$ (with continuous compounding), or $F_*(T) = S_0(1+r)^T = 5226.75$ (with simple compounding). We will use the continuous compounding result in the following.

i) For this case the market forward price $F(T) = 5200$ is smaller than either of them, so a trading strategy which realizes the arbitrage is as follows:

At time $t = 0$, take a long position in the forward contract. Short the gold: this requires that we borrow the gold (the strategy assumes that gold can be shorted) and sell it for S_0. Put this cash in the bank.

At maturity receive the gold (from the forward contract) and return it to the gold lender. Pay 5200 to the forward contract short side. We have $S_0 e^{rT} = 5229.88$ in the bank, from which we pay 5200 to the forward contract short side. We have \$29.88 left over, which is the profit of the strategy.

ii) For this case the market forward price is larger than the fair forward price. An arbitrage strategy can be realized by reversing the actions in the previous case.

Take the short side in the forward contract. Borrow S_0 from the bank, and use it to purchase gold.

At maturity, deliver the gold, and receive 5300. Pay the bank the borrowed amount plus interest $S_0 e^{rT} = 5229.88$ and we still have something left over.

Net profit is $F(T) - F_*(T) = 5300 - 5229.88 = 70.12$.

Problem 1.2
Assume that an interest rate is quoted as 3.25% with semi-annual compounding. What is the rate when expressed with:
i) annual compounding.
ii) quarterly compounding?
iii) continuous compounding.

Solution. Denote r_n the interest rate with discrete compounding and frequency n, and r_c the interest rate with continuous compounding.

We are given $r_2 = 0.0325$. Recall that we can convert to discrete compounding rates with any frequency m using the formula

$$e^{r_c} = \left(1 + \frac{r_m}{m}\right)^m$$

i) Take $m = 1$. We get $r_1 = (1 + r_2/2)^2 - 1 = 0.0327641$ or 3.276%
ii) Take $m = 4$. We get $r_4 = 4[(1 + r_2/2)^{1/2} - 1] = 0.032369$ or 3.237%
iii) Continuous compounding rate. We have $r_c = 2\ln(1+r_2/2) = 0.03224$ or 3.224%

Problem 1.3
Credit card companies quote the APR on the outstanding balance. APR means Annual Percentage Rate and is the interest rate with annualized compounding. Typical APRs are shown in the Table below. However, credit card interest rate is compounded *daily*, for 365 days a year.

Suppose the balance on a credit card is \$10,000. What is the total balance including interest after 30 days, if the customer has Excellent Credit?

Solution. The total balance after 30 days is obtained by compounding the initial amount \$1000 with daily compounding at the daily rate

$$r_{365} = 365[(1 + 0.1715)^{1/365} - 1] = 0.158319 \tag{1}$$

which is thus 15.832%.

The total balance is

$$10,000(1 + r_{365}/365)^{30} = 10,130.90\,. \tag{2}$$

In practice credit card companies approximate this as $(1 + APY/365)^{30}$ but this is not exactly correct. Using this formula would give 10,146.90, which is slightly larger than the correct result.

Table 1: Current Credit Card Interest Rates. As of Jan 2026, from `https://wallethub.com/edu/cc/credit-card-landscape-report/24927`.

Category	Interest Rate
Excellent Credit	17.15%
Good Credit	23.33%
Fair Credit	26.55%
Student Credit Cards	19.04%

Problem 1.4

We are given the zero rates $R(T)$ for several maturities in Table 2. These zero rates apply for all maturities in the ranges shown.

1) Price a bond B_1 with annual coupon 5.0% paid twice a year with maturity 2Y.

2) Price a bond B_2 with annual coupon 6.0% paid twice a year, with maturity 10Y.

3) Compute the bond yields of the two bonds in points 1) and 2).

Solutions.

We use discounted cash flow valuation and use the zero rates from Table 2 for computing the discount factors $D(T) = e^{-R(T)T}$.

1) The price of the 2Y bond is $B_1 = 102.831$

2) The price of the 10Y bond is $B_2 = 111.843$

3) The bond yields y are computed such that the bond prices computed above are reproduced when computing them with constant zero rate equal to y, as $B = \sum_{i=1}^{n} C_i D(T_i)$ with C_i the cash flow paid at time T_i.

The yield of the bond B_1 is $y_1 = 3.491\%$, and the yield of the bond B_2 is $y_2 = 4.465\%$.

Table 2: Data for Problem 1.4.

T	$R(T)$
[0,1Y]	3.00%
(1Y,2Y]	3.50%
(2Y,5Y]	4.25%
(5Y,10Y]	4.50%

Problem 1.5

A US company is due to make a payment of 1.0m Euros in 6 months. They plan to hedge this payment by taking a long position in a forward contract for 1.0m Euros with maturity 6 months, at a forward exchange rate 1.150. The current EUR/USD rate is $X_0 = 1.100$, and the actual exchange rate realized at maturity is $X(6M) = 1.175$.

i) What is the gain or loss of the company at maturity?

ii) What is the interest rate differential $r_{USD} - r_{EUR}$ for maturity $6M$ implied by the quoted forward FX rate?

Solution.

i) At maturity the company pays $1.0m \times 1.150 = 1.15m$ USD for 1.0m EUR. If they were purchasing the 1.0m EUR at the realized exchange rate at maturity they would have paid $1.0m \times 1.175 = 1.175m$ USD which is more. The realized gain is

$$1.0m \times (1.175 - 1.150) = 0.025m \text{ USD} \tag{3}$$

ii) The forward EUR-USD FX rate depends on the interest rate differential as

$$X_{fwd}(T) = X(0)e^{(r_{USD} - r_{EUR}) \cdot T} \tag{4}$$

Substituting here $X_{fwd}(T) = 1.150, X(0) = 1.100$ gives

$$r_{USD} - r_{EUR} = \frac{1}{0.5} \ln \frac{1.150}{1.100} = 0.0816 \tag{5}$$

which is 8.890%.