Lab 2: Fourier Methods

September 20, 2021

1 Introduction

1.1 Background Reading

1. "Fourier Methods Instructor's Manual", TeachSpin, Inc.

2. "Spectrum and Spectral Density Estimation by the Discrete Fourier transform (DFT), Including a Comprehensive List of Window Functions and Some New Flat-Top Windows", G. Heinzel, A. Rüdiger, and R. Schilling (2002).

3. "Measurements and their Uncertainties", Ifan G. Hughes and Thomas P. A. Hase.

1.2 Motivation

The purpose of this lab is for you to become familiar with thinking in the frequency (Fourier) domain. While working with signals in the time domain may seem more intuitive, a wide range of technologies, including wireless communications (radio, television, WiFi, cellular phones, etc), radar, fiber optics, precision clocks, etc. are often more easily understood and measured in the frequency domain.

This lab includes several "model" systems for you to explore. The frequency-domain techniques you learn with these experiments will be used for the remainder of the semester with more complex physics systems.

2 Objectives

These objectives are intended to guide your experiments. They are not a step-by-step procedure.

1. Introduction to spectrum analyzers. View signals on an oscilloscope and spectrum analyzer; record both. Use a FFT to transform the oscilloscope data to the spectral data using numpy or matlab (as well as you can). Explain the effect of the frequency resolution (measurement time), windows, and linear versus log scale. Give a mathematical description of the origin of the frequency domain signatures you observe for each signal.

   (a) Sine, square, and triangle waves. For the sine wave, record data with both a "nice" frequency (such as 10,000 Hz) and a more random frequency (such as 10,003 Hz), each with a "uniform" and "Hanning" window. Explain your results.

   (b) Sum of two sine waves (using "Summer" module). Test and explain the conditions under which you can and cannot resolve the two peaks in the frequency domain.

   (c) Amplitude modulated wave (using "Multiplier" module).

(d) Sine wave buried in noise (using “Buried Treasure” module).
(e) OPTIONAL: Frequency modulated wave (using “Voltage Controlled Osc.” module).
(f) OPTIONAL: Mixed signals (using “Audio Mixer” module).

CHECKPOINT: Show one of the instructors your data plotted in the analysis software of your choice before proceeding.

2. LRC filter: Measure the response (transfer function) of an LRC filter by several methods and compare the results:
   (a) Use a single-frequency drive as an input and an oscilloscope and lock-in amplifier to measure the output at several frequencies.
   (b) Use a noise drive as the input and the spectrum analyzer to measure the output.
   (c) Use the Arduino-based network analyzer to sweep over a wide range of frequencies with fine steps to get the amplitude and phase response.
   (d) Measure the transient response using a square wave. You will take the Fourier transform of this to get the transfer function.

   CHECKPOINT: Show one of the instructors your data plotted in the analysis software of your choice before proceeding.

3. Acoustical cavity: Measure the response (transfer function) of the acoustical cavity by several methods and compare the results:
   (a) Use a single-frequency drive as an input and an oscilloscope and lock-in amplifier to measure the output at several frequencies.
   (b) Use a noise drive as the input and the spectrum analyzer to measure the output.
   (c) Use the Arduino-based network analyzer to sweep over a wide range of frequencies with fine steps to get the amplitude and phase response.
   (d) Measure the transient response using a square wave. You will take the Fourier transform of this to get the transfer function.

3 Questions

These questions should be specifically answered in your lab notebook, and can also serve as a guide for discussion in your lab report analysis.

1. How does the SRS spectrum analyzer normalize the signals it displays?
2. Use Fourier transforms and plots to show that the different driving and detection methods give compatible results with both the LRC circuit and acoustical cavity.
3. In which measurements did you find frequency domain analysis to be most useful? Could you do or interpret these experiments in the time domain alone?

4 Hints

1. Don't forget the basic relationships in Fourier transforms:
   - Time step: Δt
   - Total time of record: t_{max}
   - Frequency step: Δf
   - Maximum frequency: f_{max}

   $\Rightarrow \quad \Delta f = \frac{1}{t_{max}} \neq \frac{1}{\Delta t} \quad$ and $\quad f_{max} = \frac{1}{2\Delta t}$

2. The SRS spectrum analyzer needs a specially-formatted flash drive and file names should be limited to 8 characters. The file names will not retain the case of letters.
3. Aim for getting your signal of interest in the center of the frequency range, both on the spectrum analyzer and oscilloscope data. Note that this may not mean you should use the same time span for each.
4. Check the data file from the oscilloscope to see how many points it actually gives you over the time window shown. This will be important for determining the maximum frequency you will get from an FFT of the oscilloscope data.
5. The time sample recorded (the total horizontal axis range) on the oscilloscope will determine the frequency resolution you get from the FFT of the oscilloscope data.
6. Check your spectrum analyzer and oscilloscope data files before moving to the next step - they have both been known to have problems!
7. The oscilloscope may not have an anti-aliasing filter.
8. The noise floor of the oscilloscope is probably much higher than the spectrum analyzer. You will probably see this after taking the FFTs.
9. The SRS spectrum analyzer can be set to give an amplitude spectrum or power spectral density (PSD). Refer back to Lab 0 for a power spectral density example.
10. In the transient measurements, try to choose an square wave period and oscilloscope horizontal scale so you get all of one (and only one) transient response on the screen.
11. Don't worry about getting the amplitudes to match with the different methods in parts 2 and 3, but do compare the overall shapes and phases as a function of frequency. The easiest way is to plot the different methods together, with arbitrary vertical scaling as needed to get them to match in height.
12. If your results from different methods don't match in any part of the lab, don't worry about it. Just explain what you could have done to make them more comparable.
13. Error analysis is not very important in this lab – it is all about getting comfortable with Fourier transforms.