1 Introduction and Motivation

GDP, which stands for Gross Domestic Product, is widely used as a reference point for the health of national and global economies. When GDP is growing, especially if inflation is not a problem, workers and businesses are generally better off than when it is not. GDP measures the monetary value of final goods and service. That is, the counts of all output generated within the borders of a country in a specific time period - monthly, quarterly or annually.

The goal of this analysis is to provide a realistic forecast based on latest available data to reflect the current state of the economy of Switzerland, supported by information in the study providing an adequate justification for economic growth and trends in standards of living over time.

The aim here is to develop a model that can accurately predict the economic growth rate of Switzerland using the dataset that is available from the dataseries.org website.

2 Data source identification and description

Data Range : From 01-01-1980 to 04-01-2019.
Datasource Description : The dataset contains details of quarterly GDP of Switzerland. It is a non-federal data set downloaded from the dataseries.org website. The dataset consists of 158 rows and 2 columns which contains the following variables.
Time : This is the variable used as the date variable for the time series analysis.
GDP : This variable is used to represent the quarterly output generated within the borders of Switzerland.

3 Time Series Analysis

We perform exploratory data analysis for the dataset, then we shall select an appropriate model for forecasting. We then use the best possible model to predict GDP growth rate. Finally, we will predict for the next 24 quarters ahead using the data from Jan - 1980 to April - 2019 with 95% confidence interval.

```
p-value smaller than printed p-value
        Augmented Dickey-Fuller Test

data:  diff(log(swiss.gdp))
Dickey-Fuller = -4.1639, Lag order = 5, p-value = 0.01
alternative hypothesis: stationary
```

Figure 1: Augmented Dickey-Fuller (ADF) Test

4 Stationarity Test of Growth Rate

Here, we examine the stationarity of the growth rate of GDP. We study this because, we observe an upward trend in the original GDP data. To be certain that the growth rate of GDP is stationary we perform an Augmented Dickey-Fuller (ADF) Test with the following hypotheses:

$$H_o : \text{The growth rate of GDP has a unit root}$$
$$H_a : \text{The growth rate of GDP is stationary}$$

We observe a smaller p-value, one that is less than the significance level of 5%. Thus, we have enough evidence to reject the null hypothesis that the series has a unit root and conclude that the growth rate of Swiss GDP is stationary.

5 Dependence Order and Best Model Fit

We observe that almost all the PACF at the lags lie in the confidence interval with the exception of a significant PACF at lag 1. This indicates the growth rate of GDP data is a reasonably stationary series without even examining the Augmented Dickey-Fuller (ADF) Test.
From the plots, we observe that the ACF tails off but PACF cuts off after lag 1 which suggests the growth rate of GDP follows an *AR(1) or ARMA (1, 0, 0)* .

We now fit the AR(1) model to the growth rate of GDP and observe the results. The $AR(1)$ parameter estimate is 0.5168 (with standard error of 0.0678). This parameter is significant because its p-value (0) is less than the significance level of 5%. The estimated white noise variance is 2.519e-05. Also, the results of the numerical iterations converges and there are no warning signs. This gives us some confidence that $AR(1)$ may be the best model but we continue our analysis by investigating the residuals.

```
Call:
stats::arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D,
    Q), period = S), xreg = constant, transform.pars = trans, fixed = fixed,
    optim.control = list(trace = trc, REPORT = 1, reltol = tol))

Coefficients:
         ar1  constant
      0.5168    0.0043
s.e.  0.0678    0.0008

sigma^2 estimated as 2.519e-05:  log likelihood = 608.31,  aic = -1210.62

$degrees_of_freedom
[1] 155

$ttable
         Estimate     SE t.value p.value
ar1        0.5168 0.0678  7.6230       0
constant   0.0043 0.0008  5.2109       0

$AIC
[1] -7.710982

$AICc
[1] -7.710486

$BIC
[1] -7.652582
```

Figure 2: Fitting ARMA(1, 0, 0) to Growth Rate of GDP

6 Residual Diagnosis

No obvious patterns are observed by inspecting the time plot of the standardized residuals of the growth rate of GDP. Well, there are a few outliers because a few standardized residuals exceed 3 standard deviations in magnitude. However, there are no values that are exceedingly large in magnitude.

We also observe from the Normal Q-Q plot of the Residuals that eventhough there are a few outliers (especially, one large outlier), it still suggests that the assumption of normality of the residual is not unreasonable.

From the sample autocorrelations of the residuals, all ACFs of the model lie in the confidence interval. This indicates that there is no apparent departure from the model assumptions.

We also perform a **Test of Whiteness (Using Ljung-Box-Pierce Q-statistic)** : The graph shows that the p-values for the tests based on the lags $H = 2$ through $H = 20$ (with their corresponding degrees of freedom). The dashed horizontal line on the bottom indicates 5% significance level of the test.

In a nutshell, since all the p-values of the $AR(1)$ model exceed .05, we do not have enough evidence to reject the null hypothesis that the residuals are white and conclude that the standardized residuals are white.

7 Forecasting

The forecast plot indicates a forecast of 24 quarters ahead. We observe that as time passes the growth rate of the GDP of Switzerland approaches the average growth rate of about 0.43% per quarter and does not fluctuate as it use to in the past.

In other words, the growth rate of GDP of Switzerland will be steady for the next 6 years (24 quarters). Below is the predictions of the growth rate of the GDP of Switzerland:

```
$pred
            Qtr1        Qtr2        Qtr3        Qtr4
2019                         0.003559084 0.003922399
2020 0.004110166 0.004207207 0.004257359 0.004283278
2021 0.004296674 0.004303597 0.004307175 0.004309024
2022 0.004309980 0.004310474 0.004310729 0.004310861
2023 0.004310929 0.004310964 0.004310982 0.004310992
2024 0.004310997 0.004310999 0.004311001 0.004311001
2025 0.004311002 0.004311002
```

Figure 3: Growth Rate Predictions for coming years

8 Spectral Analysis

The purpose of estimating the spectral density is to detect any periodicities in the growth rate of GDP by observing peaks at the frequencies corresponding to these periodicities.

Here, we examine both non-parametric (Periodogram) and parametric (AR - estimator) spectral density estimation techniques to observe how well each method examines periodicities in the series.

8.1 Non Parametric Estimator - Periodogram

The non-parametric approaches like the periodogram explicitly estimate the covariance or the spectrum of the process without assuming that the process has any particular structure.

After plotting the periodogram and printing a sample of generated results, we observe predominant periods (indicated by dashed lines) at 10 year cycle (with spectrum of 1e - 4), approximately 6 year cycle (with spectrum of 0), approximately 3 year cycle (with spectrum of 1e - 4) and a 2 year cycle (with spectrum of 1e - 4).

All these cycles appear as predominant periods when you take a quick glance at the periodogram, most of them are actually harmonics because they vanish after smoothing. Only the approximately 6 year cycle turns out to be a predominant period at the frequency of 0.175.

8.2 Parametric Estimator - AR Spectral Estimator

The periodogram as an estimator is susceptible to large uncertainties. This is attributed to the fact that the periodogram uses only two pieces of information at each frequency no matter how many observations are available. To cater for the shortcomings of the Non-Parametric Estimator, we use a parametric technique in which we obtain the spectral density estimator by fitting an AR(ρ) to the data where the order ρ is determined by one of the model selection criteria, such as AIC, AICc and BIC.

We observe that both the AIC and BIC are very definite about the model it chooses. The minimum AIC/BIC is very distinct (at $\rho = 1$). This goes to show that the model AR(1) is the best model fit selected by AIC, AICc and BIC.

9 Figures

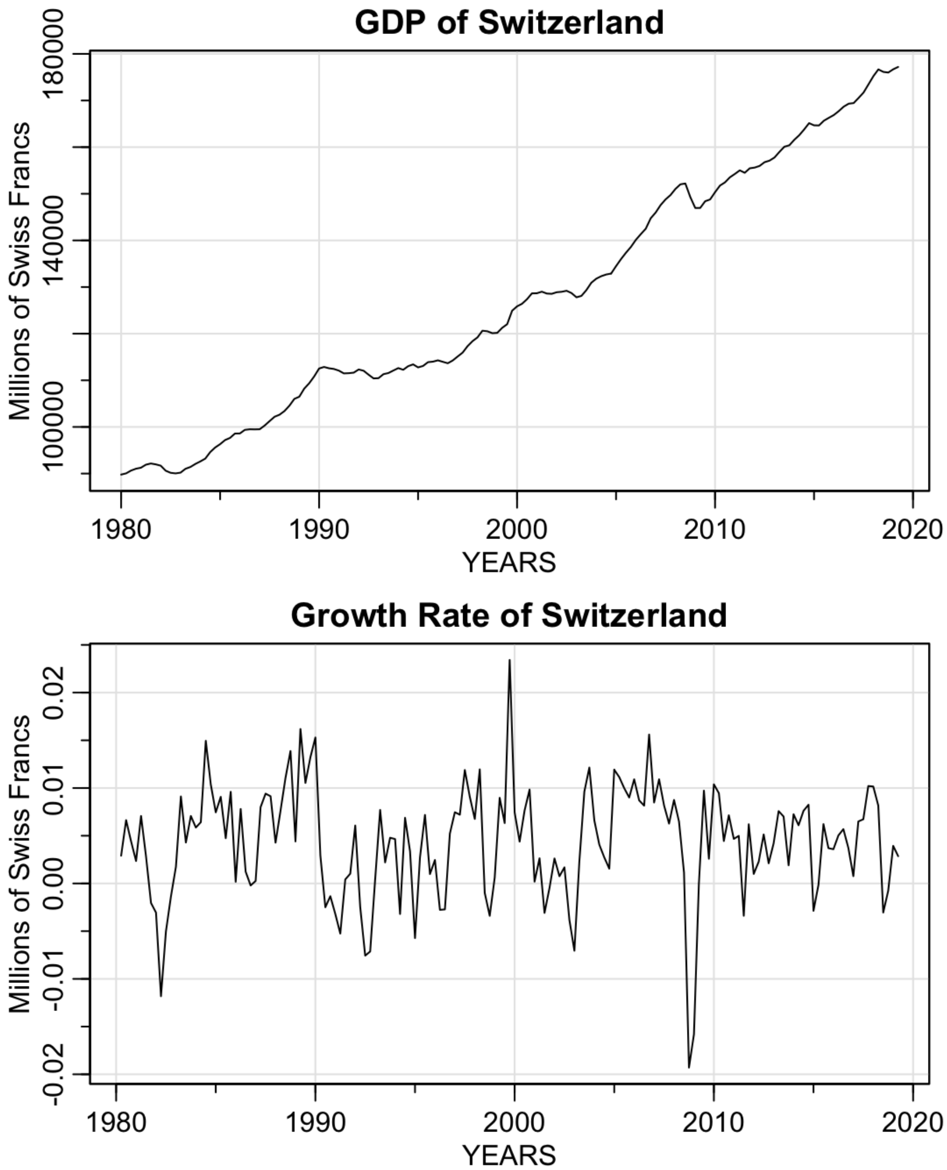

Figure 4: Plots of original GDP data and Growth Rate of GDP

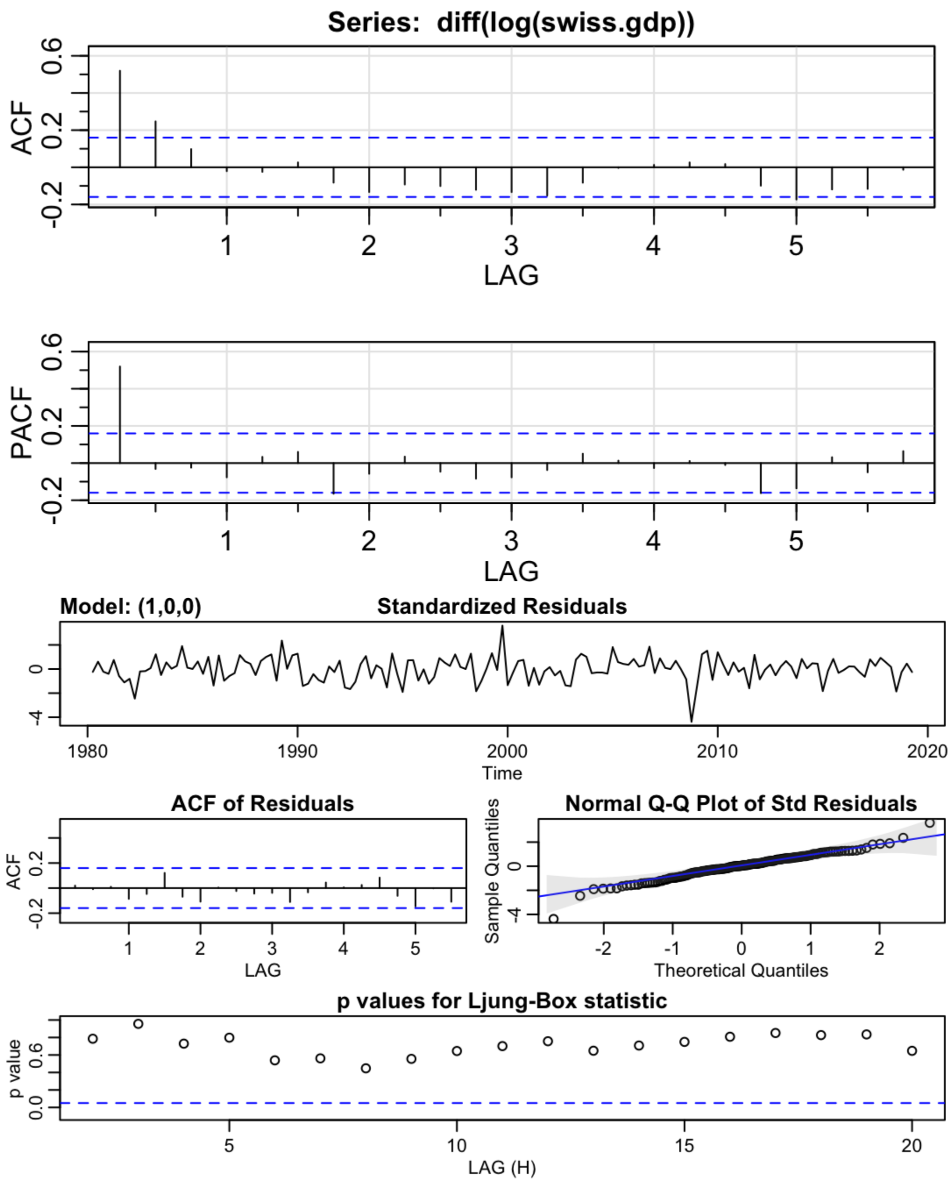

Figure 5: ACF and PACF of Growth Rate of GDP

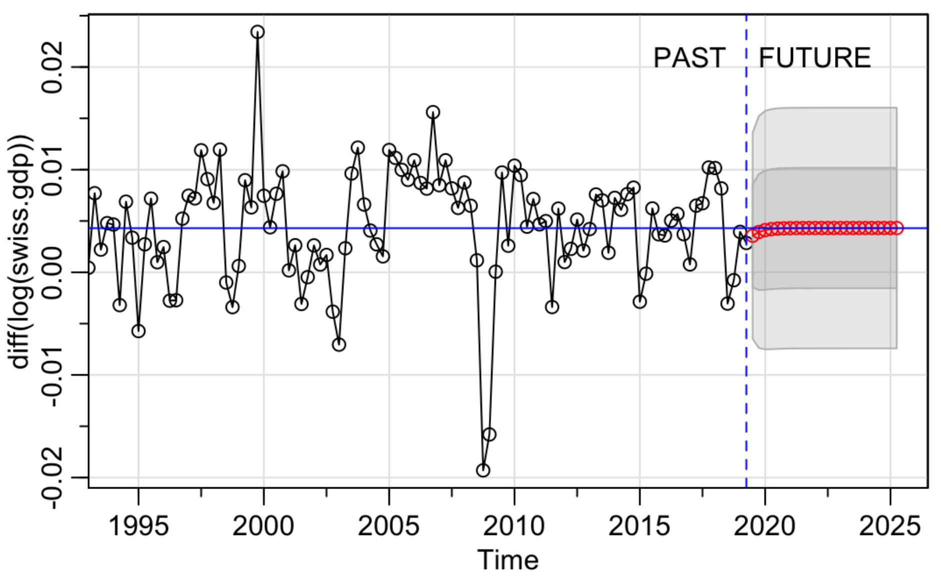

Figure 6: 6-year Forecast of Growth Rate of GDP of Switzerland

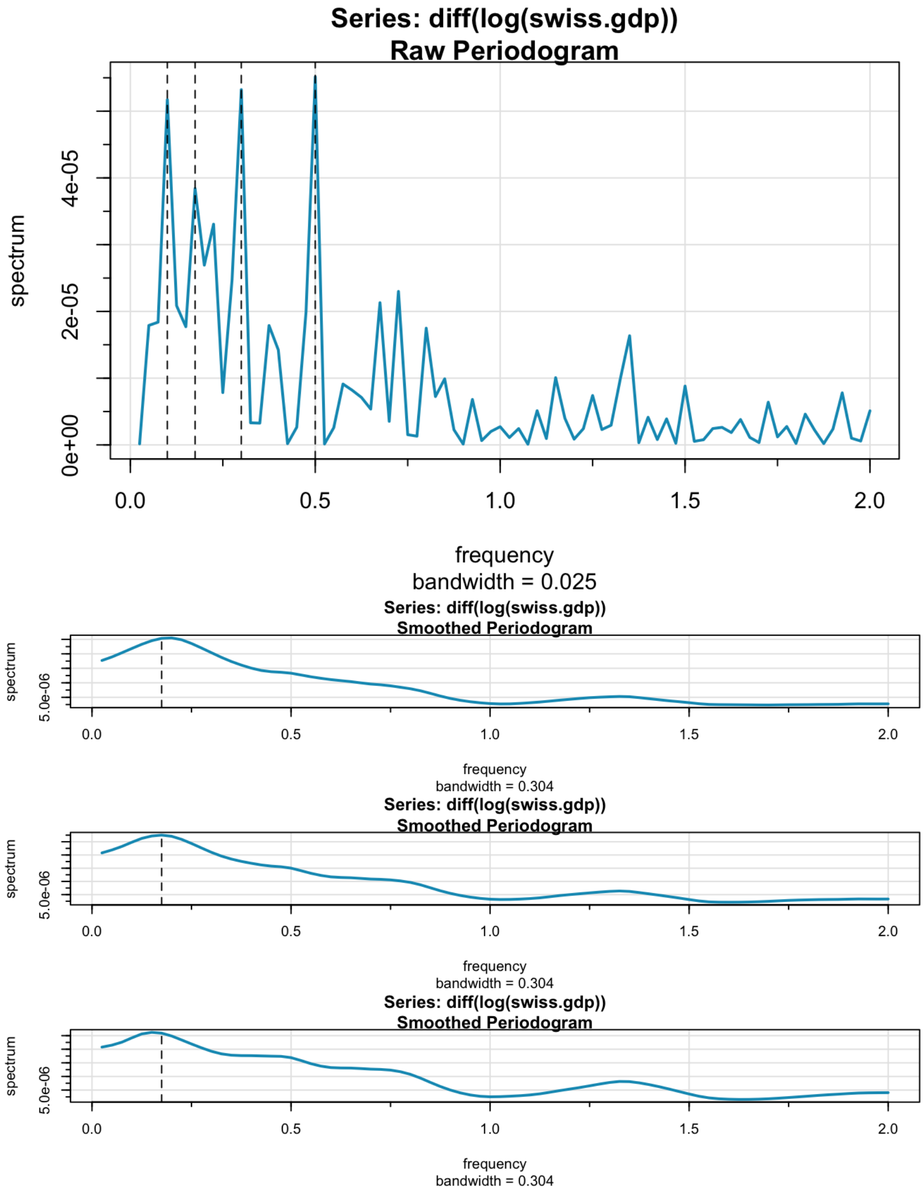

Figure 7: Raw and Smoothed Periodogram of Growth Rate of GDP

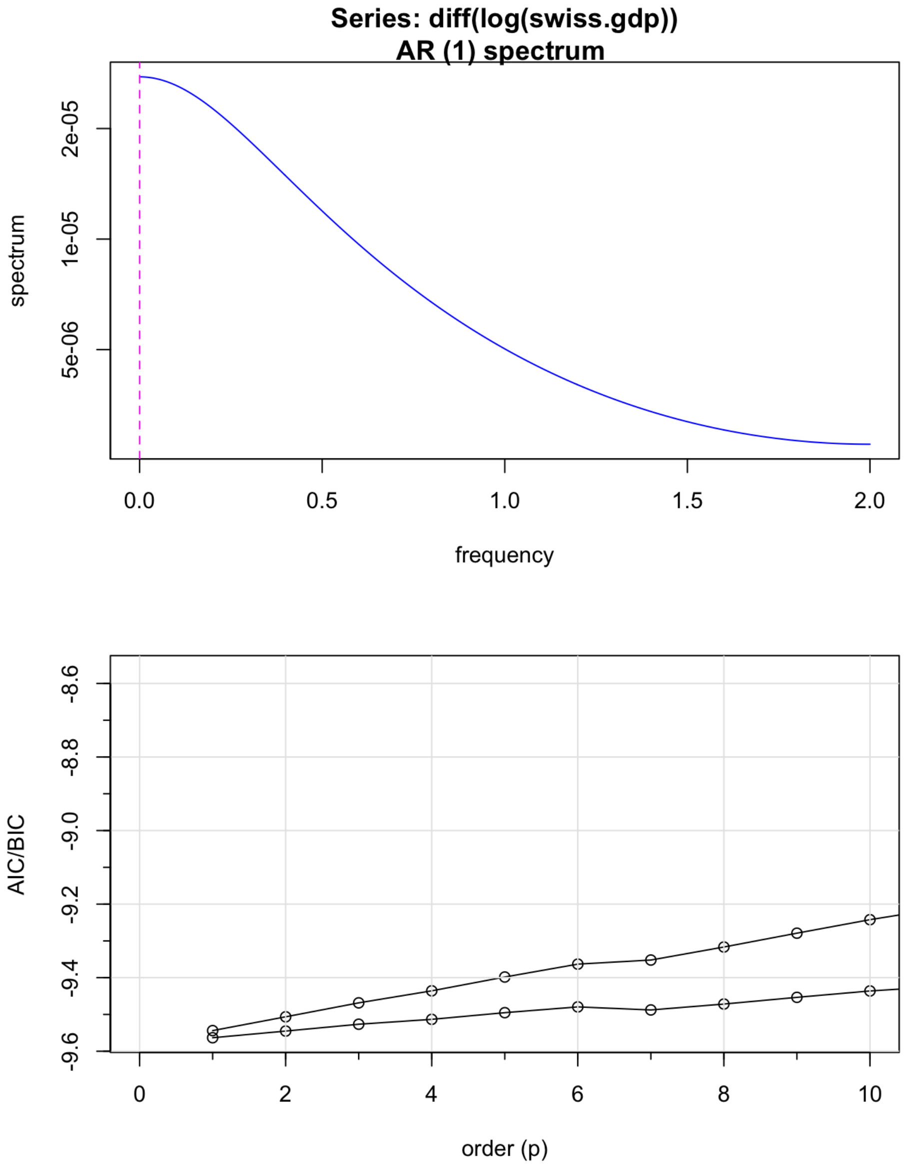

Figure 8: AR Fit and AIC/BIC of Growth Rate of GDP