Viscous and Coulomb Damping of Bicycle Wheel Pendulum Using Dashpot

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Group #2

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Experimental Set-Up

Our experiment was constructed based on the dashpot we were provided by Professor Mansour. It was an Airpot Dashpot Model 2KS160 with a 0.627" bore diameter and 2" long stroke. For the driving element we used an 6061 T6 aluminum rod attached to the bicycle wheel with Micro Measurements tape. The dashpot was anchored a 1/16" thick 6061 T6 Aluminum Alloy plate that was cantilevered in a vise.

Figure 1: 1/16" Thick 6061 T6 Aluminum Alloy Plate Dimensions with Strain Gage Location

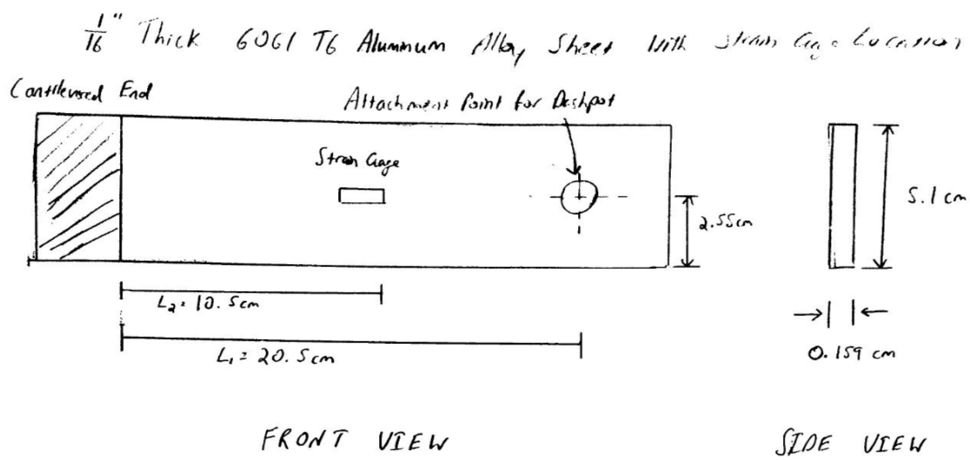


Figure 2: Airpot Dashpot with Manufacturer's Specifications

Stock Dashpot Model 160

MODEL: 2KS160

1. **Damping Coefficient**

0-5 lb/(in/s) adjustable

0-0.88 N/(mm/s)

2. **Force Guidelines**

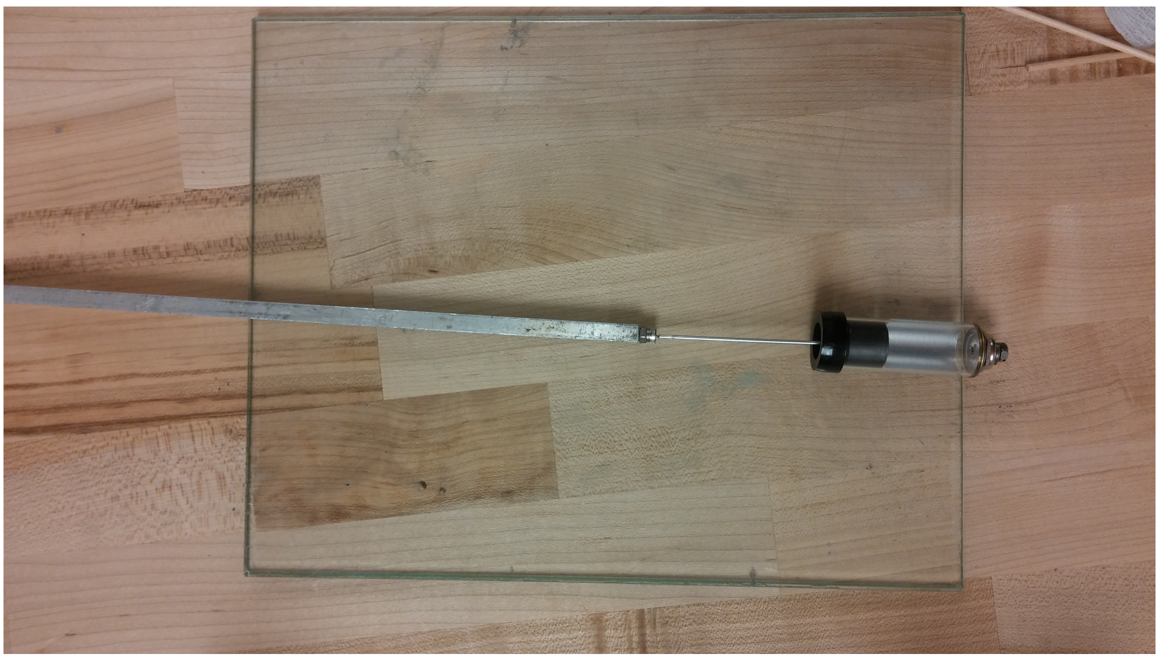
Pull Damping: 4 lb max (18 N)

Push Damping: 3 lb max (13 N)

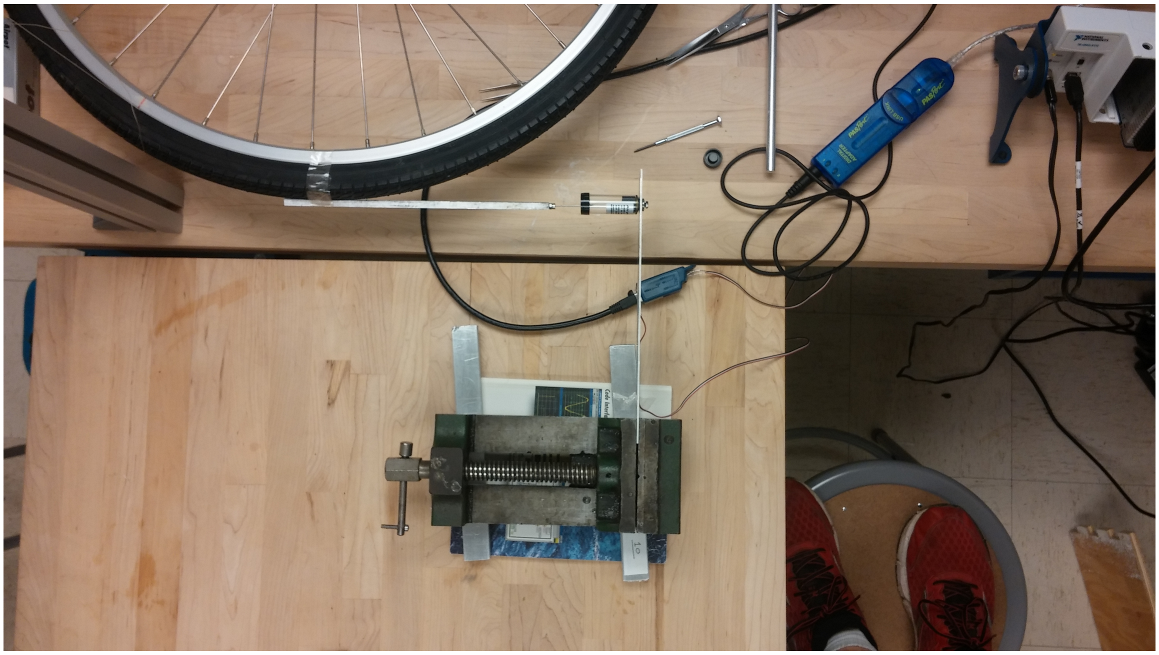
3. **Friction**

Coefficient: .2

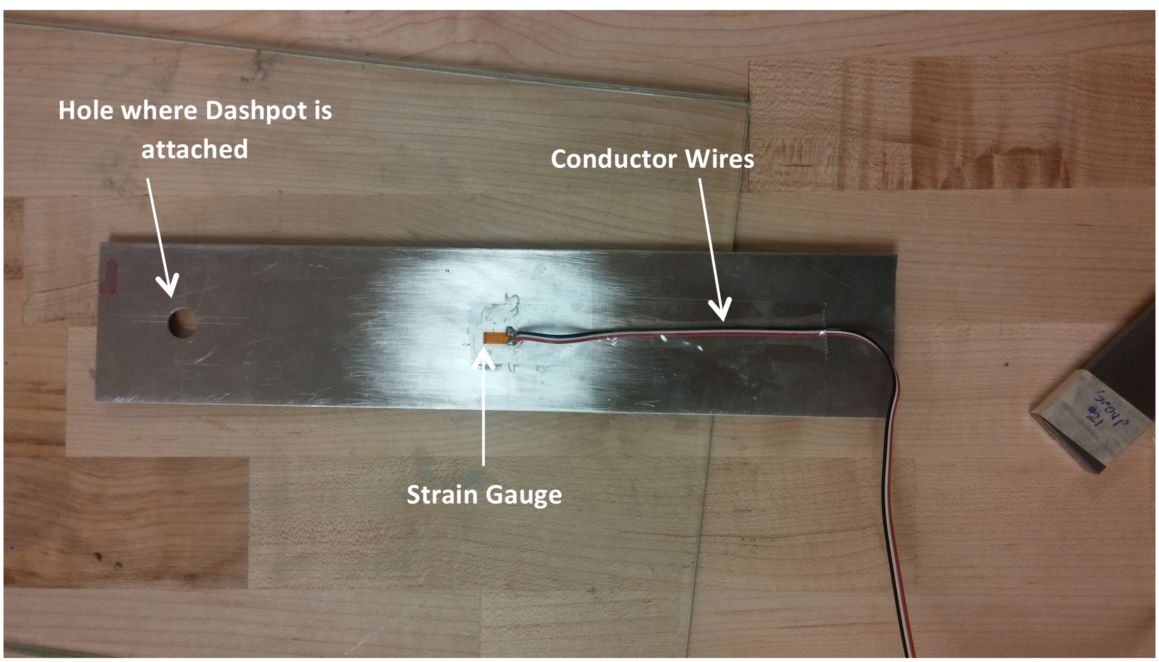
Force without side load: < 1g



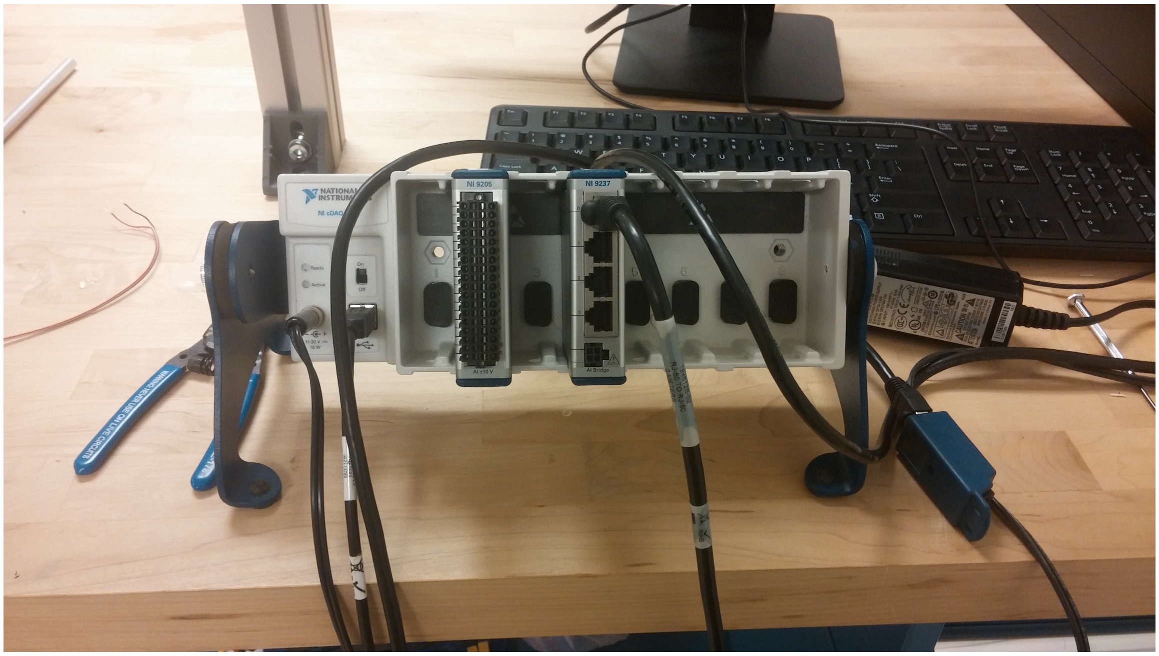
To measure the damping forces we used Vishay Precision Group Micro Measurements Strain Gage (item number EA-13-240LZ-120/E) and connected it to a 1/16" thick 6061 T6 Aluminum Alloy Sheet with Micro Measurement 3 conductor wire 26 AWG. We decided to use this relatively thin sheet of aluminum in order to measure the bending moment caused by damping, which we can use to analyze the force of our damping.



At first we attached one strain gauge in the middle of the aluminum alloy sheet and connected the wires to a National Instruments quarter bridge completion module (item number NI 9944). The quarter bridge completion module was connected via a NI RJ-50 connector to the bridge analog input card in the DAQ chassis.

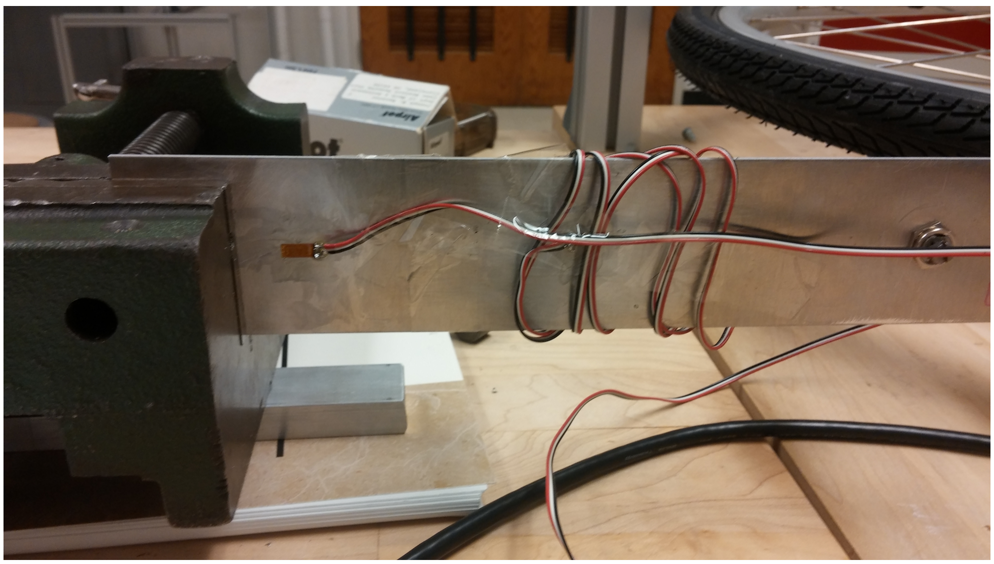


The bridge analog input card was a NI 9237 installed in the DAQ chassis. The DAQ (Data Acquisition System) was a National Instruments cDAQ-9172 where it was connected via USB cable to the computer.



In order to measure the period we taped a thin piece of wood to the bicycle wheel underneath a Pasco Scientific Photogate (item number ME-9204B).

When the bicycle wheel oscillates the piece of wood will also oscillate back and forth and the period will be read by the photogate. The photogate was connected to a Pasco digital adapter with a USB link that was connected to the computer. The software we used to acquire our data for the photogate was Pasco Capstone software. The software used for the strain gage attached to the DAQ was National Instruments Labview 2014. After testing our system we weren't getting extremely readable data, so we initially thought we could get better data if we placed our strain gauge at a different position on the sheet. We attached a strain gauge close to the cantilevered end of the sheet since the max bending moment occurs at that position.



However the strain gauge at the fixed end was not reading accurate data. We consulted with Michael Butler, and he was not able to provide a remedy for the problem. Therefore we decided to use data from the single strain gage attached at approximately the center of the beam. The picture above shows the strain gage at the cantilevered end that was not functioning.

Expected Damping Calculations

The dashpot we decided upon for this lab was the Airpot 2KS160 with a 15.93 mm bore size and a 50.8 mm stroke length. The damping force for this dashpot is proportional to velocity meaning that the pendulum is experiencing viscous damping. However, because the friction coefficient of the dashpot is not zero, we expected to have some amount of friction, or Coulomb damping. Coulomb damping dissipates energy constantly, and is independent of surface area or velocity and the frictional force always opposes the motion of the oscillator.

We therefore needed to calculate both the expected friction damping and the expected one-way viscous damping.

We were not sure where to begin, so we started off with an energy approach. The pendulum has maximum potential energy at its furthest displacement, and the kinetic energy at this point is 0:

$$U_{max} = mgl(1 - \cos\theta_{max})$$

Where U is the potential energy, m is the mass of the bicycle wheel, l is the length of the pendulum string, and $\boldsymbol{\theta}_{\text{max}}$ is the maximum displacement in degrees of the string from the vertical. In the lab, we measured the maximum displacement of our set-up to be 8.6 degrees. Therefore, the maximum energy of the pendulum is:

$$U_{max} = 1.689\ kg * \frac{9.81m}{s^2} * 0.457\ m\ \left(1 - \cos\left(7.8 * \frac{\pi}{180}\right)\right) = 0.0705\ J$$

Next, we wanted to figure out how much damping force we would need to completely stop the pendulum in a single push stroke of the dashpot. The work done by the dashpot can be compared to the maximum energy of the pendulum to find the required force to bring the pendulum to rest. The bore length of the dashpot is 0.0508m but we determined that the piston can only move a distance of 0.0444 m in the bore cylinder :

$$U_{max} = F * d, \qquad F = \frac{0.0705\ J}{0.0444m} = 1.588\ N$$

This preliminary calculation showed that we needed a total damping force (viscous plus Coulomb damping) of 1.588 N to bring the pendulum to rest within a single push stroke of the dashpot, or in other words, within a half cycle of the pendulum. This value also represents the force that will cause critical damping and will bring the pendulum to rest the quickest. From this, we decided to run one of our trials at a force above this value, to see if we could observe overdamped behavior, one of the trials at this value to achieve critical damping, and two trials below this value to demonstrate underdamping. We looked at the Manufacturer's Data for the Airpot 2KS160 dashpot given to us by Professor Mansour, and saw that it has a maximum damping force on the push stroke of 13 N and a minimum of 0 N, so it was acceptable for the experiment.

This brief calculation gave us the basis for our expected damping force without having to deal with the problem of friction damping and the issue that the dashpot displays viscous damping on the push stroke only. However, we then had to deal with friction damping. As we were dealing with low speeds, and the friction that was damping the pendulum was only between the walls of the cylinder and the piston, we decided to treat the friction force as a constant, independent of velocity. We next tried to develop a model for experimental set-up that would account for both the Coulomb and viscous damping. We first needed an idea of the magnitude of the friction in relation to the viscous damping. Once again, we looked at the Manufacturer's Specifications and saw that the friction force without a side load was 1 gram or 0.01 N. Based on this information and the fact that we expected the viscous damping to be two orders of magnitude greater than 0.01 N, we decided to ignore the friction damping in our preliminary calculations for the push stroke. The pull stroke would have only friction damping. We expect to tell whether or not these assumptions are valid based on the experimental results.

We started our derivation like many other analyses with Newton's Second Law, F=ma.

For a simple pendulum, this can be written as

$$-\frac{mg}{l}\sin\theta = m\ddot{\theta}$$

For small angles sin(θ) = θ. When the viscous damping force is added in, this becomes:

$$-\frac{mg}{l}\theta - b\dot{\theta} = m\ddot{\theta} \quad \Rightarrow \quad \ddot{\theta} + \frac{b}{m}\dot{\theta} + \frac{g}{l}\theta = 0$$

Where b is the damping coefficient (in N/m/s), and the viscous force is proportional to the velocity of the pendulum. The minus sign represents the physical fact that the viscous damping always acts in the opposite direction of the velocity. This is a linear second-order homogeneous differential equation and can be readily solved:

$$\theta(t) = \theta_0 e^{-\frac{bt}{2m}} * \cos(\omega_d t + \phi)$$

With ω being the damped frequency, and ϕ being a phase shift designed to satisfy the initial condition that the angular velocity, $\dot{\theta}$, is zero at time=0. This equation shows that the motion of the pendulum will eventually decay to zero based on the damping coefficient. The damped frequency can be expressed as a function of the natural frequency and the damping coefficient:

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

The natural frequency can be calculated from the undamped period as $\omega_0 = \frac{2*\pi}{T}$. Therefore, the critical damping coefficient is:

$$b = 2 * m * \omega_0 = \frac{2 * 1.689\ kg * 2 * \pi}{1.217s} = 17.44 \frac{N}{\left(\frac{m}{s}\right)}$$

We should be able to experimentally measure the damped period for the underdamped case to determine the damping coefficient.

To deal with the Coulomb damping occurring during the pull stroke, we again need to turn to Newton's Second Law:

$$\ddot{\theta} + \frac{g}{l}\theta = F$$

With F being the constant friction force equal to $\mu_k N$. Solving this differential equation yields an oscillating equation for the displacement that decrements by a constant amount every half cycle. This constant decrement is equal to:

$$\delta = \frac{2 * F * l}{mg} = \frac{2 * 0.01N * 0.457m}{1.689\ kg * 9.81 \frac{m}{s^2}} = 5.516\ E - 04\ m$$

As the piston has an initial displacement in the bore of 0.0222 m, we can therefore determine that it should take 40 half cycles, or 20 full periods to bring the wheel to rest with only Coulomb damping. However, because the wheel is undergoing both viscous and Coulomb damping, we expect that the wheel will be brought to rest in sooner than 20 periods.

To conclude from our preliminary calculations, we determined that the critical damping force is 1.588 N and the critical damping coefficient is 17.44 N/m/s. Based off of these calculations, we decided to run four separate trials, one overdamped, one critically damped, and two underdamped. Furthermore, we expect the Coulomb or friction damping to be a constant force that is trivial compared to the viscous damping force.

Results

For the results, Trial 1 is overdamped, Trial 2 is critically damped, and Trials 3 and 4 are underdamped. The "push" stroke refers to when the piston is contracting and is the viscous damped stroke in the dashpot. The "pull" stroke is when the piston is being pulled out of the bore cylinder.

Strain versus Time Raw Data Graphs

Figure 3: Strain versus Time for Run 1 of Trial 1

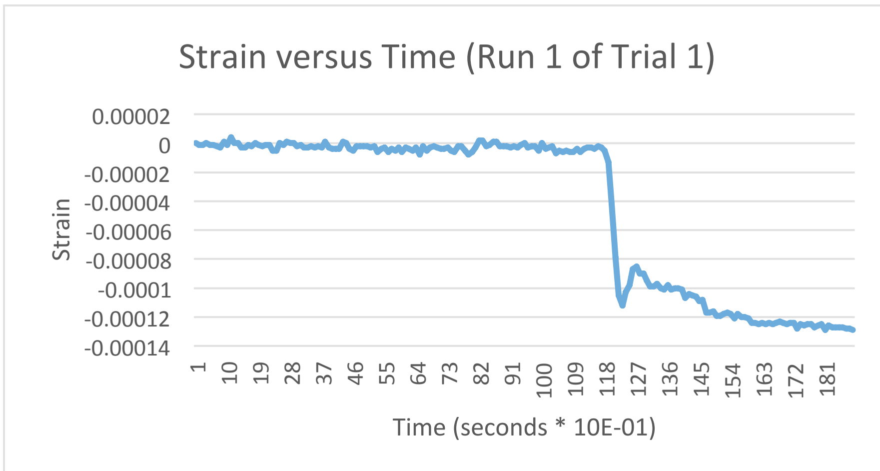


Figure 4: Strain versus Time for Run 2 of Trial 2

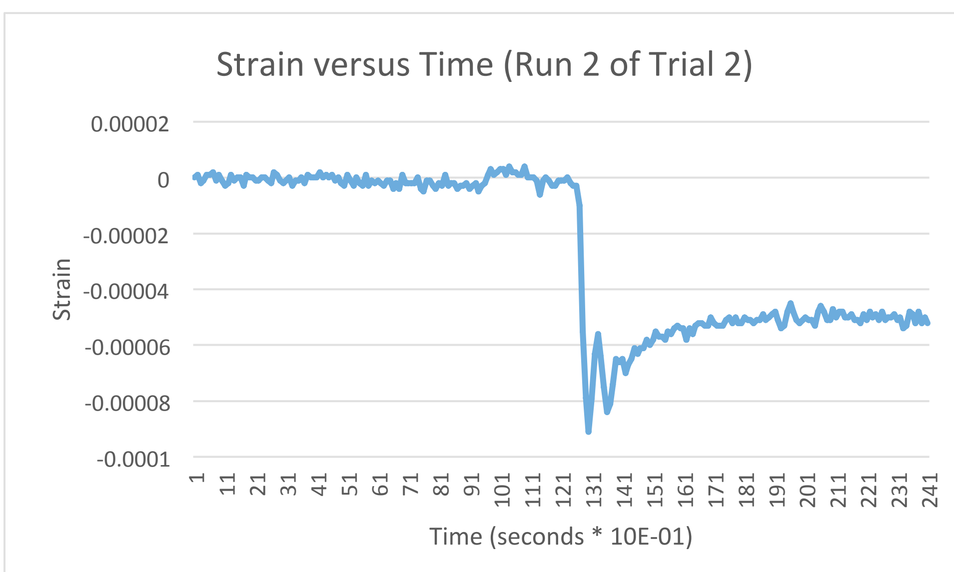


Figure 5: Strain versus Time for Run 1 of Trial 3

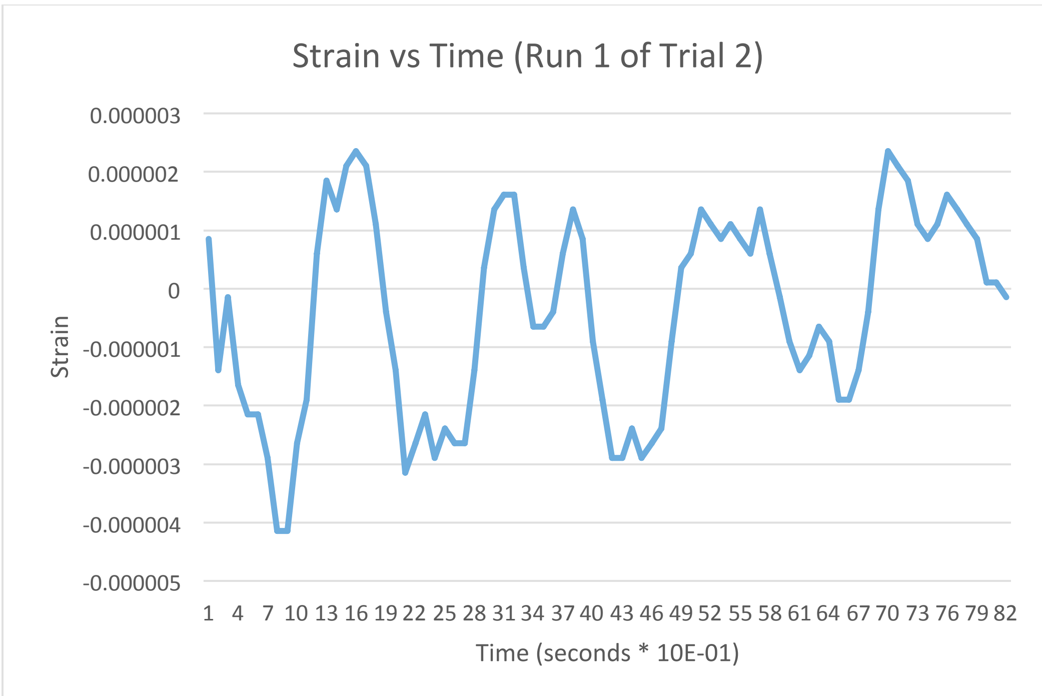
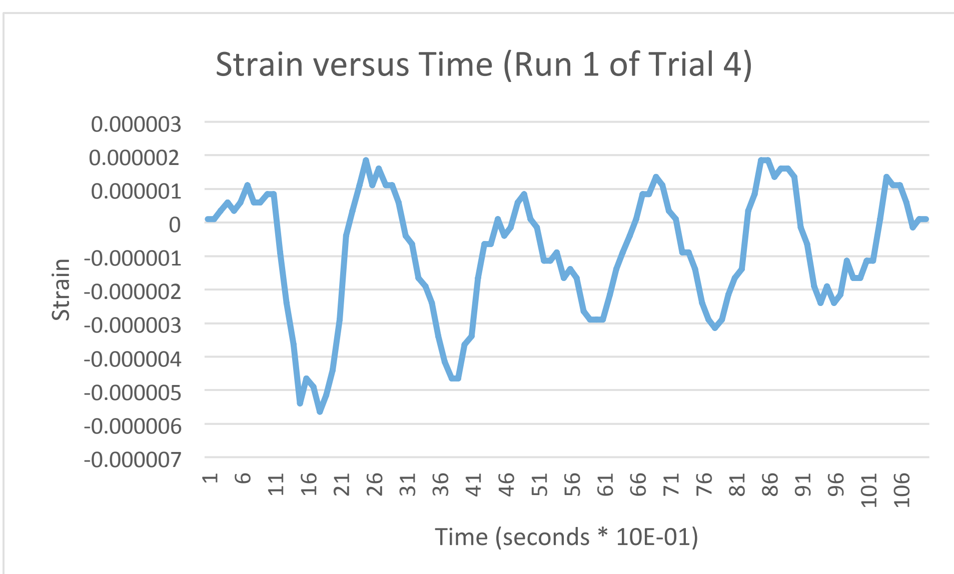


Figure 6: Strain versus Time for Run 1 of Trial 4



Force versus Time Graphs

Figure 7: Force versus Time for Run 1 of Trial 1

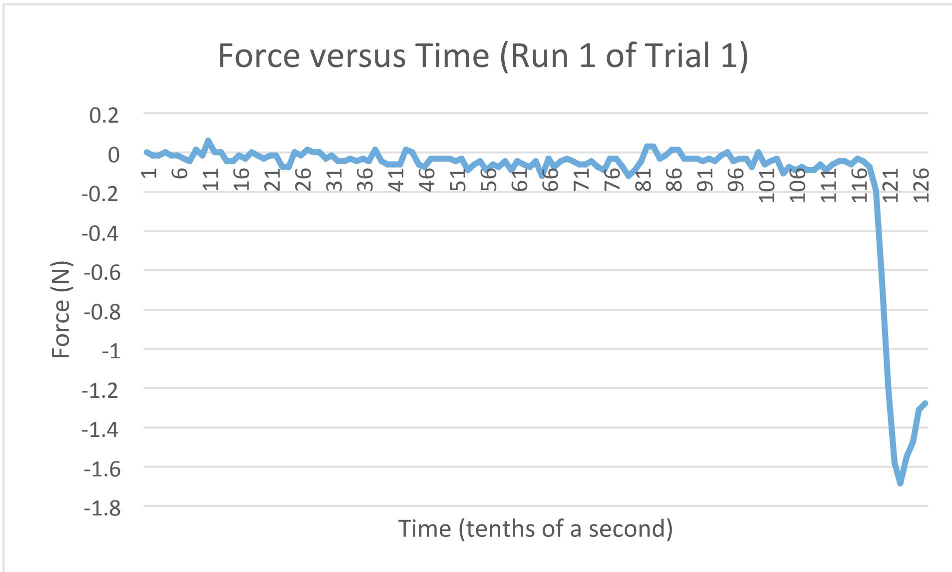


Figure 8: Force versus Time for Run 2 of Trial 2

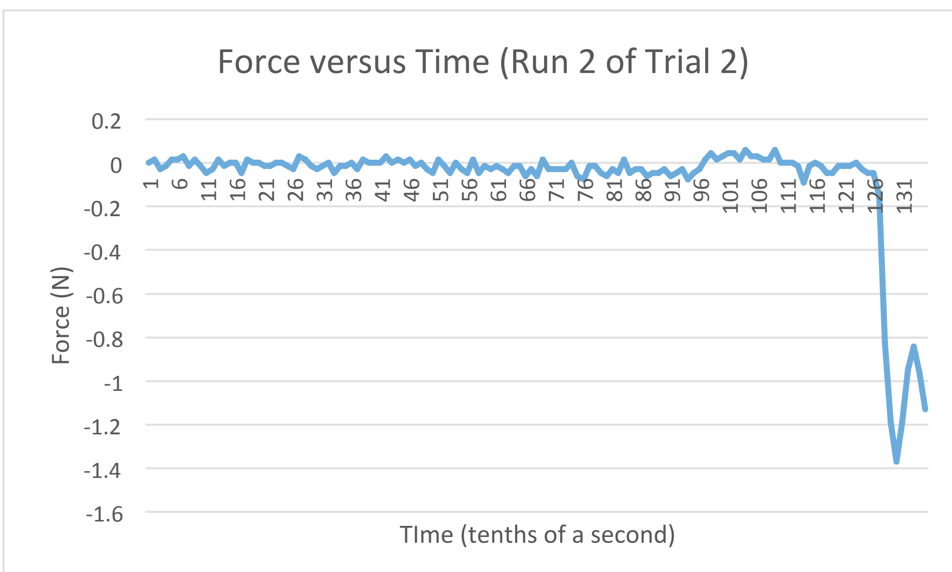


Figure 9: Force versus Time for Run 1 of Trial 3

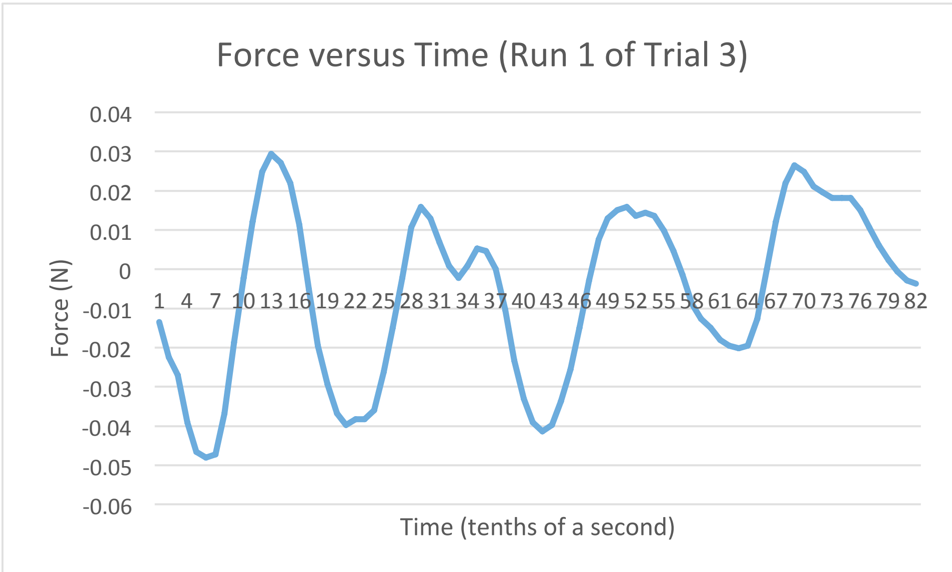
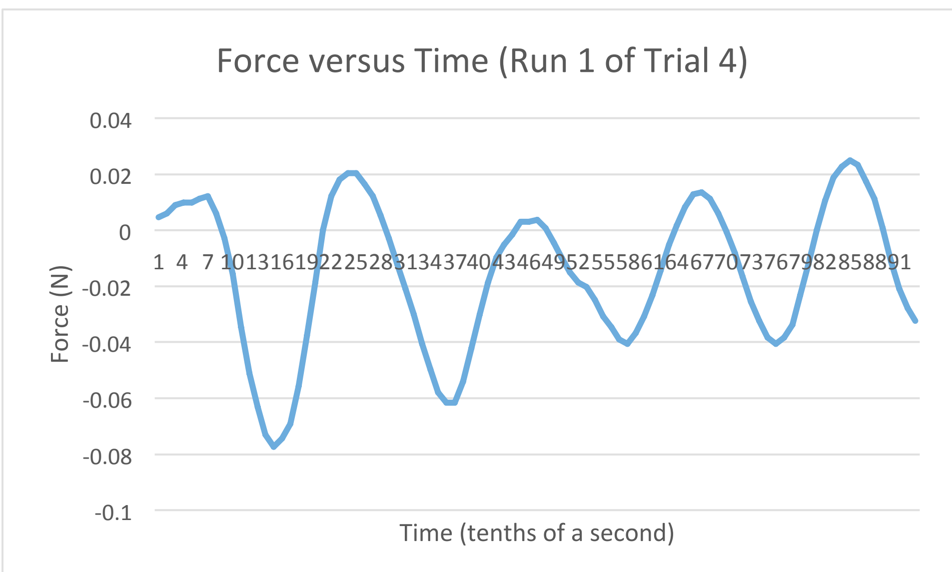


Figure 10: Force versus Time for Run 1 of Trial 4



Force Calculation

The experiment was designed so the force could be calculated using a strain gage attached to an aluminum plate that acts as a simple cantilever beam which undergoes bending from an applied point load. From Hooke's Law in the linear-elastic region of a metal we know that

$$\sigma = \varepsilon E$$

Where σ is the stress on the beam, ε is the strain on the beam, and E is Young's Modulus. For the 6061 T6 aluminum alloy metal used as the cantilever beam, Young's Modulus is 70 GPa. We also know that for a rectangular beam in bending, the stress can be expressed as:

$$\sigma = \frac{M * c}{I} = \frac{M}{S}$$

Where M is the moment at the location of interest on the beam and S is a geometric property of the cross section known as the section modulus:

$$S = \frac{b * h^3}{6} = \frac{0.051\, m * (0.00159\, m)^2}{6} = 2.149 * 10^{-8}\, m^3$$

(see Figure for dimensions of beam and location of strain gage)

The bending moment along the beam can be written as:

$$M = P(L_2 - x) = P(0.215m - 0.105m) = P * 0.1m$$

Where P is the load, L_2 is the distance away from the support at which the load is applied, and x is the distance from the support to the location of interest, in this case, the location of the strain gage. Equating the bending stress with Hooke's Law, we arrive at an expression for the force of the load as a function of strain:

$$P = 15042 \; * \; \varepsilon$$

With P in Newtons and strain unitless (see Appendix 1 for derivation of bending moment equation).

Table 1: Calculated Maximum Force for All Trials

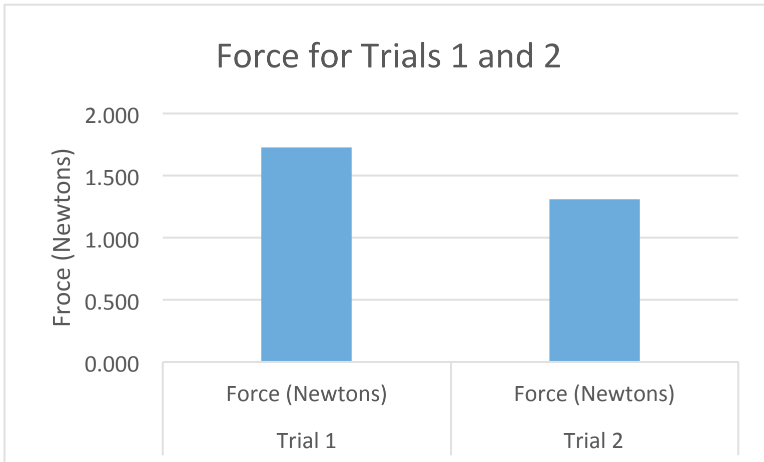
	Trial 1	**Trial 2**	**Trial 3**	**Trial 4**
Stroke	Force (Newtons)	Force (Newtons)	Force (Newtons)	Force (Newtons)
Push 1	1.727	1.312	0.0852	0.1220
Push 2			0.0732	0.0702
Push 3			0.0672	0.0702
Push 4			0.0633	0.0672
Pull 1			0.0474	0.0674
Pull 2			0.0325	0.0531
Pull 3			0.0259	0.0290
Pull 4			0.0227	0.0260

Force Graphs

For Trials 1 and 2, we determined it was acceptable to assume that all of the force was from the viscous damping of the dashpot. This is reasonable because the largest friction force we measured in Trials 2 and 3 was 0.0674 N, which is only 5.1% of the total force recorded in Trial

2. Moreover, these trials only consisted of a single push stroke. Therefore, we determined that the Coulomb damping for Trials 1 and 2 was negligible.

Figure 11: Maximum Calculated Force for Trials 1 and 2



Given that the dashpot displays no viscous damping on the pull stroke, we concluded that any strain and subsequently force registered on the pull stroke must come from Coulomb damping. This provided us with a convenient manner for determining the friction force. However, we expected to see a constant friction force and which was not observed. We conclude that the friction force is not constant because the piston rod of the dashpot was not acting in a straight line the entire time, as it moved with the driving element. Therefore, the normal force of the piston against the cylinder of the dashpot would have been changing and the friction force would have been consequently changing as well. We concluded that all of the force on the pull strokes can be attributed to the friction force. On the push strokes, the friction force was assumed to be equal to the friction force on the subsequent pull stroke. The rest of the force on the push strokes we then concluded arose from the viscous damping.

Figure 12: Maximum Calculated Force versus Stroke for Trial 3

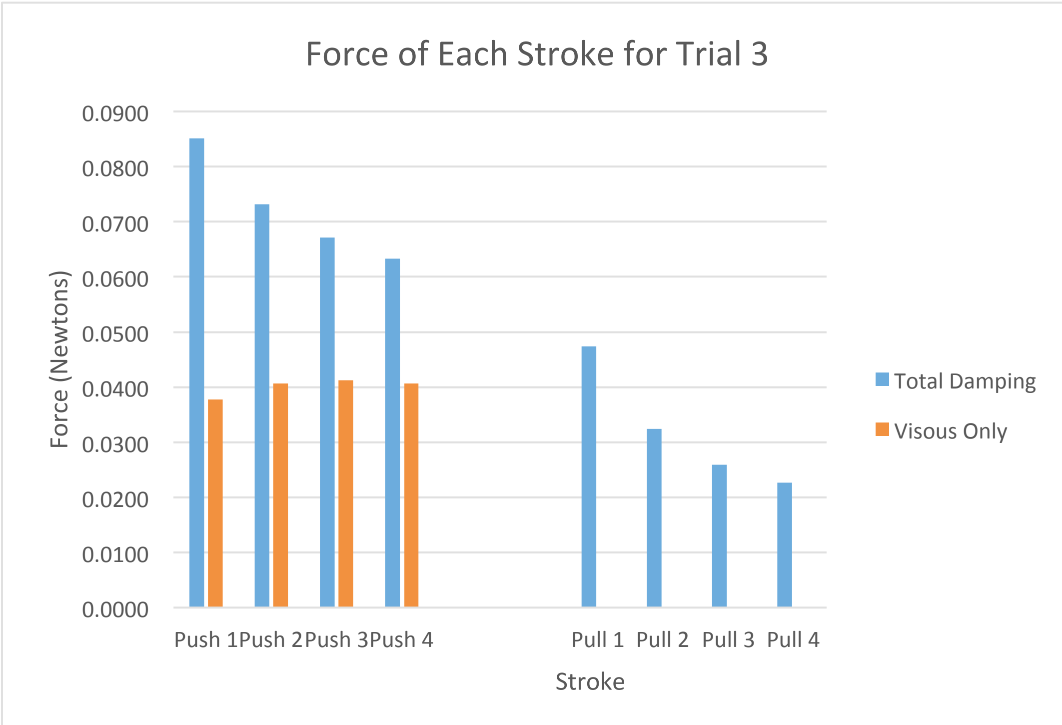
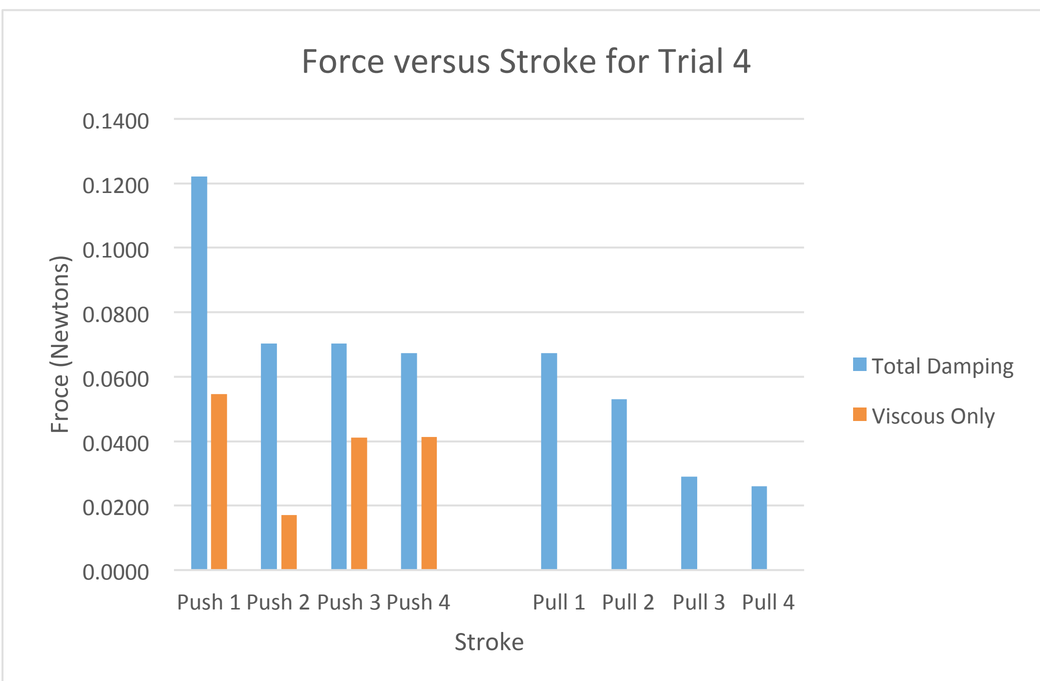


Figure 13: Maximum Calculated Force versus Stroke for Trial



Uncertainty in Force

Starting from the equation for the force, we can determine the uncertainty in the force:

$$P = \frac{\varepsilon * E * S}{L_2 - L_1}$$

$$\sigma_p = \sqrt{\left(\frac{E * S}{L_2 - L_1}\right)^2 * \sigma_\varepsilon^2 + \left(\frac{\varepsilon * E}{L_2 - L_1}\right)^2 * \sigma_S^2 + \left(\frac{\varepsilon * E * S}{(L_2 - L_1)\text{^}2}\right)^2 * \sigma_{L_2 - L_1}^2}$$

For the sake of brevity, full tables of uncertainty are not included in this report. A sample calculation for the uncertainty of the force in Trial 1 is below. The uncertainty in ε was provided by the calibration of the strain gage as 2.31%.

$$\sigma_p = \sqrt{0.0126 + 0.0325 + 0.0645} = 0.331$$

As can be seen from this calculation, the greatest uncertainty in the force arose from the distance between the strain gage placement and the force applied to the aluminum beam.

Period and Time Measurements

First, we measured the period of our set-up without any damping to ascertain the natural frequency of the system. The period of Trials 3 and 4 were measured with a Pasco Photogate. However, Trials 1 and 2 stopped oscillating after half a cycle, and no reliable period could therefore be measured with the photogate. We therefore decided to measure the time it took in Trials 1 and 2 for the bicycle wheel to come to a stop after completing half a cycle. We used stopwatches and timed by hand for this data.

Table 2: Undamped Period

	Run #1	Run #2	Run #3	Run #4	Run #5
	Period (s)	Period (s)	Period (s)	Period (s)	Period (s)
	1.225	1.207	1.231	1.215	1.224
	1.222	1.206	1.235	1.211	1.211
	1.218	1.209	1.237	1.216	1.21
	1.213	1.207	1.232	1.22	1.212
	1.213	1.211	1.226	1.218	1.217
	1.209	1.213	1.22	1.218	1.217
	1.208	1.216	1.219	1.221	1.216
	1.21	1.217	1.21	1.218	1.22
	1.21	1.216	1.213	1.217	1.219
	1.215	1.219	1.209	1.218	1.22
Overall Average	1.21668				
Standard Deviation	0.007038				

Table 3: Time for Half Cycle Trial 1

	First Run	Second Run	Third Run	Fourth Run	Fifth Run		Average	Standard Deviation
Time (s)	1.50	1.41	1.52	1.49	1.6		1.50	0.0680

Table 4: Time for Half Cycle Trial 2

	First Run	Second Run	Third Run	Fourth Run	Fifth Run		Average	Standard Deviation
Time (s)	0.95	0.79	0.84	0.92	0.85		0.87	0.0644

Table 5: Measured Period Trial 3

	Run #1	Run #2	Run #3	Run #4	Run #5
	Period (s)	Period (s)	Period (s)	Period (s)	Period (s)
	1.229	1.229	1.229	1.232	1.227
	1.238	1.236	1.242	1.239	1.236
	1.241	1.239	1.252	1.244	1.238
	1.257	1.233	1.254	1.248	1.235
	1.241	1.248	1.281	1.265	1.242
Average (s)	1.2412	1.237	1.2516	1.2456	1.2356
Standard Deviation	0.010109	0.007176	0.019191	0.012381	0.005505

Table 6: Measured Period Trial 4

	Run #1	Run #2	Run #3	Run #4	Run #5
	Period (s)	Period (s)	Period (s)	Period (s)	Period (s)
	1.229	1.228	1.226	1.225	1.235
	1.232	1.233	1.235	1.234	1.244
	1.236	1.242	1.24	1.237	1.252
	1.239	1.245	1.243	1.237	1.256
	1.252	1.245	1.254	1.223	1.278
Average (s)	1.2376	1.2386	1.2396	1.2312	1.253
Standard Deviation	0.008905	0.007701	0.01031	0.006723	0.016125

From the damped period the frequency and the damping coefficient can be calculated:

$$\omega_d = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \; ; \quad b = 2m * \sqrt{\omega_0^2 - \left(\frac{2 * \pi}{T}\right)^2}$$

This calculation of the damping coefficient assumes viscous damping the entire time. However, because the friction force varied and appeared to be proportional to the velocity, it would be more accurate to model the system with both viscous damping and friction damping that is proportional to the velocity. Thus for Trials 3 and 4, we could combine the two damping coefficients into a single damping coefficient. Compare the damping coefficients to the critical calculated damping coefficient of 17.44 N/m/s.

Table 7: Calculated Damping Coefficients

b (N/m/s)					
	Run #1	Run #2	Run #3	Run #4	Run #5
Trial 1	15.94	15.73	15.98	15.92	16.13
Trial 2	17.14	17.23	17.21	17.16	17.20
Trial 3	3.43	3.12	4.07	3.72	3.01
Trial 4	3.17	3.24	3.32	2.64	4.15

Discussion

1. Do the predicted forces produce the measured damped oscillation?

Yes, the predictions line up accurately with the measured damping forces and oscillations. We predicted the critical damping force to be 1.588 N and we measured it to be 1.312 N. Likewise, we predicted that the critical damping coefficient would be 17.44 N/m/s and it was calculated from the results as 17.19 N/m/s. Additionally, the forces we predicted would produce overdamped and underdamped motion did in the experiment. However, there was disagreement in our assumptions regarding the friction force. We expected it to be a constant in all situations, but it turned out to decrease over the oscillations and appeared to be proportional to velocity. Moreover, the Coulomb damping was much larger in magnitude than had been predicted and was close to even in terms of magnitude with the viscous damping effects.

2. Do the measured forces produce the measured damped oscillation?

Yes, the measured forces produced measured oscillations that were in line with those obtained from the experiment. The measured forces suggested we should have observed overdamping in the first trial which was demonstrated. This trial had the time to settle as expected because overdamping prevents the system from returning to equilibrium in the quickest fashion. The critical damping in Trial 2 lined up with the measurements, and the system returned to equilibrium the quickest as was expected.

3. Explain differences and similarities for both of the conditions above.

The largest difference between our preliminary calculations and the actual results was that the frictional damping was not a constant and was larger in magnitude than expected. We underestimated the magnitude of the frictional force because we took the Manufacturer's specification which applied only when there was no side load. However, in our experiment, the rod of the piston was not always acting in a straight line and was often at an angle because it swiveled on a ball joint when the wheel rotated. Therefore, the assumption that there was no side load is inaccurate and the friction force was much larger than expected. Moreover, we did not have a firm based for estimating how long it would take for the underdamp case to come to a rest, but we predicted under 20 cycles. We observed that the pendulum came to a rest after 5-6 periods, which was in line with what we expected but we were not able to predict it exactly. Overall, the predictions based on our calculations proved to be in line with the laboratory results.

Appendix 1: Derivation of Bending Moment Equation