Application of fuzzy logic to layout of integrated circuits

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Introduction

There are many applications of fuzzy logic[1] in such diverse fields as process control, neural networks, circuit and logic design, artificial intelligence, speech, pattern and image processing, and even social and economic studies see for example[2,3,4]. In some areas like design and development of control systems, it has been exploited extensively in Japan and is fast emerging as one of the most successful of today' technologies. Fuzzy logic is also being used in areas of information technology , where it can provide expert and decision-support systems with powerful approximate reasoning capabilities bounded by a minimum set of rules. Silicon implementation of fuzzy logic circuits using CMOS and BiCMOS technologies especially for analog circuits is also becoming a promising area of research and development for VLSI design. The use of fuzzy sets for placement problems in VLSI design was first proposed by Razaz et al[5].

This paper describes the application of fuzzy logic to the placement of integrated circuits. The placement problem is introduced and a methodology for applying fuzzy set theory to it is presented. Several fuzzy placement algorithms are outlined and experimental results are then presented and discussed.

The Placement Problem

The layout of an integrated circuit involves partitioning and assignment of logic circuits to physical modules (i.e. functional blocks), placement of the modules onto designated locations or slots on the chip, and finally interconnecting (or routing) these modules. Due to the complexity involved in this design process, the layout problem is treated in two different stages, namely placement and routing, and each problem is tackled separately. In this paper we look at the application of fuzzy logic to the placement problem; it is equally applicable to the routing. Various heuristic placement algorithms have been reported in the literature, see for example[6,7].

Mathematically the placement problem can be defined as follows. We are given a set of *modules* (functional blocks) $B = \{b_1, b_2, \ldots\ldots, b_N\}$, a set of input / output pins or *signals* $S=\{s_1, s_2, \ldots\ldots, s_J\}$, and a set of *cells* (i.e. specific locations or slots on a chip) $L = \{L_1, L_2, \ldots\ldots, L_P\}$, where $P \geq N$. We associate with each module a (sub)set of signals S_s , and with each signal a subset of modules B_b , called a *signal set (or net)*. The placement problem is to assign each module, say, b_i to a unique cell L_j on the chip in order to optimise some objective. In practice, a subset of these modules are initially placed (these are called seeds or nucleus) within some distance from each other or in fixed positions on a chip; the aim is to satisfy certain constraints such as elimination of cross-talk, minimisation of power or heat dissipation level, and minimisation of routing length of a signal set. After all modules are assigned to cells, the resulting circuit configuration is routed(i.e. interconnected). The ultimate objective of placement is to achieve 100 % routing for a given chip area. In practice, simpler and more precise objectives are used such as total routing length[8], maximum number of cut lines(in graph theoretic sense)[9,10,11] and maximum density[12,13]. Finding an optimum solution for a given objective is considered to be a NP hard problem (or simply put it, the computation time grows exponentially with the problem size). Therefore heuristic algorithms are used instead. There are a variety of rules for interconnecting modules which are often dictated by the specific technology and design styles used for chip fabrication. These rules are defined based on such criteria as minimum spanning (or Steiner) tree, minimum chain, and fully connected graph[14].

The modules and their interconnections in a placement problem are related through a *connection matrix* which is equivalent to the *cost matrix* of the associated quadratic assignment problem[15] [see Hanan and Kurtzberg]. This matrix can also be represented as a module-connection graph which is sometimes referred to as the placement configuration. For example Fig.1 shows the connection matrix of a placement problem with 9 modules {p, q, r, s, t, u, v, w, y}, each matrix element represents the connectivity between two elements in the set. In graph theoretic sense, each matrix element represents the weight of an edge connecting two nodes (i.e. elements).

$$
\begin{array}{cc}
 & \begin{array}{ccccccccc} p & q & r & s & t & u & v & w & y \end{array} \\
\begin{array}{c} p \\ q \\ r \\ s \\ t \\ u \\ v \\ w \\ y \end{array} &
\begin{bmatrix}
0 & 5 & 0 & 2 & 2 & 0 & 0 & 1 & 0 \\
5 & 0 & 4 & 3 & 3 & 0 & 4 & 0 & 0 \\
0 & 4 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\
2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
2 & 3 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

Fig. 1 Connection matrix of associated quadratic assignment problem

Fuzzy Relation and connection matrix

A *crisp relation* in the context of set theory represents the absence or presence of interaction or association between the elements of two or more crisp sets. However a *fuzzy relation* extends this concept of crisp relation to permit various strengths or degrees between elements of sets, and it indicates this degrees of association by its membership grades in the same way as membership grades are represented in a fuzzy set. When the fuzzy relation is defined on two sets X and Y, i.e. R(X, Y) it is called a binary relation and can be represented by a membership matrix whose elements are membership grades of the relation. It follows that a fuzzy binary relation R(X, X) defined on a single set X shows the degrees of association between elements of the same set X.

When the fuzzy relation R(X, X) is reflexive, symmetric and transitive[16], it becomes a fuzzy *similarity relation* (this is the counter part of *equivalence relation* in crisp set theory). The interpretation of this fuzzy similarity relation is that it can effectively group elements into crisp sets, the members of which are 'similar' to each other to certain degree specified by the membership grades. If this degree of similarity is 1, the grouping then becomes an *equivalence class* as is the case for a crisp relation.

In order to be able to apply fuzzy set theory to a placement problem we have appropriately modified the associated connection matrix to represent a fuzzy relation on the set of modules B to be placed. The elements of this modified matrix M' are then the membership grades of the fuzzy relation. By using the transitive *closure*[17] of M', we have also introduced an alternative connection matrix M" whose elements are membership grades of a fuzzy similarity relation with its distinct and desirable mathematical properties such as reflexivity, symmetry and transitivity.

Fuzzy placement algorithms

We have developed a number of algorithms using the connection matrix of a placement problem as a fuzzy relation. The simplest algorithm is to keep the same selection and positioning rules as in conventional placement algorithms but exploit the property of M". This fuzzy algorithm generally results in a better final placement configuration than a conventional counter part algorithm.

A second approach would be to use the *resolution form* [18,19] of the similarity relation to partition the latter into a series of crisp *equivalence* relations R_a [20], and then use these partitions to construct appropriately the final placement configuration. The main advantage of this algorithm is in its efficiency and capability to partition modules with strong similarity into the same subset, and by varying the value of a we can control the number of elements in each partition. This allows the algorithm to be flexible and hierarchically structured.

A third fuzzy placement algorithm would be to combine the fuzzy similarity relation M" with a k-means clustering method followed by a linear ordering technique. An outline of this placement algorithm follows the steps indicated below, each of which is generated by a separate software module :

1) Construct M" for a given connection matrix of P elements.
2) Partition the P elements into H horizontal subsets using a fuzzy k-means clustering method.
3) Minimise the total routing length among the H subsets in step (2) using a linear ordering technique.
4) Ensure that elements in the same horizontal subset do not appear in a vertical subset generated in step 5. This can be done by appropriately modifying M".
5) Generate V vertical subsets from the P elements as in step (2) such that the elements in the same horizontal subset cluster in different vertical subsets.
6) Generate the final placement configuration.

Results and discussion

These fuzzy algorithms have been successfully applied to a variety of placement problems. Here we present some typical experimental results. Figure 2 shows the best placement configuration (with the total routing length of 42)[21] constructed for the connection matrix shown in Fig. 1. This configuration is obtained using a conventional placement algorithm in which a pair of modules with maximum connectivity is chosen as the nucleus, the selection rule is based on the choice of a module x with the highest connectivity to the nucleus, and the positioning rule uses the criterion that x is placed in that position which results in the minimum connection cost. If we apply the first fuzzy algorithm (i.e. keeping the same selection and positioning rules but using the fuzzy similarity relation M") we obtain in the worst case a placement with the total routing length of 40 (see Fig.3), and in the best case (using the technique of looking ahead one move) the placement shown in Fig.4 with the total routing length of 36.

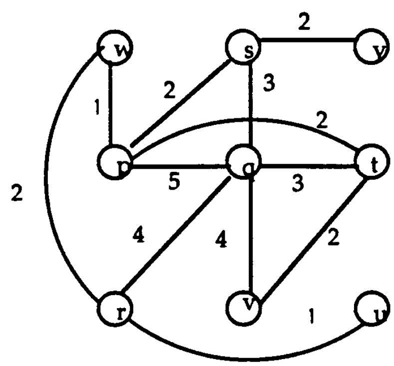

Fig. 2 The best placement configuration generated by the conventional approach. Total routing length is 42.

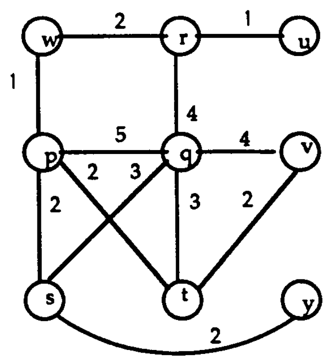

Fig. 3 The worst placement configuration generated by the first fuzzy algorithm. Total routing length is 40.

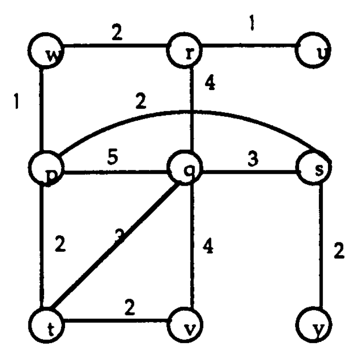

Fig. 4 The best placement configuaration generated by the first fuzzy algorithm. Total routing length is 36.

$$
C = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 2 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 \\
0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 2 & 0 & 0
\end{bmatrix}
\end{array}
$$

Figure 5: Connection matrix

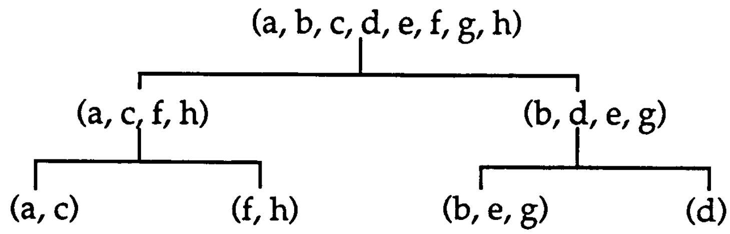

Figure 6: Partition tree using the third fuzzy algorithm

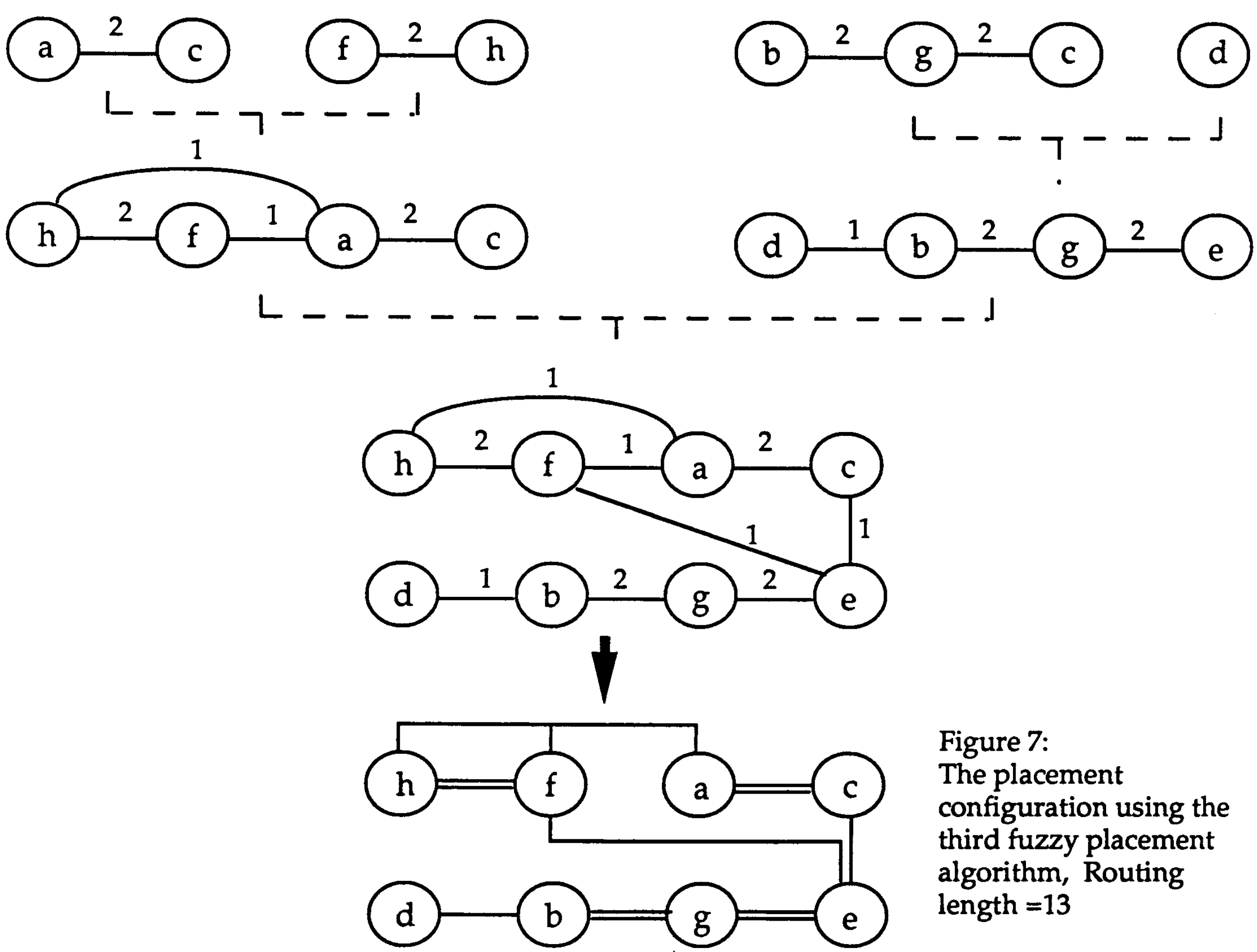

Figure 7:
The placement configuration using the third fuzzy placement algorithm, Routing length =13

Figure 5 shows the connection matrix of a 9-module placement problem, and Figures 6 and 7 present the resulting partition and merging process and final placement configuration obtained after application of the third algorithm i.e. the fuzzy k-means clustering technique. We have also compared this algorithm with work in the literature such as the placement problems reported by Goto et al (Ref. 8) and Blanks[22], and our algorithm has consistently performed better. This algorithm is flexible, hierarchical and efficient, and hence quite suitable for placement in VLSI design.

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12 (Segment)density is defined as the ratio of a segment(i.e. the boundary between two cells) to the number of signal sets that the segment can accommodate. (The whole chip is subdivided into a rectangular array of cells, and the routing path of a signal set is defined as a sequence of segments).

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14 M. Hanan, P. K. Wolf, Sr and B. J. Agule "Some experimental results on placement techniques," Proc. 13th DA Conf., 1973, pp. 214-224.

15 A quadratic assignment problem is a special case of a placement problem in which each signal set consists of a pair of modules only.

16 A fuzzy relation R(X, X) is *reflexive* if and only if (iff) its membership grade $\mu_R(x, x) = 1$ for all x being a member of the set X. R(X, X) is *symmetric* iff $\mu_R(x, y) = \mu_R(y, x)$, and it is *max-product transitive* iff $\mu_R(x, z) \geq \max [\mu_R(x, y). \mu_R(y, z)]$ for all (x, y) being a member of X^2.

17 The *transitive closure* of a fuzzy relation R(X, X) a) is transitive, b) contains R(X, X), and c) has the smallest possible membership grades that still allow conditions a and b to be satisfied.

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