

# A FUZZY LOGIC AND VARIABLE STRUCTURE BASED CONTROLLER FOR pH CONTROL

Luis E. Zárate  
Pontifical Catholic University  
of Minas Gerais,  
Brazil,

Peterson Resende  
Federal University of  
Minas Gerais,  
Brazil

Benjamin Menezes  
Federal University of  
Minas Gerais,  
Brazil

**Abstract** -The pH control is a difficult problem to solve. That happens due strong nonlinearities and extreme sensitivity to disturbances in the process. The possible combination of the variable structure system with fuzzy logic theory for the pH problem is treated in this paper. Will be showed how this theory can improve a variable structure controller applied for this process. A Smith predictor is utilized to account for the pH process time delay. This paper purposes the controller's gains adjustment by means of fuzzy logic. Simulation results are presented.

## I. INTRODUCTION

The pH process in Continuous Stirred Tank Reactors (CSTR), are widely utilized in a large variety of applications. These process are of great importance in chemical industry and in the waste water treatment. The pH regulation problem has been treated extensively in the literature [1-4], [6-7] and [9]. The main difficulty in control is its great sensitivity to disturbance and parameters uncertainties.

The variable structure systems in sliding modes are known to be insensitive to variations in plant parameters and external disturbance, as reported in several applications [16-18]. In [5] and [7], variable structure controllers (CEV) for neutralization and regulation problems were proposed respectively. The controllers shown a good performance for pH control, more presenting some limitations mainly due to the fixed gains of the CEV.

As the Fuzzy Sets theory [13] is particularly useful for applications in control with uncertainties [14], in this work an adjustment of the gains of the CEV by fuzzy logic is presented. This permit to improve the response of the controller proposed in [7].

By other wise, the description of the pH process contains strong nonlinear relationships, since the concept of pH is related to the concentration of hydrogen ions,  $[H^+]$  by:

$$pH = -\log[H^+] \quad (1)$$

As the pH process is highly nonlinear, the logarithmic relationship turn this process too sensitivity to small variations in the neighbor of pH = 7 (Fig. 1)

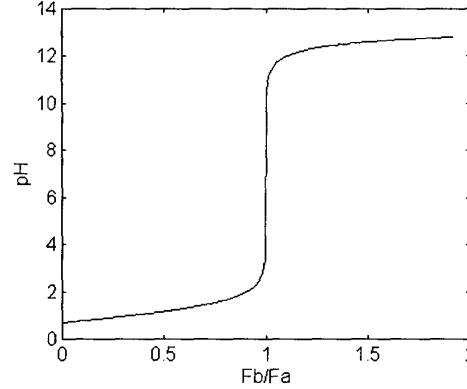


Figure 1. Typical Curve of pH

The pH system is a pH regulation process in a CSTR. The influent is a strong acid with unknown flow and concentration. The control purpose is the pH regulation in the outlet stream from the tank, by manipulating the flow of a strong base with known concentration.

Defining  $Q$  as the distance from neutrality,

$$Q = 10^{-pH} - K_w [10^{pH}] \quad (2)$$

where  $K_w$  is the water ionic product.

In [7], the model for the pH process is presented and discussed:

$$\dot{\omega}_{(t+\delta)} = A_1 10^{-\omega_{(t+\delta)}} - A_2 - A_3 U_{c(t)} \quad (3)$$

with:

$$A_1 = \frac{F_{a(t)}}{V} \left(1 + \frac{C_{a(t)}}{C_{b(t)}}\right) \left(\frac{1}{\tau_f V \ln(10)} - \dot{\omega}_{(t+\delta)}\right) \quad (4a)$$

$$A_2 = \frac{F_{a(t)}}{\tau_f V \ln(10)} + \frac{\dot{\omega}_{(t+\delta)}}{\tau_f} \quad (4b)$$

$$A_3 = \frac{1}{\tau_f V \ln(10)} \quad (4c)$$

$$\omega(t) = \log \left( \frac{Q(t) + C_b}{C_b} \right) \quad (5)$$

where  $V$  is the tank volume (considered constant).  $F_a$ ,  $C_a$  and  $F_b$ ,  $C_b$  are flow and concentration of acid na base, respectively.  $C_b$  is fixed and knew.  $\delta$  is the time delay for transport and mixing delays, considered  $0.05\tau$  [2,5,9], where  $\tau$  is time-constant of the process, represented by:

$$\tau = \frac{V}{F_a(t) + F_b(t)} \quad (6)$$

Since variable structure controllers lead to high frequency switching in the control variable, it is not practical to use the base flow variable for this purpose. To overcome this difficulty, the control variable is taken as the input of a low-pass first order filter, with time-constant  $\tau_f$  (considered 1.5 min.)

The pH regulation is performed by manipulating the base flow in the following range:

$$0 \leq F_b \leq F_b^{max} \quad (7)$$

Is considered that the acid flow and concentration,  $F_a$  and  $C_a$  are not accessible for measurement and are arbitrary within the ranges:

$$F_a^{min} \leq F_a \leq F_a^{max} \quad (8a)$$

$$C_a^{min} \leq C_a \leq C_a^{max} \quad (8b)$$

The schematic diagram of the pH process is shwon in Fig. 2.

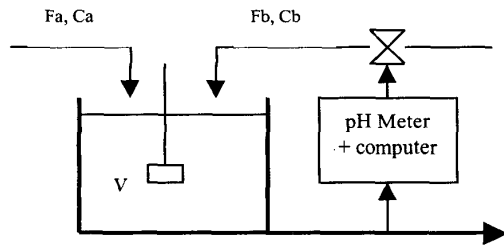


Figure 2. Schematic diagram of the pH process

This paper is organized in 3 sections. The Section 2, variable structure systems based in Fuzzy sets is presented. In the Section 3, results of simulation are discussed and finally the conclusions of the work are presented.

## II. VARIABLE STRUCTURE CONTROLLER BASED IN FUZZY SETS

### A. The Variable Structure Regulator

The variable structure regulator is proposed in [7]. The system is designed such that all trajectories in the state space

are directed toward the switching surface. Once on this surface, the system state cannot move along new trajectory adjacent to this surface. The system response depend only on the switching surface characteristics and remains insensitive for disturbances [10].

Considering as process model the Eq. (3), the sliding line  $S$  is defined by:

$$S = \sigma (\omega_{(t+\delta)} - \omega_d) + \dot{\omega}_{(t+\delta)} \quad (9)$$

where  $\omega_d$  is the set-point and  $\sigma$  is a positive constant, considered 0.5.

A well known sufficient condition to assure that the phase trajectories move toward the sliding line can be obtained from the application of the Lyapunov method [11].

$$S \dot{S} < 0 \quad (10)$$

Therefore, choosing the following control law:

$$U_{c(t)} = K_1 10^{-\omega_{(t+\delta)}} - K_2 \quad (11)$$

and using Eq. (3), after differentiating Eq. (9) with respect to time, is obtained:

$$[\sigma \omega_{(t)} - A_2 + A_3 K_2] S + 10^{-\omega_{(t)}} [A_1 - A_3 K_1] S < 0 \quad (12)$$

The conditions the switching are:

$$K_1 < \min [A_1 / A_3] \quad \text{se } S < 0 \quad (13a)$$

$$K_1 > \max [A_1 / A_3] \quad \text{se } S > 0 \quad (13a)$$

$$K_2 > \max [(A_2 - \sigma \omega') / A_3] \quad \text{se } S < 0 \quad (13b)$$

$$K_2 < \min [(A_2 - \sigma \omega') / A_3] \quad \text{se } S > 0 \quad (13b)$$

Note that, in the Eq. (11) the control variable is determined from the state variable  $\omega$ , that is expected to be at time  $t+\delta$ . For this purpose, a Smith Predictor is constructed using the nonlinear model of the system given in [5], [8] (Fig. 3).

A variable  $U_i(t)$  is introduced as integral action, with integral-constant  $K_i = 0.74$ .

### B. Influence of the Adjustment of the Controller's gain

The energy with which the trajectory of states approaches and crosses the sliding surface  $S$ , depends on the value of the effective gains  $K_1$  and  $K_2$ . Note in the Eq. (13a,b), for smaller  $K_1$  and bigger  $K_2$ , considering  $S < 0$  and for  $K_1$  bigger and smaller  $K_2$ , considering  $S > 0$ , is bigger energy to the process. The best values of  $K_1$  and  $K_2$  are those values

that considering fast response and the minimum error in stationary-state with minimum energy consumption.

Inappropriate value of K1 and K2 can produce big overshoots and/or errors of stationary-state. In pH processes, to choose the controller's gain constants can introduce low performance in the controller.

For process with delays was proposed in [12] a optimal variable structure control based in the variation of the controller's gains.

In this work, the variation of the controller's gains through of fuzzy logic, to reach the objective mentioned will be proposed. To adjust the gains, the distance of the representative point (PR) of the trajectory, in the space of states to sliding surface S will be considered (it Figs 3)

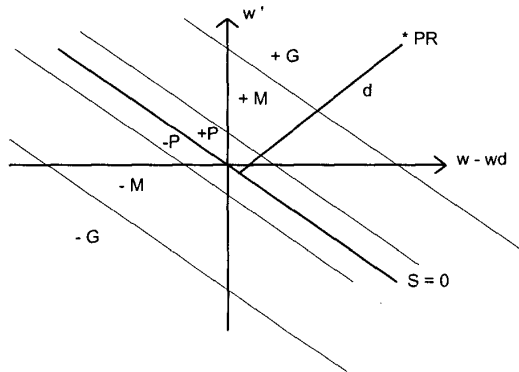


Figura 3 – Adjustment of the gains through fuzzy logic

### C. Adjustment of the gains by Fuzzy Logic

The scheme of the CEV with fuzzy adjustment is showed in the Fig. 4. The distance between PR and switching line is given by:

$$d = \frac{1}{\sqrt{1+\sigma^2}} \left| \dot{w} + \sigma (w - w_d) \right| \quad (14)$$

where  $\sigma$  is the inclination of the switching surface.

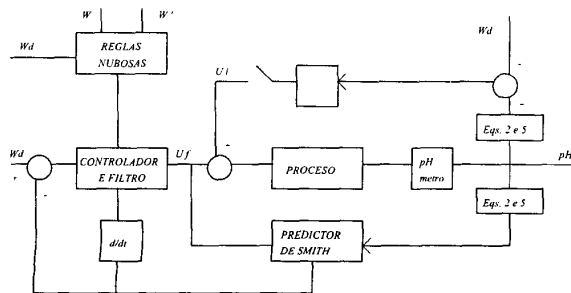


Figure 4 – Structure of the CEV and Fuzzy Logic

The rules linguistic proposals are:

- 1.If the distance is big then the control action (scale factor) is big.
- 2.If the distance is medium then the control action (scale factor) is medium.
- 3.If the distance is small then the control action (scale factor) is small.

Usually the pH control systems do not possess both acid and base reagents. Considering only the base flow, the pH controls exists in the quadrants 1 and 4 of the phase plane, where  $\omega - \omega_d > 0 - d > 0$  (Fig. 5).

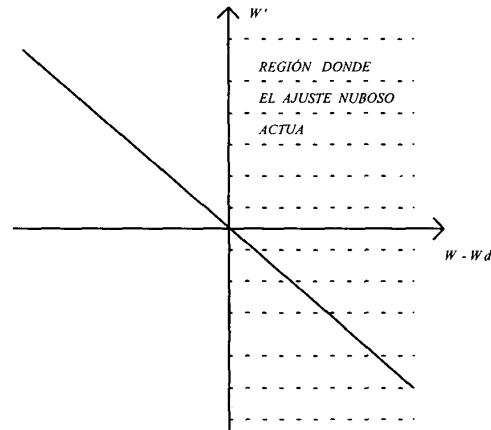


Figure 5 – Quadrants for adjustment via Fuzzy Logic

In the Table I and II, the pertinence level of the primary terms for input (distance) and output (scale factor, Fe) is showed.

In [14] a method to adjust a fuzzy controller is proposed. The method is based on the variation of the discrete levels in the universe of the variables. The scale factor, Fe, it is obtained multiplying the centroid for a discrete factor.

TABLE I  
PRIMARY TERMS FOR INPUT "DISTANCE"

Discrete Level	Range: $d \cdot [10^{-5}]$	Small (S)	Medium (M)	Large (G)
0	$d \leq 1.0$	0.0	0.0	1.0
1	$1 < d \leq 2$	0.0	0.0	1.0
2	$2 < d \leq 3$	0.0	0.0	1.0
3	$3 < d \leq 4$	0.0	0.0	1.0
4	$4 < d \leq 5$	0.0	0.3	0.7
5	$5 < d \leq 6$	0.0	0.7	0.3
6	$6 < d \leq 7$	0.0	1.0	0.0
7	$7 < d \leq 8$	0.0	0.7	0.0
8	$8 < d \leq 9$	0.0	0.3	0.0
9	$9 < d \leq 10$	0.3	0.0	0.0
10	$10 < d \leq 11$	0.7	0.0	0.0
11	$11 < d \leq 12$	1.0	0.0	0.0
12	$d > 12$	1.0	0.0	0.0

TABLE II  
 PRIMARY TERMS FOR OUTPUT "SCALE FACTOR"

Discrete Level	Small (S)	Medium (M)	Large (G)
0	0.0	0.0	1.0
1	0.0	0.0	1.0
2	0.0	0.0	1.0
3	0.0	0.0	0.7
4	0.0	0.3	0.3
5	0.0	0.7	0.0
6	0.0	1.0	0.0
7	0.0	0.7	0.0
8	0.3	0.3	0.0
9	0.7	0.0	0.0
10	1.0	0.0	0.0
11	1.0	0.0	0.0

For this process, the discrete level was of 1.0, for a satisfactory response. The scale factor was saturated in 1.005 for smaller values.

When  $W > 0$ , the gains are given by the equations:

$$K_1 < (1 - F_e) \min[A_1 / A_3] \quad \text{se } S < 0 \quad (15 a)$$

$$K_1 > F_e \max[A_1 / A_3] \quad \text{se } S > 0 \quad (15 a)$$

$$K_2 > F_e \min[(A_2 - \sigma \omega') / A_3] \quad \text{se } S < 0 \quad (15 b)$$

$$K_2 < (1 - F_e) \max[(A_2 - \sigma \omega') / A_3] \quad \text{se } S > 0 \quad (15 b)$$

For  $W - W_d < 0$  was considered fixed gains, with  $F_e = 1.65$ .

### III. SIMULATION AND CONCLUSION

In this work the variable structure controller with and without adjustment of the gains through fuzzy logic are presented.

The control systems are applied to a well stirred tank with volume  $V = 1$  l. and  $\delta = 0.5$  min., for such acid (hydrochloric acid) and base (sodium hydroxide) are fed.

$$\begin{aligned} 0.1 &\leq F_a \leq 0.1 && \text{l/min} \\ 0.2 \times 10^{-3} &\leq C_a \leq 1.0 \times 10^{-3} && \text{mol/l} \\ 0 &\leq F_b \leq 0.2 && \text{l/min} \\ C_b &= 1.0 \times 10^{-3} && \text{mol/l} \end{aligned} \quad (16)$$

Considering the values of (16) is possible to calculate the variation range for  $Q$ ,  $pH$ ,  $\omega$ , and  $\dot{\omega}$ :

$$\begin{aligned} -0.6 \times 10^{-3} &\leq Q \leq 1.0 \times 10^{-3} \\ 3.0 &\leq pH \leq 10.9 \\ -3.88 &\leq \omega \leq 0.30 \\ -0.11 &\leq \dot{\omega} \leq 0.22 \end{aligned} \quad (17)$$

Considering, Eq. (16), Eq. (13) can be calculate of gains  $K_1$  y  $K_2$ :

$$K_1 < 0.002883 \quad \text{se } S < 0$$

$$K_1 > 0.34495 \quad \text{se } S > 0$$

$$K_2 > 0.25164 \quad \text{se } S < 0$$

$$K_2 < 0.03668 \quad \text{se } S > 0$$

The sample period was chosen as 60 ms.

The Figs. 6, 7 and 8 show the pH response on a set-point from  $pH = 5$  to  $pH = 7$ ; from  $pH = 6$  to  $pH = 7$  and from  $pH = 7$  to  $pH = 8$  respectively. Observe as the variable structure controller with adjustment of the gains through fuzzy logic, CEV+NUB, has a better performance that the CEV structure.

Note in the Figs. 6a, 7 and 8, for the CEV+NUB structure the error in stationary-state is null, with a response time smaller that the CEV structure.

As it was mentioned in the section 1, the pH control in the pH neighbor of  $pH = 7$  is extremely sensitivity. The Fig. 7 shown the best performance of the combination CEV+NUB.

Fig. 6b and 6c show the control actions for both controllers.

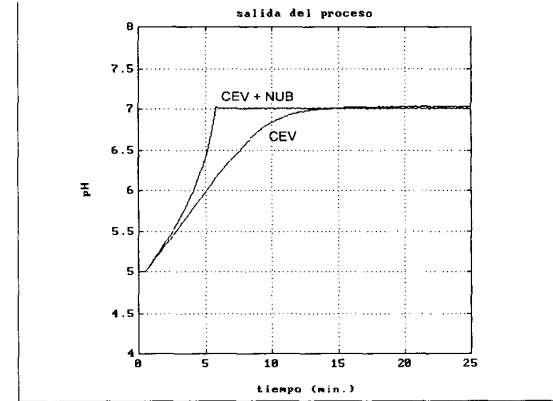


Figure 6a – Output of the process

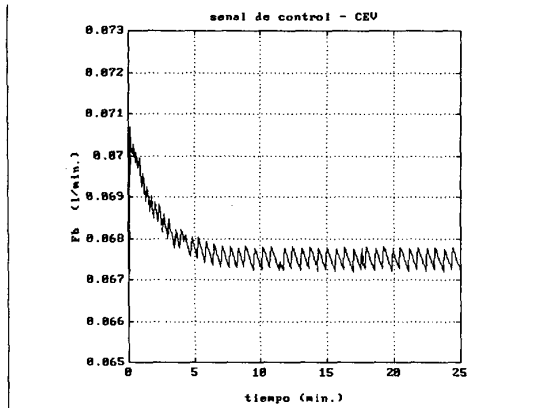


Figure 6b - Control Action - CEV

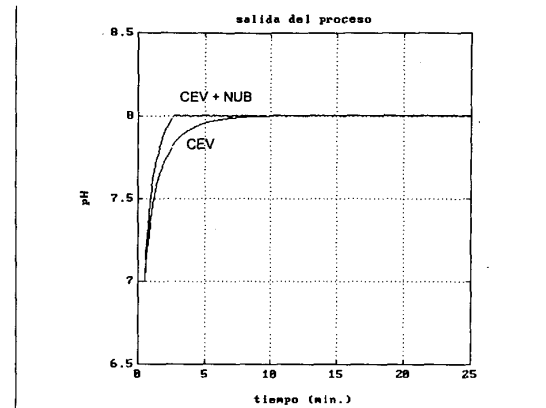


Figure 8 - pH response on a set-point from pH = 7 to pH = 8

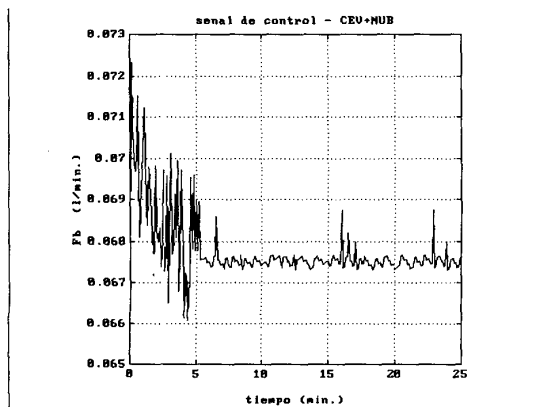


Figura 6c - Control Action - CEV + NUB

Figure 6 - pH response on a "set-point" from pH=5 to pH=7

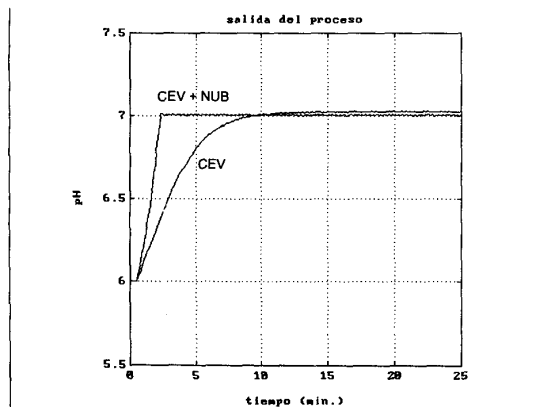


Figure 7 - pH response on a set-point from pH = 6 to pH = 7

In this paper a new control system based in variable structure system and fuzzy logic theory for pH control was presented. Was showed how this combination can improve the controllers of variable structure applied in these process.

#### IV. REFERENCES

- [1] Silver, S.L. e Palmor, Z.J. & Gutman, S. , (1990). "Model following Min-Max Tracking Controller for pH Neutralization and Regulation", XI IFAC World Congress, Tallin, Estonia, URSS, Vol. II : 199-204.
- [2] Jacobs, O. & Hewkin, P. , & While, C. , (1980). "Online Computer Control of pH in a Industrial Process.", IEE Proc. , Vol. 127, Pt. D, #4 : 161-168.
- [3] Jacobs, O. , & Badran, W. , & Proudfoot, C. & While, C. , (1987). "On Controlling pH, "IEE Proc. , Vol. 134, Pt.D, May, #3 : 196-200
- [4] Buchholt, F. & Kümmel, M. , (1979). "Self-Tuning Control of a pH Neutralization Process", Automatica, Vol. 15, pp. 665-671
- [5] Young, G. E. & Rao, S. , (1987), "Robust Sliding Mode Control of a Nonlinear Process with Uncertainty and Delay", J. of Dynamic Systems Measurement and Control, Vol. 109: 203-208
- [6] Jutila, P. , (1983), "Application of Adaptive pH-Control Algorithms", Int. J. Control, Vol. 38 : 639-655.
- [7] Resende, P. & Zárate, L. E. , (1991). "Control of a pH Process using Variable Structure Regulator and Smith Predictor", IECON '91, Kobe, Japan.
- [8] Donoghue, J.F., (1977). "A comparison of the Smith Predictor and Optimal Design Approaches for Systems with Delay in the Control", IEEE Trans. on Ind. Elec. and Contr. Inst., Vol. 24, n. 1:109:117.
- [9] Proudfoot, C. G. & Gawthrop, P. J. , (1983). "Self-tuning PI Control of a pH Neutralization Process", IEE Proc. , Vol. 130, Pt. D. , #5 : 267-272.
- [10] Utkin, V. , (1977). "Variable Structure Systems with Sliding Modes", IEEE Trans. On Automatic Control, Vol. 22 : 212-222.

- [11] Gough, N. E. & Ismail, Z.M. & King, R.E., (1984), "Analysis of Variable Structure Systems with Sliding Modes", Int. J. Systems Science, Vol. 15 : 401-409.
- [12] Itkis, U. , (1976), "Control Systems of Variable Structure", ISRAEL Universities Press, Jerusalem.
- [13] Zadeh, L.A. , (1965). "Fuzzy Sets". Information and Control, Vol. 8, pp. 338-353.
- [14] King, P.J. & Mamdani, E. H. , (1977). "The Application of Fuzzy Control Systems to Industrial Processes", Automatica, Vol 13, pp. 235-242.
- [16] J. Erschler, F. Roubellat and J.P. Vernhes, "Automation of hydroelectric power station using variable structure control systems", Automática, vol. 10, pp.31-36, 1974.
- [17] Sabanovic and D.B. Izosimov, "Application of sliding modes to induction motors control", IEEE Trans. On Industry Applications, Vol 17, pp. 41-49, 1981.
- [18] H. Sira-Ramirez and T.<sup>a</sup> Dwyer, "Variable structure controller design for spacecraft mutation damping", IEEE Trans. On Automatic Control, vol. 32, pp. 435-438, 1987.