Announcements

Make sure to sign in on the google form (I send a list of which section questions are useful for which pset questions afterwards)

Pset 1 due Friday 9/22

Introductions (again)

- Name
- One question or thought related to lecture last week (t-test, z-test, ANOVA, F-test)

Country demographics

We'll start by making last week's exploratory data analysis a bit more precise. These problems will deal with a data set of country-level statistics from UNdata and Varieties of Democracy.

1. We speculated that the Western African and Eastern African countries probably did not have a significant difference in means. Perform a formal t-test for the difference in population means between Western African and Eastern African countries. Recall that a formal test includes (1) the hypotheses, (2) the test statistic, (3) the p-value, and (4) the conclusion in the context of the problem.

```
##
##  Welch Two Sample t-test
##
## data:  west_african and east_african
## t = 0.12188, df = 24.688, p-value = 0.904
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -19.98061  22.49249
## sample estimates:
## mean of x mean of y
##  18.39294  17.13700
```

2. Perform a formal z-test for the difference in the proportions of the populations that are nurses or midwives in the US versus the UK in 2010.

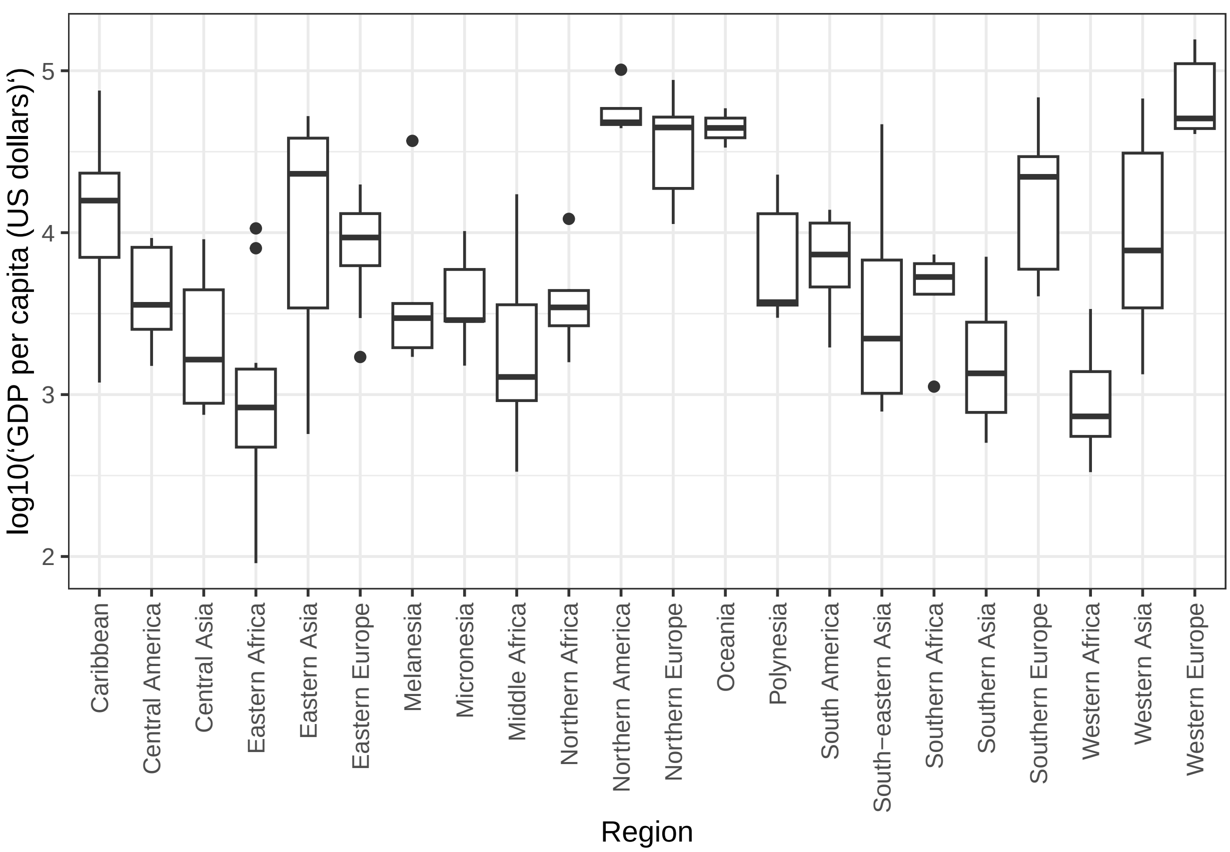
```
##
##  2-sample test for equality of proportions without continuity correction
##
## data:  c(us_nurses_midwives, uk_nurses_midwives) out of c(us_pop, uk_pop)
## X-squared = 57941, df = 1, p-value < 2.2e-16
## alternative hypothesis: two.sided
## 95 percent confidence interval:
##  0.003585282 0.003637892
## sample estimates:
##      prop 1      prop 2
## 0.012504975 0.008893388
```

3. Suppose we wanted to test whether there was a change in the mean number of doctors per country between 2019 and 2020 (e.g., in response to COVID-19). What would be a good way to do so?

4. Perform a formal analysis of variance for the difference in 2010 log GDP per capita by world region.

```
##              Df    Sum Sq   Mean Sq F value Pr(>F)
## Region       21 6.977e+10 3.322e+09   12.84 <2e-16 ***
## Residuals   187 4.839e+10 2.588e+08
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 23 observations deleted due to missingness
```

5. Comment on the assumptions of the test.




```
##                Region Variance Number of countries
## 1           Caribbean     0.80                  22
## 2     Central America     0.49                   8
## 3        Central Asia     1.15                   5
## 4      Eastern Africa     1.28                  18
```

```
## 5         Eastern Asia      2.95                   7
## 6       Eastern Europe      0.60                  10
## 7            Melanesia      1.56                   5
## 8           Micronesia      0.55                   5
## 9        Middle Africa      1.72                   9
## 10     Northern Africa      0.47                   6
## 11    Northern America      0.15                   4
## 12     Northern Europe      0.52                  10
## 13             Oceania      0.16                   2
## 14           Polynesia      0.84                   5
## 15       South America      0.35                  12
## 16  South-eastern Asia      2.17                  11
## 17     Southern Africa      0.57                   5
## 18       Southern Asia      0.96                   9
## 19     Southern Europe      0.89                  14
## 20      Western Africa      0.43                  16
## 21        Western Asia      1.44                  17
## 22      Western Europe      0.31                   9
```

Manipulating new distributions

Let $T_n \sim t_n$. Find the following:

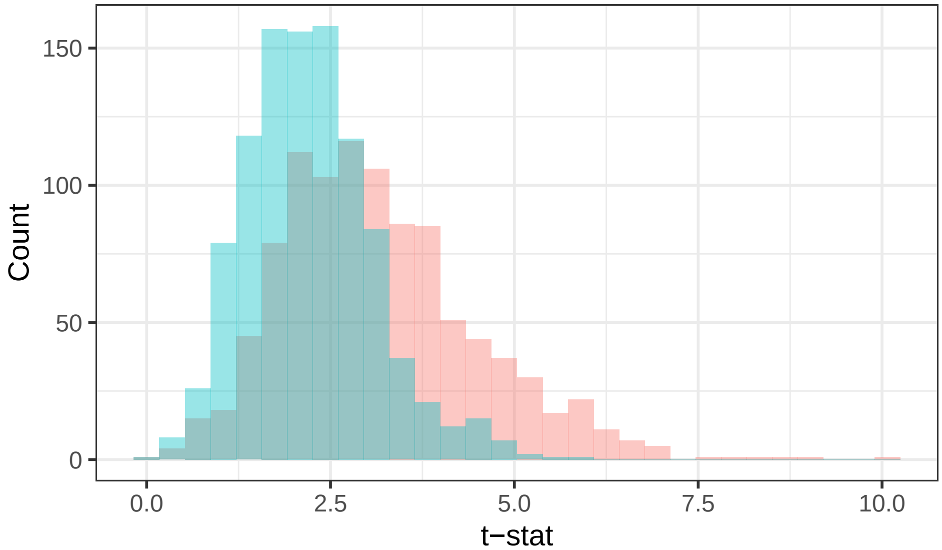
1. Distribution of T_n^2. Hint: Think about the representation of T_n.

2. Distribution of T^{-2}

3. Let $X_1, ..., X_n \sim \text{Expo}(\alpha)$. Find the k (in terms of α) such that $k\sum_{i=1}^n X_i \sim \chi^2_{2n}$.

Simulations

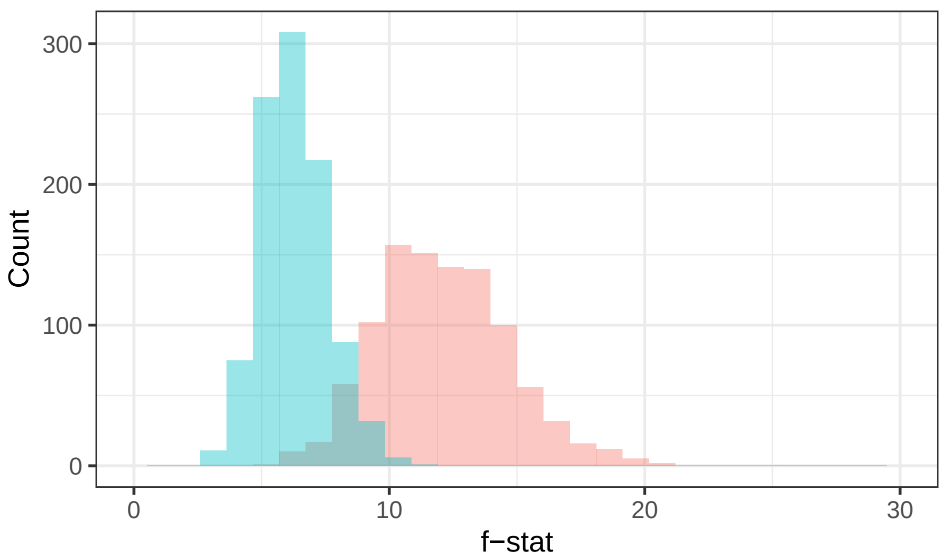
1. Let $X_1, X_2, \dots, X_n \sim \mathcal{N}(\mu, \sigma^2)$. Then, let $X_{i,1} = X_i + \epsilon_{i,1}$ and $X_{i,2} = X_i + \beta + \epsilon_{i,2}$ with $\epsilon_{i,j} \sim \mathcal{N}(0, \sigma^2)$. Suppose we simulate many paired and unpaired t-tests for the difference in the mean of the $X_{i,1}$s vs. the mean of the $X_{i,2}$s. If β is non-zero, which color is the paired t-test?



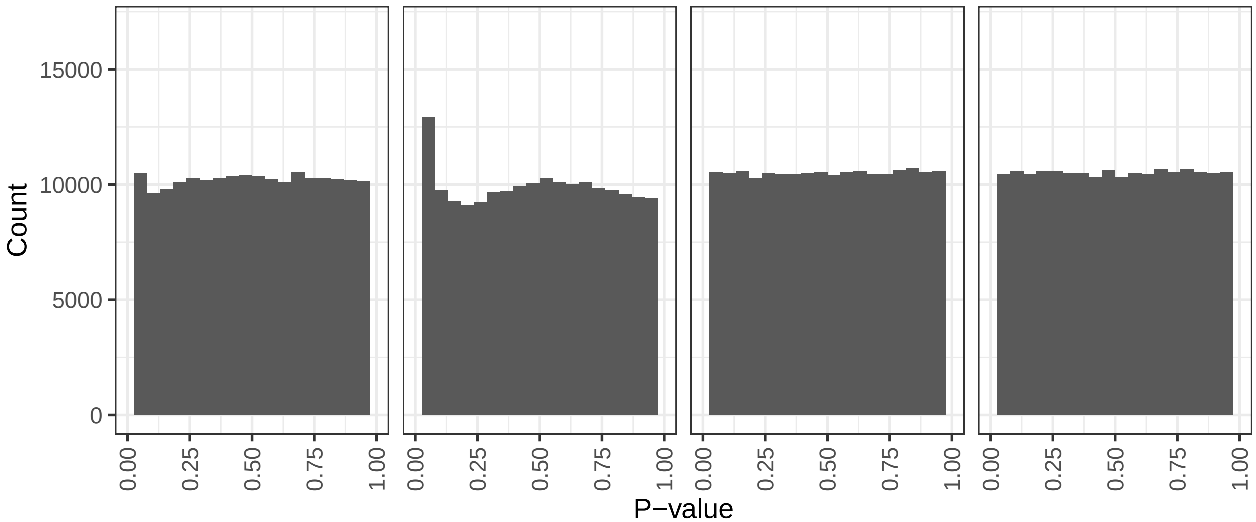
2. Suppose we have some β_i for $i \in \{1, ..., n_\beta\}$ that are not equal. Let $X_{i,j} = \beta_i + \epsilon_{i,j}$ for $j = 1$ to n with $\epsilon_{i,j} \sim \mathcal{N}(0, \sigma^2)$. We want to test whether $\beta_1 = \beta_2 = \dots = \beta_{n_\beta}$. We'll run a simulation in which we consider two cases:

- In the first case, we use the proper groupings of the $X_{i,j}$; that is, there are n observations in each group, all with the same β_i.
- In the second case, we'll subdivide each of these groups into 2 so that there are $n/2$ observations in each group with two groups for each β_i.

We'll run an ANOVA in each case and repeat this many times. Which color is which case?



3. Let $X_i \sim \mathcal{N}(0, 1)$ for i from 1 to n. Let $Y_i \sim -1 + \text{Expo}(1)$ for i from 1 to n. Suppose we conduct a two-sided, one-sample t-test for $H_0 : \mu = 0$ vs. $H_a : \mu \neq 0$ and record the p-value. The plots below show p-values from simulations repeating this many times for the two distributions and $n = 5$ or $n = 20$. Identify which is which.



4. Which of the two comparisons do you expect to have the lower p-value? The one with a larger difference in sample means or the one with more data points (40 vs 400)?