CS6351 September 22, 2022
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Homework 2
Due Date: October 13, 2022

Problem 1: (20 points)
Consider an array $x[0{:}\,n-1]$ of ascii characters. The Burrows-Wheeler Transform (BWT) of *x* works as follows:

1. It forms a matrix *A* of all the *n* rotations of *x*; (ex: if *x* =[a,b,c], the rotations of *x* are [a,b,c], [b,c,a], and [c,a,b])
2. It sorts the rows of *A* lexicographically where every row is treated as a word; the outcome is a matrix *B*;
3. It returns (*y*,*L*) where *y* is the last column of *B*, and *L* is the location of the original *x* in *B*;

a. Apply BWT on the string $ababbccab$, and also on the string “the dog in the fog” ', where the blank character ranks lexicographically before any alphabetic character. Note that you can use Matlab “sortrows” command to sort the rows of A.

b. Give a general algorithm that constructs *B* from (*y*,*L*).

c. Give an algorithm to reconstruct *x* from (*y*,*L*) using the algorithm in (b).

d. Illustrate how the reconstruction algorithm works on the (*y*,L) derived in (a) for $ababbccab$.

Problem 2: (20 points)
Let *x* and *y* be two column vectors:

$$x_k = \frac{(k-8)^2}{4}, \text{ for } k=0,1,\ldots,15$$

$$y_k = \cos\left(k\frac{\pi}{16}+1\right)+\sin\left((2k+1)\frac{\pi}{16}\right), \text{ for } k=0,1,\ldots,15.$$

a. Give in a table of 3 columns the values of $[X, |X|, \hat{X}]$, in a 2^{nd} table the values of $[Y, |Y|, \hat{Y}]$, in a 3^{rd} table of 2 columns the values of $[x, \hat{x}]$, and in a 4^{th} table the values of $[y, \hat{y}]$, where:
   - *X* is the Fourier transform of *x*, *Y* is the Fourier transform of *y*.
   - $|X|$ is the column of the magnitudes of the elements of *X*, and $|Y|$ is the column of the magnitudes of the elements of *Y*, where magnitude of a real number *t* is its absolute value $|t|$, and the magnitude of a complex number $z=a+ib$ is $|z| = \sqrt{a^2+b^2}$.
   - $\hat{X}$ is a column derived from *X* by replacing each of the 11 smallest-magnitude elements of *X* by 0, and leaving the other elements intact. $\hat{Y}$ is defined similarly.
   - $\hat{x}$ is the inverse Fourier transform of $\hat{X}$, and $\hat{y}$ is the inverse Fourier transform of $\hat{Y}$.

b. Plot *x* and $\hat{x}$ in one figure, and *y* and $\hat{y}$ in another figure.

c. Compute the mean square error $MSE(x, \hat{x})$ between *x* and $\hat{x}$, and also $MSE(y, \hat{y})$.

d. Compute $SNR(x, \hat{x})$ and $SNR(y, \hat{y})$.

Problem 3: (20 points)

Let x and y be as in Problem 2. Let X be the DCT of x, and Y the DCT of y. Let $\hat{X}$ be derived from X by replacing the last 11 elements of X by zeros while keeping the rest of the elements the same, and define $\hat{Y}$ similarly from Y. Finally, let $\hat{x}$ be the inverse DCT of $\hat{X}$, and $\hat{y}$ the inverse DCT of $\hat{Y}$.

a. Give the following four tables: $[X, \hat{X}]$, $[Y, \hat{Y}]$, $[x, \hat{x}]$, and $[y, \hat{y}]$.

b. Plot x and $\hat{x}$ in one figure, and y and $\hat{y}$ in another figure.

c. Compute the mean square error MSE(x, $\hat{x}$) between x and $\hat{x}$, and also MSE(y, $\hat{y}$).

d. Compute SNR(x, $\hat{x}$) and SNR(y, $\hat{y}$).

Problem 4: (20 points)

a. Let x and y be as in Problem 2, and use this time the Hadamard transform, so X and Y are the Hadamard transforms of x and y, respectively. $\hat{X}$ is derived from X by replacing the 11 smallest-magnitude elements of X by zeros while keeping the rest of the elements the same. $\hat{Y}$ is defined similarly. Report the four tables: $[X, |X|, \hat{X}]$, $[Y, |Y|, \hat{Y}]$, $[x, \hat{x}]$, and $[y, \hat{y}]$. (Use the Matlab/Octave ``hadamard" command to generate the Hadamard matrix.)

b. Compute the mean square error MSE(x, $\hat{x}$) between x and $\hat{x}$, and also MSE(y, $\hat{y}$).

c. Put in one figure the plots of x and the three $\hat{x}$'s of Problems 2, 3 and 4(a), and in another figure the plots of y and the three $\hat{y}$'s of Problems 2, 3 and 4(a).

d. Compare the MSE(x, $\hat{x}$)'s for all three problems, also the MSE(y, $\hat{y}$)'s. Which transform gives the best MSE(x, $\hat{x}$), and which gives the best MSE(y, $\hat{y}$).

Problem 5: (20 points)

Let x and X be as in Problem 3. For each $n = 1, 2, \dots, 15$, let $\hat{X}_n$be derived from X by replacing the last n elements of X by zeros while keeping the rest of the elements the same, and let $\hat{x}_n$ be the inverse DCT of $\hat{X}_n$. Let also $MSE(n)$ be MSE(x,$\hat{x}_n$). (Don't report the values of $\hat{X}_n$ and $\hat{x}_n$, but only compute them for yourself so you can answer parts a and b of this problem.)

a. Compute the $MSE(n)$ for $n = 1, 2, \dots, 15$, show them in a table, and plot the array $MSE(1{:}\,15)$ as a function of n.

b. What can you observe about the trend of MSE as n grows from 1 to 15.

c. Repeat (a) and (b) but this time use Hadamard instead of DCT.