Research Statement

Summary

My research interests are stochastic differential equations, heavy-tailed stochastic processes, time series analysis and the applications of these subjects to climate modelling. In climate modelling, there are many problems that require expertise in stochastic modelling techniques as simplified climate models typically include a stochastic component to account for complex or unresolved dynamics that are not of primary interest, but have an effect on the results of a particular project. My current work is focused on heavy-tailed noise processes and an approximation technique known as stochastic averaging in which dynamical models with two or more distinct time scales and noisy forcing are reduced to a smaller set of equations that can be used to approximate the slow variables of the original system in the distributional sense. The field of climate modelling stands to benefit from this work as there is increasing evidence that heavy-tailed stochastic processes are appropriate for modelling some aspects of climate [2, 3, 7]. Additionally, several climate problems are studied using stochastic averaging to negate the need to explicitly model fast time scale processes (i.e weather). My collaborators and I have derived new stochastic averaging approximations for stochastic dynamical systems where the stochastic forcing is a particular type of heavy-tailed noise known as α stable noise. Further related research aims to explain the appearance of α stable noise in climate data time series, such as the Greenland ice core project (GRIP) [2]. Additionally, I have worked on using a novel variational parameter estimation method for stochastic differential equation models on real wind speed data. In this project, my collaborators and I applied this method to estimate the parameters of a wind model derived from atmospheric physics allowing us to reconstruct the global parameter fields and identified deficiencies with the model itself.

Current Work

Stochastic averaging and α stable noise.

My current research focuses on problems related to stochastic averaging and homogenization of stochastic differential equation (SDE) models experiencing heavy-tailed stochastic forcing. Stochastic averaging methods are important in climate modelling and many simplified models for climate dynamics are found using these methods (e.g. [5]). The goal of stochastic averaging and homogenization is to reduce a system of stochastic dynamical equations evolving on multiple time scales to a smaller set of equations evolving on fewer time scales (typically just the longest one). Subsequently, the reduced system is a good weak-sense approximation of the dynamics of a subset of variables of the full system which are of interest to the researcher. If the reduction can be done in a systematic and consistent way, the reduced system retains key features of the full system (i.e. moments) but is also easier to analyze and simulate on the time scales of interest. The theory of stochastic averaging and homogenization has been explored in many written works and is now well-established for dynamics where the stochasticity is introduced through a Gaussian white noise forcing.

Stochastic averaging theory is limited for systems with heavy-tailed forcing processes, but at least one researcher has begun to investigate this problem [8]. Considering more general fast-slow systems forced, our question of interest is whether or not a more general stochastic averaging formula can be written for this situation. Previously derived stochastic averaging methods rely on computation of the second moment and autocovariance functions of the perturbations from the mean dynamics of the slow variable. However, this is not possible in the case of α stable noise because the second moment is undefined. Hence, I take a different approach with an analysis of the fast-slow system via characteristic functions [10], which exist for any random variables with normalizable probability density functions and can be determined by solving a partial differential equation (PDE) given by the Fourier transform of the appropriate Fokker-Planck equation. The solution to this PDE can be expressed in terms of an integral which we evaluate using an asymptotic analysis in the limit of infinite time-scale separation, showing that the perturbations away from the mean dynamics or the current state can be approximated by an integrated α stable noise process when the separation of time scales is great. Thus, we can approximate the dynamics of the slow variable of the system as a mean drift plus a stochastic perturbation given by an appropriately chosen Ornstein-Uhlenbeck-Lévy process, which approximates the effect of the fast dynamics. This approximation can be evaluated for both linear and nonlinear systems where the approximation of the latter may include multiplicative stochastic forcing, requiring us to interpret the stochastic averaging approximations in the sense of Marcus (the analogue of Stratonovich calculus for jump processes), rather than Itō. Numerical simulations show good agreement between the full system and the approximations.

Parameter estimation

In projects where process modelling is required, there are often associated model parameters that cannot be computed from theory (e.g. complexity, non-physicality, etc.). Often parameters for continuous time models are estimated from data assuming the target model is deterministic and that any randomness in the data is due to measurement error. When stochasticity is an intrinsic dynamical component of the model, the problem of parameter estimation is more complicated.

I worked on a parameter estimation project for SDEs using a more recent method [1] which optimizes the infinitesimal generator of a stochastic differential equation model relative to the eigenstruture of the conditional expectation operator which is estimated from data. This variational method is fast compared to other parameter estimation schemes, such as Monte Carlo methods, as it requires only the minimization of a quadratic objective function to determine optimal parameters. We applied this method to sea surface wind data and fit the data to a sea surface wind model derived from atmospheric physics [6]. Applying this method required us to resolve inconsistencies between the data and the requirements of the estimation method. For example, the data are inherently non-Markovian, while the parameter estimation method assumes the data are given by a Markov process. There were also difficulties surrounding the model in question such as the fact that the wind model did not allow for positively skewed data in the mean wind direction and assumes local isotropy of autocorrelation time scales in the zonal and meridional wind speeds, neither of which strictly hold considering statistical analysis of the data. To resolve the inconsistency related to the non-Markovian nature of the data, we applied temporal subsampling of the data, such that the stochastic forcing could be considered almost Markovian while not de-resolving the drift dynamics of the wind model. To resolve the difficulties associated with the inconsistencies of the model and the data we applied constrained optimization to search regions of parameter space where the lack of positive skewness in the mean wind direction is not violated (as not placing this restriction results in non-physical parameter estimates) and used a rescaling of the estimated parameters such that the distribution of the wind speeds remained invariant, but the autocorrelation times of the zonal and meridional wind speeds were rescaled to match the geometric mean of the two. After resolving these inconsistencies we were able to compute global parameter fields for mean wind forcing, noise parameters, and other parameters given by the model

[11]. The estimated parameters allowed us to generate simulated data from the wind model that had the same statistics as the original data. Moreover, this parameter estimation project allowed us to identify possible deficiencies and improvements to the wind model and provided insight into how to model wind flow in different geographical regions (i.e. storm tracks should be modelled with strong stochastic forcing).

M.Sc. Work

During my Master's degree, I analyzed phase synchronization of uncoupled oscillators forced with a combination of shared and intrinsic Gaussian white noise forcing [9]. This work is distinct from other studies of noise-induced phase synchronization because the oscillators in question did not have attracting limit cycles and the goal of this project was to determine whether noise-induced phase synchronization occurs in these oscillators. The parameters in this study were selected so that each oscillator was below, but near, a subcritical Hopf bifurcation point and could be excited by stochastic forcing resulting in coherent oscillations about a stable equilibrium point. Applying a stochastic multiple time scales analysis and a perturbation expansion analysis, we obtained approximate probability density functions for both the phase and amplitude of the oscillators. This analysis incorporated nonlinear effects into the higher order corrections, allowing prediction of the effects of amplitude-dependent phase evolution, the ratio of different intrinsic noises, and the ratio of extrinsic to common noise forcings on the probability density for the phase differences.

Future Work

I plan to continue studying stochastic averaging of multiple time scale systems where the dynamics of the fast process and/or their effects on the slow dynamics are nonlinear. I have preliminary results in the case where such a system is forced by Gaussian white noise in the fast component and the dynamics of the fast component influence the slow component through the square of their values. I have found that using a scaled Gaussian white noise forcing to approximate the dynamical effect of the square of an Ornstein-Uhlenbeck is insufficient in the sense that the full and averaged systems display a significant difference in distribution. Although, this is likely because the difference in time scales is not large enough to allow for sufficient convergence of the perturbations to Gaussianity.

I will continue to analyze the dynamical origins of α stable noise which are of interest because α stable noise has been observed in the calcium signal of the GRIP ice core [2]. Previous research has shown that a coarsely resolved dynamical system with chaotic behaviour is indistinguishable from a statistically similar stochastic system in an information-theoretic sense [4], highlighting the fact that poorly resolved fast chaotic processes can appear as Gaussian white noise in a time series. The question that we are interested in answering is whether or not there exists a time continuous model that can lead to the appearance of α stable noise forcing on a sufficiently long time scale. Systematic reductions of the Navier-Stokes equations result in simple linear dynamical systems with a combination of additive and multiplicative Gaussian white noise forcings that have been shown to have heavy-tailed stationary distributions [7]. I assert that if these linear processes are sufficiently fast, they will appear as α stable noise on a long time scale according to the Generalized Central Limit Theorem. Since the Gaussian white noise forcing in these systems can be treated as the coarsely resolved limit of a chaotic system, chaotic dynamics may lead to α stable noise appearing on very long time scales. To date, I have obtained results regarding the required separation of time scales required for convergence of the heavy-tailed noise to α stable noise within such a system. Future work on this project will include a simulation study of multiple time scale differential equation models and the application of the results of studies like [7] to predict the character of the resulting stochastic forcing approximation. One of the immediate challenges of this project will be time-efficient simulation of multiple time scale dynamical systems where the time scales are several orders of magnitude apart and several references related to this topic are available.

Ultimately, I am interested in pursuing further research in stochastic dynamics or a research project involving application of stochastic differential equations theory and am open to many possible research applications.

Last updated on October 9, 2014.

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