MAT 561 Review

1 Univariate

PDF:

$$PDF = f(x) = P(X = x)$$

Properties

$$f(x) \geq 0$$

$$\sum_{x \in R} f(x) = 1$$

CDF:

$$\begin{aligned} F(x) &= P(X \leq x) \\ &= \sum_{x \in a} P(X = a) \end{aligned}$$

Properties

$$P(X > c) = 1 - P(X \leq c)$$
$$P(X = c) = P(X \leq c) - P(X \leq c)$$

Expected Value:

$$\mu = E(x)$$
$$= \sum_{x \in R} x \cdot P(X = a)$$

more generally...

$$\mu = E(g(x))$$
$$= \sum_{i=1}^{\infty} g(x_i) \cdot P(X = x_i)$$

Variance:

$$\sigma = Var(x)$$
$$= E(X - \mu)^2$$
$$= E(X^2) - E((X)^2)$$

MGF:

$$M_x(t) = E(e^{tx})$$
$$= \sum_{x \in R} e^{tx} f(x)$$

and

$$M^n(0) = E(X^n)$$

some common cases

$$M_{x+a} = e^{at} M_x(t)$$
$$M_{bx}(t) = M_x(bt)$$
$$M_{\frac{x+a}{b}}(t) = e^{\frac{at}{b}} M_x \frac{t}{b}$$
$$M_{X+Y}(t) = M_X(t) M_Y(t) \quad \text{only by independence!}$$

Theorems:

$$E(aX + b) = aE(x) + b$$
$$Var(aX + b) = a^2 Var(x)$$

For distributions, see here:

http://matthias.vallentin.net/blog/2010/10/probability-and-statistics-cheat-sheet/

Common Series in Probability:

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x \qquad x \in R$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad -1 \leq x \leq 1$$

2 Discrete Bivariate

$$f(x,y) = P(X = x, Y = y)$$

$$f(x,y) \geq 0$$

$$\sum_{x \in R_x} \sum_{y \in R_y} f(x,y) = 1$$

Discrete Marginal Density Functions

$$f_x(x) = P(X = x) \sum_{y \in R_y} P(X = x, Y = y)$$

$$f_y(y) = P(Y = y) \sum_{x \in R_x} P(X = x, Y = y)$$

$$f(x,y) = f_x(x) f_y(y) \qquad \text{by independence}$$

Assorted Properties

$E(XY) = E(X)E(Y)$ by independence

$COV(X, Y) = E(XY) - E(X)E(Y)$

$\rho = \dfrac{COV(X, Y)}{\sigma_x \sigma_y}$

$VAR(X + Y) = VAR(X) + VAR(Y)$ by independence

$VAR(X + Y) = VAR(X) = VAR(Y) + 2COV(X, Y)$ without independence

some additional properties of covariance in handwritten notes if needed

Calculating Continuous Univariate Probabilities

$$P(a \leq x \leq b) = \int_a^b f(x)dx$$
$$P(X > c) = \int_c^\infty f(x)dx$$
$$E(g(x)) = \int_{-\infty}^\infty g(x)f(x)dx$$

CDF Method to find PDF

$\frac{d}{dy}F(y) = f(y)$ where F(y) is the CDF and f(y) is the PDF

Jointly Continuous Random Variables

$f(x,y) \geq 0$

$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)dydx = 1$

note that for the marginals, you are integrating "out" one of the vars

$f_x(x) = \int_{y=-\infty}^{\infty} f(x,y)dy$

$f_y(y) = \int_{x=-\infty}^{\infty} f(x,y)dx$

$E(XY) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xyf(x,y)dydx = 1$

Conditional notes are in handwritten set if needed

Limit Theorems

The central limit theorem states that if you have a population with mean μ and standard deviation σ and take sufficiently large random samples from the population with replacement , then the distribution of the sample means will be approximately normally distributed. Additionally, can only be applied if:

$$x_1, x_2, \text{...are iid}$$

$$\sigma^2 \leq \infty$$

$$n \geq 32$$

Also see notes on the SLLN and WLLN.