MAT 343

LAB 6 (6/23/2020)

```
A = imread('gauss.jpg');
B = double (A(: ,: ,1));
B = B /255;
[U S V] = svd(B);
```

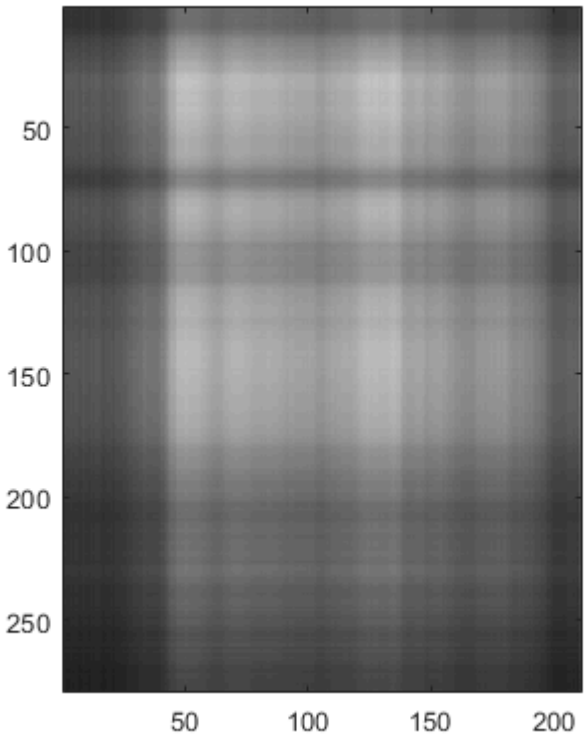
Problem 1

```
size(U), size(S), size(V)
```

```
ans = 1×2
   279   279
ans = 1×2
   279   211
ans = 1×2
   211   211
```

Problem 2 - find the rank 1 approximation of B

```
rank1 = U(:,1)*S(1,1)*V(:,1)';
C = zeros(size(A));
C(:,:,1) = rank1;
C(:,:,2) = rank1;
C(:,:,3) = rank1;
C = max(0,min(1,C));
image (C), axis image
```

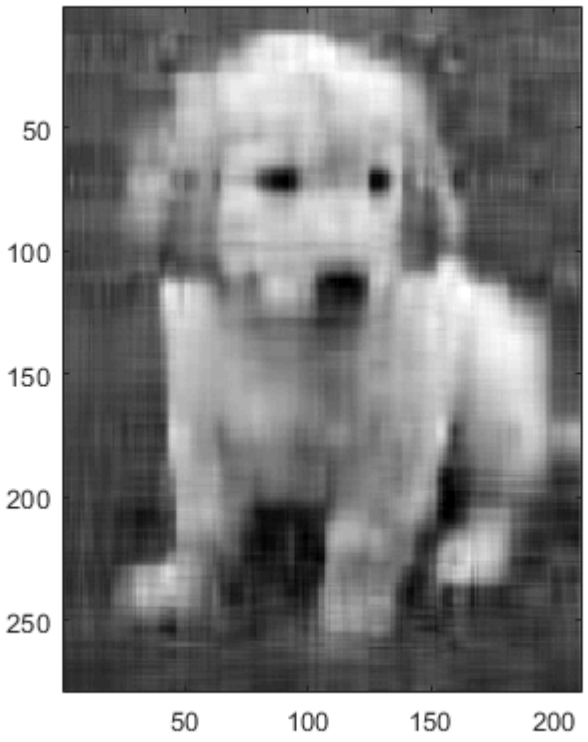


Problem 3 - rank 10 approximation

```
rank_selection = 10;

rank1 = zeros(279,211);
for i=1:rank_selection
    rank1 = rank1 + U(:,i)*S(i,i)*V(:,i)';
end

C = zeros(size(A));
C(:,:,1) = rank1;
C(:,:,2) = rank1;
C(:,:,3) = rank1;
C = max(0,min(1,C));
image (C), axis image
```



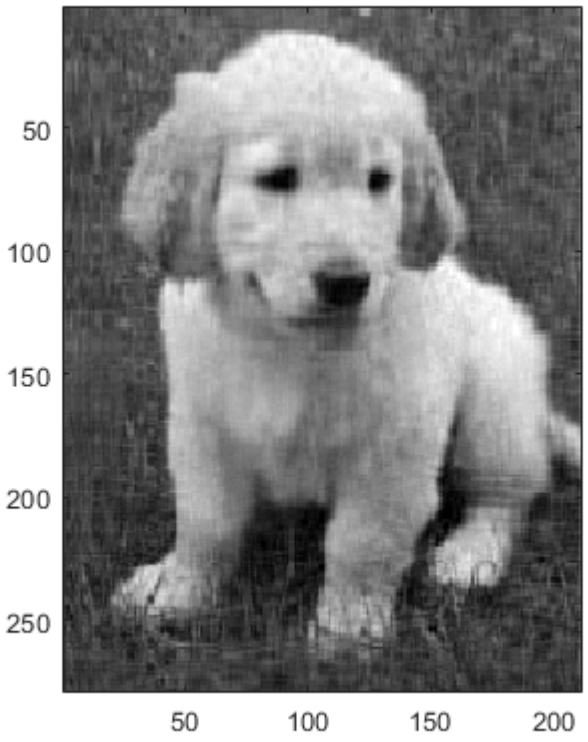
Problem 4 - higher level approximations

1. Rank 30 appears to give a good approximation. I can't notice any speed differences at this level, although when I tried a rank 200 it was noticeably slower. I'm sure if I used a timing function it would show that rank 30 is extremely fast in comparison - this seems like a good trade off between speed and image quality. While the image below is grainy, it is obviously a dog.

```
rank_selection = 30;

rank1 = zeros(279,211);
for i=1:rank_selection
    rank1 = rank1 + U(:,i)*S(i,i)*V(:,i)';
end

C = zeros(size(A));
C(:,:,1) = rank1;
C(:,:,2) = rank1;
C(:,:,3) = rank1;
C = max(0,min(1,C));
image (C), axis image
```



Problem 5 - What rank-r approximation exactly reproduces the original picture?

1. Rank 211 exactly reproduces the original image since that is the total number of singular values we have.

Problem 6 - How much data is needed for a rank-k approximation?

i) Generally:

$k + k * U_{m,m} + k * V_{n,n}$

ii) The compression rate represents the amount of data removed (or kept) from the original image by choosing k singular values. Thus, we can use about 75% less data than the original image.

```
%size of original image
279*211
```

```
ans = 58869
```

```
%size of rank 30
30+30*279+30*211
```

```
ans = 14730
```

```
compression_rate = 14730 / 58869
```

```
compression_rate = 0.2502
```

Problem 7

1. using formula 6 i), I've found that $k <= \frac{(m * n)}{(m + n + 1)}$ otherwise it will exceed the original image size.

   Solving in this example, I've found that k = 119.8961. However, because it must be an integer, and must be less than or equal to, this must be rounded down to 119.
2. In the test case below, k > 119 results in more data than size(A) (the original image).

```
m = 279;
n = 211;

%generic formula k*(m+n+1) <= m*n
%reduces to k <= (m*n) / (m + n + 1)
k = (m*n) / (m+n+1)
```

```
k = 119.8961
```

```
test_k_large = 120;

test_k_large+test_k_large*279+test_k_large*211
```

```
ans = 58920
```

```
test_k_correct = 119;

test_k_correct+test_k_correct*279+test_k_correct*211
```

```
ans = 58429
```