Induction Machines

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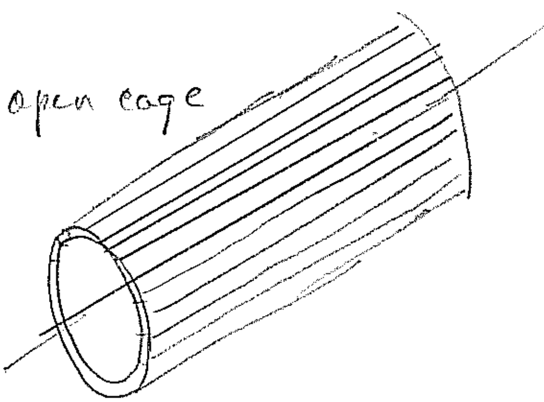
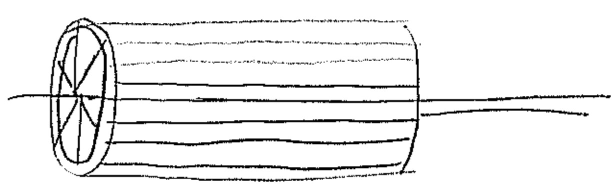
Introduction

- Most commonly used motor
- Consists of a stator (armature) and a rotor mounted in the stator on bearings.
- Rotor separated from the stator by an air gap.
- 2 types of induction motors
  - Squirrel cage – brushless
  - wound rotor – slip rings
- acts like a transformer with a rotating secondary
- runs on torque developed by interaction of induced rotor currents & air gap fields

Construction

open cage





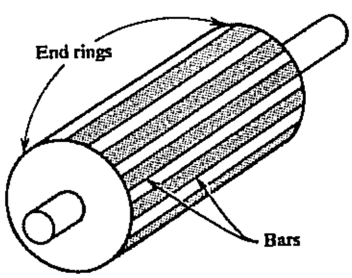
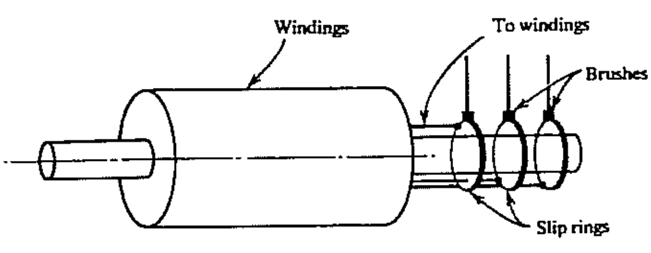
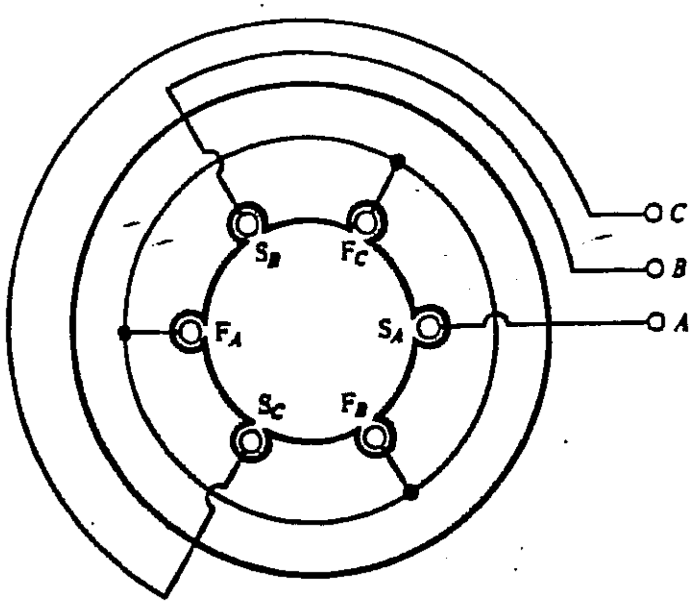


Fig. 5-1

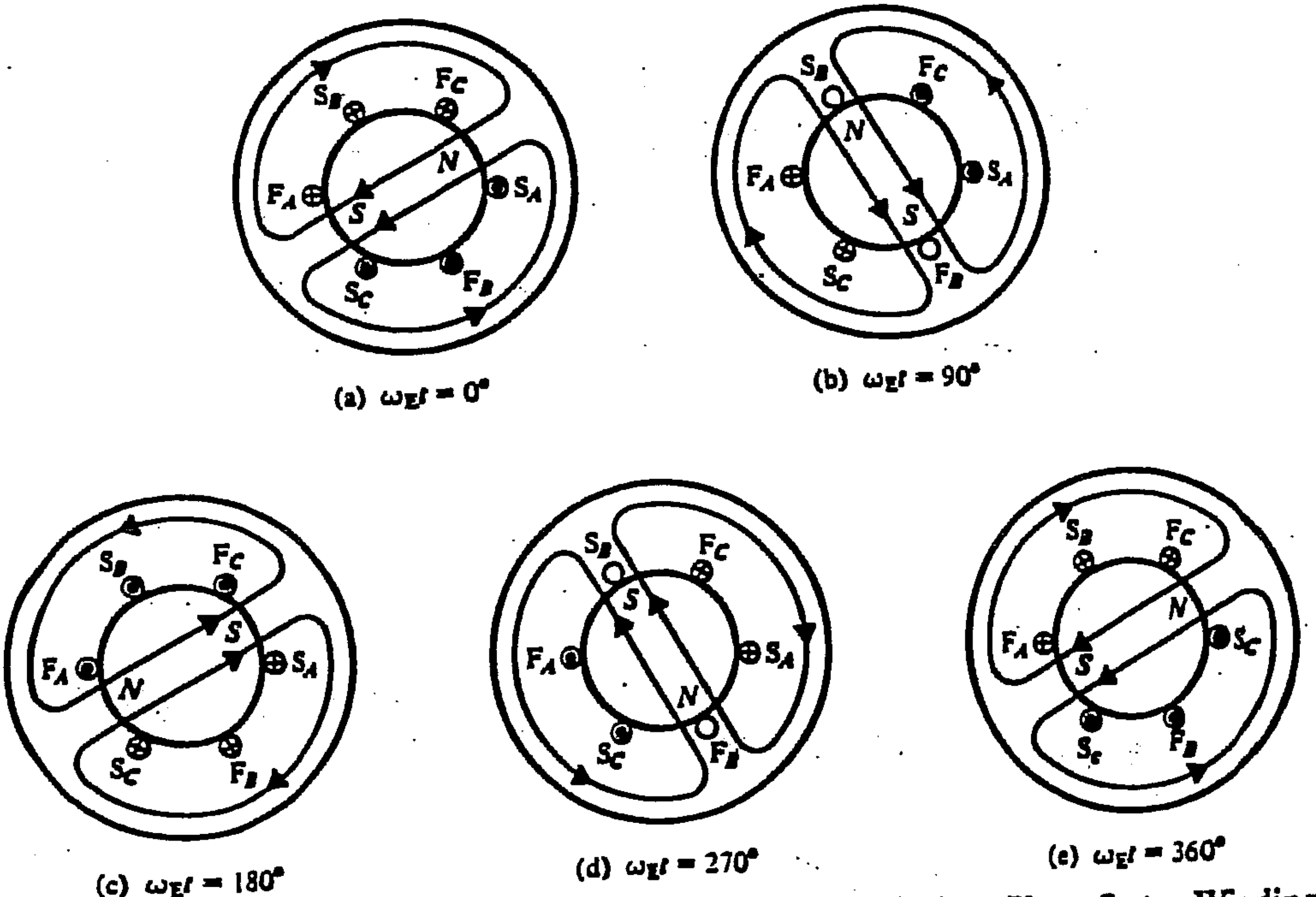


wound rotor

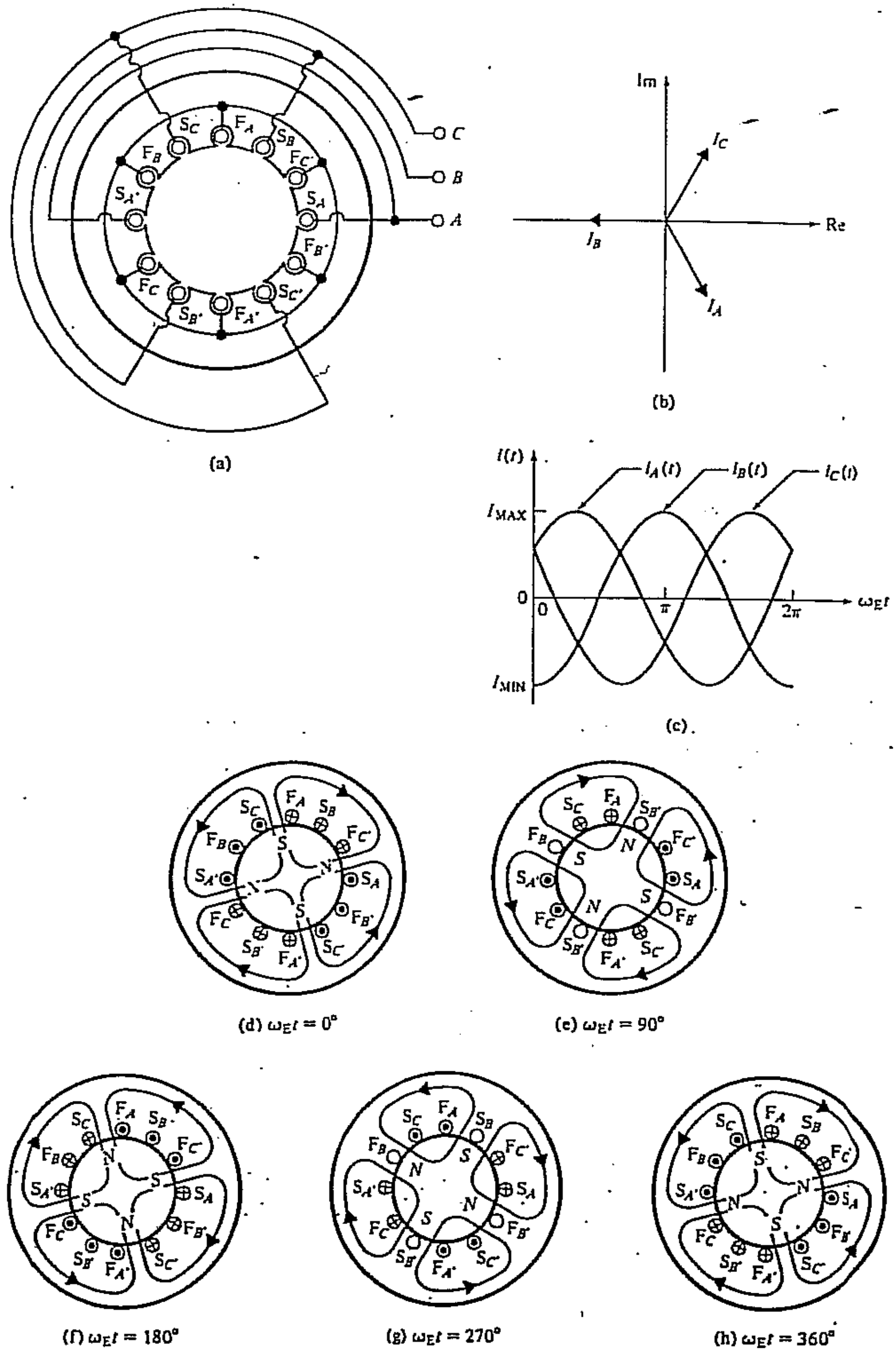
Fig. 5-2



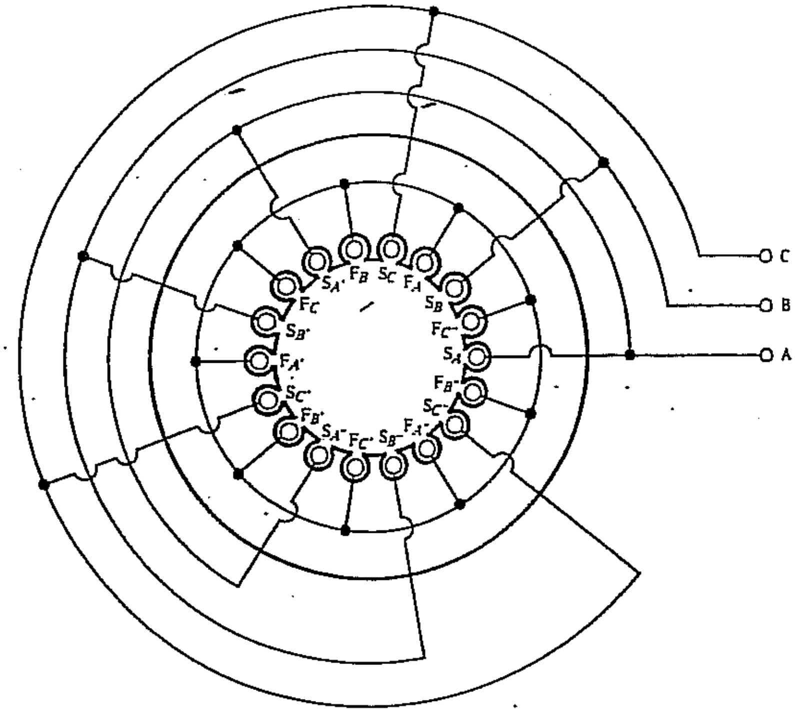
Three-Phase Stator Winding Distribution for the Generation of Two Poles in the Stator of an Induction Device



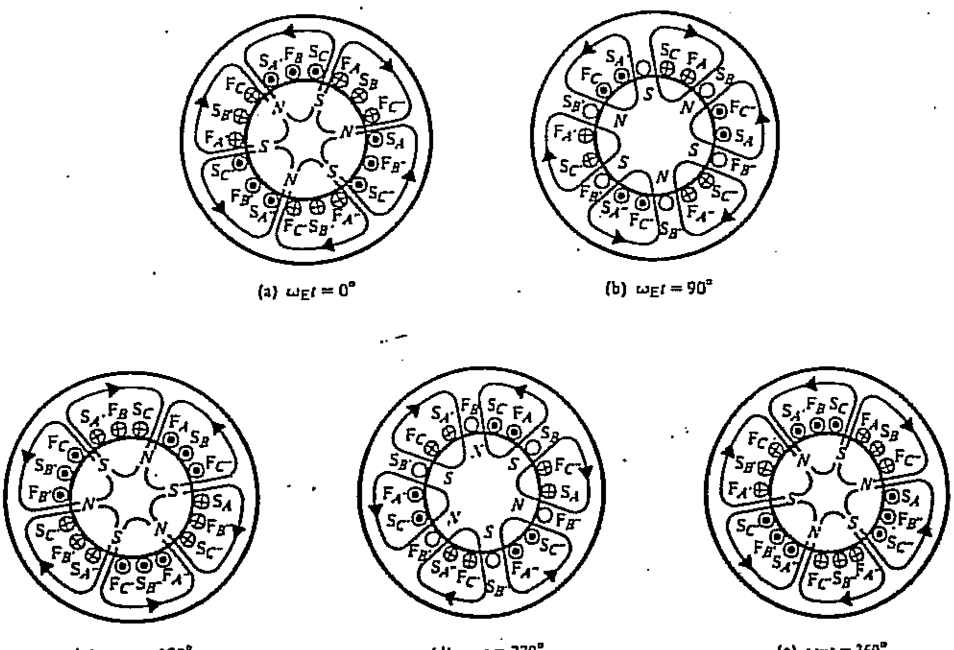
t-Domain Current Orientation in the Concentrated Three-Phase Stator Winding at $\omega_E t = 0°$, 90°, 180°, 270°, and 360° and Thus the Generation of Two Magnetic Poles Rotating Counterclockwise



Generation of a Four-Pole Rotating Field in the Stator of a Three-Phase Induction Motor: (a) Three-Phase Winding Distribution, (b) *ABC*-Sequence, Stator Phasor Currents, (c) *ABC*-Sequence, Stator *t*-Domain Currents, and (d) through (h) *t*-Domain Current Orientation in the Concentrated Three-Phase Stator Winding at $\omega_E t = 0°$, 90°, 180°, 270°, and 360° and Thus the Generation of Four Magnetic Poles Rotating Counterclockwise



Three-Phase Stator Winding Distribution for the Generation of Six Poles in the Stator of an Induction Device



t-Domain Current Orientation in the Concentrated Three-Phase Stator Winding at $\omega_E t = 0°$, $90°$, $180°$, $270°$, and $360°$ and thus the Generation of Six Magnetic Poles Rotating Counterclockwise

- SLIP

- The actual speed, n, of the rotor is expressed of the synchronous speed, n_s, using the term slip.

$$s \triangleq \frac{n_s - n_{act}}{n_s}$$

and

$$\%s \triangleq \frac{n_s - n_{act}}{n_s} \times 100\%$$

- at standstill, starting or locked rotor $s = 1$. The rotating magnetic field produced by the stator has the same speed with respect to the rotor windings as with respect to the stator windings. The frequency of the rotor currents, f_2, is the same as the frequency of the stator currents, f_1.
- At synchronous speed ($s = 0$), there is no relative motion between the rotating field and the rotor, and the frequency of rotor current is zero.
- At all speeds inbetween the rotor current frequency is proportional to the slip. $f_2 = s f_1$

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5.4. A 4-pole, 3-phase induction motor is energized from a 60-Hz supply, and is running at a load condition for which the slip is 0.03. Determine: (*a*) rotor speed, in rpm; (*b*) rotor current frequency, in Hz; (*c*) speed of the rotor rotating magnetic field with respect to the stator frame, in rpm; (*d*) speed of the rotor rotating magnetic field with respect to the stator rotating magnetic field, in rpm.

$$n_s = \frac{120f_1}{p} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

(*a*) $$n = (1-s)n_s = (1-0.03)(1800) = 1746 \text{ rpm}$$

(*b*) $$f_2 = sf_1 = (0.03)(60) = 1.8 \text{ Hz}$$

(*c*) The p poles on the stator induce an equal number of poles on the rotor. Now, the same argument that led to (*5.4*) can be applied to the rotor. Thus, the rotor produces a rotating magnetic field whose speed, *relative to the rotor*, is

$$n_r = \frac{120f_2}{p} = \frac{120sf_1}{p} = sn_s$$

But the speed of the rotor relative to the stator is $n = (1-s)n_s$. Therefore, the speed of the rotor field with respect to the stator is

$$n_s' = n_r + n = n_s$$

i.e., in this case, 1800 rpm.

(*d*) Zero.

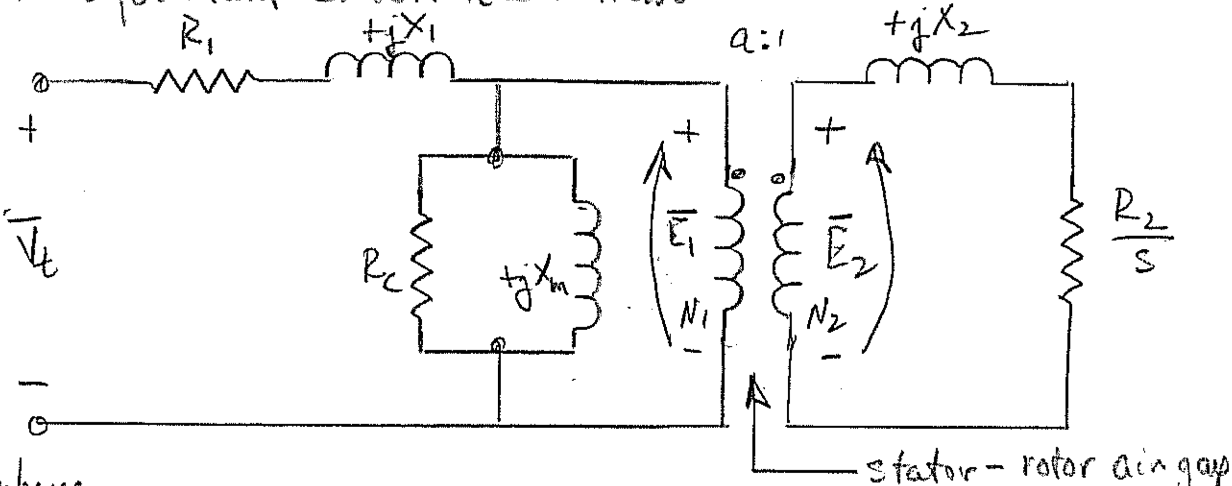
5.5. A 60-Hz induction motor has 2 poles and runs at 3510 rpm. Calculate (*a*) the synchronous speed and (*b*) the percent slip.

(*a*) $$n_s = \frac{120f_1}{s} = \frac{120(60)}{2} = 3600 \text{ rpm}$$

(*b*) $$s = \frac{n_s - n}{n_s} = \frac{3600-3510}{3600} = 0.025 = 2.5\%$$

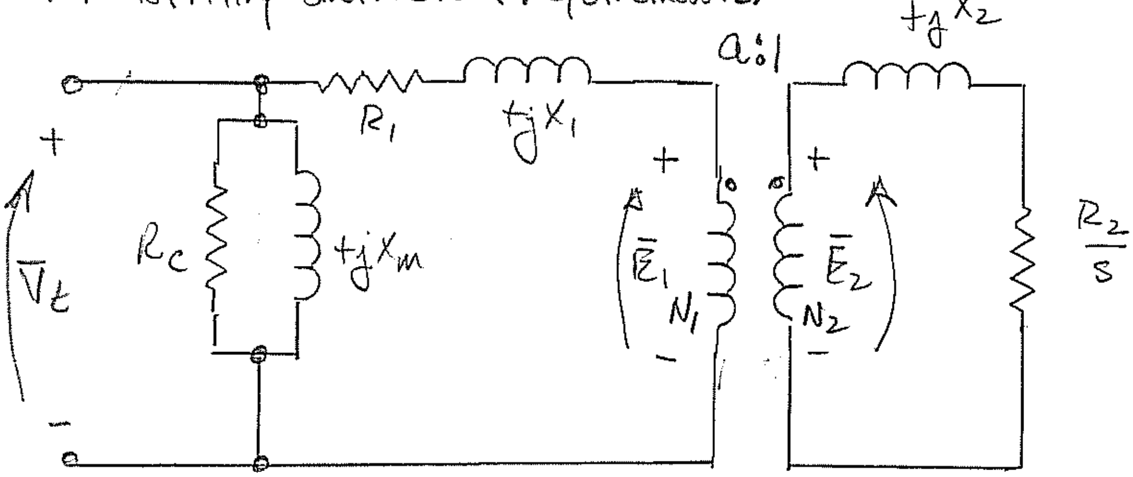
Equivalent Circuits

- Since the induction motor is an AC machine in which alternating current is supplied to the stator armature windings directly.
- From the stator windings the power is transferred to the rotor windings by electromagnetic induction or transformer action.
- The device has been called a rotating transformer.
- An induction regulator operates the same way except its rotation causes the rotor winding to boost or buck the input voltage so the output satisfies a given setpoint.
- The equivalent circuit is as follows:



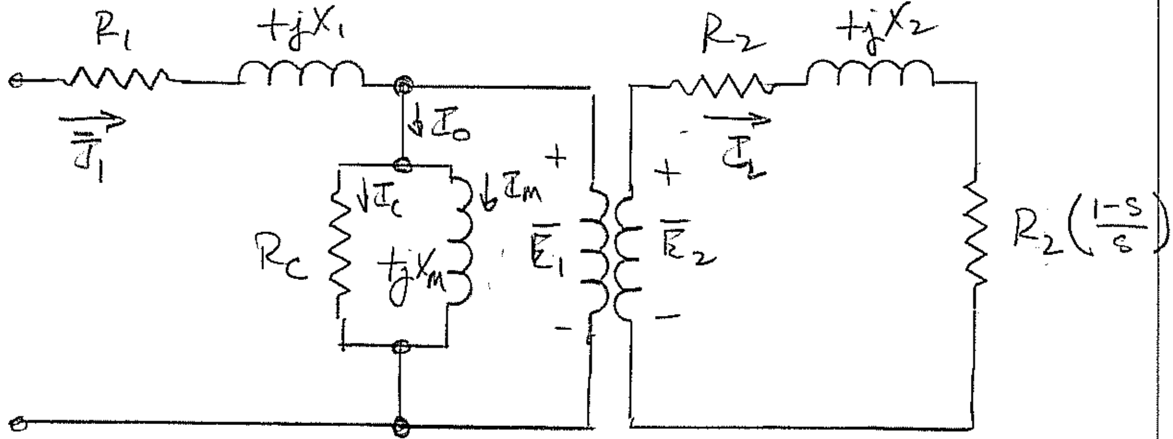
where
$\bar{V}_t$ = stator terminal voltage per phase
$\bar{E}_1$ = stator induced voltage per phase
$\bar{E}_2$ = rotor induced voltage at standstill i.e. $s = 1.0$
R_1 = stator winding resistance
X_1 = stator leakage reactance
R_c = equivalent stator/rotor core resistance
X_m = equivalent stator magnetizing reactance
R_2 = rotor winding resistance
X_2 = rotor leakage reactance
a = stator/rotor turns ratio $|\bar{E}_1|/|\bar{E}_2|$

- The "approximate" equivalent circuit shown below can be be used with a high degree of accuracy but is <u>not</u> a substitute for calculations for fulfiling contractual requirements.

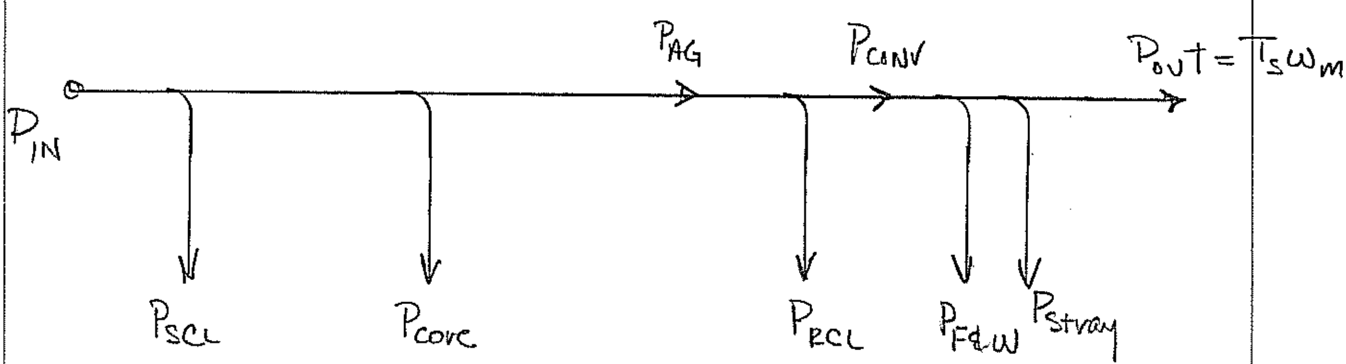


Induction motor/generator losses

- An induction motor can be basically described as a rotating transformer. For an ordinary transformer, the output is electric power from the secondary windings.
- The secondary windings in an induction motor (the rotor) are shorted out, so no electrical output exists from normal induction motors.
- Using our equivalent circuit we have as follows:



The corresponding power flow diagram is



$P_{IN} \triangleq V I \cos\theta$

$P_{SCL} \triangleq$ stator copper loss $= I_1^2 R_1$

$P_{core} \triangleq$ Core loss $= I_c^2 R_c$

$P_{AG} \triangleq$ Power Crossing the airgap

$P_{RCL} \triangleq$ rotor copper loss $= I_2^2 R_2$

P_{CONV} = power converted

$P_{F\&W} \triangleq$ friction & windage loss

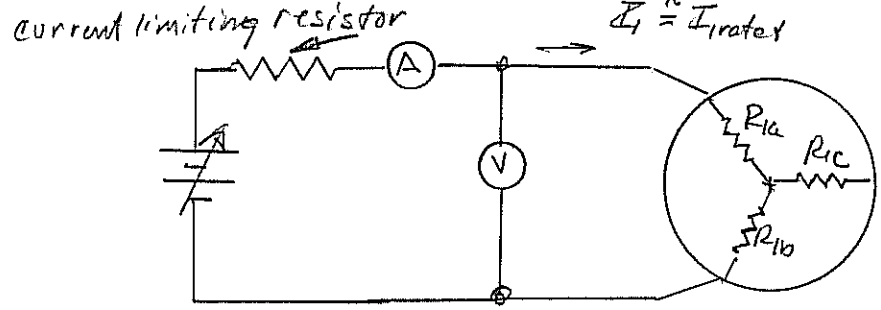
$P_{stray} \triangleq$ stray loss

$P_{out} \triangleq$ Mech power out $= T_s \omega_m$

Motor & Generator Testing and Equivalent Circuit Parameters

- The equivalent circuit of an induction motor is very useful for determining the motor's response. To determine R_c, X_m, R_1, R_2, X_1 and X_2 a series of tests must be performed. They are DC test for stator resistance, No Load Test, Locked Rotor Test.

- DC test for Stator resistance
  - The rotor resistance R_2 plays a critical role in the operation of an induction motor. R_2 determines the shape of the speed-Torque curve, determining the speed at which the pullout torque occurs. The locked-rotor test determines only total resistance.
  - There is a test for R_1 independent of R_2, X_1 and X_2. A DC voltage is applied to the stator windings of an induction motor. Because the current is DC, there is no induced voltage in the rotor circuit and no resulting current flow. The reactance of the motor is zero at DC. Thus, the only quantity limiting current flow is the stator resistance.
  - DC Test Circuit



Since the current in the windings is a function of R_1 i.e. $2R_1 = \dfrac{V_{DC}}{I_{DC}}$ and $R_1 = \dfrac{1}{2}\dfrac{V_{DC}}{I_{DC}}$

To be more rigorous we make 3 tests taking two windings at a time.

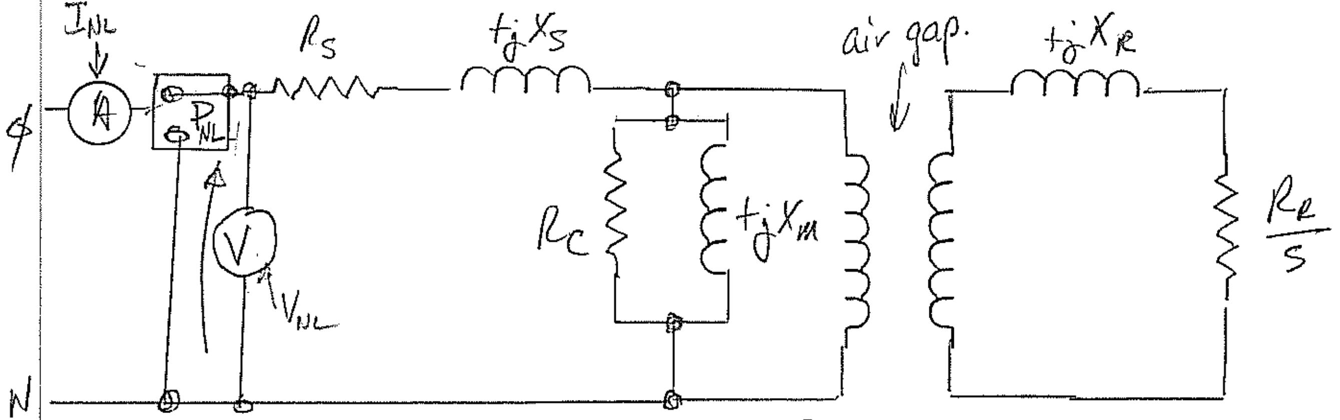
$$R_{1a} + R_{1b} = V_{DC1}/I_{DC1}$$
$$R_{1b} + R_{1c} = V_{DC2}/I_{DC2}$$
$$R_{1a} + R_{1c} = V_{DC3}/I_{DC3}$$

The true values of R_1 can be determined on each phase

+ IEEE Std 112 more details regarding these measurements

No Load Test

- Like the open circuit test on a transformer, the test is performed to obtain the shunt parameters, which represent the magnetizing current and core loss. Also included is the windage & friction. The <u>no-load</u> test is taken at rated voltage and frequency. Using our model



- When running at no load $s \approx 0$ then R_R/s = large. When the motor operates at rated voltage & frequency, the combined rotational loss including friction and windage, hysteresis & eddy current loss, and stray loss are assumed to be constant at any load.

$$P_{rot} = P_{NL} - 3I_{NL}^2 R_S \Rightarrow R_S = \frac{P_{NL}}{3I_{NL}^2}$$

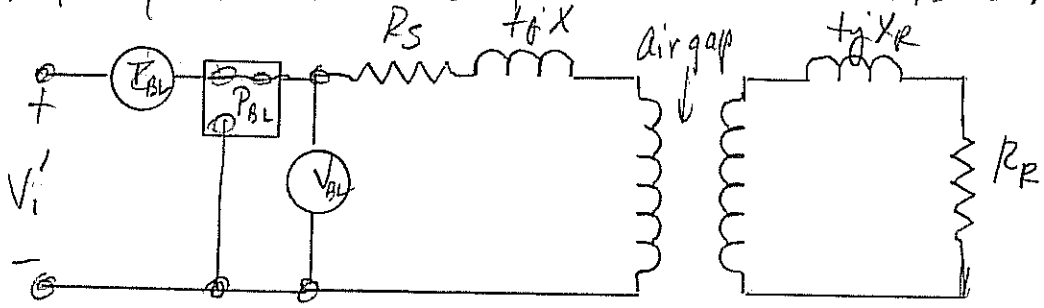
- Note $R_{NL} \neq R_1$ because R_{NL} includes <u>all</u> the no load losses.

$$Z_{NL} = \frac{V_{NL}/\sqrt{3}}{I_{NL}} = R_{NL} + jX_{NL} \quad \text{but} \quad X_{NL} = \sqrt{Z_{NL}^2 - R_{NL}^2}$$

and $X_{NL} = X_s + X_m$ and note $X_m \gg X_s$

BLOCKED ROTOR TEST

- In this test, the rotor is blocked so that it can not rotate i.e $s=1$. The motor appears to be a short circuited transformer. A reduced voltage is applied to get rated current flowing.
- During normal running conditions, the rotor frequency $\propto$ slip. Therefore, when the performance of the motor is being investigated at, or near, rated loads (low values of slip) the blocked rotor test should be taken at lower frequency. A test frequency of 25% of rated frequency is recommended by IEEE. Our circuit is as follows:



- From the readings $R_{BL} = \dfrac{P_{BL}}{3 I_{BL}^2}$. Then at the test frequency f_{test} and $V_{BL} \ll V_{rated}$ R_c and X_m are negligable. Then

$$Z_{BL} = \frac{V_{BL}}{\sqrt{3}\, I_{BL}} \quad \text{and} \quad X_{BL,test} = \sqrt{Z_{BL}^2 - R_{BL}^2}$$

Correcting $X_{BL,test}$ to X_{BL} $\quad X_{BL} = \dfrac{f_{rated}}{f_{test}} X_{BL,test}$

Then since the winding ratio $a=1$ (most of the time)

$$R_{BL} = R_S + R_R \quad \text{and} \quad X_{BL} = X_S + X_R$$

Since R_S is determined from the DC Test then $R_R = R_{BL} - R_S$

and $\quad X_S = X_R = \frac{1}{2} X_{BL}$

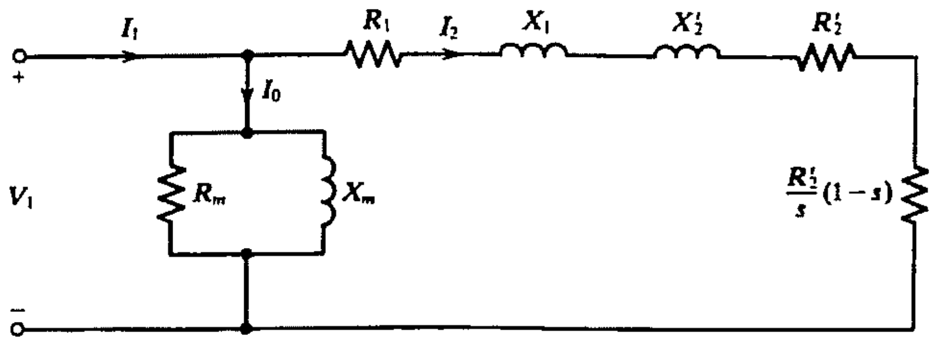


Fig. 5-9

The results of no-load and blocked-rotor tests on a 3-phase, wye-connected induction motor are as follows:

no-load test: line-to-line voltage = 400 V
input power = 1770 W
input current = 18.5 A
friction and windage loss = 600 W

blocked-rotor test: line-to-line voltage = 45 V
input power = 2700 W
input current = 63 A

Determine the parameters of the approximate equivalent circuit (Fig. 5-9).

From no-load test data:

$$V_0 = \frac{400}{\sqrt{3}} = 231 \text{ V} \qquad P_0 = \frac{1}{3}(1770 - 600) = 390 \text{ W} \qquad I_0 = 18.5 \text{ A}$$

Then, by (*5.17*) and (*5.18*),

$$R_m = \frac{(231)^2}{390} = 136.8 \ \Omega$$

$$X_m = \frac{(231)^2}{\sqrt{(231)^2(18.5)^2 - (390)^2}} = 12.5 \ \Omega$$

From blocked-rotor test data:

$$V_s = \frac{45}{\sqrt{3}} = 25.98 \text{ V} \qquad I_s = 63 \text{ A} \qquad P_s = \frac{2700}{3} = 900 \text{ W}$$

Then, by (*5.19*) and (*5.20*),

$$R_e = R_1 + a^2 R_2 = \frac{900}{(63)^2} = 0.23 \ \Omega$$

$$X_e = X_1 + a^2 X_2 = \frac{\sqrt{(25.98)^2(63)^2 - (900)^2}}{(63)^2} = 0.34 \ \Omega$$

Motor Nameplate Data

The following data are required to be on the nameplate of a motor:

- Hp – horsepower output on shaft; usually slightly more
- FLA – full load amperes
- V – voltage input rating
- Service Factor – 1.15 typical, maybe as high as 1.25
- Insulation Class – Higher the letter, the higher the temperature rating, H the highest
- Duty – continuous or intermittent
- Starting Inrush – defined by CODE
- NEMA Design Code

Locked-Rotor Indicating Code Letter

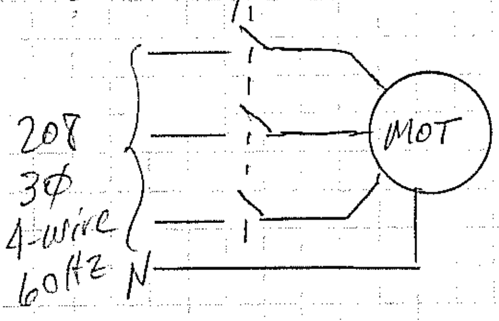
- NEC Requires code letters marked on motor nameplates to show motor input with locked rotor, Ref. NEC 430-7(B).
- Data is Letter $\Rightarrow \frac{KVA}{Hp.}$ with locked rotor condition

Locked Rotor Indicating Code Letters

Code Letter	Kilovolt-Amperes per Horsepower with Locked Rotor
A	0 – 3.14
B	3.15 – 3.54
C	3.55 – 3.99
D	4.0 – 4.49
E	4.5 – 4.99
F	5.0 – 5.59
G	5.6 – 6.29
H	6.3 – 7.09
J	7.1 – 7.99
K	8.0 – 8.99
L	9.0 – 9.99
M	10.0 – 11.19
N	11.2 – 12.49
P	12.5 – 13.99
R	14.0 – 15.99
S	16.0 – 17.99
T	18.0 – 19.99
U	20.0 – 22.39
V	22.4 and up

Example

- A 20Hp, 208VAC, 3-phase, 60Hz motor is to be start directly of the line. The locked rotor code on the nameplate is D. What is the range of inrush current that is expected?
- Our setup is as follows



For a 20Hp Motor from the NEC

$I_{FLA} = 59.4 A$

CODE LETTER "D" has a range of 4.0 - 4.49 KVA/Hp. (See next sheet)

We also know $S = \sqrt{3} V_{LL} I_L$

For the lower value of current

$S_1 = 20Hp \cdot 4.0 KVA/Hp = 80 KVA = \sqrt{3}(208) I_1 \Rightarrow \underline{I_1 = 222.05A}$

or $3.74 I_{FLA}$

For the upper number of current

$S_2 = 20Hp \cdot 4.49 KVA/Hp = 89.8 KVA = \sqrt{3}(208) I_2 \Rightarrow \underline{I_2 = 249.26 A}$

or $4.2 I_{FLA}$

Table 430.7(B) Locked-Rotor Indicating Code Letters

Code Letter	Kilovolt-Amperes per Horsepower with Locked Rotor
A	0–3.14
B	3.15–3.54
C	3.55–3.99
D	4.0–4.49
E	4.5–4.99
F	5.0–5.59
G	5.6–6.29
H	6.3–7.09
J	7.1–7.99
K	8.0–8.99
L	9.0–9.99
M	10.0–11.19
N	11.2–12.49
P	12.5–13.99
R	14.0–15.99
S	16.0–17.99
T	18.0–19.99
U	20.0–22.39
V	22.4 and up

From nameplate

Thermal Classification of insulators

Organizations such as IEEE, NEMA, and IEC have grouped insulatators into five classes depending on their ability to withstand heat. The classes correspond to the maximum temperature levels of 105°C, 130°C, 155°C, 180°C, and 220°C (A, B, F, H, and R)

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22-142 100 SHEETS
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Table 5.5.1 Thermal Classification of Electrical Insulating Materials

Class	IEEE Maximum Permissible Temperature Rise in °C Beyond the Ambient Temperature of 40°C	Materials	NEMA Maximum Permissable Temp. Rise above ambient
O	50	paper, cotton, silk	
A	65	cellulose, phenolic resins	70°C
B	90	mica, glass, asbestos with organic binder	100°C
F	115	same as above, with suitable binder	130°C
H	140	mica, glass, asbestos with slicone binder, silicone resin, teflon	155°C
C	>180	mica, porcelain, glass	

Induction Motor Design Classes

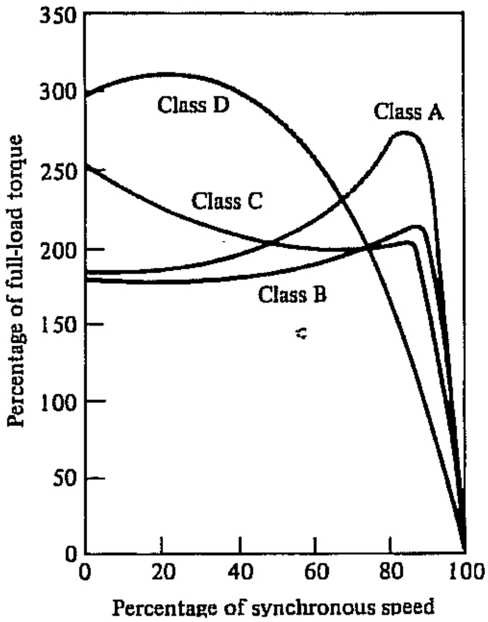


FIGURE 10-25
Typical torque-speed curves for different rotor designs.

NEMA Design	Starting Torque	Starting Current	Breakdown Torque	Full load Slip
A	Normal	Normal	High	Low
B	Normal	Low	Medium	Low
C	High	Low	Normal	Low
D	Very High	Low	—	High

- It is possible to produce a large variety of speed-torque curves by varying the rotor characteristics of induction motors.
- NEMA and the IEC have defined a series of design standards with different speed-torque curves. These are called Design Classes
- Design Class A — Full load slip is less than 5%, & less than that of a design B motor

General, 430.1 through 430.18	Part I
Motor Circuit Conductors, 430.21 through 430.29	Part II
Motor and Branch-Circuit Overload Protection, 430.31 through 430.44	Part III
Motor Branch-Circuit Short-Circuit and Ground-Fault Protection, 430.51 through 430.58	Part IV
Motor Feeder Short-Circuit and Ground-Fault Protection, 430.61 through 430.63	Part V
Motor Control Circuits, 430.71 through 430.74	Part VI
Motor Controllers, 430.81 through 430.90	Part VII
Motor Control Centers, 430.92 through 430.98	Part VIII
Disconnecting Means, 430.101 through 430.113	Part IX
Adjustable Speed Drive Systems, 430.120 through 430.128	Part X
Over 600 Volts, Nominal, 430.221 through 430.227	Part XI
Protection of Live Parts—All Voltages, 430.231 through 430.233	Part XII
Grounding—All Voltages, 430.241 through 430.245	Part XIII
Tables, Tables 430.247 through 430.251(B)	Part XIV

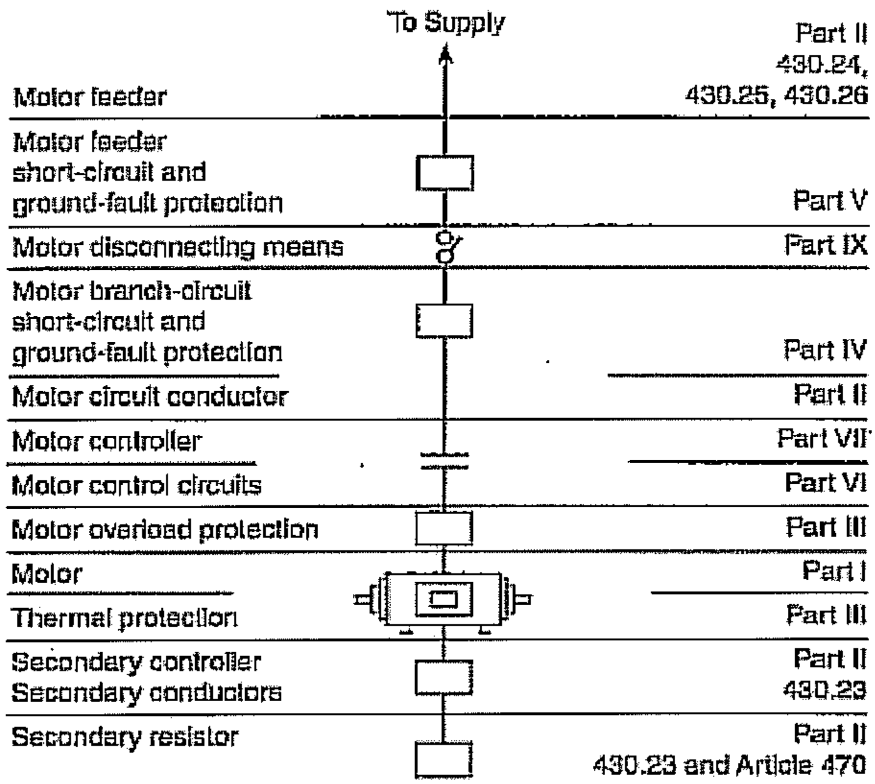


Figure 430.1 Article 430 Contents.

Example

A 50Hp, 208VAC, 3-phase, 4-wire, is running at full load with an efficiency of 80% and a power factor of 85%. The ambient temperature is 40°C and all cables are in the same raceway. Select a conductor type and size to support the load.

- $P_O = 50\ Hp \times 746 W/Hp = 37300 W$ $\quad P_{IN} = \frac{P_O}{\eta} = \frac{37300}{.80} = 46625 W$

We also know $P_{IN} = \sqrt{3}\, V_{LL}\, I_L \cos\theta$ ← pf. Then we have

$$I_L = \frac{P_{IN}}{\sqrt{3}\, V_{LL} \cos\theta} = \frac{46625 W}{\sqrt{3}(208)(.85)} = 152.25 A$$

We can choose a 90°C copper cable THHN.

The cable to be used has to be 125% of operating current then our required ampacity is $I_{Req} = (152.25A)(1.25) = 190.3 A$

- To check choice ① on the NEC Table 310.16 a 3/0 cable, 225A would have to be derated by .91 or $225 \cdot .91 = 204.7A$

This is larger than 190.3A so this will support the application. So we can use THHN, 90°C, copper, 3/0 or larger.

Table 310.16 Allowable Ampacities of Insulated Conductors Rated 0 Through 2000 Volts, 60°C Through 90°C (140°F Through 194°F), Not More Than Three Current-Carrying Conductors in Raceway, Cable, or Earth (Directly Buried), Based on Ambient Temperature of 30°C (86°F)

	Temperature Rating of Conductor [See Table 310.13(A).]						
	60°C (140°F)	75°C (167°F)	90°C (194°F)	60°C (140°F)	75°C (167°F)	90°C (194°F)	
Size AWG or kcmil	Types TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE, ZW	Types TBS, SA, SIS, FEP, FEPB, MI, RHH, RHW-2, THHN, THHW, THW-2, THWN-2, USE-2, XHH, XHHW, XHHW-2, ZW-2	Types TW, UF	Types RHW, THHW, THW, THWN, XHHW, USE	Types TBS, SA, SIS, THHN, THHW, THW-2, THWN-2, RHH, RHW-2, USE-2, XHH, XHHW, XHHW-2, ZW-2	
	COPPER			ALUMINUM OR COPPER-CLAD ALUMINUM			Size AWG or kcmil
18	—	—	14	—	—	—	—
16	—	—	18	—	—	—	—
14*	20	20	25	—	—	—	—
12*	25	25	30	20	20	25	12*
10*	30	35	40	25	30	35	10*
8	40	50	55	30	40	45	8
6	55	65	75	40	50	60	6
4	70	85	95	55	65	75	4
3	85	100	110	65	75	85	3
2	95	115	130	75	90	100	2
1	110	130	150	85	100	115	1
1/0	125	150	170	100	120	135	1/0
2/0	145	175	195	115	135	150	2/0
3/0	165	200	225	130	155	175	3/0
4/0	195	230	260	150	180	205	4/0
250	215	255	290	170	205	230	250
300	240	285	320	190	230	255	300
350	260	310	350	210	250	280	350
400	280	335	380	225	270	305	400
500	320	380	430	260	310	350	500
600	355	420	475	285	340	385	600
700	385	460	520	310	375	420	700
750	400	475	535	320	385	435	750
800	410	490	555	330	395	450	800
900	435	520	585	355	425	480	900
1000	455	545	615	375	445	500	1000
1250	495	590	665	405	485	545	1250
1500	520	625	705	435	520	585	1500
1750	545	650	735	455	545	615	1750
2000	560	665	750	470	560	630	2000
	CORRECTION FACTORS						
Ambient Temp. (°C)	For ambient temperatures other than 30°C (86°F), multiply the allowable ampacities shown above by the appropriate factor shown below.						Ambient Temp. (°F)
21–25	1.08	1.05	1.04	1.08	1.05	1.04	70–77
26–30	1.00	1.00	1.00	1.00	1.00	1.00	78–86
31–35	0.91	0.94	0.96	0.91	0.94	0.96	87–95
36–40	0.82	0.88	0.91	0.82	0.88	0.91	96–104
41–45	0.71	0.82	0.87	0.71	0.82	0.87	105–113
46–50	0.58	0.75	0.82	0.58	0.75	0.82	114–122
51–55	0.41	0.67	0.76	0.41	0.67	0.76	123–131
56–60	—	0.58	0.71	—	0.58	0.71	132–140
61–70	—	0.33	0.58	—	0.33	0.58	141–158
71–80	—	—	0.41	—	—	0.41	159–176

1

Ambient Range

* See 240.4(D).

Derating factor

MAXIMUM POWER TRANSFER THEOREMS

The following maximum power transfer theorems determine the values of the load impedances which result in maximum power transfer across the terminals of an active network.

We consider a series combination of source and fixed complex impedance delivering power to a load consisting of a variable resistance or a variable complex impedance.

Case 1. Load: Variable resistance R_L (Fig. 12-8).

The current in the circuit is

$$\mathbf{I} = \frac{\mathbf{V}_g}{(R_g + R_L) + jX_g}$$

$$I = |\mathbf{I}| = \frac{V_g}{\sqrt{(R_g + R_L)^2 + X_g^2}}$$

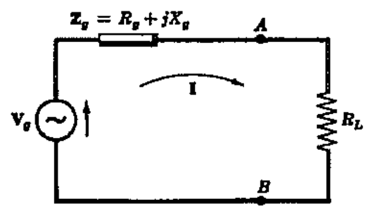


Fig. 12-8

Then the power delivered to R_L is

$$P = I^2 R_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2}$$

To determine the value of R_L for maximum power transferred to the load, set the first derivative dP/dR_L equal to zero.

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_g^2 R_L}{(R_g + R_L)^2 + X_g^2}\right] = V_g^2\left\{\frac{[(R_g + R_L)^2 + X_g^2] - R_L(2)(R_g + R_L)}{[(R_g + R_L)^2 + X_g^2]^2}\right\} = 0$$

or $$R_g^2 + 2R_gR_L + R_L^2 + X_g^2 - 2R_LR_g - 2R_L^2 = 0$$

and $$R_g^2 + X_g^2 = R_L^2$$

Hence $$R_L = \sqrt{R_g^2 + X_g^2} = |\mathbf{Z}_g|$$

With a variable pure resistance load the maximum power is delivered across the terminals of the active network if the load resistance is made equal to the absolute value of the active network impedance.

If the reactive component of the impedance in series with the source is zero, i.e. $X_g = 0$, then the maximum power is transferred to the load when the load and source resistances are equal, $R_L = R_g$.

Case 2. Load: Impedance $\mathbf{Z}_L$ with variable resistance and variable reactance (Fig. 12-9).

The circuit current is

$$\mathbf{I} = \frac{\mathbf{V}_g}{(R_g + R_L) + j(X_g + X_L)}$$

$$I = |\mathbf{I}| = \frac{V_g}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}}$$

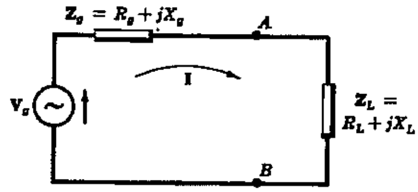


Fig. 12-9

The power delivered by the source is

$$P = I^2 R_L = \frac{V_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2} \tag{11}$$

If R_L in (*11*) is held fixed, the value of P is maximum when $X_g = -X_L$. Then equation (*11*) becomes

$$P = \frac{V_g^2 R_L}{(R_g + R_L)^2}$$

Consider now R_L to be variable. As shown in case 1, the maximum power is delivered to the load when $R_L = R_g$. If $R_L = R_g$ and $X_L = -X_g$, $\mathbf{Z}_L = \mathbf{Z}_g^*$.

With the load impedance consisting of variable resistance and variable reactance, maximum power transfer across the terminals of the active network occurs when the load impedance $\mathbf{Z}_L$ is equal to the complex conjugate of the network impedance $\mathbf{Z}_g$.

Case 3. Load: Impedance $\mathbf{Z}_L$ with variable resistance and fixed reactance (Fig. 12-10).

We obtain the same equations for current I and power P as in case 2 with the condition that X_L be kept constant.

When the first derivative of P with respect to R_L is set equal to zero, it is found that

$$R_L^2 = R_g^2 + (X_g + X_L)^2$$

and $$R_L = |\mathbf{Z}_g + jX_L|$$

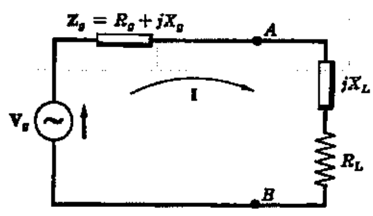
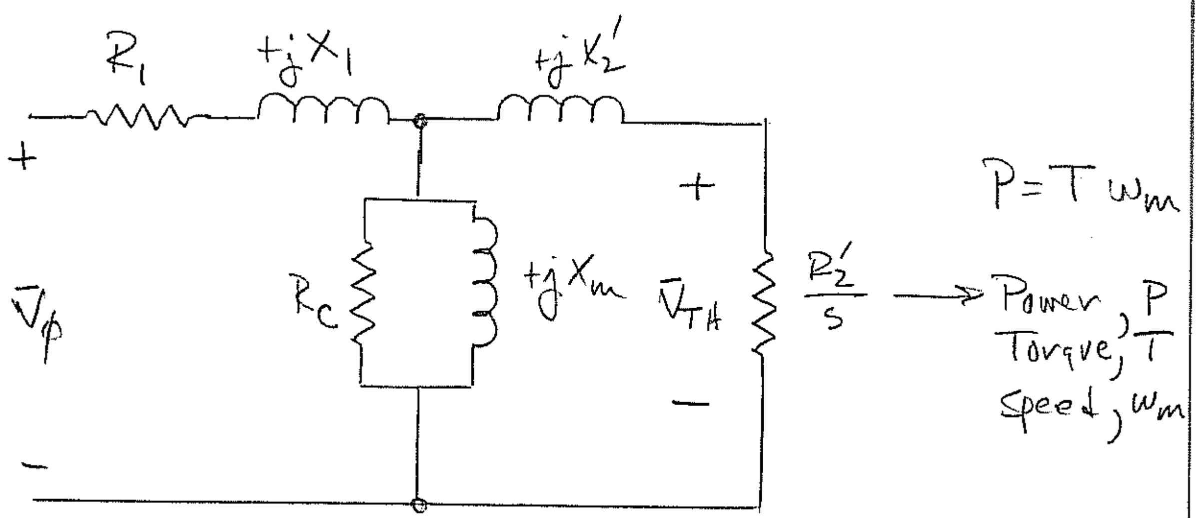


Fig. 12-10

Since $\mathbf{Z}_g$ and X_L are both fixed quantities, they could be combined into a single impedance. Then, with R_L variable, case 3 is reduced to case 1 and the maximum power results when R_L is equal to the absolute value of the network impedance.

Induction motor starting – Current, voltage, torque relationships

Referring to the equivalent circuit on a per phase basis



We know $\bar{V}_{TH} \propto \bar{V}_\phi$ because of linear network theory

But $P \propto V_{TH}^2$ and $P \propto T$

But $T \propto \bar{V}_{TH}^2$

Therefore $T \propto V_\phi^2$

Since the starting current is proportional to the Impressed voltage, and since the starting torque is proportional to the square of the impressed voltage, we can find the reduced voltage required to for any percentage of starting torque.

starting under full voltage $\quad T_{ST} = K_T T_{FL} \propto (V_\phi)^2 \Rightarrow K_T T_{FL} = K_V (V_\phi)^2$

where $T_{ST} \triangleq$ starting Torque $\quad$ starting at a reduced voltage V_ϕ'

$T_{FL} \triangleq$ FULL LOAD Torque $\quad T_{ST} = T_{FL} \propto (V_\phi')^2 \Rightarrow T_{FL} = K_V V_\phi'^2$

$$K_T (K_V V_\phi'^2) = K_V (V_\phi)^2$$

$$V_\phi' = \frac{V_\phi}{\sqrt{K_T}}$$

Since $I_0 \propto V_0$ and $T_{ST} \propto V_0^2 \Rightarrow T_{ST} \propto I_0^2$

$$T_{ST} = K_T T_{FL} \propto (I_{ST})^2 \Rightarrow K_T T_{FL} = K_I (I_{ST})^2$$

where $T_{ST} \triangleq$ starting Torque

$T_{FL} \triangleq$ FULL LOAD TORQUE

starting at a reduced current I_{FL}

$$T'_{ST} = T_{FL} \propto (I'_{ST})^2 \Rightarrow T_{FL} = K_I (I'_{ST})^2$$

$$\frac{T_{ST}}{T'_{ST}} = \frac{K_I (I_{ST})^2}{K_I (I'_{ST})^2} = \left(\frac{I_{ST}}{I'_{ST}}\right)^2$$

Example

A 4-pole, 400V, 3ϕ, 60Hz induction motor takes 150A of a current at starting and 25A while running at full load. The starting torque is 1.8 times the full load torque at full load at 400VAC. It is desired that the starting torque be the same as the full load torque, determine

a) the applied voltage

b) the corresponding line current

Torque $\propto P \propto (V)^2$ Then

a) $T_{start} = 1.8\,T_{FL} \propto (400)^2$ at some other voltage we have

$T'_{start} = T_{FL} \propto (V)^2$

Using the proportional laws above

$$\frac{T_{START}}{T'_{START}} = \frac{1.8T_F}{T_F} = \frac{(400)^2}{V^2} = 1.8 = \left(\frac{400}{V}\right)^2$$

Then $V^2 = \dfrac{400^2}{1.8} \Rightarrow V = \dfrac{400}{\sqrt{1.8}} = \underline{\underline{298.14V}}$

b) Since $T \propto P \propto (V)^2 \propto (I)^2$, then $V \propto I$

$$\frac{I}{150} = \frac{K\ 298.14}{K\ 400} \Rightarrow I = \frac{150}{400}(298.14) = \underline{\underline{111.8A}}$$

Example

An induction motor is started by a Y-Δ switch. Determine the ratio of the starting torque to the full load torque if the starting current is $5I_{FLA}$ and S_{FL} is 5%.

$S_{ST} = 1$ since $n_m = 0$ and $T_e = \frac{I_2'^2 R_2'}{s\omega_s}$

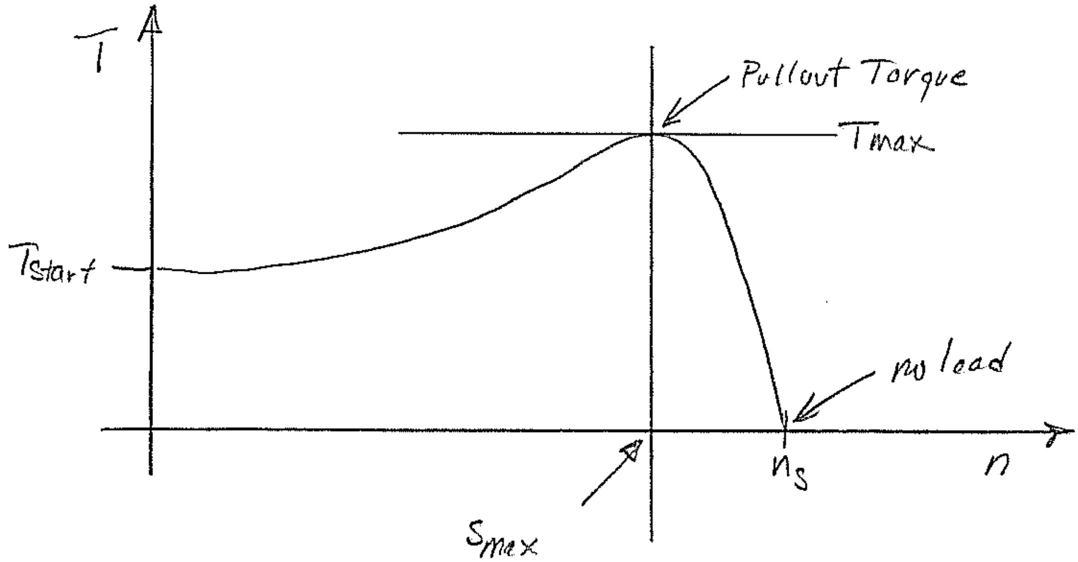
$T \propto P \propto (V)^2 \propto (I)^2$ and $V \propto I$

$$\frac{T_{ST}}{T_{FL}} = \frac{I_{ST}^2}{I_{FL}^2} \cdot \frac{S_{FL}}{S_{ST}} = \frac{(5I_{FLA})^2}{(I_{FLA})^2} \cdot \frac{.05}{1} = 1.25$$

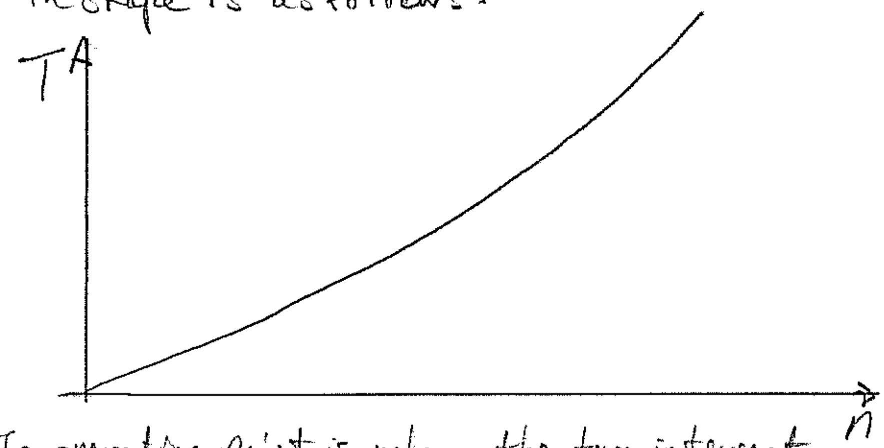
or $T_{ST} = 1.25\,T_{FL}$

Torque - Speed Characteristic and Operation

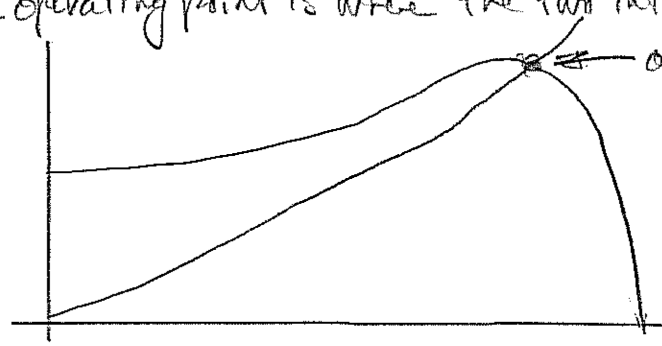
- The induction motor has a characteristic that defines load torque as a function of speed.
- The shape is a follows:



- The load characteristic defines the operating requirements of load at any given speed.
- The load driven has varying shapes but is typically a square or cubic function.
- The shape is as follows:



- The operating point is where the two intersect



operating point

- The graph of Motor T, S may be given
- The graph or equation of T, S may be given.

Protection of Induction Motors

Table 430.52 Maximum Rating or Setting of Motor Branch-Circuit Short-Circuit and Ground-Fault Protective Devices

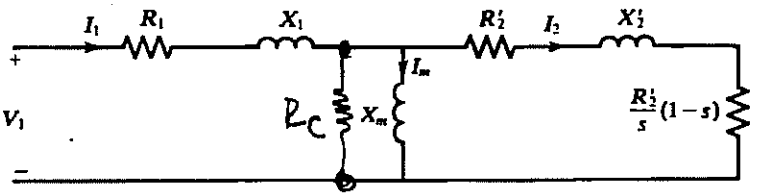
	Percentage of Full-Load Current			
Type of Motor	Nontime Delay Fuse[1]	Dual Element (Time-Delay) Fuse[1]	Instantaneous Trip Breaker	Inverse Time Breaker[2]
Single-phase motors	300	175	800	250
AC polyphase motors other than wound-rotor	300	175	800	250
Squirrel cage — other than Design B energy-efficient	300	175	800	250
Design B energy-efficient	300	175	1100	250
Synchronous[3]	300	175	800	250
Wound rotor	150	150	800	150
Direct current (constant voltage)	150	150	250	150

Note: For certain exceptions to the values specified, see 430.54.

[1]The values in the Nontime Delay Fuse column apply to Time-Delay Class CC fuses.

[2]The values given in the last column also cover the ratings of nonadjustable inverse time types of circuit breakers that may be modified as in 430.52(C)(1), Exception No. 1 and No. 2.

[3]Synchronous motors of the low-torque, low-speed type (usually 450 rpm or lower), such as are used to drive reciprocating compressors, pumps, and so forth, that start unloaded, do not require a fuse rating or circuit-breaker setting in excess of 200 percent of full-load current.



(a)

Fig 5-8. Power flow in an induction motor

5.16. (a) Replace the circuit of Fig. 5-8(a) by its Thévenin equivalent circuit and express the Thévenin voltage, $\mathbf{V}_{Th}$, and impedance, $\mathbf{Z}_{Th} = R_{Th} + jX_{Th}$, in terms of the circuit parameters of Fig. 5-8(a) and the voltage V_1. (b) The per-phase parameters for Fig. 5-8(a) are as in Problem 5.14. Other data also remain the same. Draw a Thévenin equivalent circuit for the motor.

(a) From Fig. 5-8(a),

$$\mathbf{V}_{Th} = \frac{jX_m}{R_1 + j(X_1 + X_m)} V_1 \qquad \mathbf{Z}_{Th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

(b) The Thévenin circuit is shown in Fig. 5-13, for which the numerical values are:

$$\mathbf{V}_{Th} = \frac{400}{\sqrt{3}} \frac{j20}{0.2 + j20.5} \qquad \text{or} \qquad V_{Th} = 225.3 \text{ V}$$

$$R_{Th} + jX_{Th} = \frac{j20(0.2 + j0.5)}{0.2 + j20.5} = 0.19 + j0.49 \quad \Omega$$

5.8. The rotor of a 3-phase, 60-Hz, 4-pole induction motor takes 120 kW at 3 Hz. Determine (*a*) the rotor speed and (*b*) the rotor copper losses.

$$s = \frac{f_2}{f_1} = \frac{3}{60} = 0.05 \qquad n_s = \frac{120 f_1}{p} = \frac{120(60)}{4} = 1800 \text{ rpm}$$

(*a*)
$$n = (1 - s)n_s = (1 - 0.05)(1800) = 1710 \text{ rpm}$$

(*b*) By (*5.15*),

$$\text{rotor copper loss} = s \times (\text{rotor input}) = (0.05)(120) = 6 \text{ kW}$$

√ **5.9.** The motor of Problem 5.8 has a stator copper loss of 3 kW, a mechanical loss of 2 kW, and a stator core loss of 1.7 kW. Calculate (*a*) the motor output at the shaft and (*b*) the efficiency. Neglect rotor core loss.

From Problem 5.8, the rotor input is 120 kW and the rotor copper loss is 6 kW.

(*a*)
$$\text{motor output} = 120 - 6 - 2 = 112 \text{ kW}$$

(*b*)
$$\text{motor input} = 120 + 3 + 1.7 = 124.7 \text{ kW}$$

$$\text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{112}{124.7} = 89.7\%$$

5.10 A 6-pole, 3-phase, 60-Hz induction motor takes 48 kW in power at 1140 rpm. The stator copper loss is 1.4 kW, stator core loss is 1.6 kW, and rotor mechanical losses are 1 kW. Find the motor efficiency.

$$n_s = \frac{120 f_1}{p} = \frac{120(60)}{6} = 1200 \text{ rpm} \qquad s = \frac{n_s - n}{n_s} = \frac{1200 - 1140}{1200} = 0.05$$

rotor input = stator output = (stator input) − (stator losses) = 48 − (1.4 + 1.6) = 45 kW

rotor output = $(1 - s) \times$ (rotor input) = (1 − 0.05)(45) = 42.75 kW

motor output = (rotor output) − (rotational losses) = 42.75 − 1 = 41.75 kW

$$\text{motor efficiency} = \frac{41.75}{48} = 87\%$$

4.29 A 2,500-kva three-phase 60-cycle 6,600-volt alternator has a field resistance of 0.43 ohm and an armature resistance of 0.072 between each terminal and the neutral. The windings are Y-connected. The field current at full-load unity power factor is 200 amp, and at full-load 0.80 pf lagging it is 240 amp. The friction loss is 35 kw and the core loss, 47.5 kw. Assume friction and core loss constant at either unity power factor or 0.80 pf lagging.

a. Calculate the full-load efficiency at unity power factor.

b. Calculate the full-load efficiency at 0.80 pf.

4.29 *a.* At unity pf:

Output $= 2{,}500 \times 1$ kw

Current $= \dfrac{2{,}500}{(\sqrt{3} \times 6.6 \times 1)} = 219$ amp

Armature copper loss $= \dfrac{[3 \times (219)^2 \times 0.072]}{1{,}000} = 10.4$ kw

Field loss $= \dfrac{[(200)^2 \times 0.43]}{1{,}000} = 17.2$ kw

Friction loss $= 35.0$ kw

Core loss $= 47.5$ kw

Total loss $= 110.1$ kw

Efficiency $= \dfrac{\text{output}}{(\text{output} + \text{losses})} = \dfrac{(2{,}500 \times 100)}{2{,}610.1}$

$= 95.8$ per cent

b. At 0.8 pf:

Output $= 2{,}500 \times 0.8 = 2{,}000$ kw

Current $= \dfrac{2{,}000}{(\sqrt{3} \times 6.6 \times 0.8)} = 219$ amp

Armature copper loss $= \dfrac{[3 \times (219)^2 \times 0.072]}{1{,}000} = 10.4$ kw

Field loss $= \dfrac{[(240)^2 \times 0.43]}{1{,}000} = 24.8$ kw

Friction loss $= 35.0$ kw

Core loss $= 47.5$ kw

Total loss $= 117.7$ kw

Efficiency $= \dfrac{(2{,}000 \times 100)}{2{,}117.7} = 94.4$ per cent

4.31 A 10-hp 550-volt 60-cps three-phase induction motor has a starting torque of 160 per cent of full-load torque and a starting current of 425 per cent full-load current.

a. What voltage is required to limit the starting current to full-load value?

b. If the motor is used on a 440-volt 60-cps system, what is the starting torque and starting current expressed in per cent of full-load values?

4.31 *a.* Current at start varies directly as the applied voltage. For rated current at start, $V_{start} = 550/4.25 = 130$ volts.

b. Starting torque varies as the square of the voltage. At 440 volts, $T_{start} = [(440)^2/(550)^2] \times 160$ per cent $= 102$ per cent of rated torque, and $I_{start} = (440/550) \times 425$ per cent $= 340$ per cent of rated current.

EXAMPLE 7-1

A 3-phase Y-connected 220-volt (line-to-line) 10-hp 60-Hz 6-pole induction motor has the following constants in ohms per phase referred to the stator:

$$r_1 = 0.294 \qquad r_2 = 0.144$$
$$x_1 = 0.503 \qquad x_2 = 0.209 \qquad x_\varphi = 13.25$$

The total friction, windage, and core losses may be assumed to be constant at 403 watts, independent of load.

For a slip of 2.00 percent, compute the speed, output torque and power, stator current, power factor, and efficiency when the motor is operated at rated voltage and frequency.

Solution

The impedance Z_f (Fig. 7-7*a*) represents physically the per-phase impedance presented to the stator by the air-gap field, both the reflected effect of the rotor and the effect of the exciting current being included therein. From Fig. 7-7*a*,

$$Z_f = R_f + jX_f = \frac{r_2}{s} + jx_2 \qquad \text{in parallel with } jx_\varphi$$

Substitution of numerical values gives, for $s = 0.0200$,

$$\begin{aligned} R_f + jX_f &= 5.41 + j3.11 \\ r_1 + jx_1 &= \underline{0.29 + j0.50} \\ \text{Sum} &= 5.70 + j3.61 = 6.75\underline{/32.4^\circ} \text{ ohms} \end{aligned}$$

$$\text{Applied voltage to neutral} = \frac{220}{\sqrt{3}} = 127 \text{ volts}$$

$$\text{Stator current } I_1 = \frac{127}{6.75} = 18.8 \text{ amp}$$

rotor with respect to stator

$$\text{Power factor} = \cos 32.4^\circ = 0.844$$

$$\text{Synchronous speed} = \frac{2f}{p} = \frac{120}{6} = 20 \text{ rev/sec, or 1,200 rpm}$$

$$\omega_s = 2\pi(20) = 125.6 \text{ rad/sec}$$

$$\begin{aligned} \text{Rotor speed} &= (1 - s) \times (\text{synchronous speed}) \\ &= (0.98)(1{,}200) = 1{,}176 \text{ rpm} \end{aligned}$$

From Eq. 7-12,

$$P_{g1} = q_1 I_2^2 \frac{r_2}{s} = q_1 I_1^2 R_f$$

$$= (3)(18.8)^2(5.41) = 5{,}740 \text{ watts}$$

From Eqs. 7-12 and 7-15, the internal mechanical power is

$$P = (0.98)(5{,}740) = 5{,}630 \text{ watts}$$

Deducting losses of 403 watts gives

$$\text{Output power} = 5{,}630 - 403 = 5{,}230 \text{ watts, or } 7.00 \text{ hp}$$

$$\text{Output torque} = \frac{\text{output power}}{\omega_{\text{rotor}}} = \frac{5{,}230}{(0.98)(125.6)}$$

$$= 42.5 \text{ newton-meters, or } 31.4 \text{ lb-ft}$$

The efficiency is calculated from the losses.

Total stator copper loss = $(3)(18.8)^2(0.294)$	=	312 watts
Rotor copper loss (from Eq. 7-13) = $(0.0200)(5{,}740)$	=	115
Friction, windage, and core losses	=	403
Total losses	=	830 watts
Output	=	5,230
Input	=	6,060 watts

$$\frac{\text{Losses}}{\text{Input}} = \frac{830}{6{,}060} = 0.137 \qquad \text{Efficiency} = 1.000 - 0.137 = 0.863$$

The complete performance characteristics of the motor can be determined by repeating these calculations for other assumed values of slip.

7-4 TORQUE AND POWER BY USE OF THÉVENIN'S THEOREM

When torque and power relations are to be emphasized, considerable simplification results from application of Thévenin's network theorem to the induction-motor equivalent circuit.

In its general form, Thévenin's theorem permits the replacement of