577 HW04 Problem2

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(a). we need to prove two part:

1. S is reasonable 🡪 for all integers t ≤ n, the number of homeworks in S due within t days or less is no more than t.

We prove this by contradiction, if there is a t0 <= n. such that the number of homeworks in S due within t0 days > t0. This is in contract with the statement in the problem. So this is true.

Next we need to prove second part:

1. for all integers t ≤ n, the number of homeworks in S due within t days or less is no more than t 🡪 S is reasonable.

We prove this by induction:

For t = 1: we have 0 or 1 homeworks, and we can finish it in that day, so it is correct for t = 1.

Inductive hypothesis: for t = k(k < n), it is also true.

Then we need to prove it is true for t = k+1. So we have k+1 homeworks in k + 1 days, but because our inductive hypothesis says that we can finish k homework in k days. Then we can finish the remaining homework in k+1 days and thus S is also reasonable.

So, S is reasonable iff for all integers t ≤ n, the number of homeworks in S due within t days or less is no more than t.

(b)

Algorithm:

1. sort all homework by their due dates, for homework which have the same due date, sort them by their points from largest to smallest.
2. We name homework which due on 1 day is h1, …, hk1,

Homework which due on i day is hk(i-1)+1, …, hki

1. So we create a data structure M to store which homework we plan to do.
2. For(i = 0; i < n; i++){

Add hki+1 to M;--------- (Because this is the highest point hw in ith day)

Compare other homework due on ith day with homework in M, for each homework due on i day which has higher point than any homework in M, we replace the lowest point homework in M by the homework on ith day

}

Proof of correctness:

We prove it by contradiction that there is a subset M’ which has higher score than M. Then there exists a pair <ha, hb> which ha ∈M’ but not ∈ M, and hb ∈ M but not ∈M’. and pha > phb.

There are three categories:

1. tha = thb 🡪 not impossible because in our algorithm, for homework which due in same day, we will choose the homework with highest point.
2. tha > thb 🡪 we will replace hb with ha
3. tha < thb 🡪 we will not replace ha with hb.

So in conclusion, subset M is the largest point.

Running time:

O(n2logn)