Uninformed Search

Chapter 3.1 – 3.4

Many AI Tasks can be Formulated as Search Problems

Goal is to find a *sequence of actions*

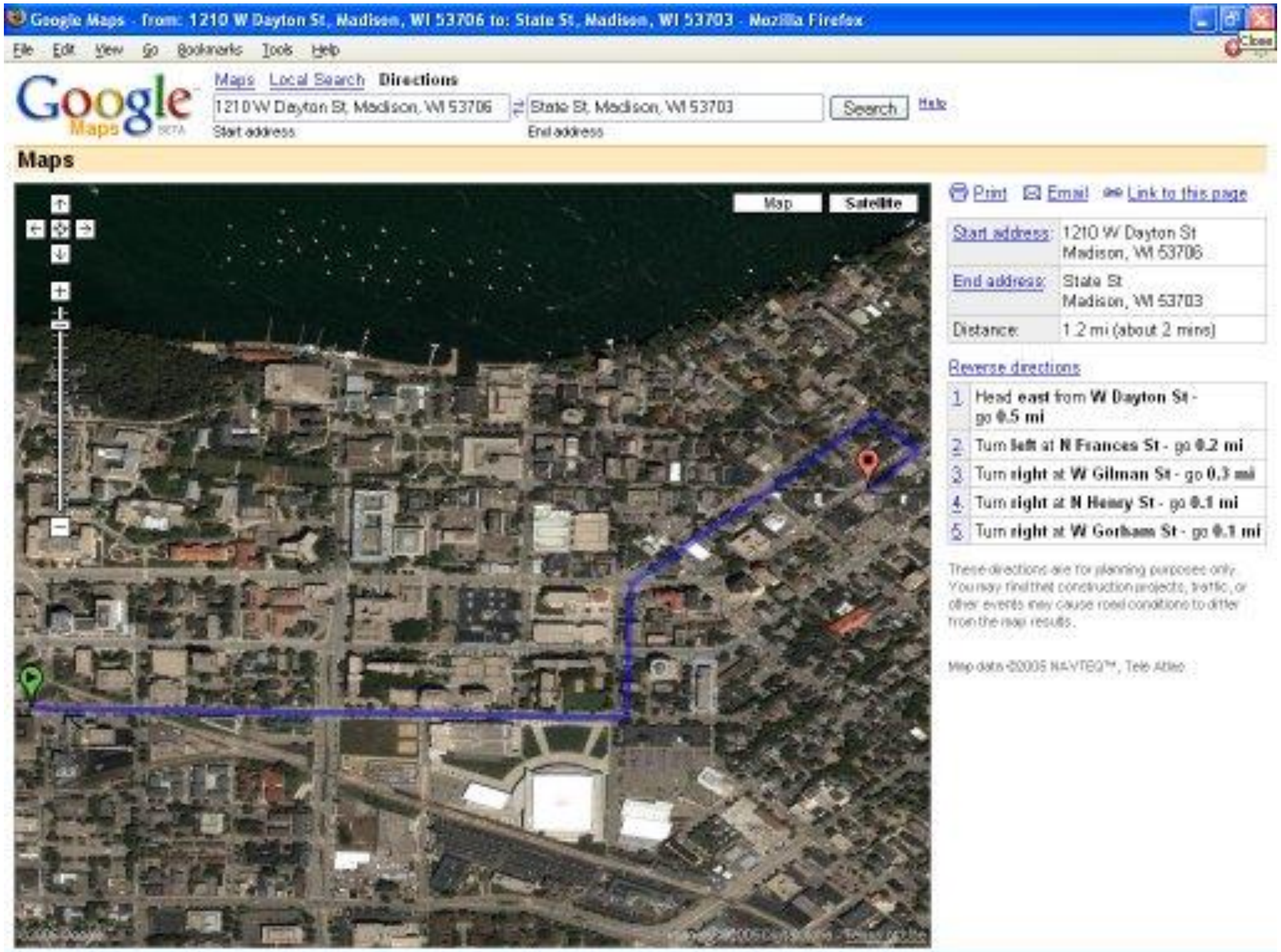
- **Puzzles**
- **Games**
- **Navigation**
- **Assignment**
- **Motion planning**
- **Scheduling**
- **Routing**

Models To Be Studied in CS 540

State-based Models

- **Model task as a graph of all possible states**
- A state captures all the relevant information about the past in order to act (optimally) in the future
- Actions correspond to transitions from one state to another
- Solutions are defined as a sequence of steps/actions (i.e., a path in the graph)
- State-space graphs

Search Example: Route Finding



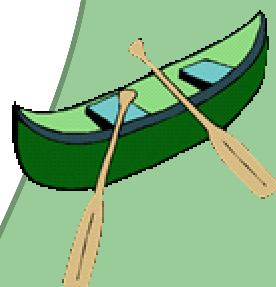
Actions: go straight, turn left, turn right

Goal: shortest? fastest? most scenic?

Search Example: River Crossing Problem







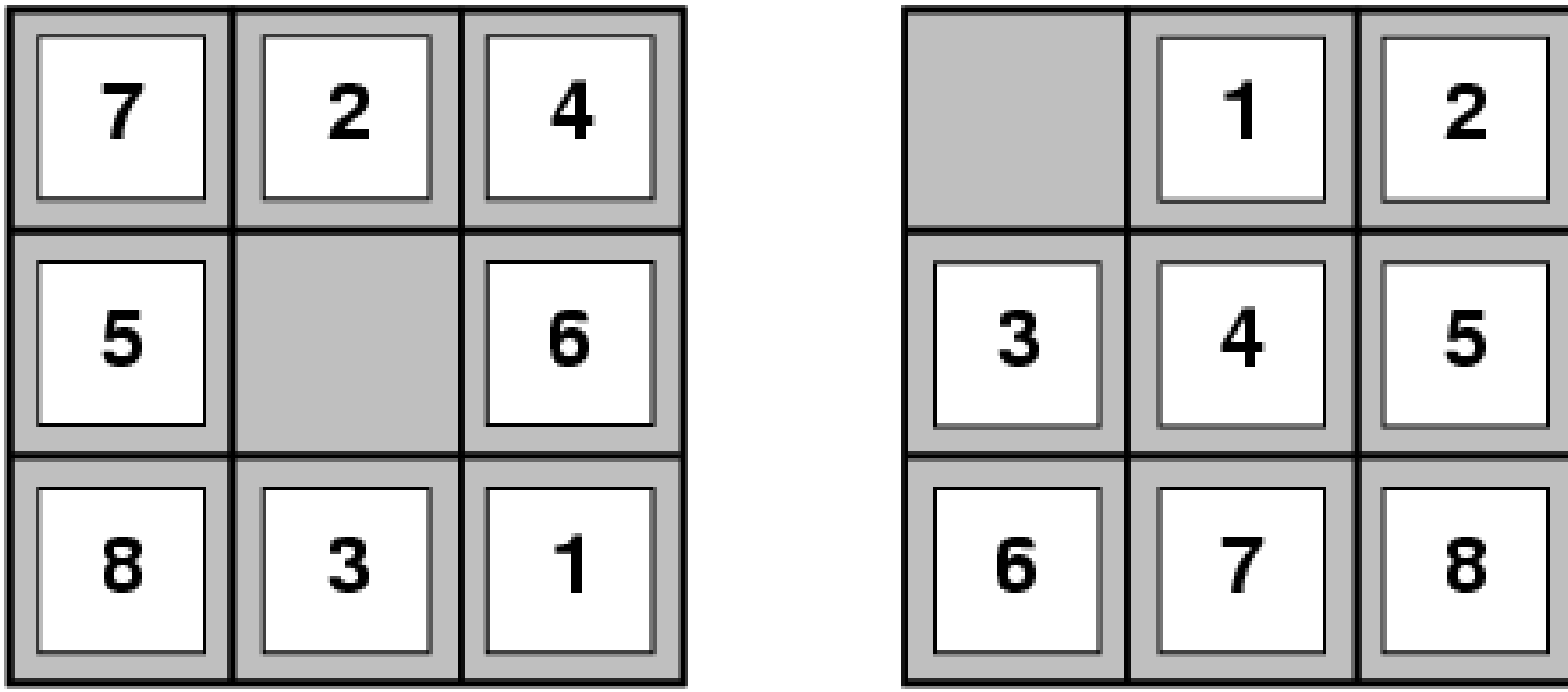

Goal: All on right side of river

Rules:

1) Farmer must row the boat
2) Only room for one other
3) Without the farmer present:
   - Dog bites sheep
   - Sheep eats cabbage

Actions: F>, F<, FC>, FC<, FD>, FD<, FS>, FS<

Search Example: 8-Puzzle

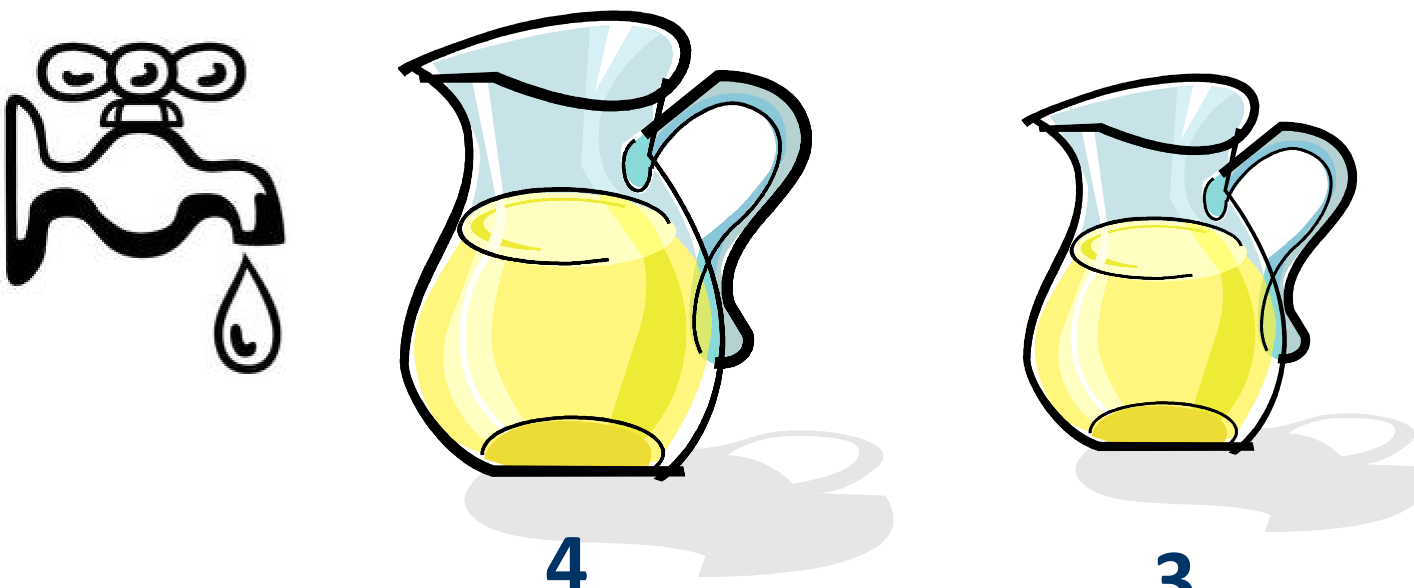


Start State Goal State

Actions: move tiles (e.g., Move2Down)
Goal: reach a certain configuration

Search Example: Water Jugs Problem

Given 4-liter and 3-liter pitchers, how do you get exactly 2 liters into the 4-liter pitcher?



Search Example: Robot Motion Planning

Actions: translate and rotate joints

Goal: fastest? most energy efficient? safest?

Search Example: Natural Language Translation

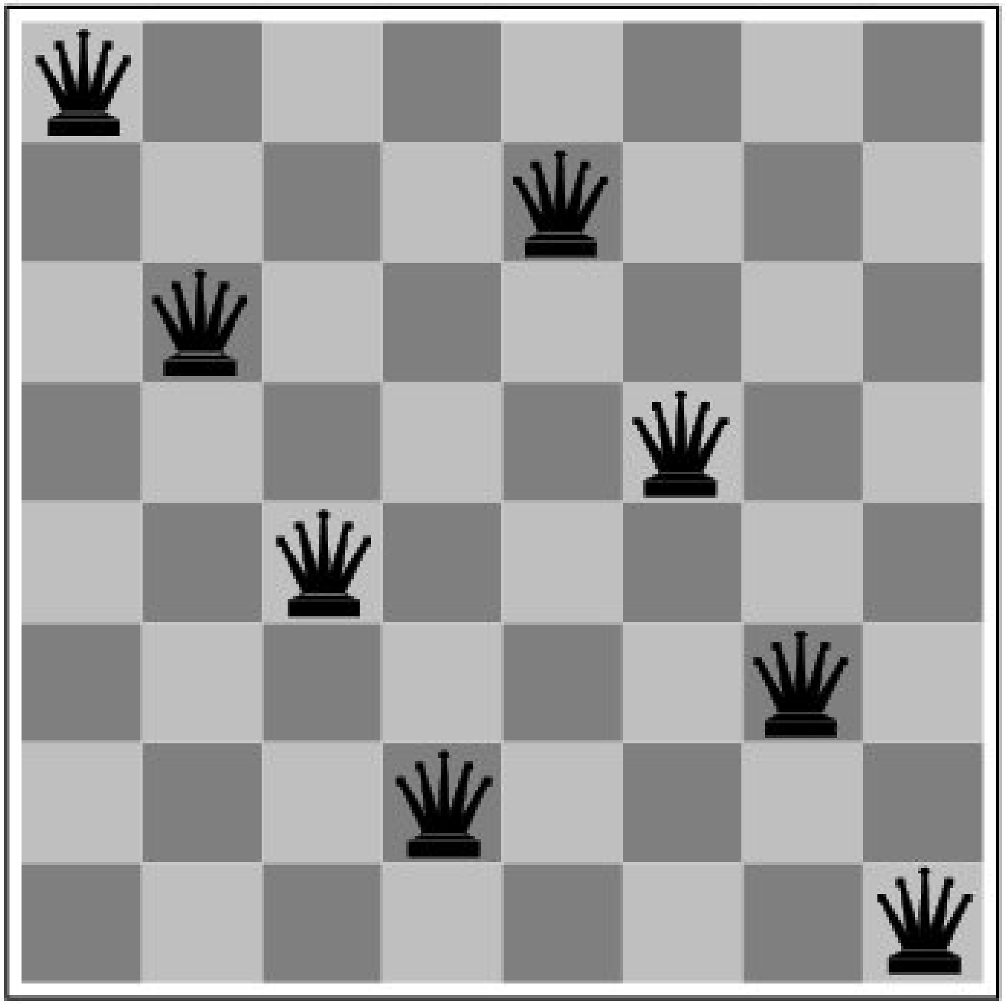
Italian → English:

la casa blu → the blue house

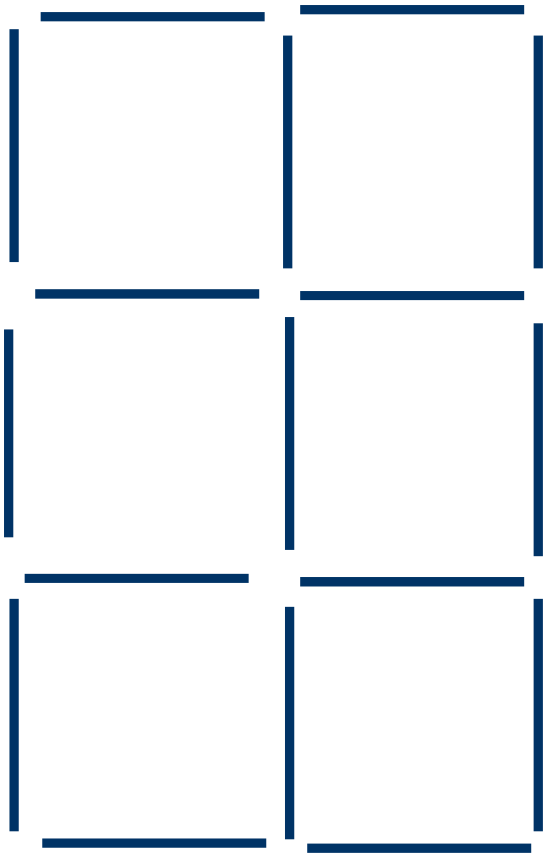
Actions: translate single words (e.g., la → the)

Goal: fluent English? preserves meaning?

Search Example: 8-Queens



Search Example:
Remove 5 Sticks Problem



Remove exactly 5 of the 17 sticks so the resulting figure forms exactly 3 squares

Basic Search Task Assumptions (usually, though not games)

- Fully observable
- Deterministic
- Static
- Discrete
- Single agent

- Solution is a sequence of actions

What Knowledge does the Agent Need?

- The information needs to be
  - sufficient to describe all relevant aspects for reaching the goal
  - adequate to describe the world ***state / situation***
- **Fully observable** assumption, also known as the ***closed world assumption***, means
  - *All necessary information about a problem domain is accessible so that each state is a complete description of the world; there is no missing information at any point in time*

How should the Environment be Represented?

- **Knowledge representation problem:**
  - What information from the sensors is relevant?
  - How to represent domain knowledge?
- ***Determining what to represent is difficult and is usually left to the system designer to specify***
- Problem ***State*** = representation of all necessary information about the environment
- ***State Space*** (aka **Problem Space**) = *all* possible valid configurations of the environment

What Goal does the Agent want to Achieve?

- **How do you describe the goal?**
  - as a task to be accomplished
  - as a state to be reached
  - as a set of properties to be satisfied
- **How do you know when the goal is reached?**
  - with a **goal test** that defines what it means to have achieved/satisfied the goal
  - or, with a set of **goal states**
- ***Determining the goal is usually left to the system designer or user to specify***

What Actions does the Agent Need?

- **Discrete and Deterministic task assumptions imply**

- **Given:**
  - an ***action*** (aka ***operator*** or ***move***)
  - a description of the current state of the world

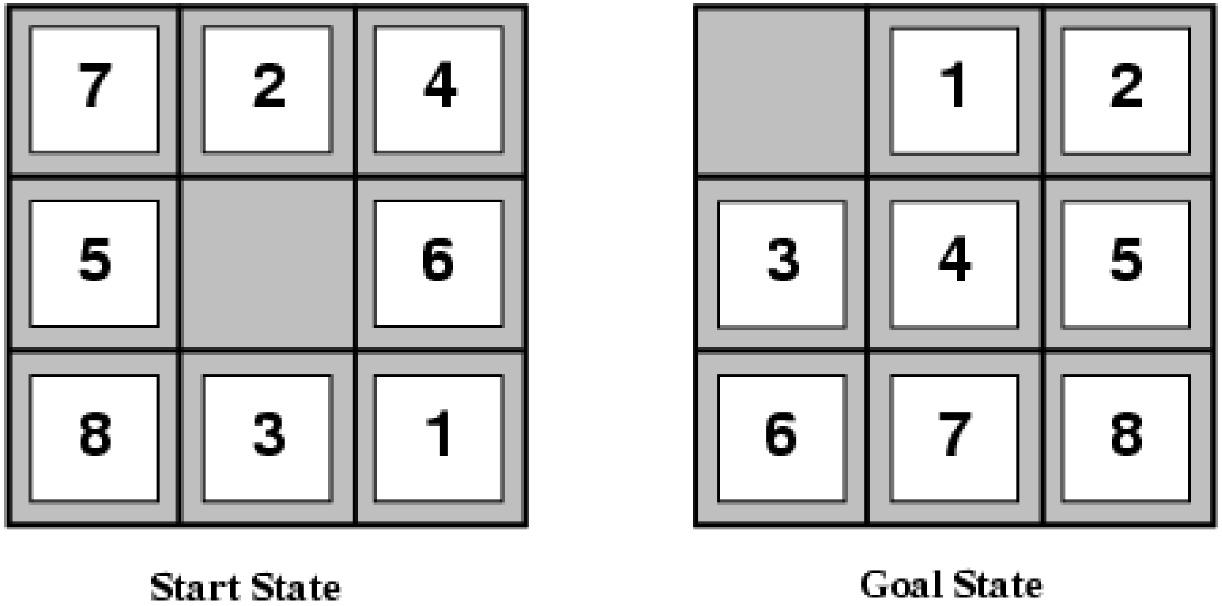
- **Action completely specifies:**
  - if that action *can* be applied (i.e., legal)
  - what the exact state of the world will be after the action is performed in the current state (no "history" information needed to compute the successor state)

What Actions does the Agent Need?

- **A finite set of actions/operators needs to be**
  - decomposed into atomic steps that are discrete and indivisible, and therefore can be treated as instantaneous
  - sufficient to describe all necessary changes

- ***The number of actions needed depends on how the world states are represented***

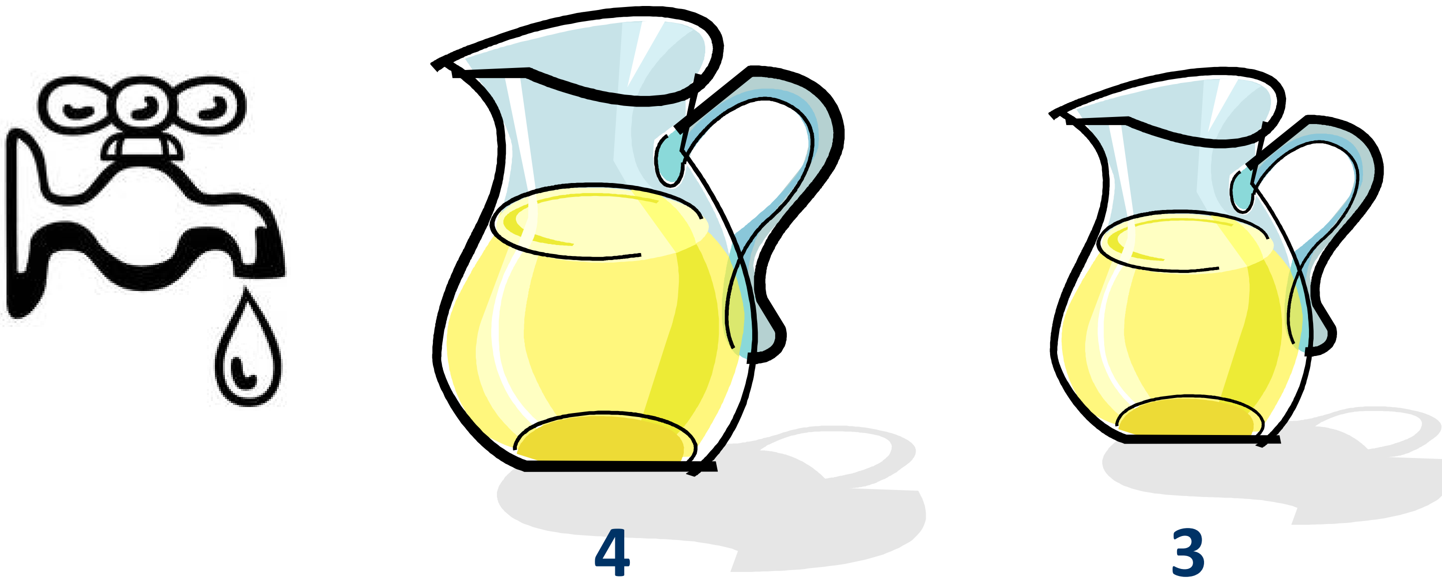
Search Example: 8-Puzzle



- States = configurations
- Actions = up to 4 kinds of moves: up, down, left, right

Water Jugs Problem

Given 4-liter and 3-liter pitchers, how do you get exactly 2 liters into the 4-liter pitcher?



State: (*x*, *y*) for # liters in 4-liter and 3-liter pitchers, respectively

Actions: empty, fill, pour water between pitchers

Initial state: (0, 0)

Goal state: (2, *)

Actions / Successor Functions

1. $(x, y \mid x < 4) \rightarrow (4, y)$ "Fill *4*"
2. $(x, y \mid y < 3) \rightarrow (x, 3)$ "Fill *3*"
3. $(x, y \mid x > 0) \rightarrow (0, y)$ "Empty *4*"
4. $(x, y \mid y > 0) \rightarrow (x, 0)$ "Empty *3*"
5. $(x, y \mid x+y \geq 4 \ \textit{and} \ y > 0) \longrightarrow (4, y - (4 - x))$

   "Pour from *3* to *4* until *4* is full"
6. $(x, y \mid x+y \geq 3 \ \textit{and} \ x > 0) \longrightarrow (x - (3 - y), 3)$

   "Pour from *4* to *3* until *3* is full"
7. $(x, y \mid x+y \leq 4 \ \textit{and} \ y > 0) \longrightarrow (x+y, 0)$

   "Pour all water from *3* to *4*"

Formalizing Search in a State Space

- **A state space is a directed *graph*: (V, E)**
  - V is a set of nodes (vertices)
  - E is a set of arcs (edges)
    each arc is *directed* from one node to another node
- **Each node is a data structure that contains:**
  - a **state** description
  - other information such as:
    - link to parent node
    - name of action that generated this node (from its parent)
    - other bookkeeping data

Formalizing Search in a State Space

- **Each arc corresponds to one of the finite number of actions:**
  - when the action is applied to the state associated with the arc's source node
  - then the resulting state is the state associated with the arc's destination node

- **Each arc has a fixed, positive cost:**
  - corresponds to the cost of the action

Formalizing Search in a State Space

- **Each node has a finite set of successor nodes:**
  - corresponds to all of the legal actions
    that can be applied at the source node's state

- **Expanding a node means:**
  - generate *all* of the successor nodes
  - add them and their associated arcs to the state-space search tree

Formalizing Search in a State Space

- **One or more nodes are designated as start nodes**
- **A goal test is applied to a node's state to determine if it is a goal node**
- **A solution is a sequence of actions associated with a path in the state space from a start to a goal node:**
  - just the goal state (e.g., cryptarithmetic)
  - a path from start to goal state (e.g., 8-puzzle)
- **The cost of a solution is the sum of the arc costs on the solution path**

Search Summary

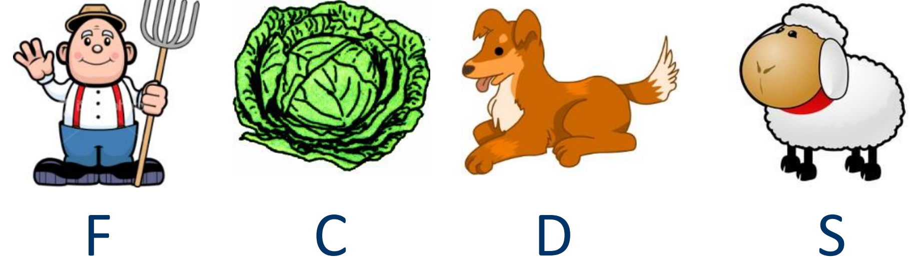
- **Solution is an ordered sequence of primitive actions (steps)**

  $f(x) = a_1, a_2, \ldots, a_n$ where x is the input
- **Model task as a graph of all possible states and actions, and a solution as a path**
- **A state captures all the relevant information about the past**

Sizes of State Spaces

Problem	Nodes	Brute-Force Search Time (10 million nodes/second)
Tic-Tac-Toe	3^9	
8 Puzzle	10^5	.01 seconds
2^3 Rubik's Cube	10^6	.2 seconds
15 Puzzle	10^{13}	6 days
3^3 Rubik's Cube	10^{19}	68,000 years
24 Puzzle	10^{25}	12 billion years
Checkers	10^{40}	
Chess	10^{120}	

Formalizing Search



A search problem has five components:

$\mathcal{S}$, I, G, actions, cost

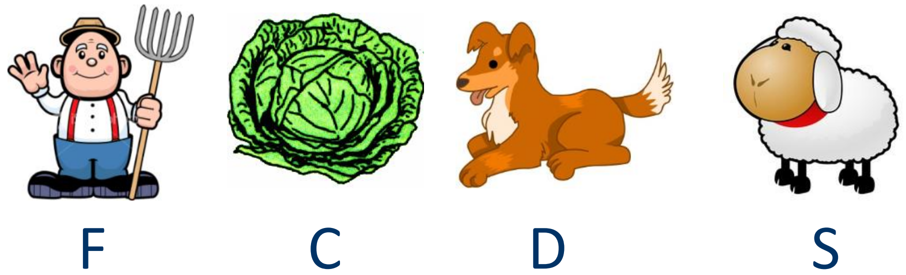
1. **State space $\mathcal{S}$: all valid configurations**
2. **Initial states *I* ⊆ $\mathcal{S}$: a set of start states** *I* = {(FCDS,)} ⊆ $\mathcal{S}$
3. **Goal states *G* ⊆ $\mathcal{S}$: a set of goal states** *G* = {(,FCDS)} ⊆ $\mathcal{S}$
4. **An action function *successors*(*s*) ⊆ $\mathcal{S}$: states reachable in one step (one arc) from *s***

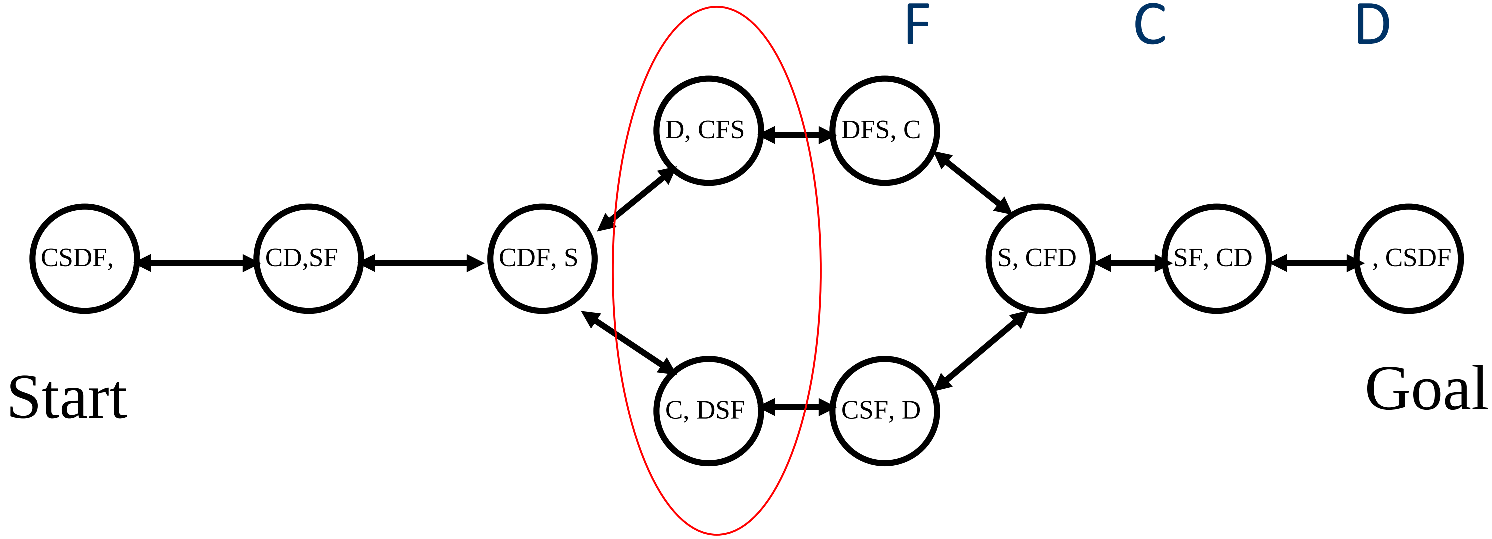
   successors((FCDS,)) = {(CD,FS)}
   successors((CDF,S)) = {(CD,FS), (D,FCS), (C,FSD)}

5. **A cost function cost(*s*, *s'*): The cost of moving from *s* to *s'***

- **The goal of search is to find a solution path from a state in *I* to a state in *G***

State Space = A Directed Graph

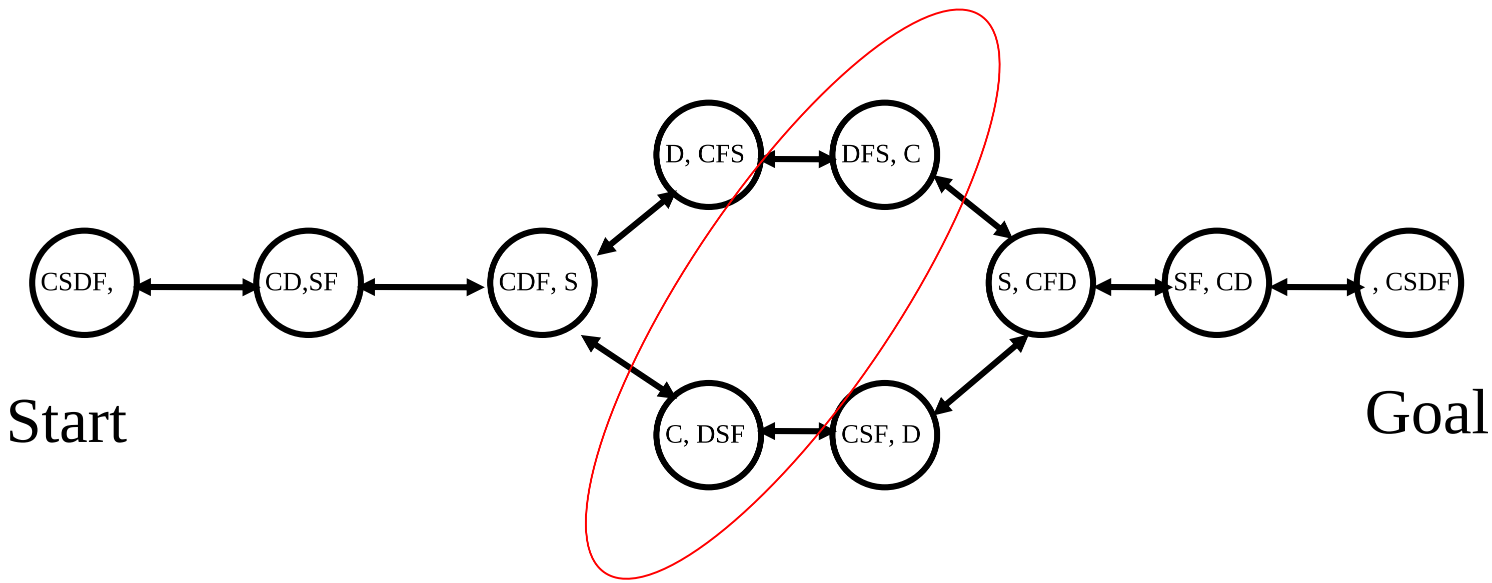




- In general there will be many generated, but un-expanded, states at any given time
- One has to choose which one to “expand” next

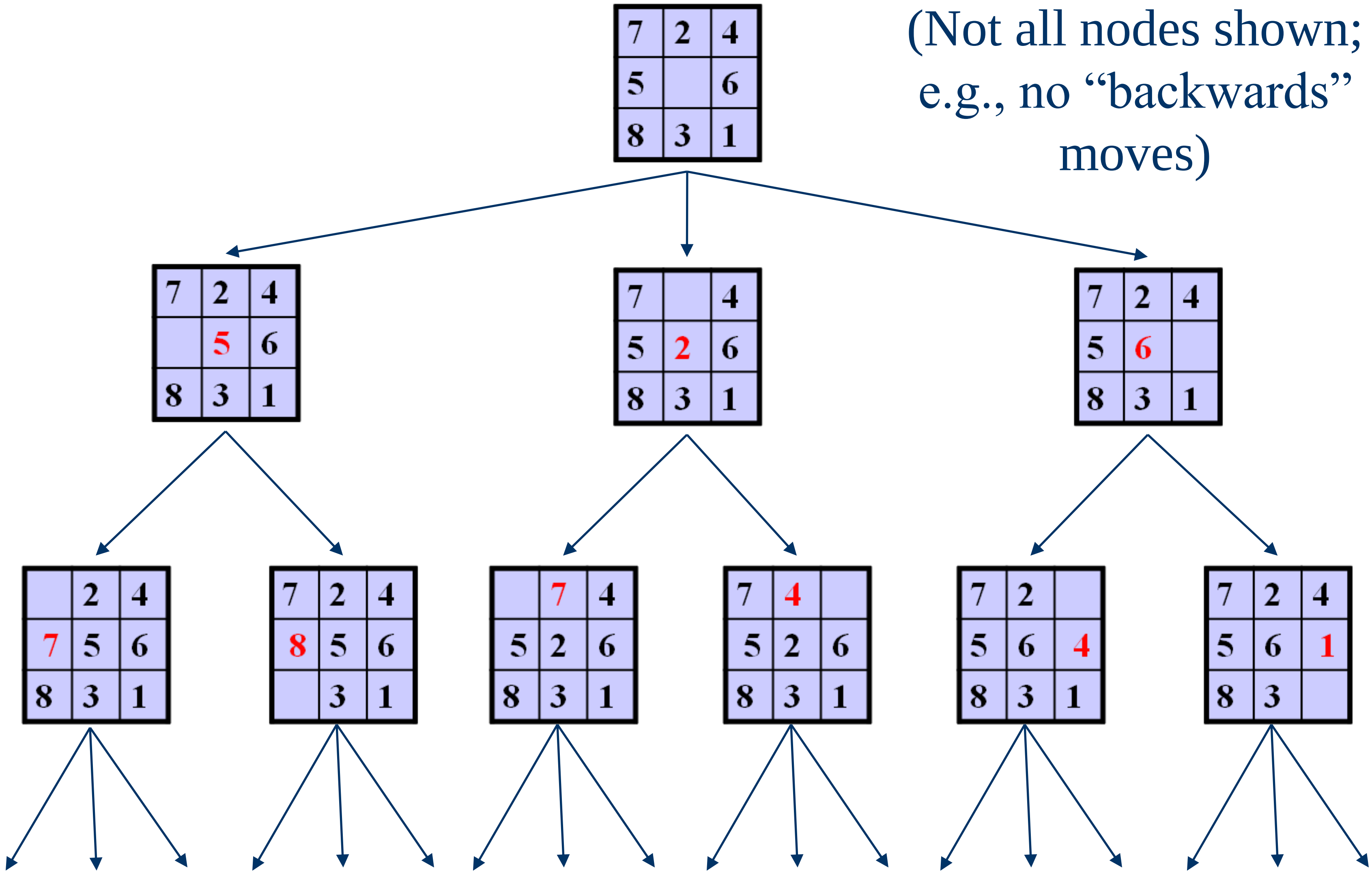
Different Search Strategies

- **The generated, but not yet expanded, states define the *Frontier* (aka *Open* or *Fringe*) set**
- **The essential difference is, which one to expand first?**



8-Puzzle State-Space Search Tree

(Not all nodes shown; e.g., no “backwards” moves)



Uninformed Search Strategies

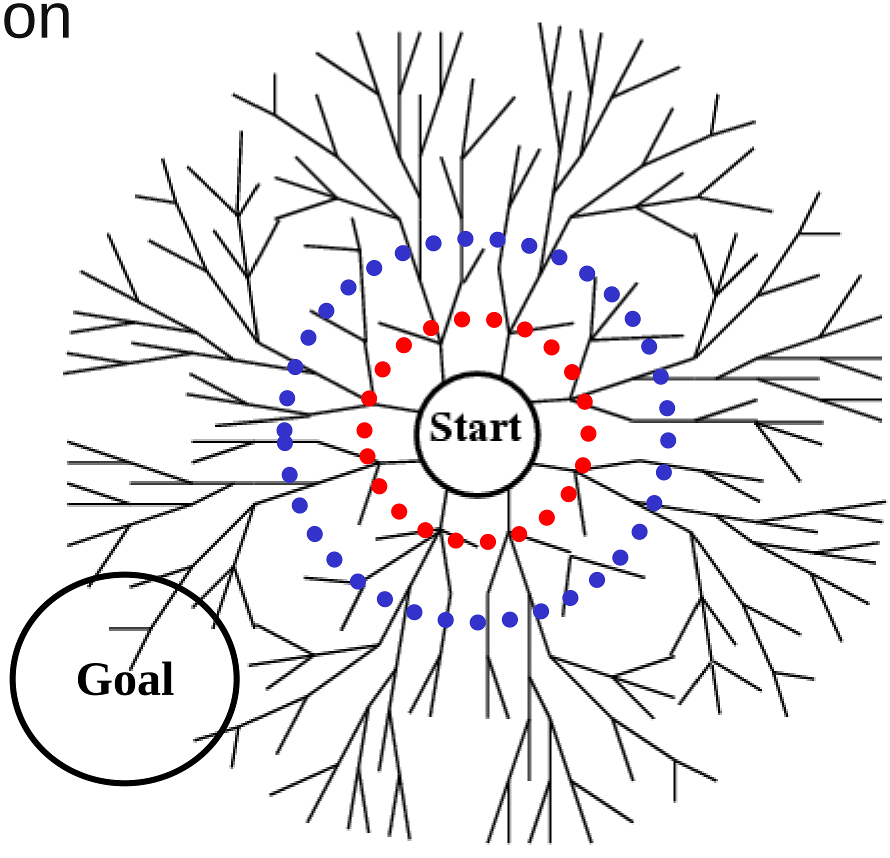
Uninformed Search: strategies that order nodes *without* using any domain specific information, i.e., don't use any information stored in a state

- **BFS: breadth-first search**
  - *Queue* (*FIFO*) *used for the Frontier*
  - remove from front, add to back
- **DFS: depth-first search**
  - *Stack* (*LIFO*) *used for the Frontier*
  - remove from front, add to front

Breadth-First Search (BFS)

Expand the shallowest node first:

1. Examine states one step away from the initial states
2. Examine states two steps away from the initial states
3. and so on

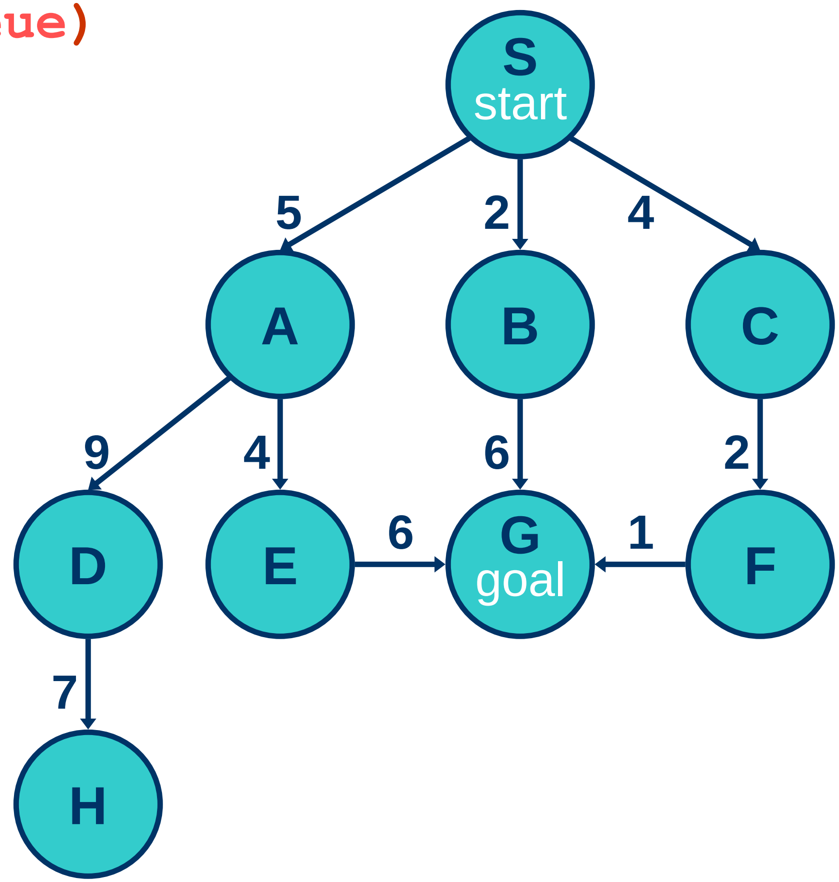


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 0, expanded: 0

expnd. node	Frontier list
	{S}

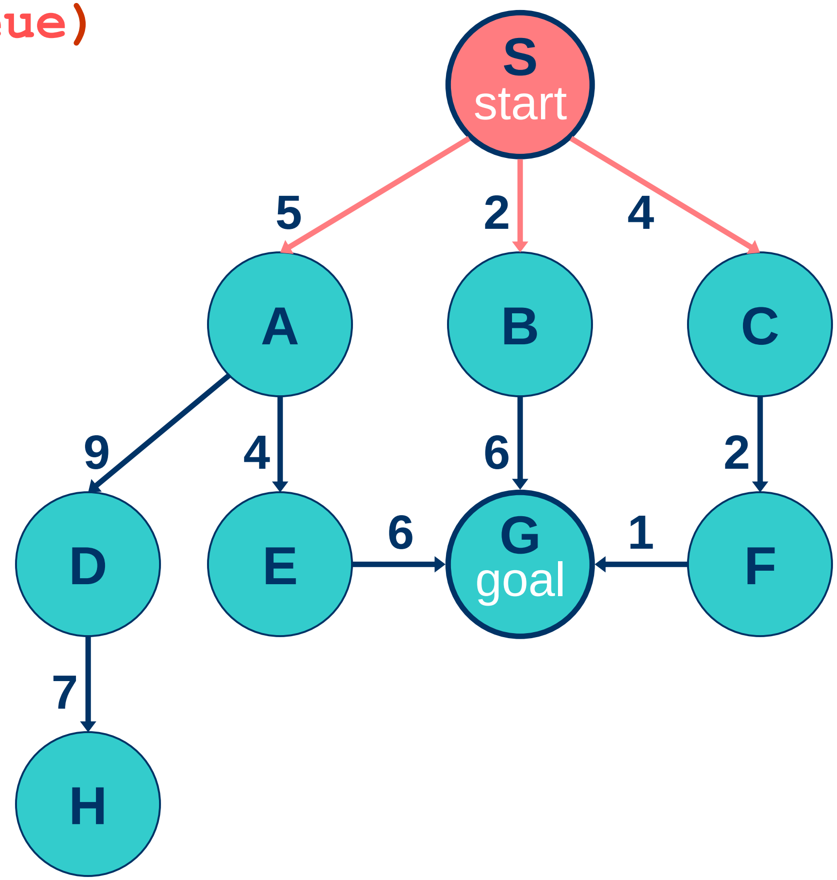


Breadth-First Search (BFS)

generalSearch(problem, queue)

\# of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S}
S not goal	{A,B,C}

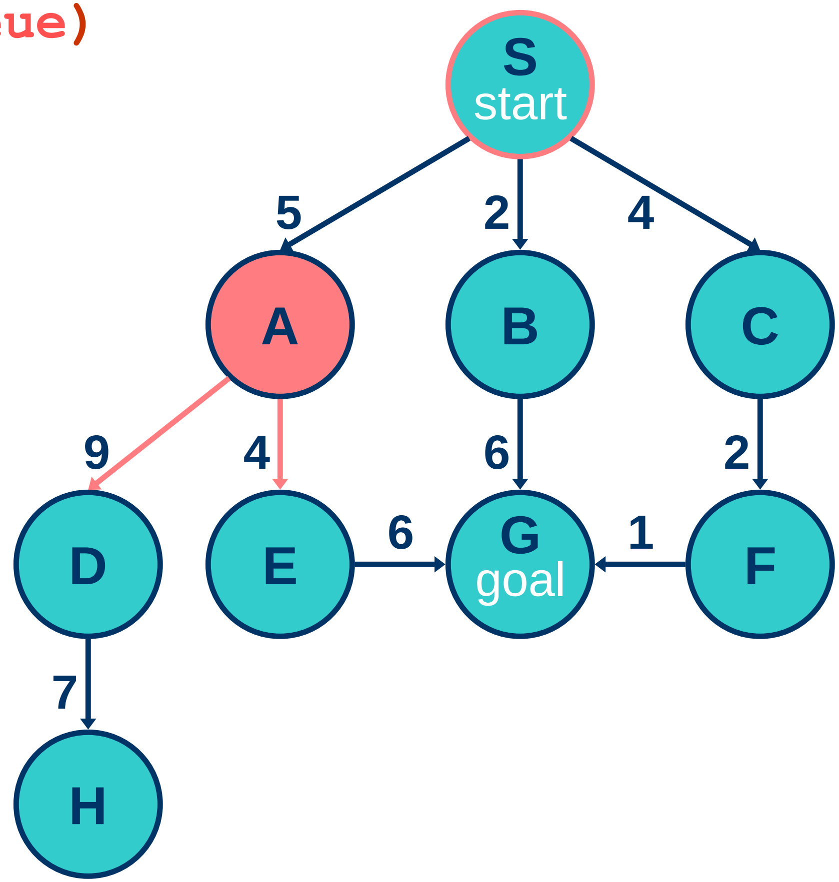


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier list
	{S}
S	{A,B,C}
A not goal	{B,C,D,E}

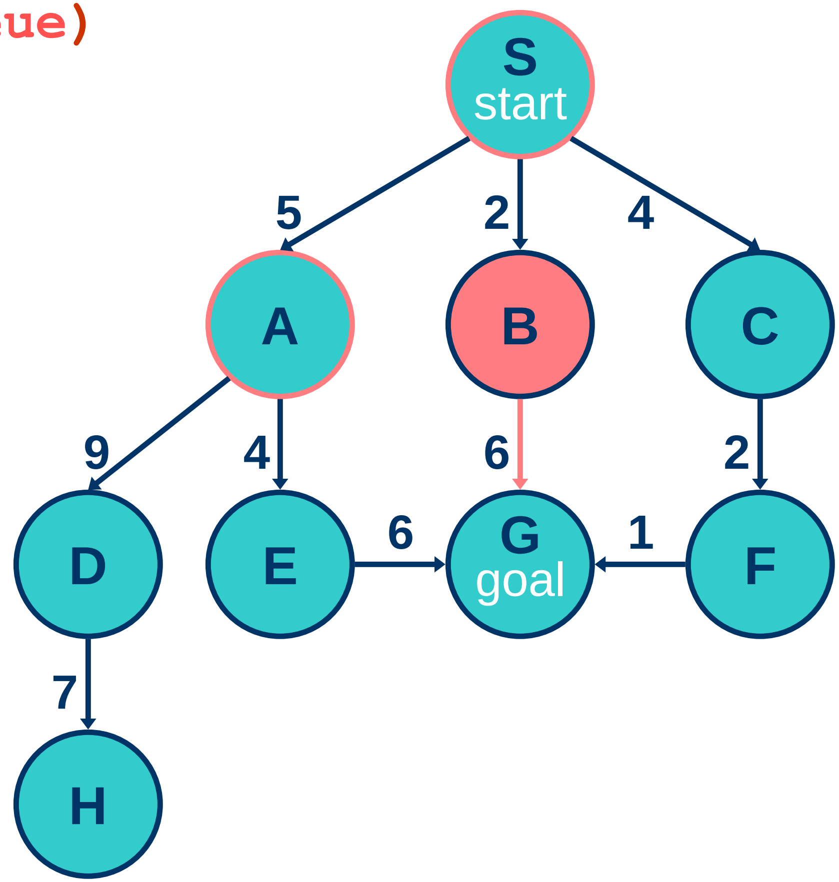


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 3, expanded: 3

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B not goal	{C,D,E,G}

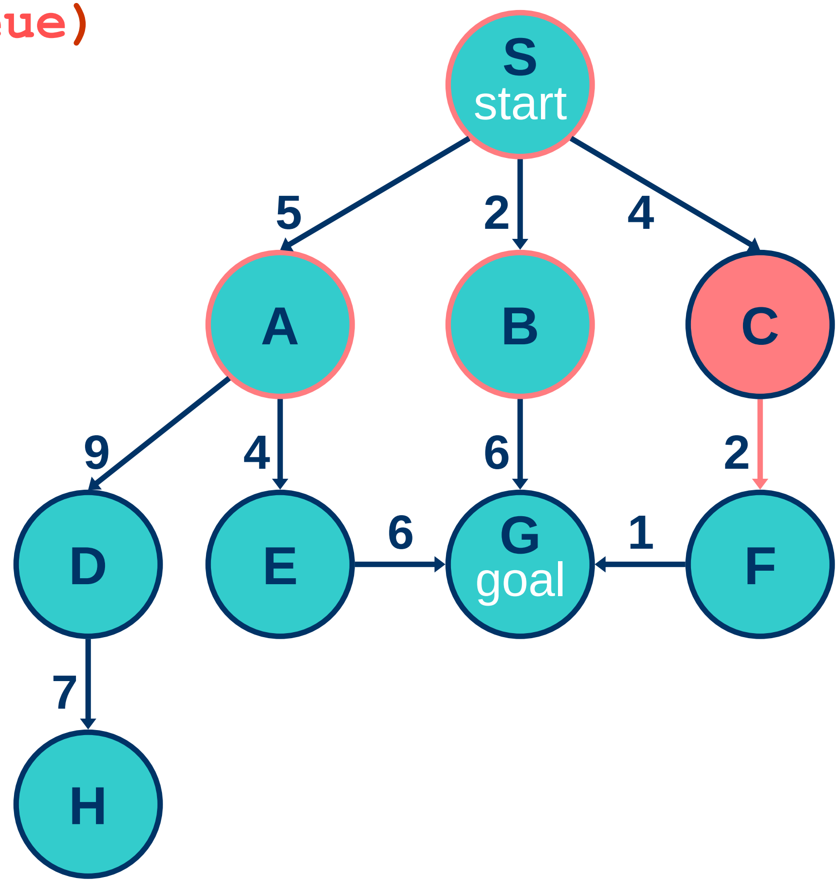


Breadth-First Search (BFS)

generalSearch(problem, queue)

# of nodes tested: 4, expanded: 4

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C not goal	{D,E,G,F}

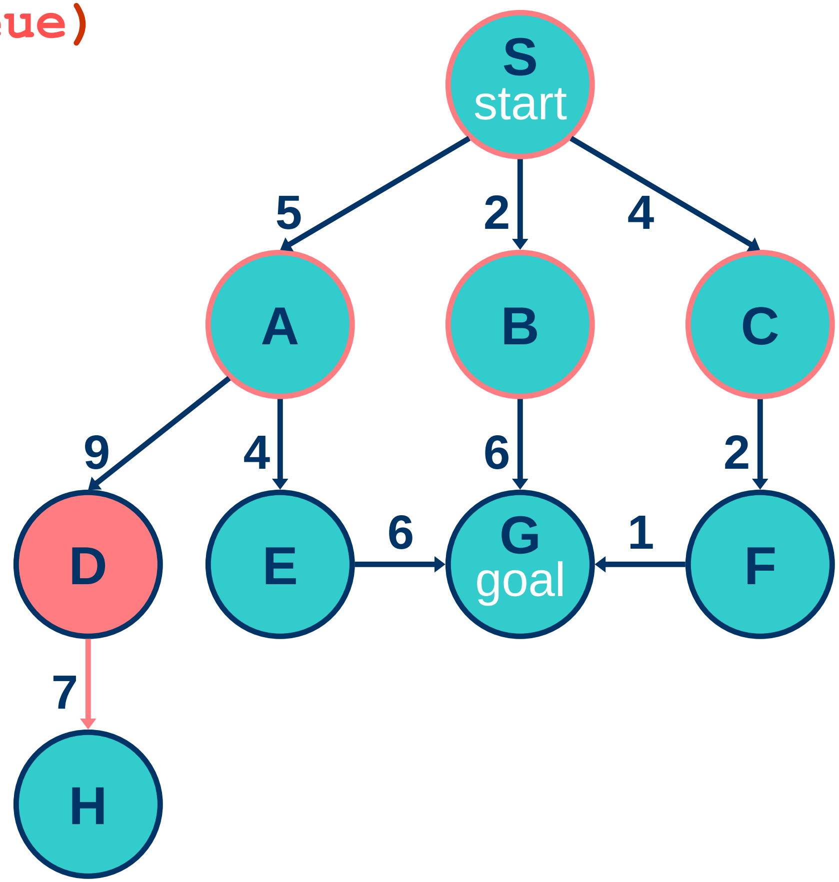


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

\# of nodes tested: 5, expanded: 5

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D not goal	{E,G,F,H}

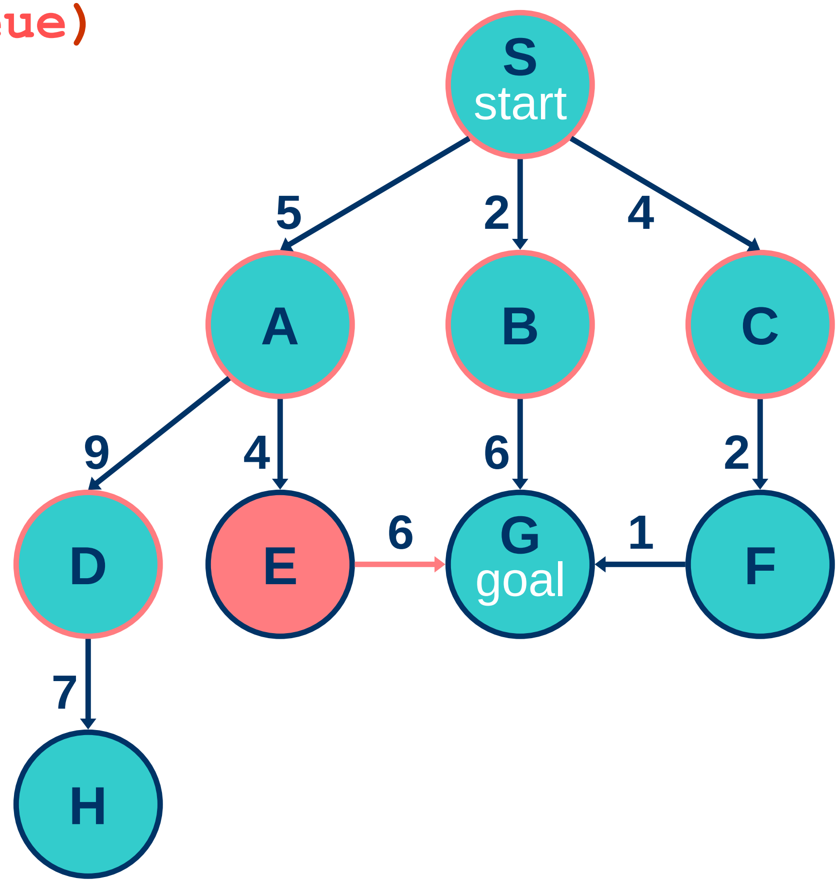


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 6, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E not goal	{G,F,H,G}

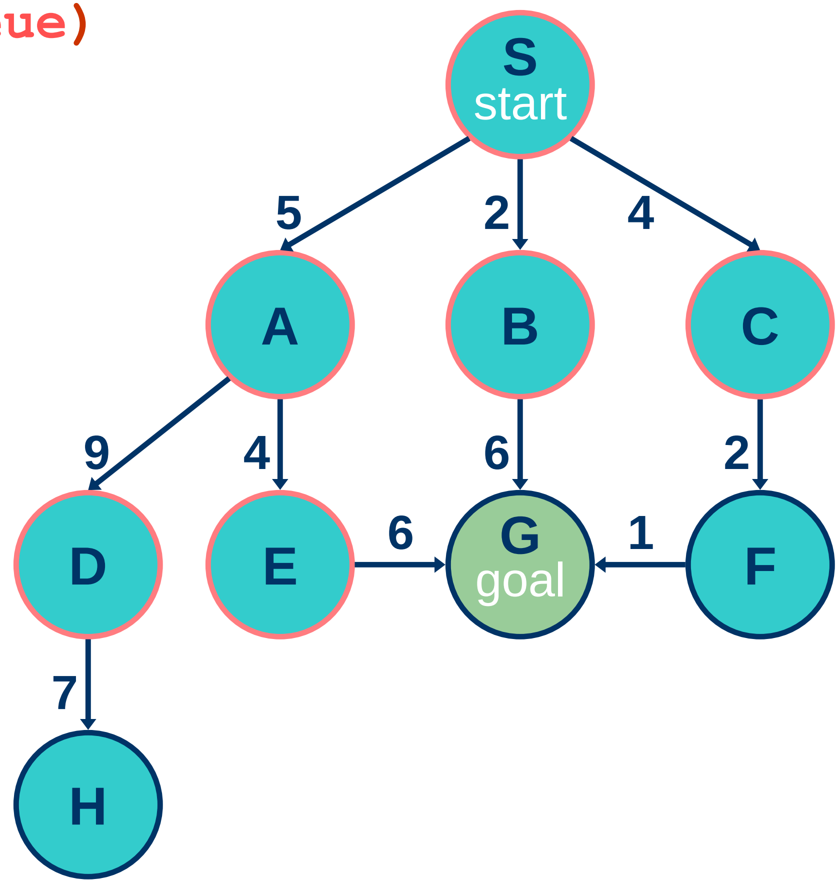


Breadth-First Search (BFS)

generalSearch(problem, queue)

# of nodes tested: 7, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E	{G,F,H,G}
G goal	{F,H,G} no expand

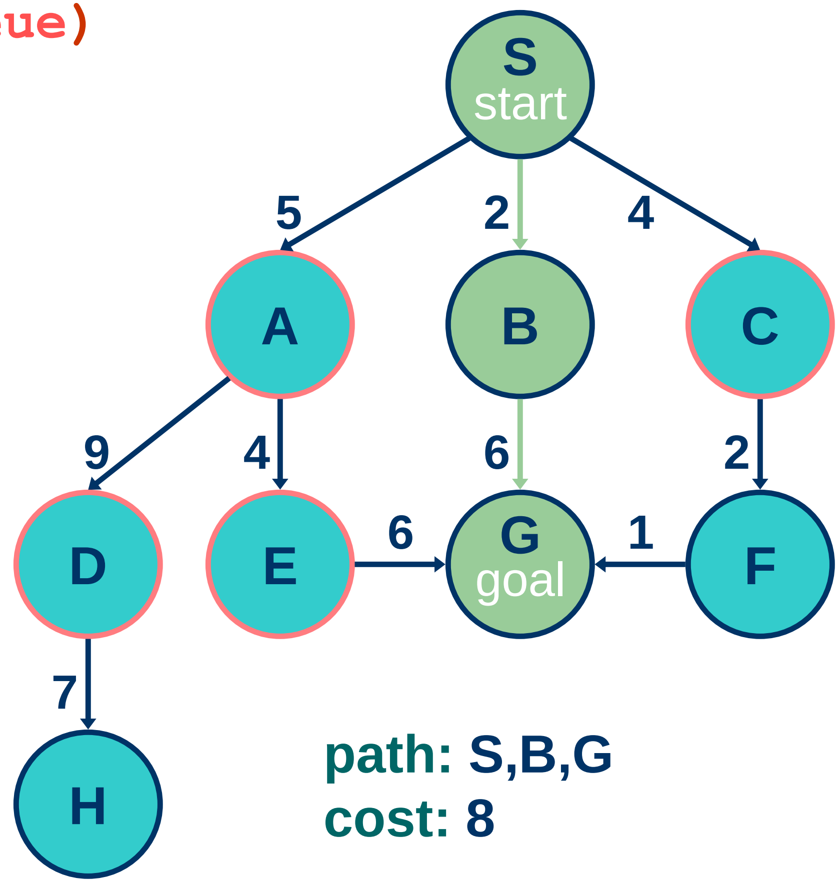


Breadth-First Search (BFS)

`generalSearch(problem, queue)`

# of nodes tested: 7, expanded: 6

expnd. node	Frontier list
	{S}
S	{A,B,C}
A	{B,C,D,E}
B	{C,D,E,G}
C	{D,E,G,F}
D	{E,G,F,H}
E	{G,F,H,G}
G	{F,H,G}



path: S,B,G
cost: 8

Evaluating Search Strategies

- **Completeness**
  If a solution exists, will it be found?
  - a complete algorithm will find ***a*** solution (not all)

- **Optimality / Admissibility**
  If a solution is found, is it guaranteed to be optimal?
  - an admissible algorithm will find a **solution with minimum cost**

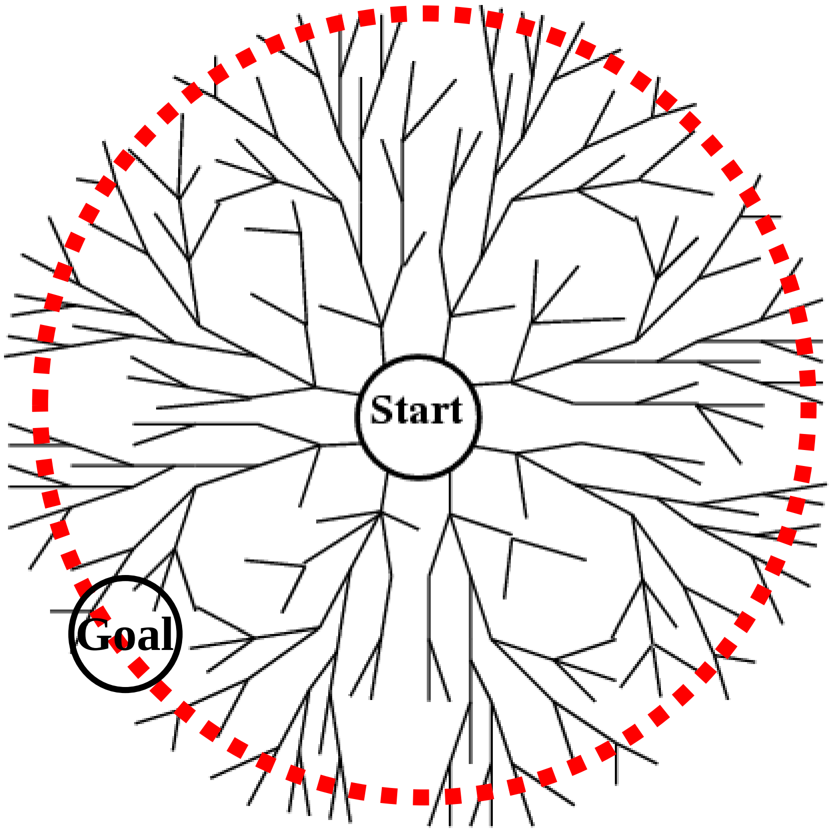
Evaluating Search Strategies

- **Time Complexity**
  How long does it take to find a solution?
  - usually measured for worst case
  - measured by counting **number of nodes expanded**

- **Space Complexity**
  How much space is used by the algorithm?
  - measured in terms of the **maximum size of the *Frontier*** during the search

What's in the Frontier for BFS?

- **If goal is at depth *d*, how big is the Frontier (worst case)?**



Breadth-First Search (BFS)

- **Complete**

- **Optimal / Admissible**
  - **Yes**, ***if*** all operators (i.e., arcs) have the same constant cost, or costs are positive, non-decreasing with depth
  - otherwise, not optimal but *does* guarantee finding solution of shortest *length* (i.e., fewest arcs)

Breadth-First Search (BFS)

- **Time and space complexity: $O(b^d)$ (i.e., exponential)**
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node

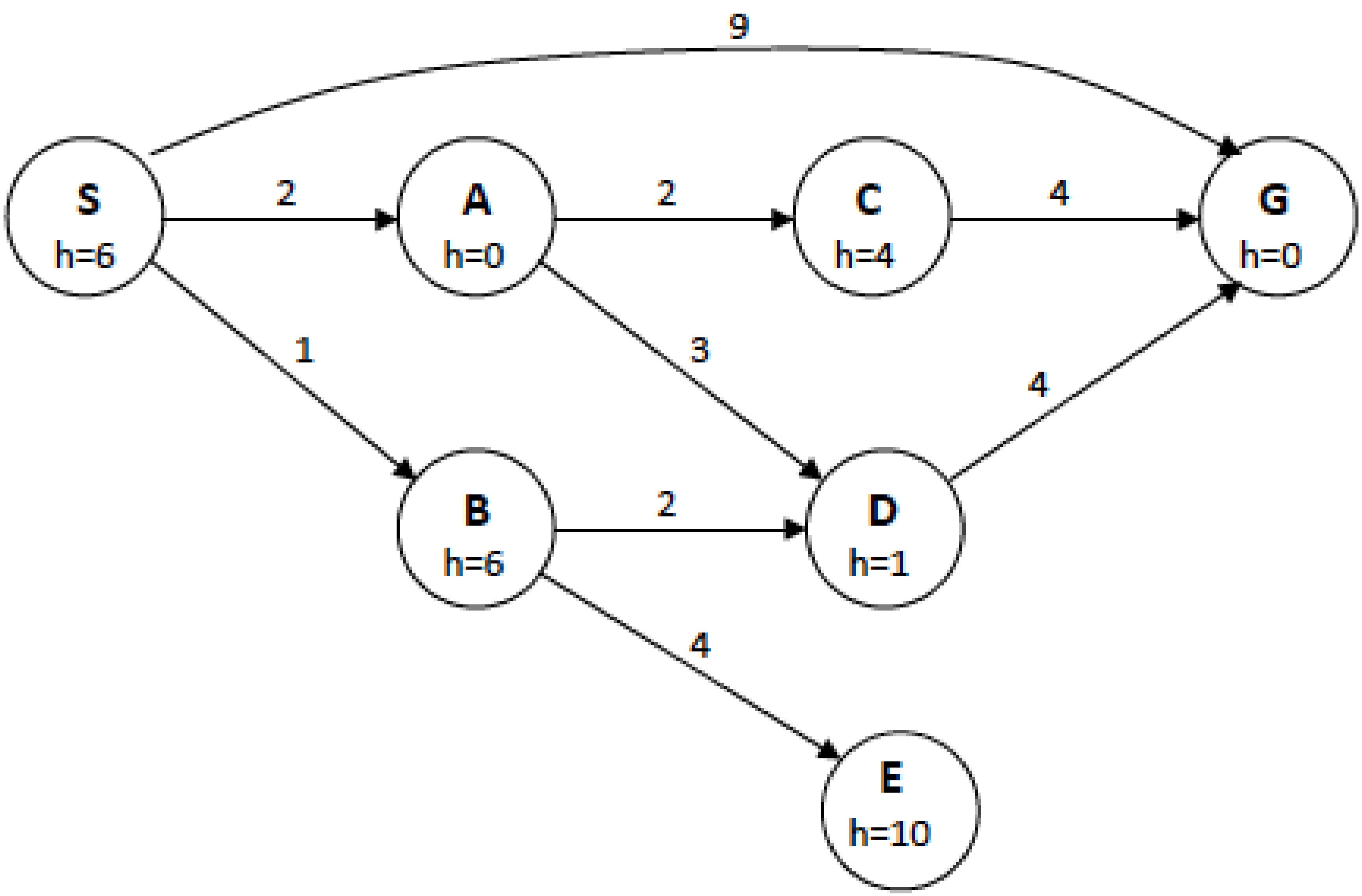
- Very slow to find solutions with a large number of steps because must look at *all* shorter length possibilities first

Breadth-First Search (BFS)

- **A complete search tree has a total # of nodes =** $\mathbf{1 + b + b^2 + \ldots + b^d = (b^{(d+1)} - 1) / (b-1)}$
  - d: the tree's depth
  - b: the branching factor at each non-leaf node
- **For example:** $\mathbf{d = 12, b = 10}$

  $1 + 10 + 100 + \ldots + 10^{12} = (10^{13} - 1)/9 = O(10^{12})$
  - If BFS expands 1,000 nodes/sec and each node uses 100 bytes of storage, then BFS will take 35 years to run in the worst case, and it will use 111 terabytes of memory!

Problem: Given State Space

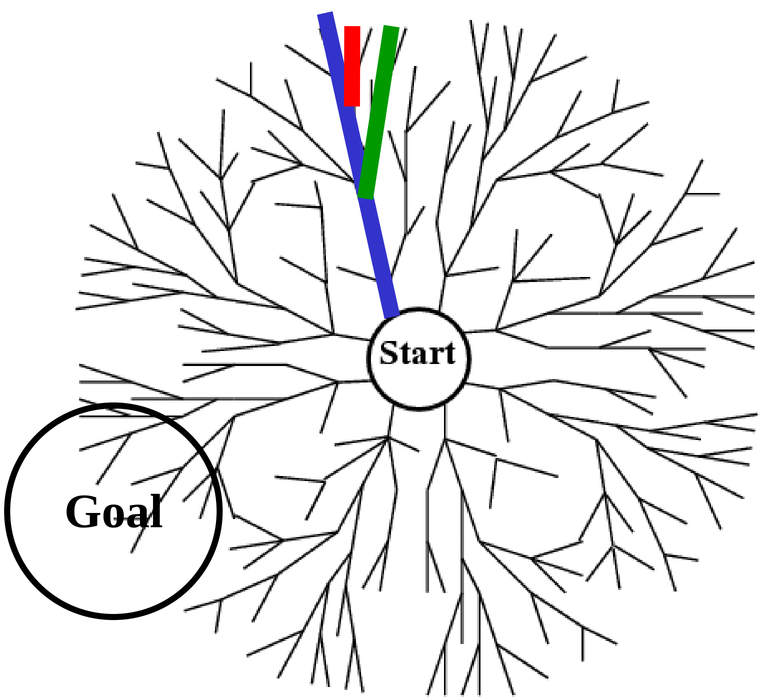


Depth-First Search

Expand the ***deepest*** node first

1. Select a direction, go deep to the end
2. Slightly change the end
3. Slightly change the end some more…

Use a Stack to order nodes on the *Frontier*

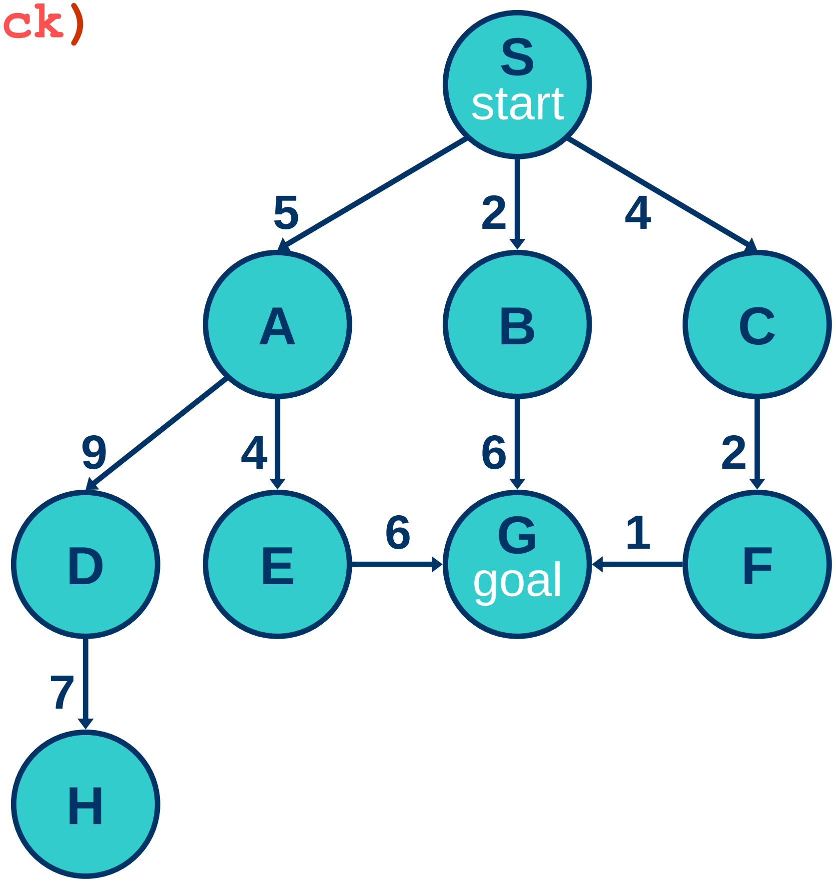


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 0, expanded: 0

expnd. node	Frontier
	{S}

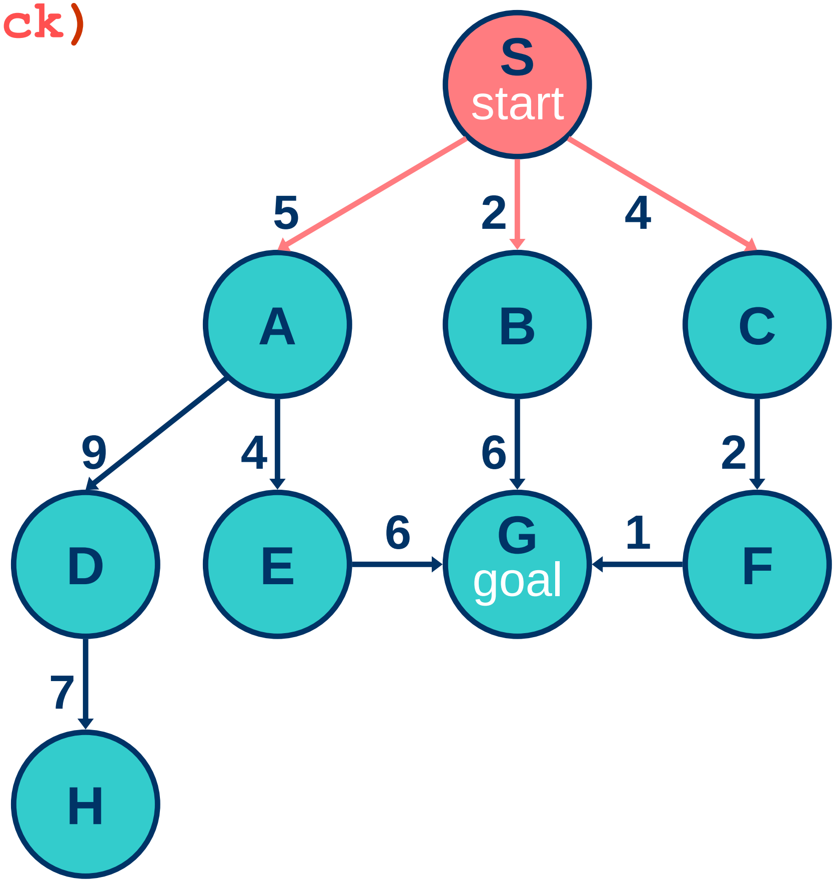


Depth-First Search (DFS)

`generalSearch(problem, stack)`

\# of nodes tested: 1, expanded: 1

expnd. node	Frontier
	{S}
S not goal	{A,B,C}

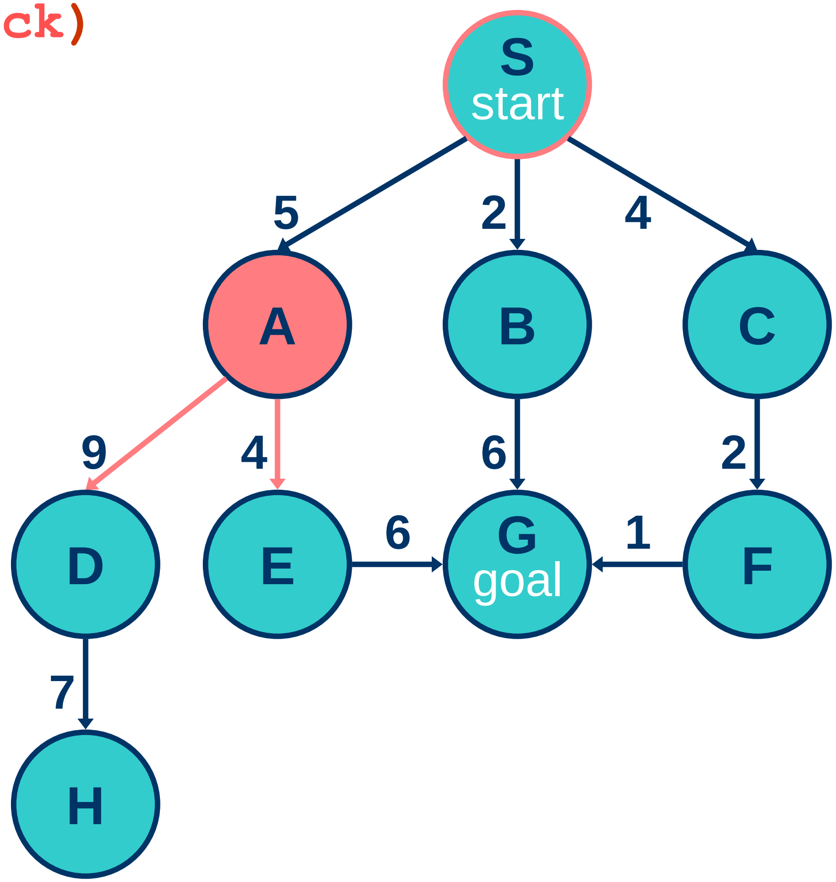


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier
	{S}
S	{A,B,C}
A not goal	{D,E,B,C}

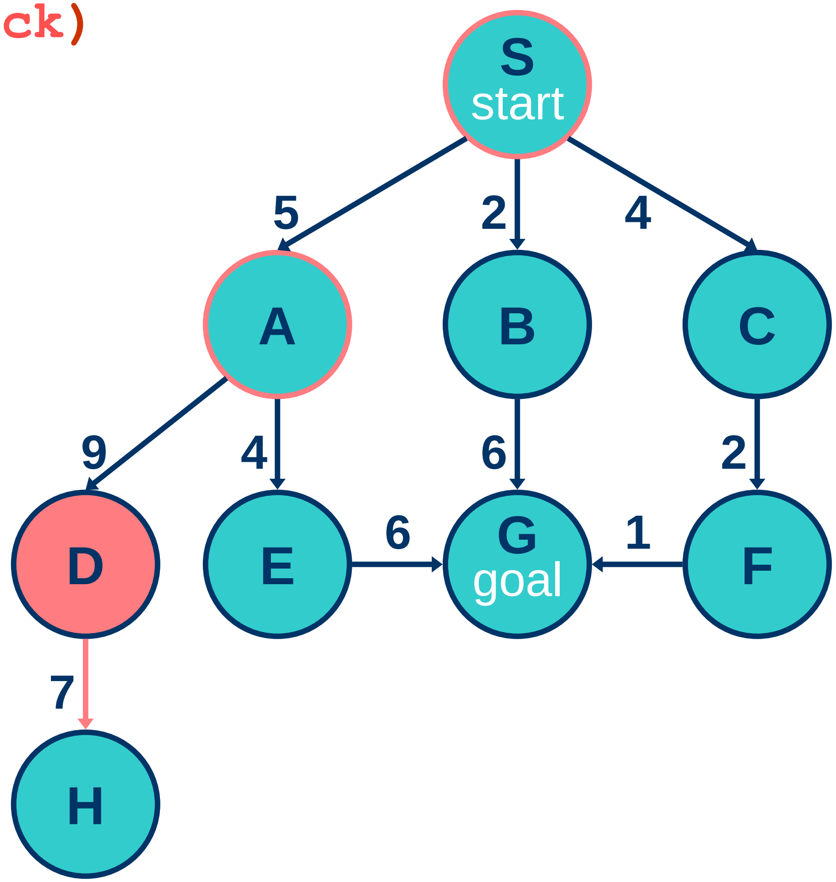


Depth-First Search (DFS)

`generalSearch(problem, stack)`

# of nodes tested: 3, expanded: 3

expnd. node	Frontier
	{S}
S	{A,B,C}
A	{D,E,B,C}
D not goal	{H,E,B,C}

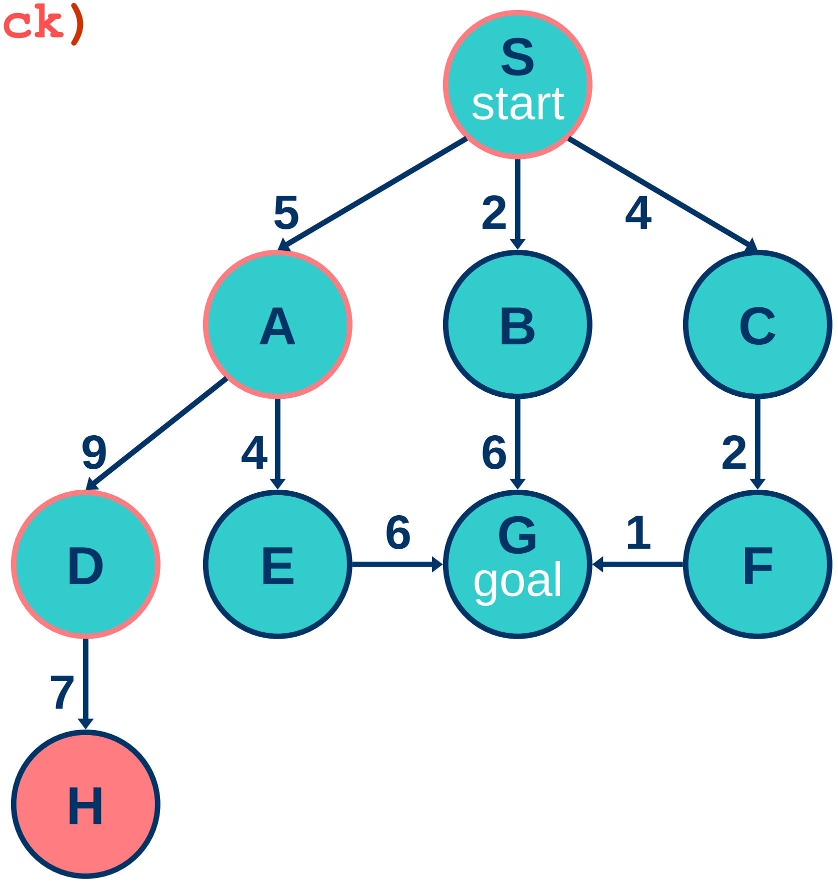


Depth-First Search (DFS)

`generalSearch(problem, stack)`

\# of nodes tested: 4, expanded: 4

expnd. node	Frontier
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H not goal	{E,B,C}

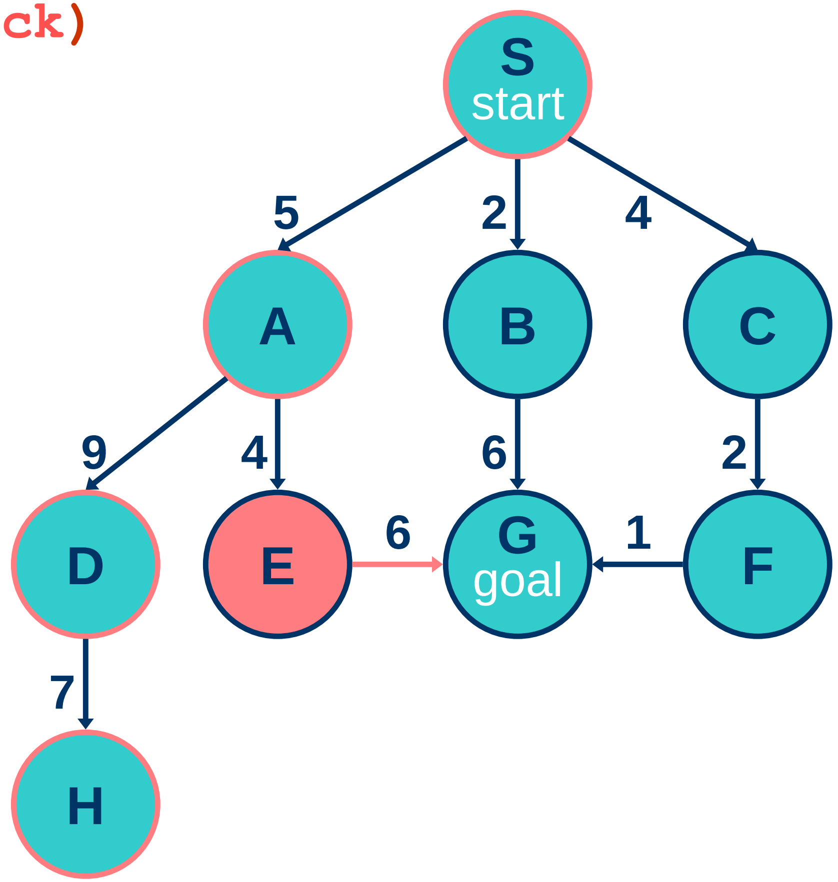


Depth-First Search (DFS)

generalSearch(problem, stack)

# of nodes tested: 5, expanded: 5

expnd. node	Frontier
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E not goal	{G,B,C}

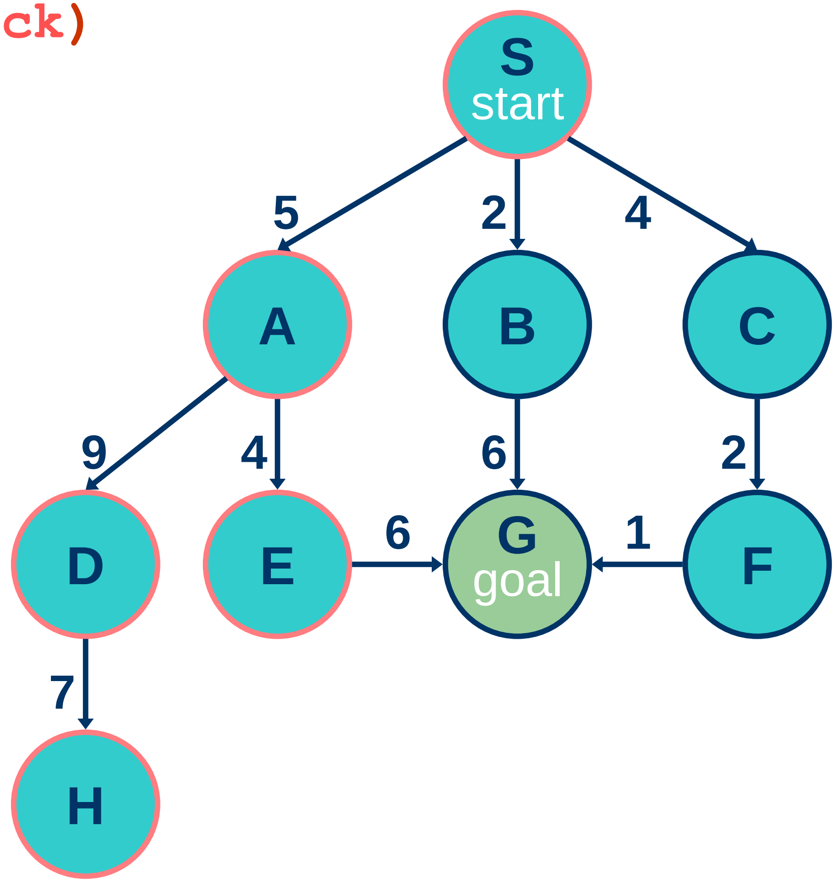


Depth-First Search (DFS)

generalSearch(problem, stack)

# of nodes tested: 6, expanded: 5

expnd. node	Frontier
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E	{G,B,C}
G goal	{B,C} no expand

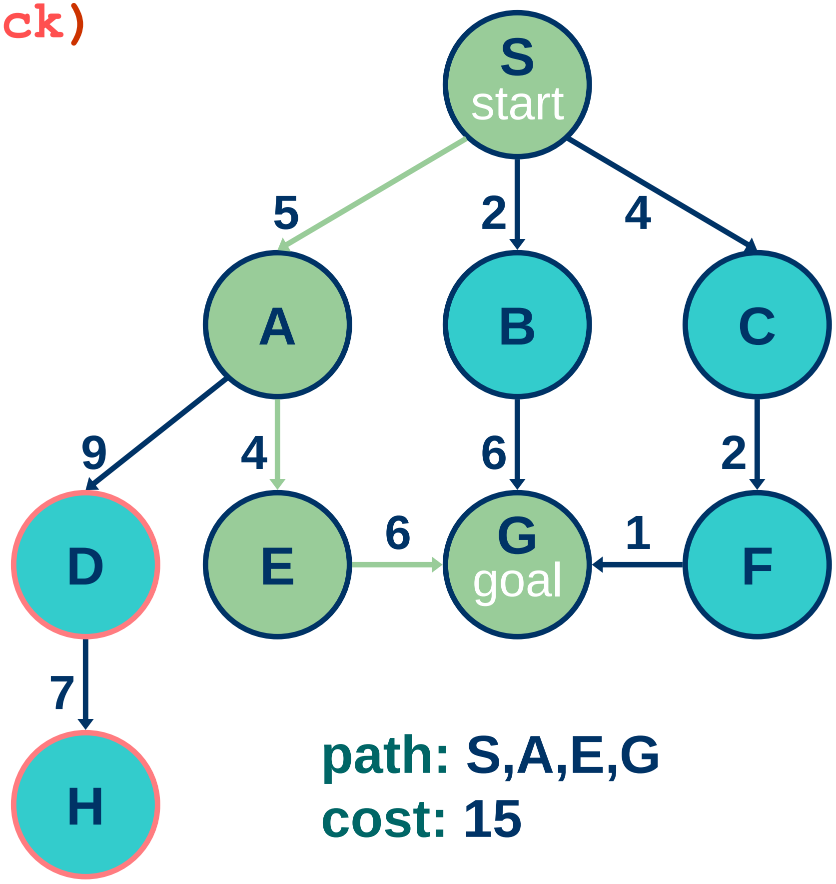


Depth-First Search (DFS)

generalSearch(problem, stack)

# of nodes tested: 6, expanded: 5

expnd. node	Frontier
	{S}
S	{A,B,C}
A	{D,E,B,C}
D	{H,E,B,C}
H	{E,B,C}
E	{G,B,C}
G	{B,C}



path: S,A,E,G
cost: 15

Depth-First Search (DFS)

- **May not terminate without a depth bound i.e., cutting off search below a fixed depth,** D

- **Not complete**
  - with or without cycle detection
  - and, with or without a depth cutoff

- **Not optimal / admissible**

- ***Can find long solutions quickly if lucky***

Depth-First Search (DFS)

- **Time complexity:** $O(b^d)$ **exponential**
  Space complexity: $O(bd)$ **linear**
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node
- Performs "**chronological backtracking**"
  - i.e., when search hits a dead end, backs up *one* level at a time
  - problematic if the mistake occurs because of a bad action choice near the top of search tree

Uniform-Cost Search (UCS)

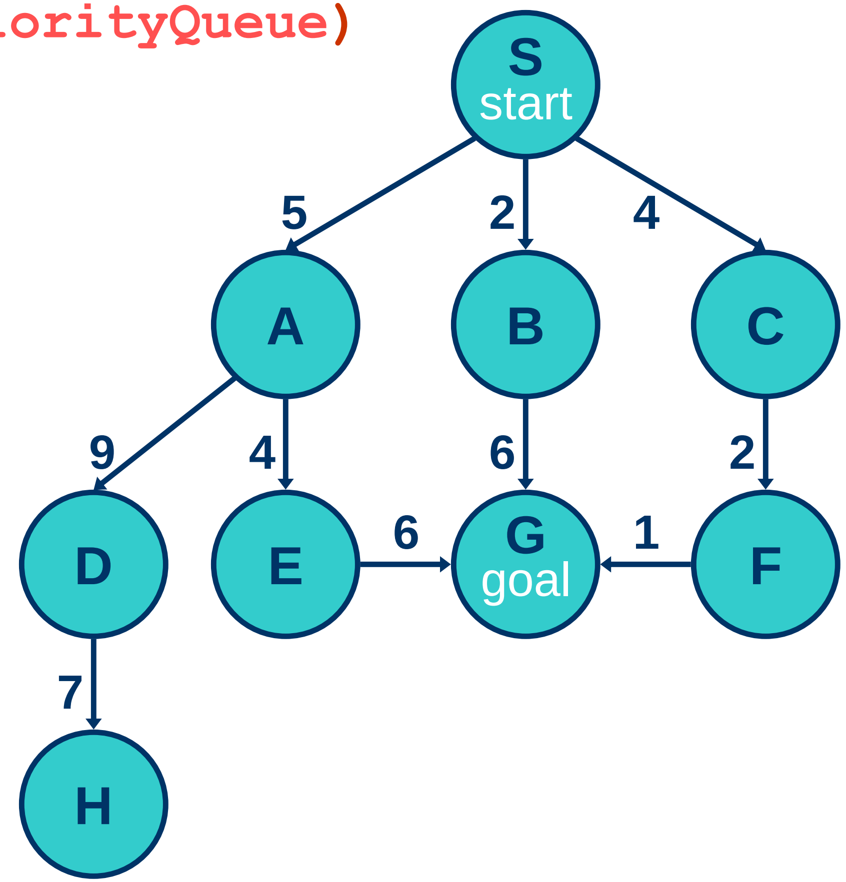
- Use a "**Priority Queue**" to order nodes on the *Frontier* list, sorted by path cost
- Let $g(n)$ = cost of path from start node s to current node n
- Sort nodes by increasing value of g

Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 0, expanded: 0

expnd. node	Frontier list
	{S}

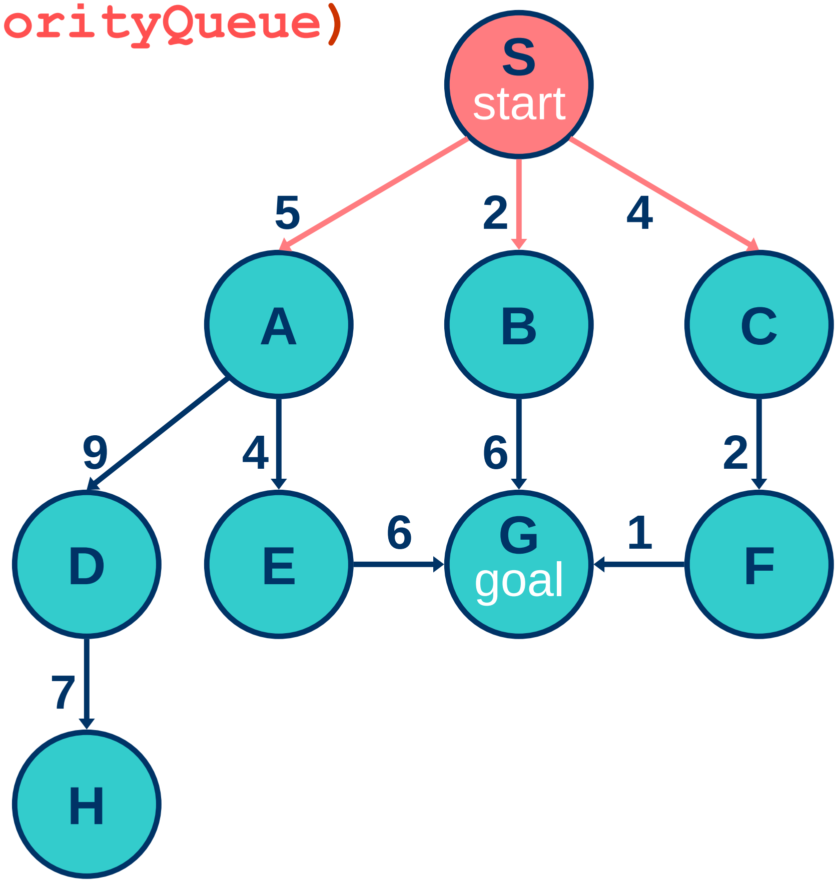


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

\# of nodes tested: 1, expanded: 1

expnd. node	Frontier list
	{S:0}
S not goal	{B:2,C:4,A:5}

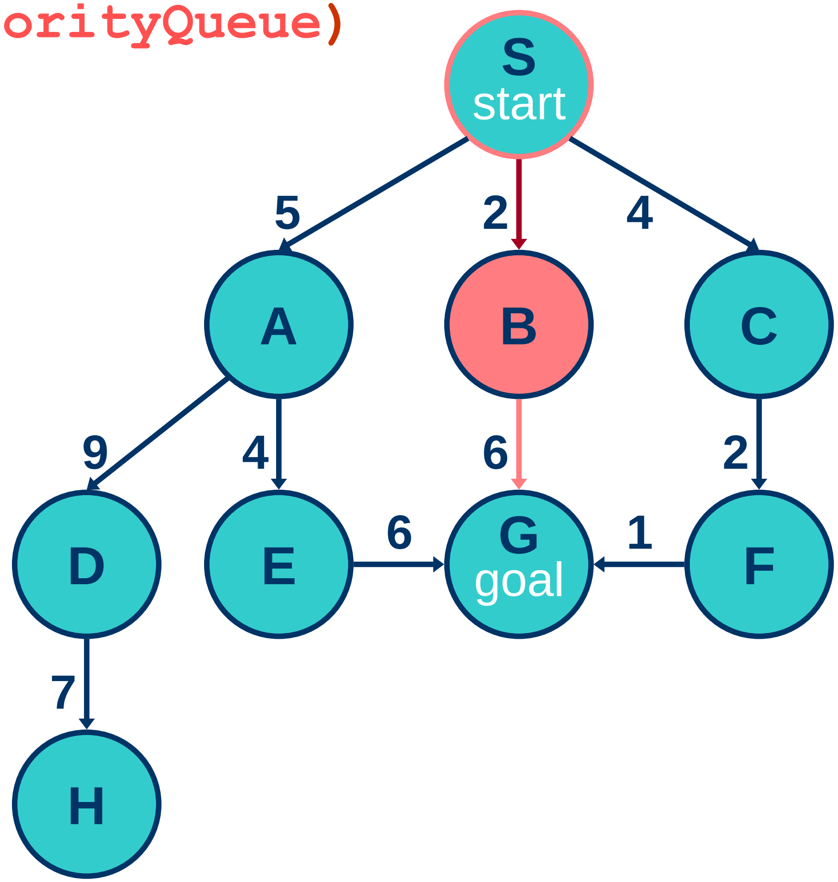


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 2, expanded: 2

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B not goal	{C:4,A:5,G:2+6}

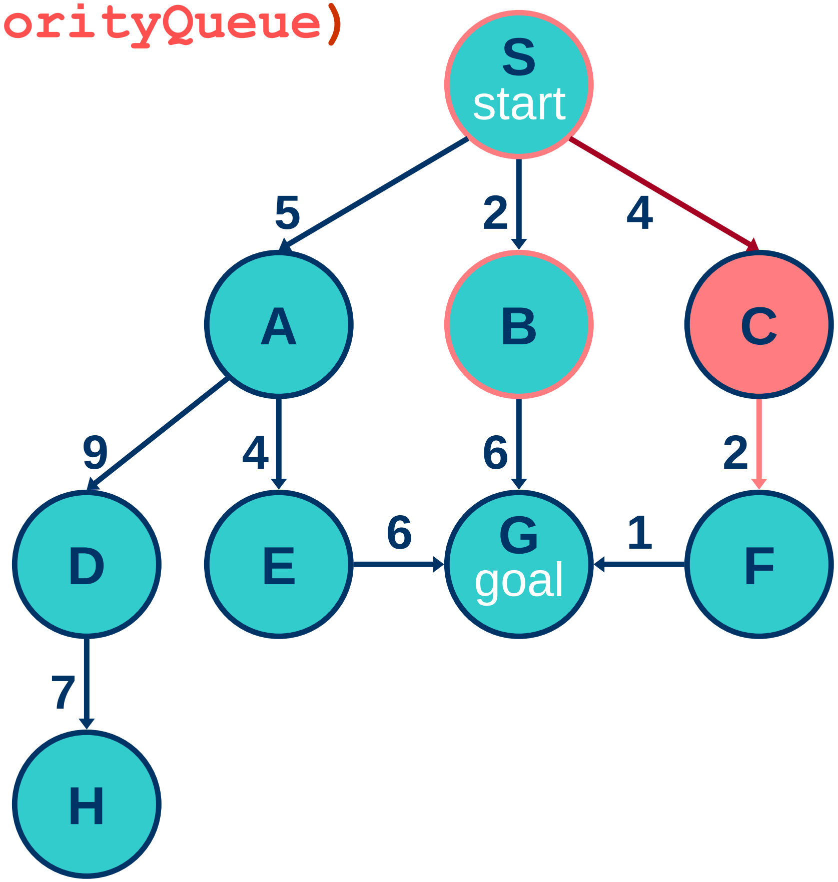


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

\# of nodes tested: 3, expanded: 3

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C not goal	{A:5,F:4+2,G:8}

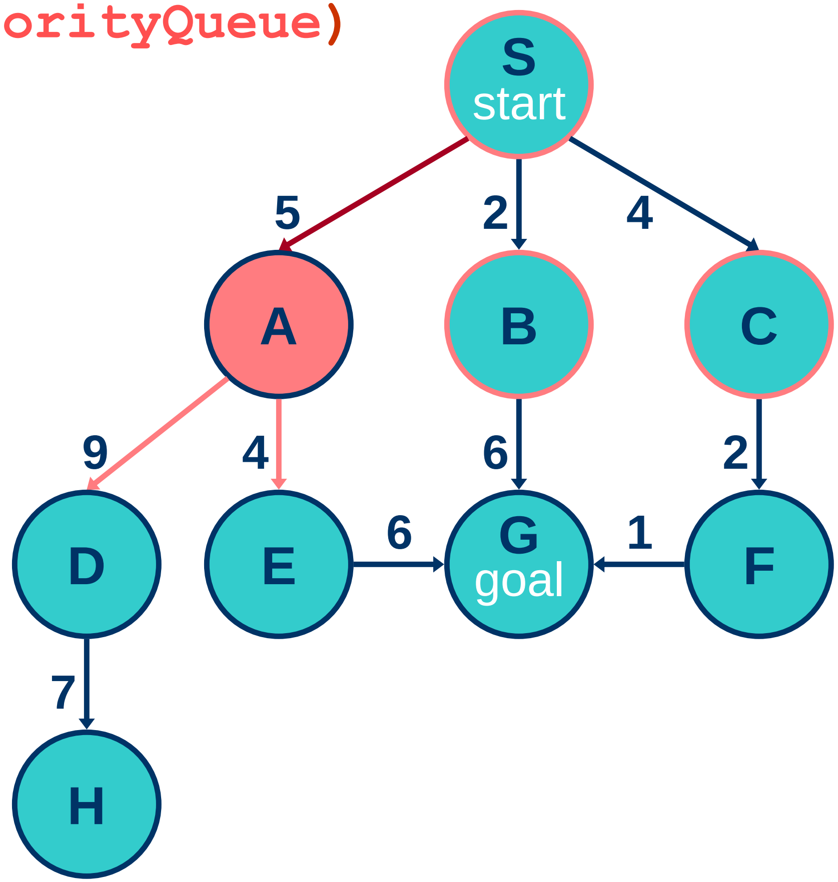


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 4, expanded: 4

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A not goal	{F:6,G:8,E:5+4, D:5+9}

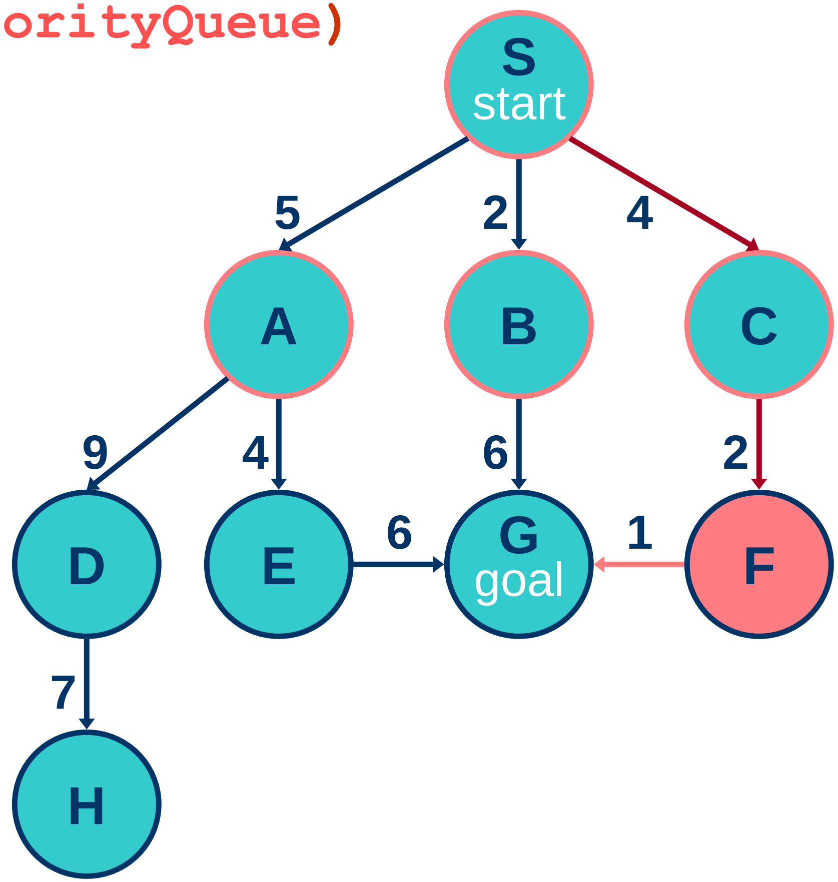


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 5, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F not goal	{G:4+2+1,G:8,E:9, D:14}

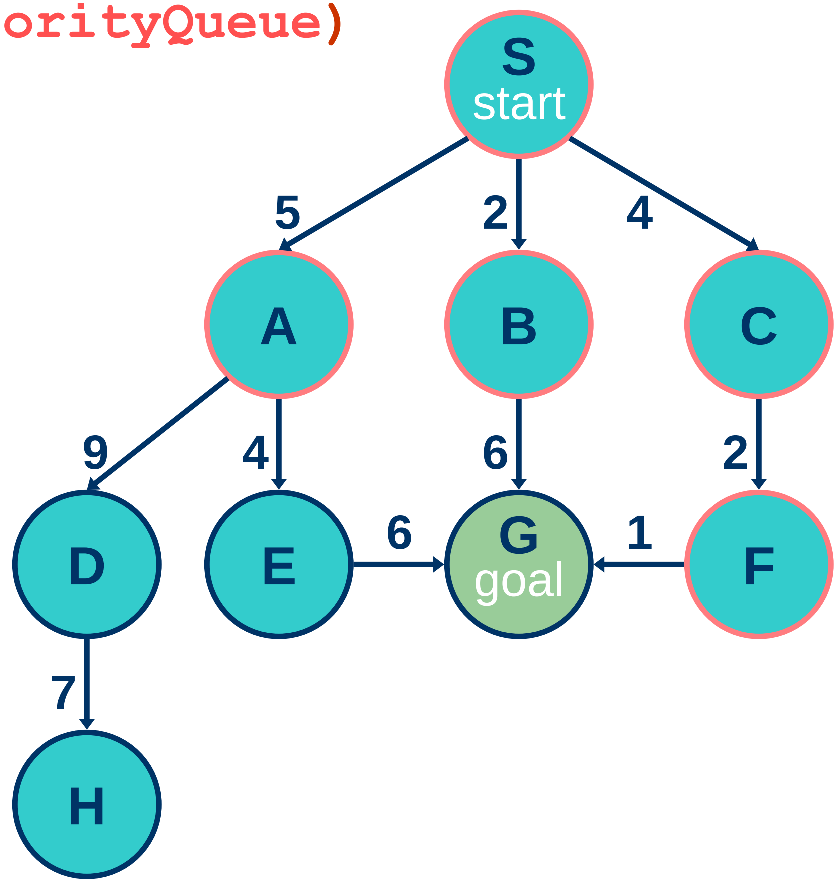


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F	{G:7,G:8,E:9,D:14}
G goal	{G:8,E:9,D:14} no expand

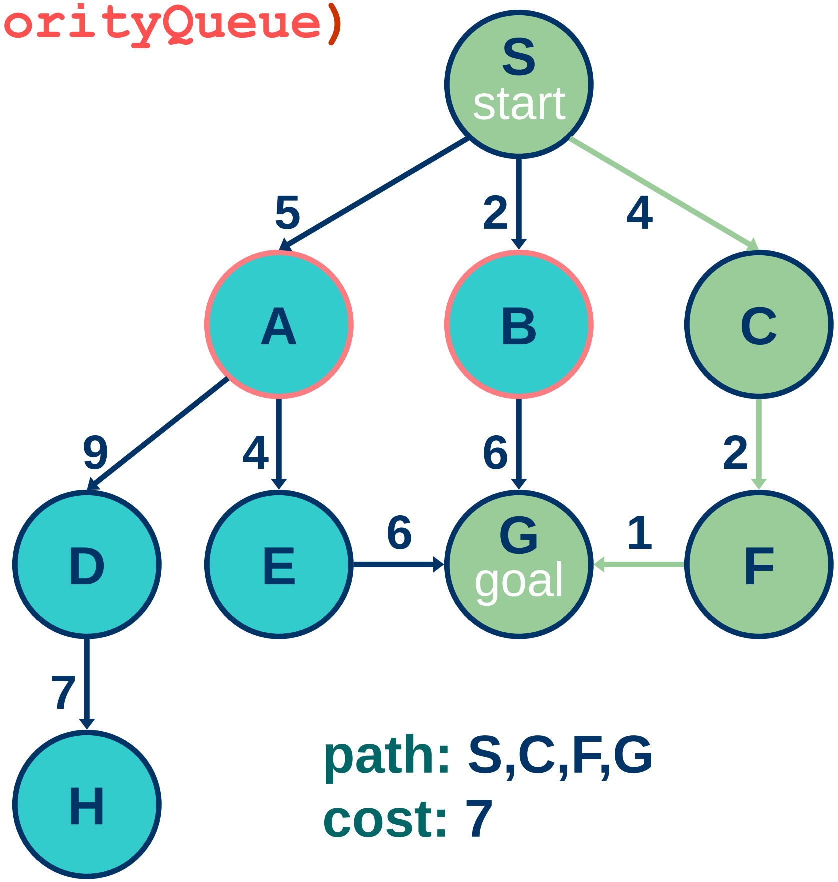


Uniform-Cost Search (UCS)

`generalSearch(problem, priorityQueue)`

# of nodes tested: 6, expanded: 5

expnd. node	Frontier list
	{S}
S	{B:2,C:4,A:5}
B	{C:4,A:5,G:8}
C	{A:5,F:6,G:8}
A	{F:6,G:8,E:9,D:14}
F	{G:7,G:8,E:9,D:14}
G	{G:8,E:9,D:14}



path: S,C,F,G
cost: 7

Uniform-Cost Search (UCS)

- **Called *Dijkstra's Algorithm* in the algorithms literature**
- **Similar to *Branch and Bound Algorithm* in Operations Research literature**

- **Complete**
- **Optimal / Admissible**
  - requires that the goal test is done when a node is ***removed*** from the *Frontier* rather than when the node is generated by its parent node

Uniform-Cost Search (UCS)

- **Time and space complexity:** $O(b^d)$ **(i.e., exponential)**
  - d is the depth of the solution
  - b is the branching factor at each non-leaf node

- **More precisely, time and space complexity is** $O(b^{C^*/\varepsilon})$ **where all edge costs** $\varepsilon \sum > 0$**, and** C^* **is the best goal path cost**