Programming Assignment 3
STAT 311

Please complete the following problems and submit a file named `STAT311-HW3.R` to Gradescope. You should start from the provided `STAT311-HW3.R` file on Canvas.

Some question answers are hidden and students are encouraged to think carefully about their answers in their final submissions.

Remember:

- Do not destroy or overwrite any requested variables in your program. I check them only after I have run your entire program from start to finish.
- Check to make sure you do not have any syntax errors. Reset the working environment and rerun your entire assignment to ensure it runs without errors using the source command.
- Do not include any plotting commands in your submission, or the autograder will fail.

Question 1

Consider a vase (or vase if you prefer that pronunciation) containing rubber balls of 3 different colors, `red`, `blue`, and `green`. The number of balls of each color is random, but will be saved in the variables `red`, `blue`, and `green`.

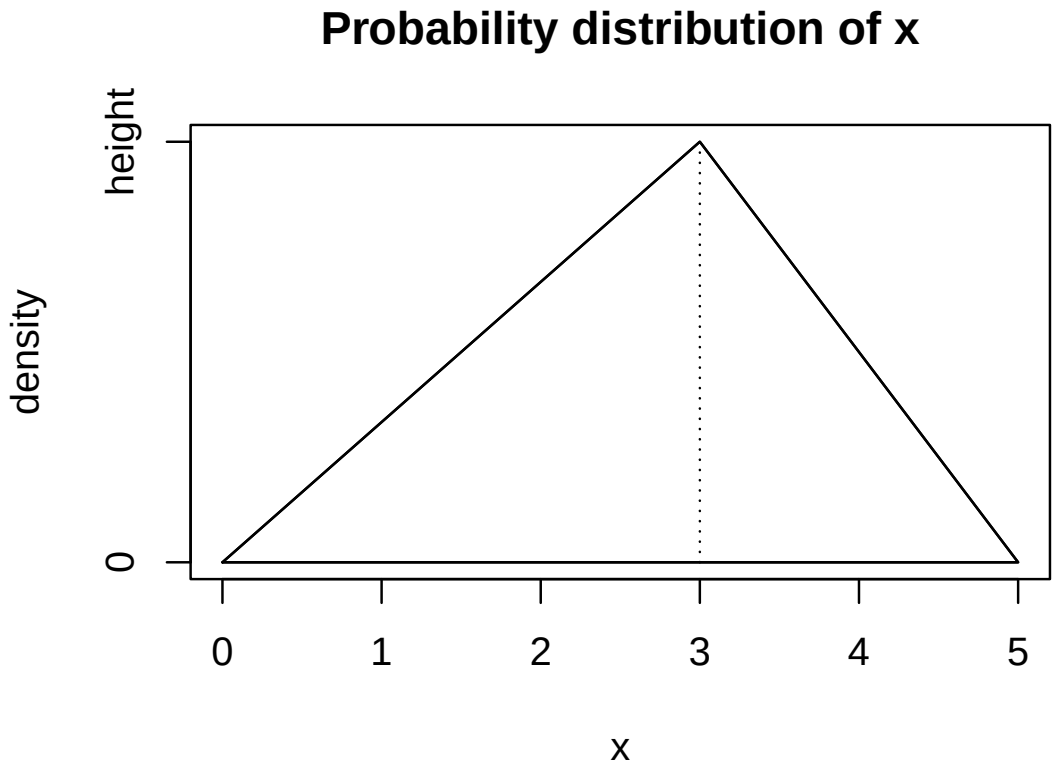
You will need to provide 'solutions', not 'answers', to these problems. IE: your solution should use the variables `red`, `blue`, and `green`, rather than numbers, which will be subject to change.

- Question 1.a - If we draw a single ball at random, what is the probability that we draw a red ball? Save your answer in the variable `q1.a`
- Question 1.b - If we drew a red ball and do not return it to the vase, what is the probability that the next draw is blue? Save your answer in the variable `q1.b`
- Question 1.c - If we draw 4 balls at random, replacing them each time we draw one, what is the probability that we draw 4 green balls? Save your answer in the variable `q1.c`
- Question 1.d - If we draw 4 balls at random, not replacing them when we draw, what is the probability that we draw 4 green balls? Save your answer in the variable `q1.d`
- Question 1.e - In how many unique orders can we draw 1 blue, 1 green, and 2 red balls? Save your answer in the variable `q1.e`
- Question 1.f - What is the probability of drawing (with replacement) 2 red balls, followed by 1 blue ball, followed by 1 green ball. Save your answer in the variable `q1.f`
- Question 1.g - What is the probability of drawing (without replacement) 2 red balls, 1 blue ball, and 1 green ball in any order? Save your answer in the variable `q1.g`

- Question 1.h (Answer hidden, consider carefully) - If we add a single yellow ball to the urn, what is the probability of drawing (without replacement) 2 red balls, 1 blue ball, and 1 yellow ball in any order? Save your answer in the variable `q1.h`
- Question 1.i (Answer hidden, consider carefully) - If we add a single yellow ball to the urn, what is the probability of drawing (without replacement) 2 red balls, 1 blue ball, and 1 other colored ball (ie not red or blue) in any order? (Note: One ball total, not in addition to any added in previous questions) Save your answer in the variable `q1.i`

Question 2

Consider generating a random number continuously from the range [0,5] following a triangular distribution peaking at $x = 3$, as can be seen if running the command `showplot()` defined in the provided code. NOTE: You must NOT include a call to `showplot()` in your final submission as gradescope autograder will crash when attempting to make plots.



- Question 2.a - What is the height of the triangle at its peak, such that it is a valid probability distribution? Save your answer in the variable `height`
- Question 2.b - Create a function `area_to_the_left`, which takes a single variable ($x1$) and calculates the area to the left, returning a value between 0 and 1. The basic setup of the

function is defined in the provided code. Note that you will need to use a slightly different equation if $x1$ is less than or greater than 3.

- Question 2.c - Create a function `area_to_the_right`, which takes a single variable ($x1$) and calculates the area to the right, returning a value between 0 and 1. The skeleton of the function is defined in the provided code. You can attempt to copy the setup used in the previous function, but students may find it easier to utilize `area_to_the_left()` already defined.
- Question 2.d - Create a function `area_between`, which takes two numeric variables ($x1$ and $x2$) and calculates the area between them, returning a value between 0 and 1. You can assume that $x1 < x2$. The skeleton of the function is defined in the code, and again students may find it easiest to utilize the previously defined functions `area_to_the_left()` and/or `area_to_the_right()`.
- Question 2.e - If we generate a random number utilizing this distribution, what is the probability that we get a number less than 1 or greater than 4? Save your answer in the variable `q2.e`
- Question 2.f - If we generate a random number utilizing this distribution, what is the probability that we get a number between 2.5 and 3.75? Save your answer in the variable `q2.f`
- Question 2.g - If we generate a random number utilizing this distribution, but we discard any value greater than 4, what is the probability that we get a value greater than 1.5? Save your answer in the variable `q2.g`
- Question 2.h - If we generate two random numbers utilizing this distribution, what is the probability that one of the numbers is greater than 2.5, and one is less than 2.5? Save your answer in the variable `q2.h`
- Question 2.i (Answer hidden, consider carefully) - If we generate two random numbers utilizing this distribution, what is the probability that the first number generated is less than the second number? Save your answer in the variable `q2.i`
- Question 2.j (Answer hidden, consider carefully) - If we generate a random number X and calculate the value $Y = (X - 3)^2$, what is the probability that Y is greater than 1? Save your answer in the variable `q2.j`

Question 3

A researcher at a streaming service is assessing user preferences and finds the following facts:

- 63% of users liked action movies.
- 78% of users liked comedy movies.
- 36% of users liked horror movies.
- 42% of users liked romance movies.
- 26% of users liked comedy and romance movies.
- 48% of users that enjoyed action movies also enjoyed horror movies.

Answer each of the following:

- Question 3.a - What proportion of users that enjoyed romance movies also enjoy comedy movies? Save your answer in the variable `q3.a`
- Question 3.b - What proportion of users enjoyed action and horror movies? Save your answer in the variable `q3.b`
- Question 3.c - If a user enjoys horror movies, what is the probability that they enjoy action movies? Save your answer in the variable `q3.c`
- Question 3.d - If liking action and comedy movies was independent, what would be the probability of a user liking both action and comedy movies? Save your answer in the variable `q3.d`
- Question 3.e - TRUE or FALSE, If 40% of users like action and comedy movies, users that likes action movies like comedy movies at a higher probability than users in general. Save your answer in the variable `q3.e`