Group 4: Opinions Advertising

Alannah Hounat, Aysu Ismayilova, Muyang Liu, Ata Tuna, Will Boyd

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The purpose of this report is to explain the underlying mechanisms behind how opinions of a group of people evolve, how they can be influenced by "advertisers", how the results are affected if people only know others' opinions through polling, and to recommend strategies for advertising.

We have constructed an Agent Based Bounded Confidence Model. We implemented this model numerically, where we introduced advertisers, polling, and random noise; experimented with deterministic and random initializations, different ranges of influence, and explored various advertiser strategies.

Results of our investigation suggest that the collective behaviour of people heavily depend on the number of people in the group one's interested in, initial distribution of opinions in this group, whether people influence each other via polling or not, presence and behaviour of advertisers, and the range of influence of the given people in the model.

We've concluded that errors due to polling are significant and hinder a realistic model, a larger range of influence per person and noise result in more consensus, uniformly random initialization is favourable for small groups and tend towards the behaviour that of linearly spaced initialized groups, and non-constant advertiser opinion tend to gather a larger following.

We recommend Mumbles Marketing to have a non-constant yet not too volatile opinion stance on the product they intend to market, and invest in gathering more information regarding the initial state of the opinions of the target audience.

1 The Model

Bounded Confidence Model: People tend to form opinions more heavily influenced by other people with similar opinions, ie. Biased Assimilation [16]. Motivated by this, we will follow an agent-based modelling approach with bounded confidence. The following describes our simple initial model [12].

- **Environment:** We have an opinion space regarding a subject (e.g. a product), which is simply a number in the set $[0, 1]$. 0 and 1 represent opposite extreme opinions.
- **Agents:** We have n number of people who have an opinion on the subject matter. We label the ith agent's opinion at some discrete time step t by $x_i(t) \in [0, 1]$. People form their opinions based on inter-group communication [7]. To simulate this exchange of ideas, these agents interact with each other as per some predefined rule and change their position in the opinion space. The following algorithm describes that.
- **The Algorithm:**
  1. **Determine** average opinion around i at time t:
     We first determine the agents who are R-close to i at time t, that is:
     $$I(i, x(t)) = \{1 \leq j \leq n : |x_j(t) - x_i(t)| \leq R\}. \tag{1}$$
     Compute the average opinion around i based on the average opinion of agents who are R-close to i:
     $$A_i = |I(i, x(t))|^{-1} \times \sum_{j \in I(i,x(t))} x_j(t) \quad \text{for} \quad t \in T. \tag{2}$$
  2. **Update** the opinion of agent i at time t+1 as:
     $$x_i(t+1) = A_i. \tag{3}$$
  3. **Repeat** for each $i \in n$.

Constant Signal Theorem: Concerning the behaviour of agents as described by the BC Model above, Constant Signal Theorem states that all the agents will converge to an opinion, which agrees with our numerical simulations. We will not prove this theorem, however, the proof and more details can be found here [12].

2 Numerical Implementation

We base our numerical model on the BC algorithm described in section 1.1. The model initialises N_x floating point decimals distributed between 0 and 1. At each time step, for each agent x, the opinion value of that agent is updated to be the mean of those agent's opinions that are R-close. Floating point errors are unfortunately unavoidable[12]. We find that, unsurprisingly, the value of this R has a pronounced effect on the behaviour we see. We discuss these findings later. Due to the nature of the model, we expect to see convergence to tight-knit groups, or *clusters*. Referring to sociology literature on opinion evolution [19], [6], a reasonable estimation of the timestep units is weeks.

Agent Initialisation: We can initialise our agents in multiple different ways and see significantly different behaviour. The model can initialise agents in a linearly spaced distribution; a random uniform distribution; a random normal distribution; or a bimodal random normal distribution (i.e. 2 normal distributions of different mean). See figure 1. As would be expected, the more equally spaced the initialisation the slower convergence to clusters.

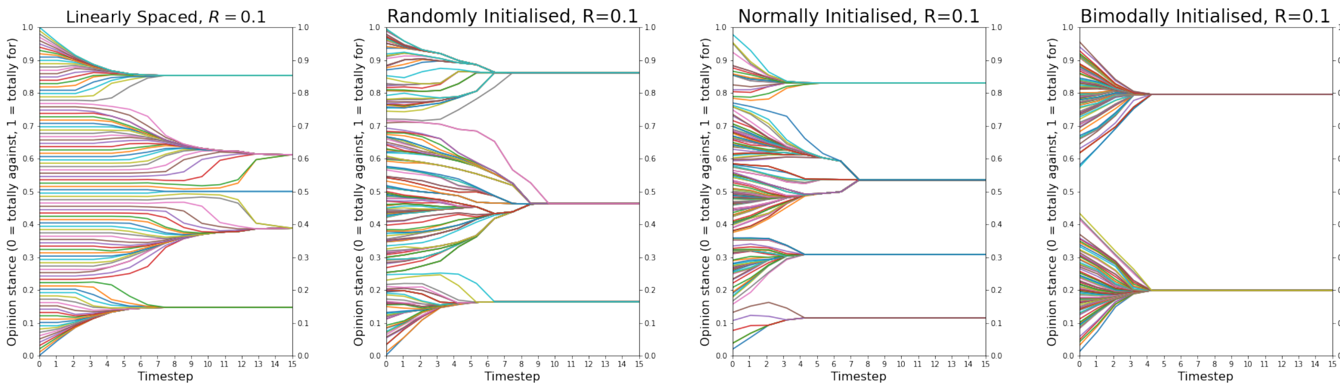

Figure 1: *Different agent initialisation distributions. From left to right: linearly spaced; random uniform; random normal; bimodal random normal.*

We suggest that both normal and bimodal distributions can be accurate representations of societal opinion arrangement - see this 1928 paper discussing political opinion distributions of legislative bodies [20]. We found however that these distributions yield very predictable and thus less interesting behaviour - with the largest clusters forming at or around the mean(s). Based on the results of our model (see figure 2) we conjecture that, for large enough N_x and fixed R, the behaviour of a system with randomly uniformly distributed agents converges towards that of a system with linearly spaced agents.

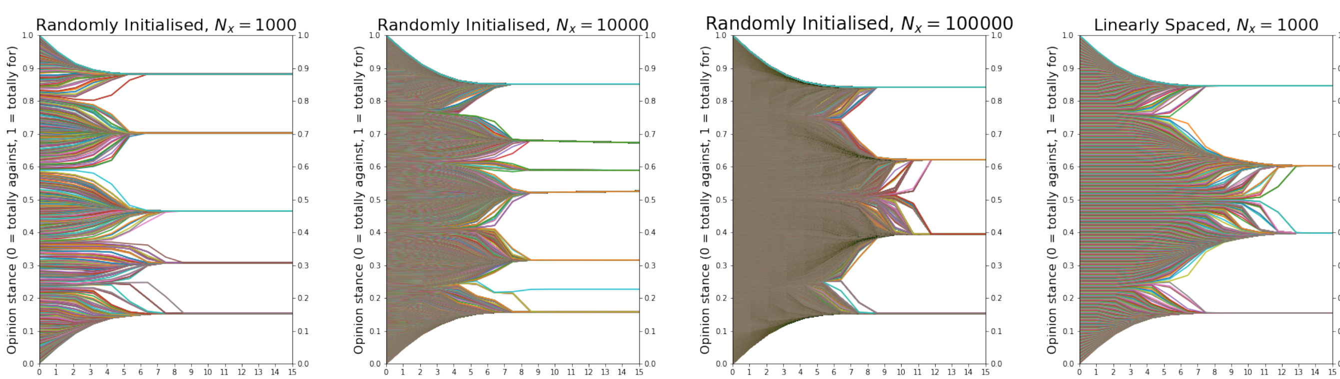

Figure 2: *Convergence of random initialisation behaviour to that of linearly spaced as* N_x *increases.*

Thus we conclude, with reasonable confidence, that the macro behaviour of the opinions of a very large population (of order > 4) can be accurately simulated by our model with *linearly spaced* initialisation and a far smaller value of N_x - gaining the advantage of far superior computational efficiency and lesser floating point error. In this case individual agents in fact represent pre-formed clusters of individuals. It is obvious, however, that, for smaller populations, random initialisation results in the most true-to-life simulation, where agents are true individual opinions. To best suit the purposes of a small firm such as Mumbles Marketing, we proceed with random uniform initialisation, using $N_x = 200$ for clear visualisation of behaviour.

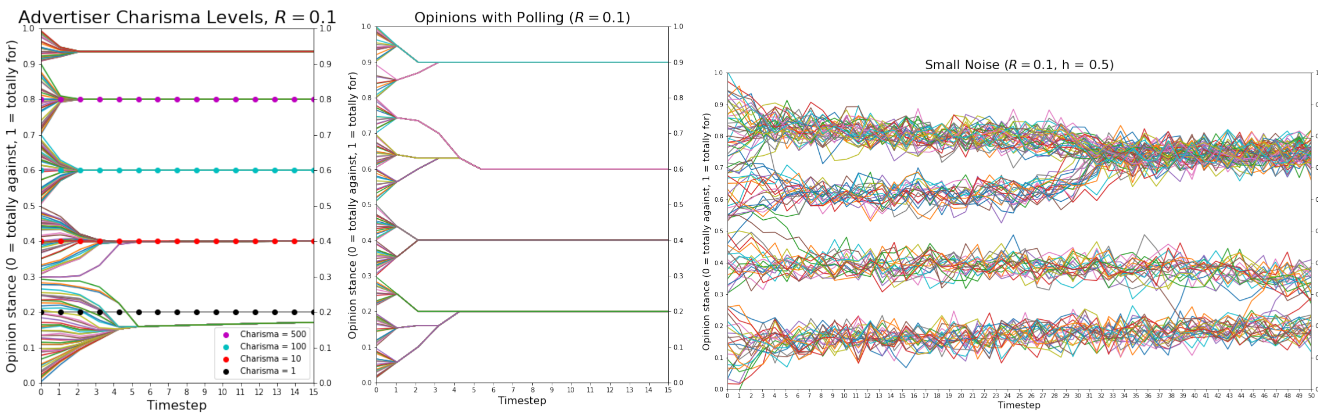

Figure 3: Left, *Advertisers and illustration of the effect of charisma on convergence.* Centre, *Opinion updates based on discrete polling.* Right, *Noise in the system.*

Advertisers: The first addition to the model was that of "advertisers", often referred to in the literature as *radicals* [12][14]. These agents are *stubborn* [14] - i.e. they have fixed opinions - and in this case aim to simulate the addition of an element such as Mumbles Marketing. Additionally, in our model each advertiser has assigned a *degree of charismaticity* [12] - this is a measure of their "influence" on agents close in opinion. It is expressed numerically as the number of times the radical agent is included in the mean if R-close - regular, non-radical agents have charisma equal to one. A reasonable charisma range is between 1 and 500, with the relative effect on convergence fast decaying with charisma increasing. As one would expect, the model shows that higher charisma results in faster convergence to the opinion of the advertiser of R-close agents, see figure 3 a). As will become clear, the effect of advertising can be highly influential on the final opinion state - given careful strategy and moderate charisma.

Noise: Realistically, no one exactly knows someone else's opinion. One highly influential paper we've found states "...the motivation for including noise comes from agents' 'free will', or uncertainty in measurement and communication." [11]. One way to simulate this phenomenon is to introduce an independent Wiener Processes to each agent's trajectory at each time step. Hence at each time step, we update agent i's opinion by $x_i(t+1) = A_i + hW^{(i)}$ instead of (3). Just like Ben-Naim [3], for high h, randomness dominates and we see no clustering. Without diffusion, we see convergence into clusters as BC would require. Interestingly, for weak diffusion, clusters form and furthermore, just like in [3], the opinion landscape continually evolves as small cluster trajectories merge into larger ones - see figure 3 c).

Polling: In our literature research, we have not come across any papers which used polling in computing x_j's at (2). Hence we tried a model which updated opinions in the same fashion, but via data rounded to 1 decimal place. We observed trivial and unrealistic behaviour where agents very quickly converge towards the closest discretized (polled) opinion. We deem this to be unrealistic because of the reasons outlined in the noise section above - namely that people never exactly convey their opinion neither survey each other on a regular basis. See figure 3 b).

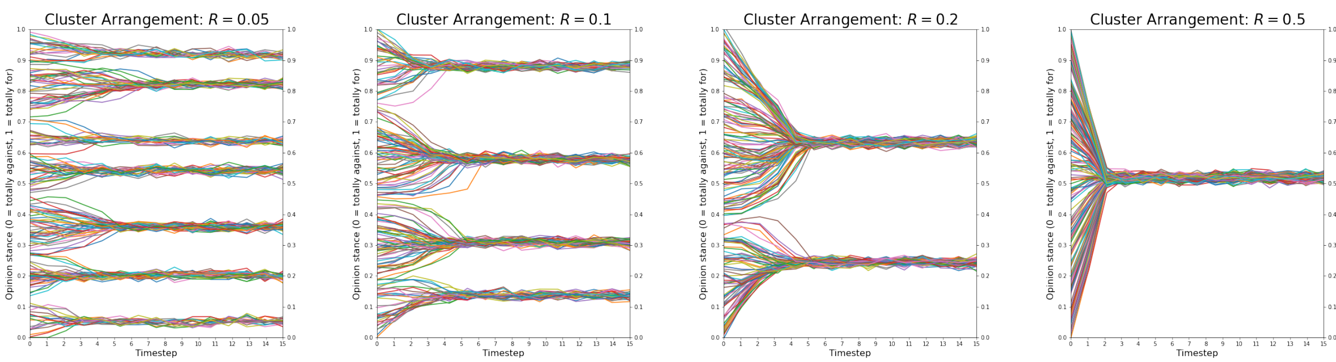

Figure 4: *Final cluster arrangement for different values of R. From left to right* $\frac{1}{2R} = 10, \quad 5, \quad 2.25, \quad 1 \quad .$

Confidence Bound: According to our model, the most influential parameter on final number of opinion clusters to be the confidence boundary radius, R 4. From the literature, for BC models, the number of clusters is approximately $\frac{1}{2R}$ [22]. Although tending to over-estimate, we found this rule also holds approximately for our model. Thus, despite the randomness present, R remains highly deterministic of cluster arrangement.

2.1 Recommended Strategy

Our model suggests that a dynamic advertiser opinion has a "sweeping" effect over the normal agents. Given the mechanics of our model, this makes sense; the wider the territory an advertiser covers, the more agent's intervals of confidence the advertiser enters, thus encouraging a larger cluster. The more agents cluster with the advertiser, the more attractive the cluster is. Hence we have a positive feedback mechanism. As seen in Figure 5, a high amplitude sinusoidal advertising projectile manages to sweep all of

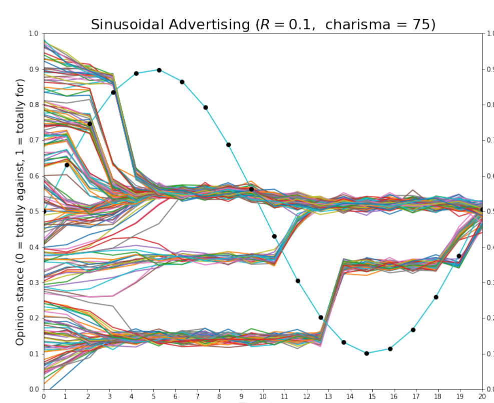

Figure 5: Recommended Strategy

the agents. One issue our model overlooks is that of "trust". Our model appears to reward bigotry over honesty. In real life such a wide range of "sweeping" may not be tolerated. However, we still see this type of "populist" behaviour in election campaigns where candidates "appear" to deviate in ideology throughout [9].

3 Results & Conclusion

A key take from our analysis is that variability in an advertiser's opinion prevails over consistency and high charisma. To elaborate, in order to affect the opinions of a population on a specific topic, it is by far the most effective strategy to run campaigns which allow entry to as wide a range of the confidence's of the populous as possible- i.e. to gain the trust of the majority - rather than focusing on being as influential as possible to a smaller group. The most important and influential parameter to consider when simulating BC opinion dynamics is the size of the confidence bound itself - this will determine the number of resulting opinion clusters in the absence of advertising. However, many other factors and model decisions are important in deciding the accuracy and suitability of the model for specific situations - including agent initialisation and additional noise.

4 Further Work

DeGroot Model and Repulsion: DeGroot model allows us to assign different weights to any agent we please rather than those within a fixed radius [8]. One extension is the that agents impact others' opinions negatively. If an agent shares a similar enough opinion to another but they do not trust one another this will 'repel' the agents away from one another [21]. This can be implemented as in [18] [5].

Accurate Parameters: In order to be able to use our model to accurately predict how populous opinions will behave, research must be undertaken to accurately estimate influential and fundamental parameters such as R - as its lacking in literature.

Initial Distribution of Opinions: In our analysis, we have opted for a uniform random initialization. However, the convergence of opinions and the ideal strategy depends on the initial distribution of these opinions. For instance, opinions of people are sometimes unimodally or bimodally distributed on non-controversial topics. Therefore extensive market research is required to come up with an accurate initial opinion distribution, and then new strategies need to be devised [2].

Multiple Advertisers: Our model included only one advertiser. In reality, one can expect numerous competitors to try to influence the market. In such a case, the optimum strategy is likely to depend on the opponents' strategy. Even with in a single advertiser model, the current sinusoidal strategy is unlikely to be "optimal" as it was merely used to demonstrate one way to gather all the agents under one opinion cluster. Further analysis needs to be conducted to come up with an optimal strategy to influence the agents in some desired way.

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