Executive Summary

Quenching is a common technique used to cool and strengthen metals. Bridles and Blades, a company that produces a variety of metal objects, are interested in quenching multiple metals efficiently in the same container. The choice of quenching media, initial and final metal temperature, and quench times, all affect the quality of the metal. We first provide a simplified model of a one-dimensional heat equation which includes no heat loss from the system. The problem is solved analytically under reasonable assumptions, then numerically using a backward Euler method. Additional features are added to the model in one and two dimensions, which is then compared to experimental data. We discuss the previously researched effects of various quenchants and ideal temperatures for steel strength. Based on the simulation results, the temperature is near-uniform throughout the system after sufficient time has passed, with the metal and fluid reaching an equilibrium temperature hotter than that of the initial fluid. We conclude that water is not a good medium for quenching, and quenching multiple objects sequentially in the same bath greatly reduces efficacy. Further work should focus on validating the model for metals and fluids of other thermal diffusivities, and adding capability in the model for liquid diffusivity to vary with temperature, and to capture the effects of quenchant evaporation.

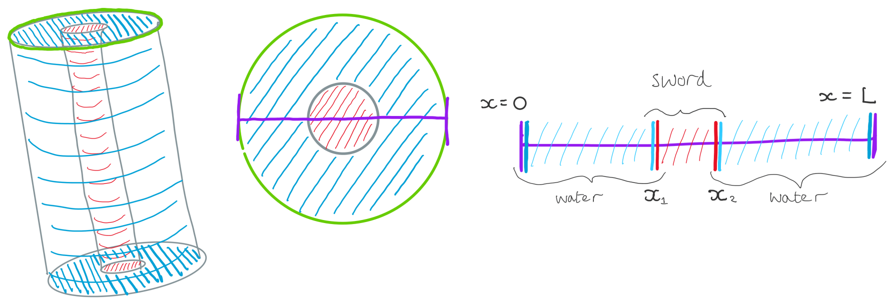

Figure 1: The 1D system we have modelled. We have assumed the quenching trough to be cylindrical - the water is shown in blue, and the hot, quenched sword in red.

1 The Model

To model quenching a single metal object, (e.g. a sword) we first assume both the quenching trough and quenched object to be cylindrical. Thus, we are able to analyse the behaviour of the whole 3D system by only modelling the (one-dimensional) diameter of the cross section, as described in Figure 1. To model this system, we use the (1D) heat equation. The application of models such as this are standard in industry [1]. Our equation is

$$\partial_t u(x,t) = \partial_x[D(x)\partial_x u(x,t)], \quad x \in (0,L), \tag{1}$$

where $0 \le x \le L$ is the position in the container, $t \ge 0$ is time, and $D(x)$ is the heat diffusion coefficient at x [2]. Our initial conditions are $u(x,0) = A\chi_{[x_1,x_2]}(x)$, where A is the initial temperature difference between the sword and the water, $\chi_{[x_1,x_2]}(x) = \begin{cases} 0, & \text{if } x \notin [x_1,x_2], \\ 1, & \text{if } x \in [x_1,x_2], \end{cases}$ and x_1 and x_2 are the edges of the sword (see Figure 1).
At first, we will assume the quenching trough is fully insulated, i.e that no heat can enter or leave. This leads to boundary conditions $u_x(0,t) = 0$ and $u_x(L,t) = 0$. We will also assume initially that the diffusivity is constant throughout the system, i.e that $D(x) = D \in \mathbb{R}$. This results in (1) simplifying to $\partial_t u(x,t) = D\partial_{xx}u(x,t)$.

2 Analytical Solution

We want to solve (1), assuming constant diffusivity, with boundary and initial conditions

$$u_x(0,t) = 0, \quad u_x(L,t) = 0, \quad u(x,0) = A\chi_{[x_1,x_2]}(x). \tag{2}$$

We proceed by separation of variables, meaning we assume that the solution can be written as $u(x,t) = X(x)T(t)$. This gives us $\partial_t u(x,t) = X(x)T'(t)$ and $\partial_{xx}u(x,t) = X''(x)T(t)$. As a result, from (2), $X(x)T'(t) = DX''(x)T(t)$. Assuming the solution is nonzero, this can be arranged as

$$D\frac{X''(x)}{X(x)} = \frac{T'(t)}{T(t)} = -\lambda,$$

where we have set this equal to a constant $-\lambda$ as we now have two equations, one constant in x and the other in t.
We now have the following two ODEs to solve,

$$T'(t) + \lambda T(t) = 0, \tag{3}$$
$$DX''(x) + \lambda X(x) = 0. \tag{4}$$

Equation (3) has solution $T(t) = ce^{-\lambda t}$, for some constant c.
To solve (4), we need to look at three cases: when $\lambda < 0$, $\lambda = 0$ and $\lambda > 0$. The case where $\lambda < 0$ is irrelevant as it results in the trivial solution. When $\lambda = 0$, we find that the solution is given by $X(x) = d_0$, for some constant d_0. Finally, for $\lambda > 0$, we let $\lambda = -k^2$, and find that the general solution is given by

$$X(x) = a\cos\left(\frac{k}{\sqrt{D}}x\right) + b\sin\left(\frac{k}{\sqrt{D}}x\right),$$

where the constants a and b are determined by the initial and boundary conditions.
Applying the boundary conditions gives

$$X'(0) = 0 \implies b = 0 \quad \therefore X(x) = a\cos\left(\frac{k}{\sqrt{D}}x\right),$$

and hence also

$$X'(L) = \frac{ak}{\sqrt{D}}\sin\left(\frac{kL}{\sqrt{D}}\right) = 0.$$

The case where $a = 0$ is irrelevant as it results in the trivial solution. We must therefore have

$$\sin\left(\frac{kL}{\sqrt{D}}\right) = 0 \implies k = \frac{n\pi\sqrt{D}}{L}, n \in \mathbb{N}_0.$$

These correspond to solutions of the PDE with the form

$$u_n(x,t) = X_n(x)T_n(t) = d_n\cos\left(\frac{n\pi}{L}x\right)e^{-\frac{n^2\pi^2 D}{L^2}t}.$$

Based on the superposition principle and using $\int_0^L \cos\left(\frac{n\pi}{L}x\right)\cos\left(\frac{m\pi}{L}x\right)\mathrm{d}x = \begin{cases} 0, & \text{if } n \neq m, \\ \frac{L}{2}, & \text{if } n = m, \end{cases}$
we can write a general solution as

$$u(x,t) = d_0 + \sum_{n=1}^{\infty} d_n\cos\left(\frac{n\pi}{L}x\right)e^{-\frac{n^2\pi^2 D}{L^2}t}, \tag{5a}$$
$$d_0 = A(x_2 - x_1), \tag{5b}$$
$$d_n = \frac{2}{L}A\int_{x_1}^{x_2}\cos\left(\frac{n\pi}{L}x\right)\mathrm{d}x, n > 0. \tag{5c}$$

3 Numerical Solution

To obtain a numerical solution, we discretize the heat equation in time and space, using centred finite difference in space and backward Euler scheme in time. We were able to verify that our numerical and analytic solutions match, (assuming constant diffusivity) with very little error for sufficiently small step sizes. However, to compare the model to experimental quench data, we enhance the model presented in the analytical solution. While the simple model assumed they are identical, metal and quenchants have very different thermal diffusivities, which greatly affects the heat flow. Swords are often made from high-carbon steel. Thus we take

$$D(x) = \begin{cases} 3.423 \times 10^{-3} m/s, & \text{if } x \notin [x_1, x_2] (\text{water}), \\ 0.378 \times 10^{-3} m/s, & \text{if } x \in [x_1, x_2] (1\% \text{ carbon}). \end{cases}$$

These 1D diffusivities are for 1D from the corresponding 2D values found at [3].

4 Results

Next, we consider quenching multiple metal objects. In 1D, according to our model, quenching 3 swords one after the other, results in the first taking 8.41s to reach 250°C, the second 22.3s, and the third never dropping below. Since the model fully conserves heat, consecutively quenching traps residual heat from previous quenches in the system. Thus, with each quench, the system tends to higher a uniform temperature. Now consider the case where multiple quenches happen by placing consecutive swords in different positions. Figure 4 shows the case where the second hot metal object is added to water that already contains one metal object. It shows that under the same time period, placing the second rod further away from the previous rod results in quicker cooling.

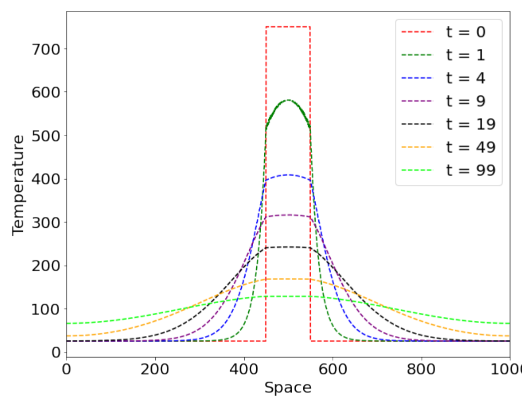

Figure 2: 1D quench simulation. x-axis (mm), y-axis (°C).

5 Real-world Data & Model Limitations

One major issue with our model is that no realistic quenching trough is perfectly insulated. However, having included flux boundary conditions, modelled based on Newton's law of heat transfer [4], we concluded that heat loss of realistic order [5] has a negligible effect on the behaviour of the overall system, more relevant we suspect is the effect of energy loss from the system via water evaporation, we recommend further research in this area.

There are several other properties of water that render our model inaccurate, one of the more notable being the Leidenfrost effect. After a very hot metal is introduced to water, it vaporizes the surrounding liquid, forming an insulating layer of gas. This means that the actual rate of heat transfer varies significantly with temperature [3][6]. Different ionic solutions can be used as quenchants, with the type and concentration of solutes used affecting the diffusivity [6] and temperature at which the Leidenfrost effect takes place [3][7][8][9].

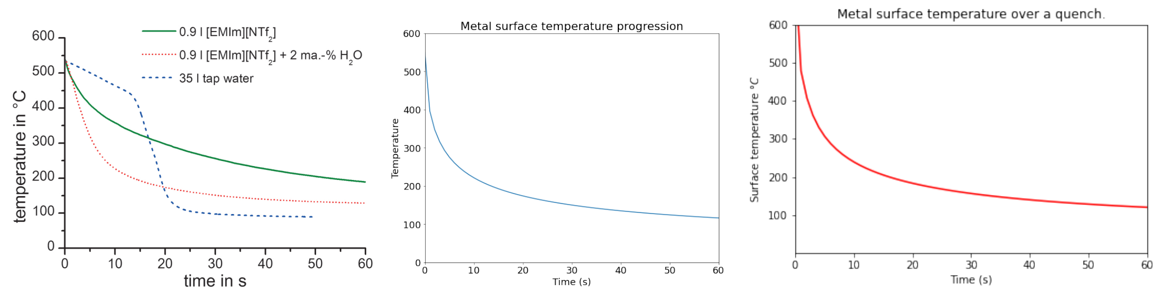

Figure 3: Left: Aluminum surface temperature over time in various liquids. [10]
Center: Simulated metal surface temperature over time (no flux).
Right: Simulated metal surface temperature over time (flux).

In the experiment [9], they measured the surface temperature over time of the aluminum that was quenched in several media. After our parameters are tweaked to reflect the diffusivity of aluminium and the ionic liquid, our model is able to closely approximate their results. Compare our model in Figure 3 (center, right) to the red experimental curve in Figure 3 (left). The steepness of the curve will depend on the exact diffusivity of the liquid used.

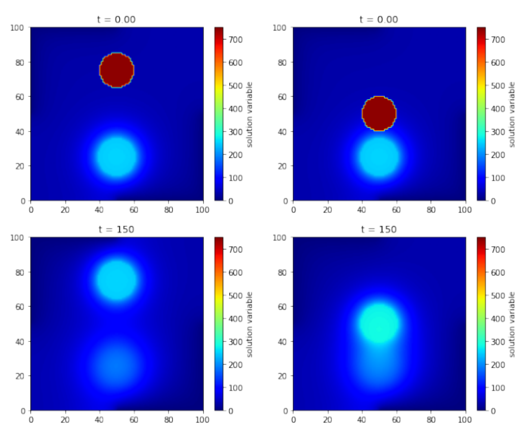

Figure 4: Left: Distance = 50 units, temp. after 150 steps is 251.97. Right: Distance = 15 units, centre temp. is 280.66.

There are several critical temperatures at which the structure of the metal can change and affect its quality. At very high temperatures, steel attains the structure of austenite, and undergoes a phase transition as it cools into martensite [11]. For low-alloy steel, the temperature where all iron is transformed into austenite, $Ac3$, is about 700°C. Transformation into martensite begins at about 350°C (Ms) and is finalized by about 250°C (Mf) [12]. The speed at which these temperatures are attained has a great influence on the strength, malleability, and hardness of the metal. Removing the metal before it has reached Mf compromises the hardness of the metal, since the martensite transformation is incomplete. The quicker the metal drops to the required temperature, the greater the proportion of martensite and the stronger the resulting metal [13].

Based on our 1D simulation, dropping a metal object with an initial temperature of 750°C into 25C water results in the the metal temperature decreasing faster than the rate at which the water temperature increases.The high diffusivity of the metal means that it is very efficient at sharing heat across its width, and so it approaches a uniform temperature faster than the water. It is limited in its cooling by the water. The water close to the sword heats up very quickly, but is slow to transfer that heat to its edges. After sufficient time has passed, the temperature is near-uniform throughout the system. See Figure 2 for a one-dimensional quench with realistic diffusivities [2] and initial temperatures of 750°C and 25°C.

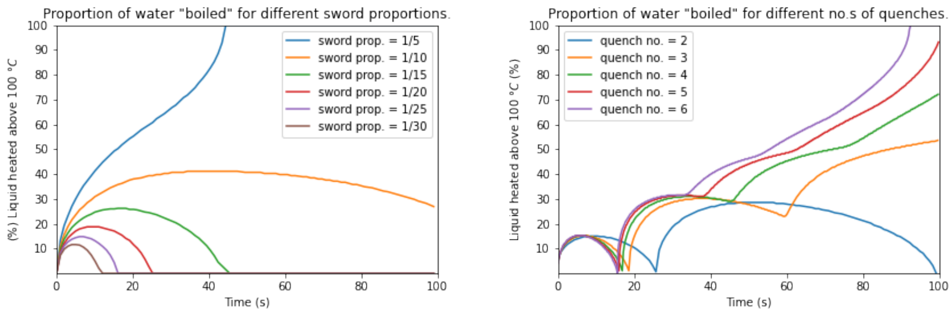

Figure 5: Left: The proportion of the liquid above 100°C plotted against time, for different proportions of sword and liquid in the quenching trough. Right: The proportion of the liquid above 100°C plotted against time, for different numbers of consecutive quenches over the 100 seconds.

The relative size of the metal to the basin also has a large effect on the quench. Larger objects will introduce more heat into the system, and as seen in Figure 5 will bring more of the fluid (in this case, water) to boiling point. To increase efficacy, the quenchant should stay in the liquid state, as the phase change to gas prevents the liquid from increasing in temperature further. So, Figure 5 shows that it is beneficial to minimize the volume of the metal relative to the quenching medium, as well as limit the number of successive quenches.

6 Conclusion & Recommendations

We have found that when a hot metal object is placed into a fluid, the rate of cooling is rapid immediately after the quench and tapers off as the temperature across this system tends toward a uniform equilibrium. Between quenches, the fluid will retain much of the heat from the previous metal, and the effectiveness on subsequent quenches will be diminished. This effect is even more pronounced with an insulated container like those used by Bridles and Blades. For maximum quenching effectiveness, greater martensite proportions thus and harder, higher quality products, we recommend using different troughs for consecutive quenches, or allowing sufficient time for the fluid to cool in between. The literature review and simulations have found that the most influential factor in the speed and quality of the quench is the choice of fluid. Due to its relatively low Leidenfrost temperature and strong variations in thermal conductivity with temperature, water is not an ideal choice, despite its affordability and accessibility.

We recommend funding further work to model the effects of state changes (evaporation) on the heat of different quenchants, for we believe that this is the key to far more accurately predicting what happens during the quenching process. Future work should also identify fluids with desirable properties such as high boiling point, high thermal conductivity without significant variation with temperature, and add functionality to the model to account for the real-life variation in diffusivity with temperature.

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