

Solutions to the Problems in Lab 1

Problem 1. Let p, q, r be propositions as follows.

p : Kids have an outdoor recess.

q : The temperature is over 40 degrees.

r : The ground is wet.

Translate the following compound propositions to English.

a) $(q \wedge \neg r) \rightarrow p$

If the temperature is over 40 degrees and the ground is not wet, then the kids have an outdoor recess.

Note: There are alternative ways to translate. We discussed a bit during the lecture that $a \rightarrow b$ can be read in many other ways such as “ a implies b ”, “ b when a ”, “ b whenever a ”, “ a is sufficient for b ”, ... We can write 1 or 2 alternatives. The one in b will be easier for this purpose since its shorter.

b) $p \rightarrow \neg r$

If the kids have an outdoor recess, then the ground is not wet.

Some alternatives: *The ground is not wet when the kids have an outdoor recess.*

The kids having an outdoor recess implies that the ground is not wet.

...

Now work in reverse order and write the following sentences in logical (symbolic) notation.

c) The ground is wet if and only if the temperature is not over 40 degrees.

$$r \leftrightarrow \neg q$$

d) If the temperature is over 40 degrees and the kids have an outdoor recess, then the ground is not wet.

$$(q \wedge p) \rightarrow \neg r$$

Problem 2. Construct truth tables for the following compound propositions.

$$p \vee (q \wedge r)$$

$$(p \vee q) \wedge r$$

Are these two compound propositions equivalent?

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	F	T	F
T	F	T	F	T	T	F	T	T	T
T	F	F	F	T	T	F	F	T	F
F	T	T	T	T	F	T	T	T	T
F	T	F	F	F	F	T	F	T	F
F	F	T	F	F	F	F	T	F	F
F	F	F	F	F	F	F	F	F	F

We see that the two compound propositions do not have the same truth values in every row, thus, they are not logically equivalent.

Problem 3. Solve the logic puzzle below using logical formalism.

On an island, there are two kinds of robots. One kind, type H (H for honest), is programmed to always tell the truth. The other kind, type L (L for liar), always lies. You encounter two robots R1 and R2. What are the types of R1 and R2 if R1 says "R2 is type H" and R2 says "The two of us are opposite types"?

- *Let p be "R1 is type H" and let's assume p is true.*
- *If p is true, then what R1 says ("R2 is type H") has to be true, and let's call this proposition q .*
- *If q is true, then what R2 says ("The two of us are opposite types") has to be true.*
- *But we already have p and q above to be true saying that both R1 and R2 are type H. We have a contradiction! So, our initial assumption that p is true must be wrong.*
- *Now, let's start with opposite, that p is false (or $\neg p$ is true), meaning "R1 is type L".*
- *If "R1 is type L", then what it says ("R2 is type H") must be a lie and therefore R2 must be type L too.*
- *If R2 is also type L, what it says ("The two of us are opposite types") has to be a lie, meaning that the two robots has to be the same type. And this now confirms what we inferred with both being type L. No contradictions, everything checks out, so, both R1 and R2 are type L.*