COT 3100C: INTRODUCTION TO DISCRETE STRUCTURES

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Basic Structures: Sets and Functions

Part-1

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Outline

Sets

- The Language of Sets.
- Set Operations.
- Set Identities.

Functions

- Types of Functions.
- Operations on Functions.
- Computability.

*Mc Graw Hill

Section

Sets

Section Summary 1

Definition of sets.

Describing Sets.

- Roster Method.
- Set-Builder Notation.

Some Important Sets in Mathematics.

Empty Set and Universal Set.

Subsets and Set Equality.

Cardinality of Sets.

Tuples.

Cartesian Product.

Introduction[1]

Sets are one of the basic building blocks for the types of objects considered in discrete mathematics.

- Important for counting.
- Programming languages have set operations.

```
myset = {"apple", "banana", "cherry", "apple"}

print(myset)                # OUTPUT:
{'banana', 'cherry', 'apple'}
```

Set theory is an important branch of mathematics.

Definition: A *set* is an unordered collection of objects.

The objects in a set are called the *elements*, or *members* of the set. A set is said to *contain* its elements.

The notation $a \in A$ denotes that a is an element of the set A.

If a is not a member of A, write $a \notin A$.

Describing a Set: Roster Method

$$S = \{a,b,c,d\}$$

Order not important.

$$S = \{a,b,c,d\} = \{b,c,a,d\}$$

Each distinct object is either a member or not; listing more than once does not change the set.

$$S = \{a,b,c,d\} = \{a,b,c,b,c,d\}$$

Ellipses (…) are used to describe a set without listing all of the members when the pattern is clear.

$$S = \{1, 2, 3, ..., 99\}$$

Roster Method

Set of all vowels in the English alphabet:

$$V = \{a,e,i,o,u\}$$

Set of all odd positive integers less than 10:

$$O = \{1,3,5,7,9\}$$

Set of all integers less than 0:

$$S = \{\ldots, -3,-2,-1\}$$

Set-Builder Notation

Specify the property or properties that all members must satisfy:

$$S = \{x \mid x \text{ is a positive integer less than } 100\}$$
$$O = \{x \mid x \text{ is an odd positive integer less than } 10\}$$
$$O = \{x \in \mathbf{Z}^{+} \mid x \text{ is odd and } x < 10\}$$

A predicate may be used:

$$S = \{x \mid P(x)\}$$

Example: $S = \{x \mid \text{Prime}(x)\}$

Positive rational numbers:

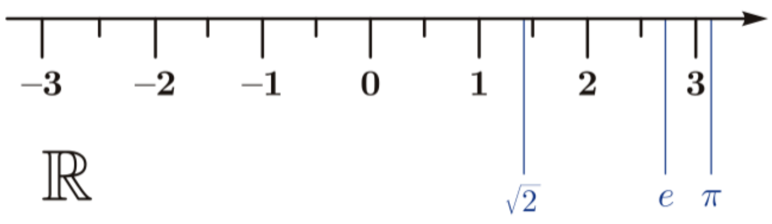
$$\mathbf{Q}^{+} = \{x \in \mathbf{R} \mid x = p/q, \text{ for some positive integers } p,q\}$$

Some Important Sets

N = *natural numbers* = $\{0,1,2,3\ldots.\}$

Z = *integers* = $\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}$

Z$^+$ = *positive integers* = $\{1,2,3,\ldots..\}$



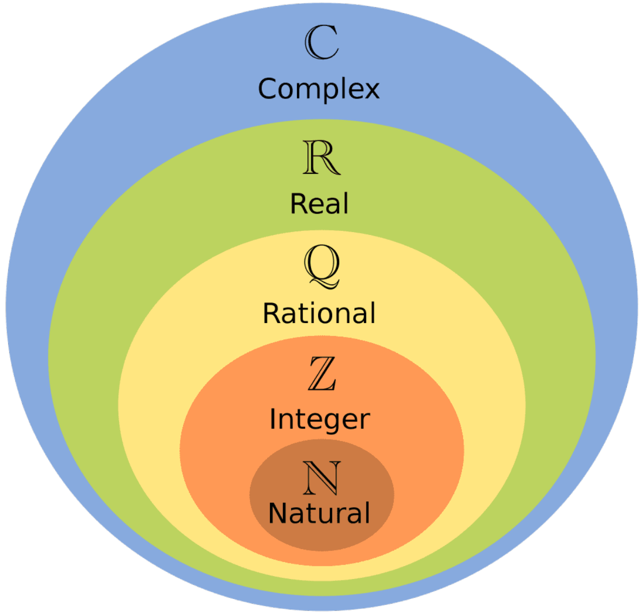
R = set of *real numbers.*

R$^+$ = set of *positive real numbers.*

C = set of *complex numbers*.

Q = set of rational numbers.

Q = {p/q | p ∈ Z, q ∈ Z, and q != 0}



Interval Notation

$$[a,b] = \{x \mid a \le x \le b\}$$
$$[a,b) = \{x \mid a \le x < b\}$$
$$(a,b] = \{x \mid a < x \le b\}$$
$$(a,b) = \{x \mid a < x < b\}$$

closed interval $[\mathrm{a,b}]$

open interval $(\mathrm{a,b})$

Example: Negative integers in (-5,-2]?
-4, -3, -2.

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- Therefore, if A and B are sets, then A and B are equal if and only if

$$\forall x\left(x \in A \leftrightarrow x \in B\right)$$

  - We write $A = B$ if A and B are equal sets.

$$\{1,3,5\} = \{3,\ 5,\ 1\}$$

$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Universal Set and Empty Set

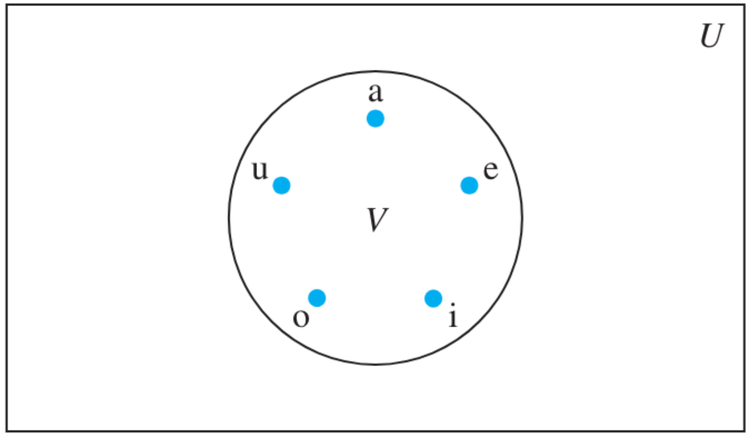
The ***universal set*** U is the set containing everything currently under consideration.

- Sometimes implicit.
- Sometimes explicitly stated.
- Contents depend on the context.

The ***empty set*** is the set with no elements.

Symbolized ∅, but {} also used.

Venn Diagram for the Set of Vowels.



John Venn (1834-1923)
Cambridge, UK

Some things to remember

Sets can be elements of sets.

$$\{\{1,2,3\},a,\ \{b,c\}\}$$

$$\{\mathbf{N},\mathbf{Z},\mathbf{Q},\mathbf{R}\}$$

The empty set is different from a set containing the empty set.

$$\emptyset \neq \{\emptyset\}$$

Russell’s Paradox

Let S be the set of all sets which are not members of themselves. A paradox results from trying to answer the question “Is S a member of itself?”

Related Paradox:

- Henry is a barber who shaves all people who do not shave themselves. A paradox results from trying to answer the question “Does Henry shave himself?”

Bertrand Russell (1872-1970)
Cambridge, UK
Nobel Prize Winner

Subsets

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.
- $A \subseteq B$ holds if and only if $\forall x \left(x \in A \rightarrow x \in B \right)$ is true.

1. Because $a \in \emptyset$ is always false, $\emptyset \subseteq S$, for every set S.
2. Because $a \in S \rightarrow a \in S$, $S \subseteq S$, for every set S.

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