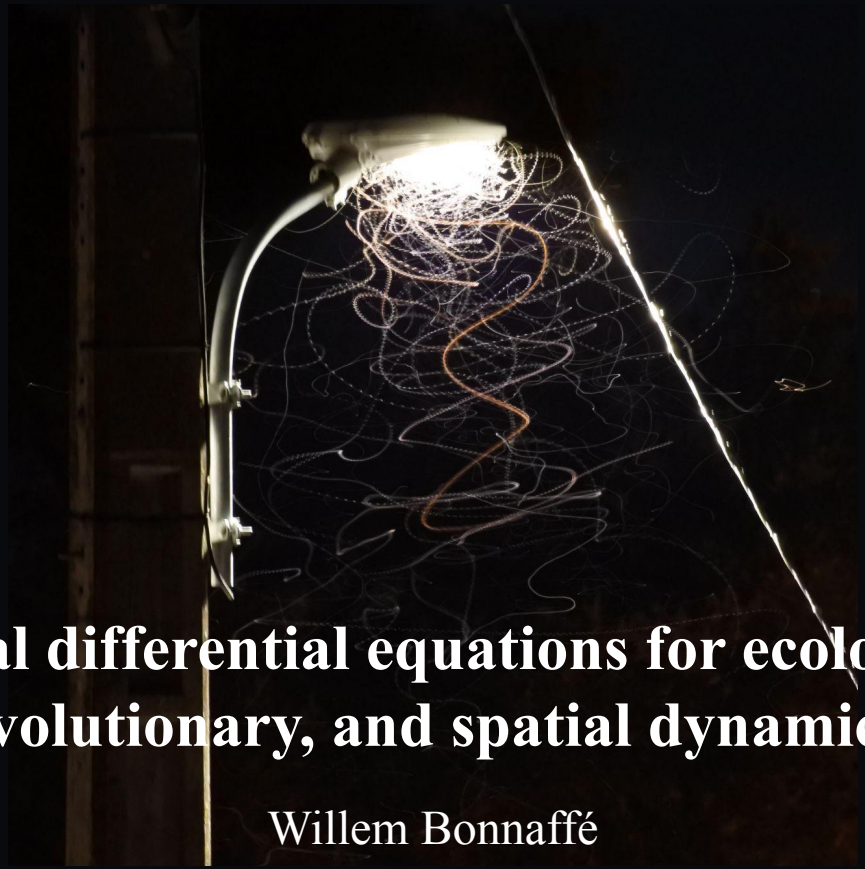
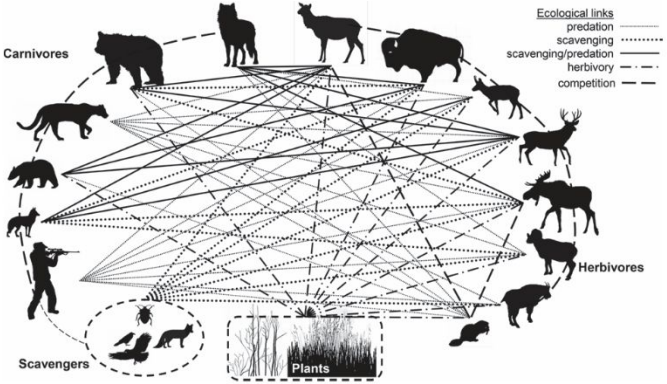


Neural differential equations for ecological, evolutionary, and spatial dynamics

Willem Bonnaffé

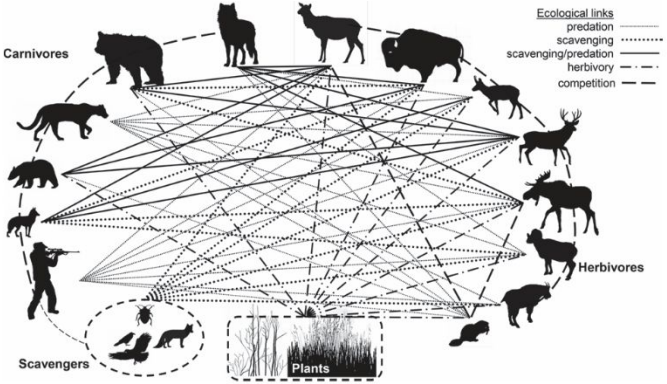
Department of Biology, University of Oxford

Ecological interactions determine the survival and reproduction of individuals



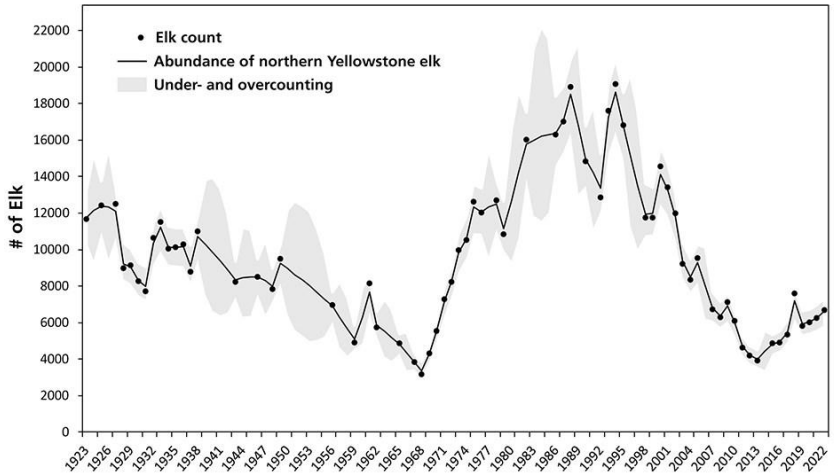
Ecological interactions determine the survival and reproduction of individuals

Smith et al. 2020



Ecological interactions determine the survival and reproduction of individuals

Survival and reproduction underpin changes in population size



Smith et al. 2020

$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t); \Omega)$$

Dynamics

$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t); \Omega)$$

States

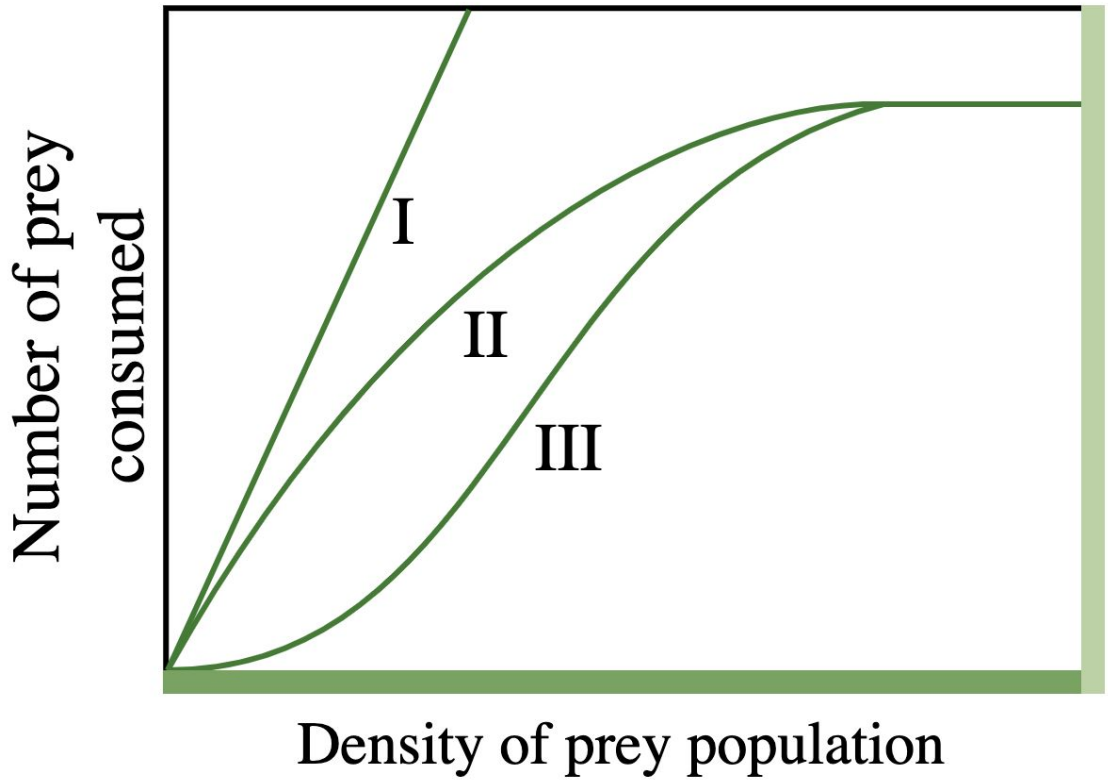
$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t); \Omega)$$

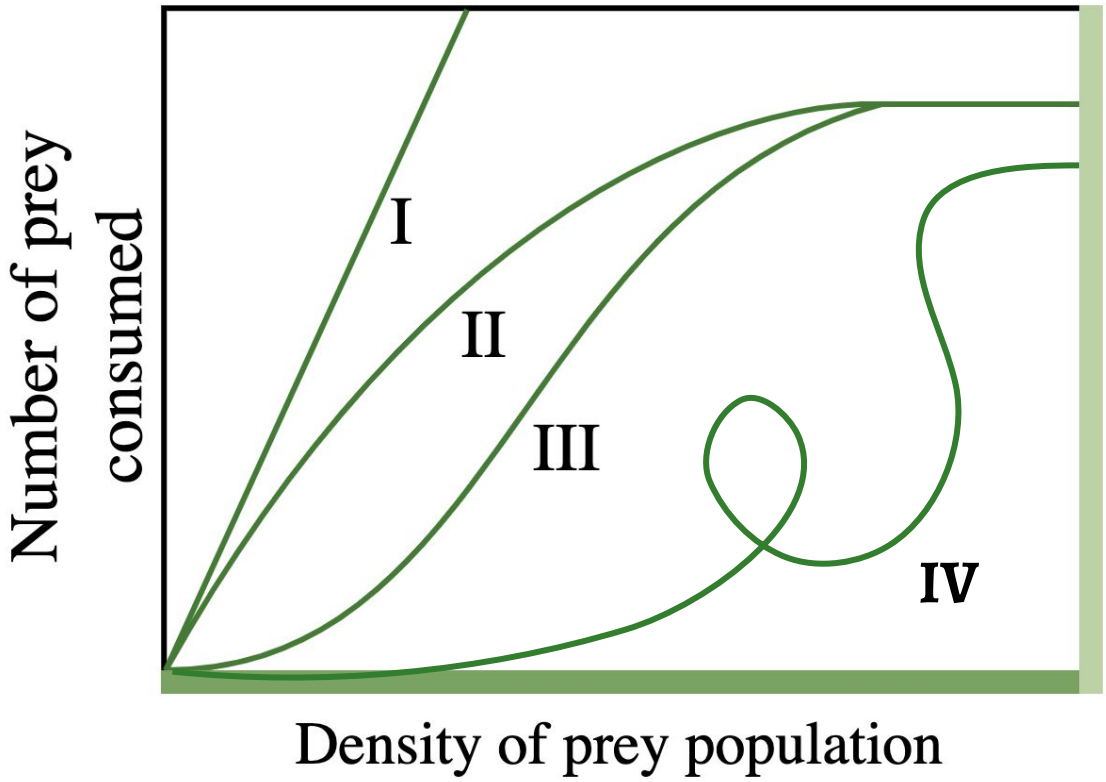
Parameters

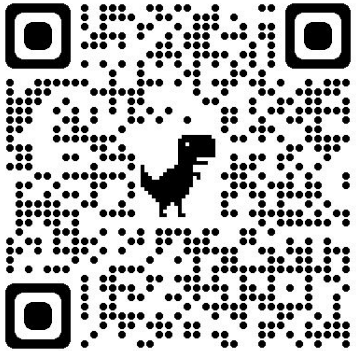
$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t); \Omega)$$

Process

$$\dot{X}(t) = f(X(t); \Omega)$$


Number of prey consumed
I
II
III
Density of prey population


Number of prey consumed
I
II
III
IV
Density of prey population



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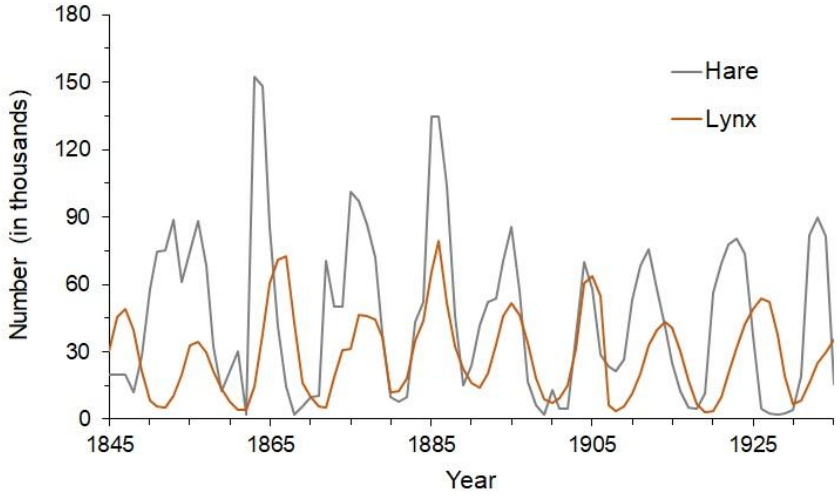
RESEARCH ARTICLE

Methods in Ecology and Evolution BRITISH ECOLOGICAL SOCIETY

Neural ordinary differential equations for ecological and evolutionary time-series analysis

Willem Bonnaffé[1,2] | Ben C. Sheldon[1] | Tim Coulson[2]

Bonnaffé, Sheldon, & Coulson 2021



Odum and Barrett 1973

Lotka-Volterra model:

$$\begin{cases} \dot{H}(t) = r_H(H(t), L(t))\, H(t) \\ \dot{L}(t) = r_L(H(t), L(t))\, L(t) \end{cases}$$

Lotka-Volterra model:

$$\begin{cases} \dot{H}(t) = (\alpha - \beta L(t))\, H(t) \\ \dot{L}(t) = (\beta H(t) - \delta)\, L(t) \end{cases}$$

Lotka-Volterra model:

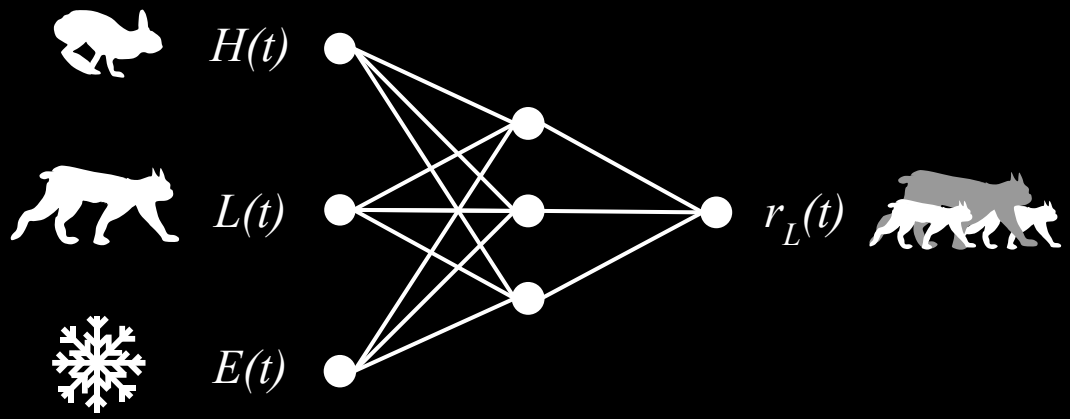
$$\begin{cases} \dot{H}(t) = (\alpha - \beta L(t)) H(t) \\ \dot{L}(t) = (\beta H(t) - \delta) L(t) \end{cases}$$

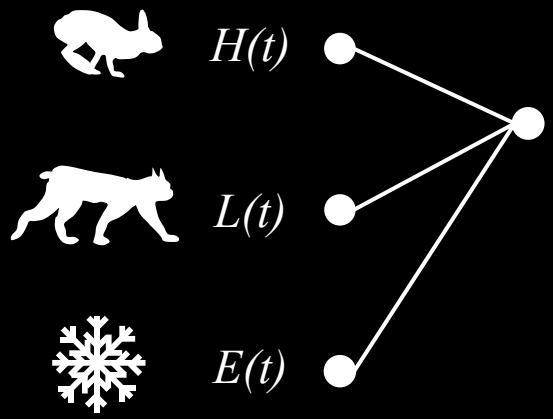
Di-trophic NODE model:

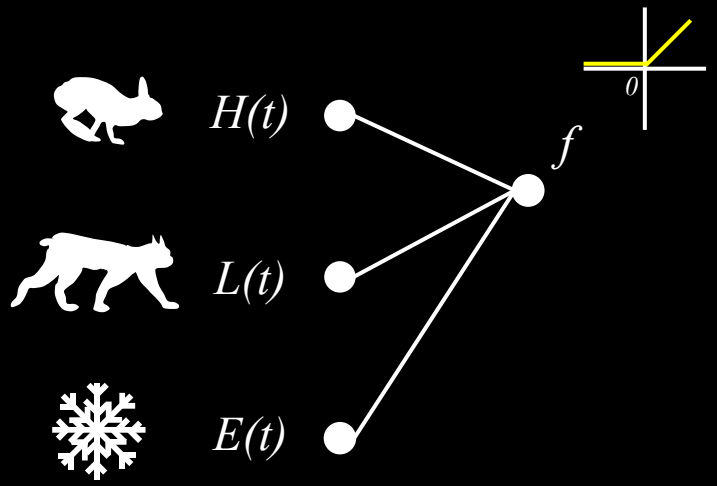
$$\begin{cases} \dot{H}(t) = r_H\big(H(t), L(t)\big)\, H(t) \\ \dot{L}(t) = r_L\big(H(t), L(t)\big)\, L(t) \end{cases}$$

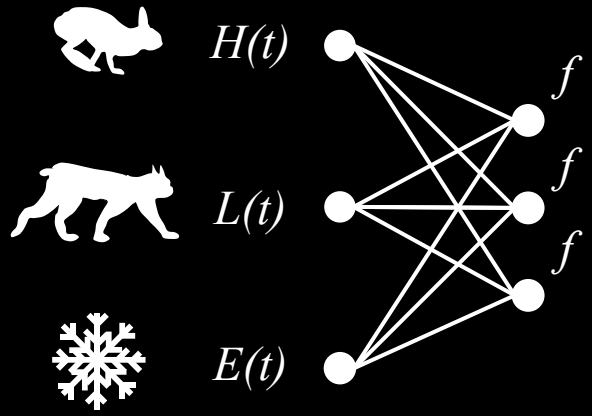
Di-trophic NODE model:

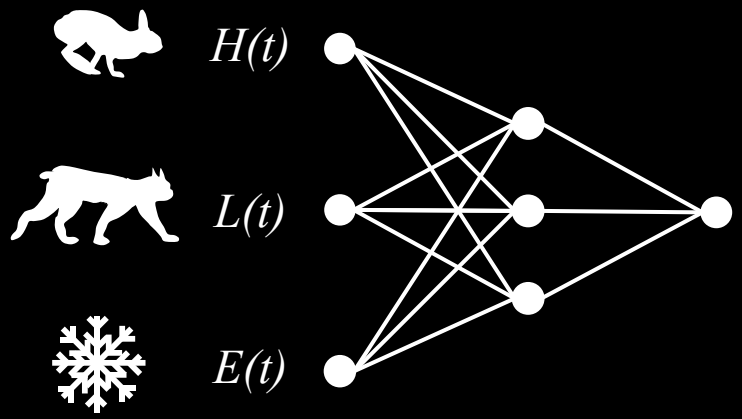
$$\begin{cases} \dot{H}(t) = r_H(H(t), L(t))\, H(t) \\ \dot{L}(t) = r_L(H(t), L(t))\, L(t) \end{cases}$$

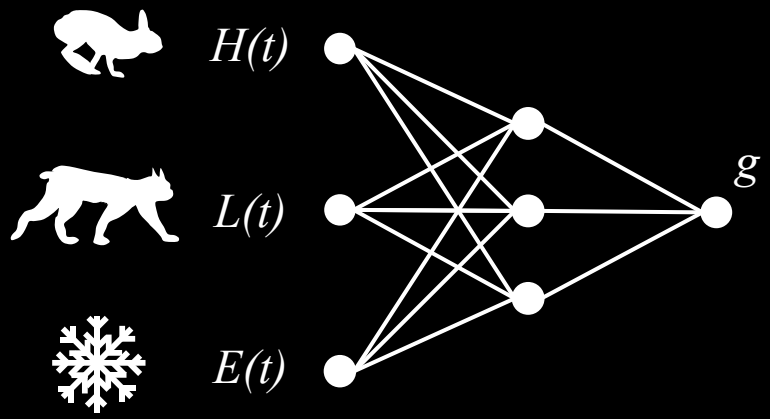

H(t)
L(t)
E(t)
$r_L(t)$

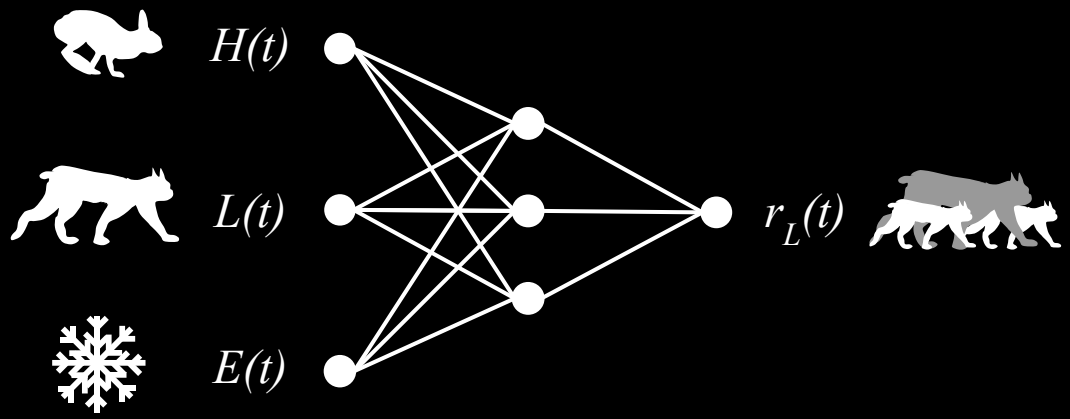

H(t)
L(t)
E(t)

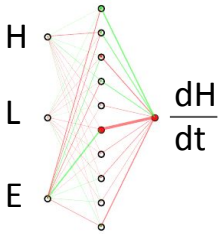

H(t)
L(t)
E(t)
f
0

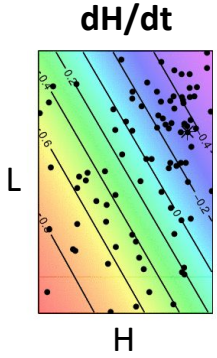

H(t)
L(t)
E(t)
f
f
f

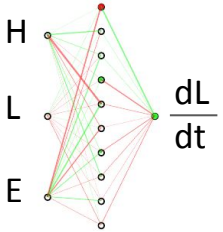

H(t)
L(t)
E(t)

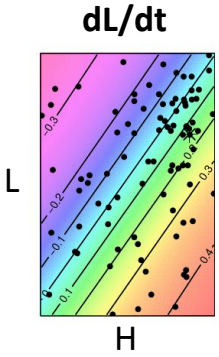

H(t)
L(t)
E(t)
g


H(t)
L(t)
E(t)
$r_L(t)$

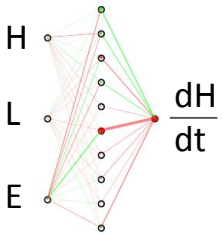


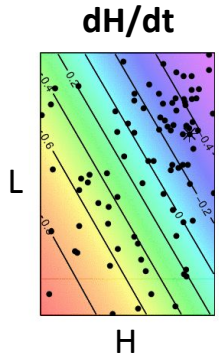


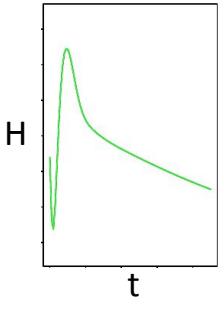


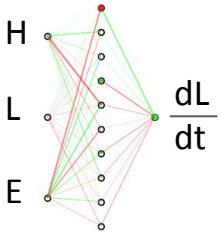


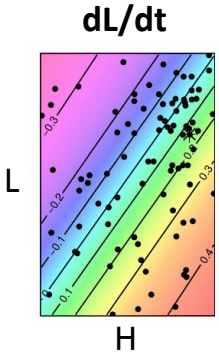
Bonnaffé, Sheldon, & Coulson 2021

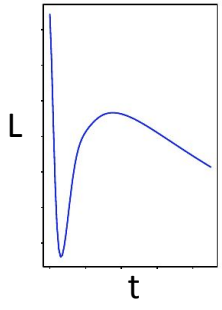












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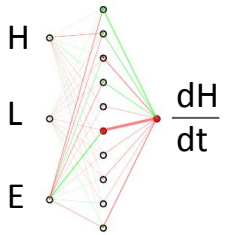
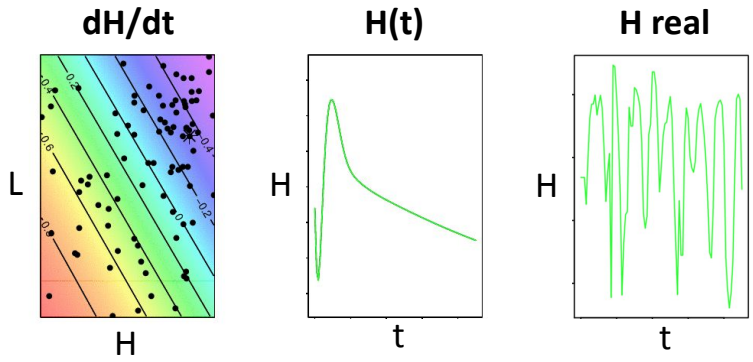
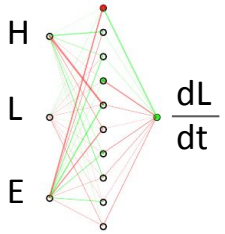
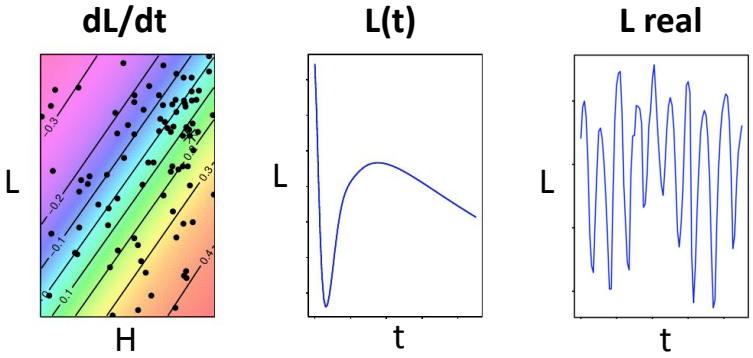
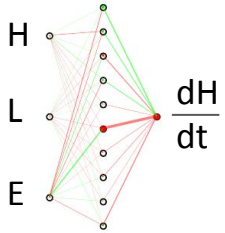


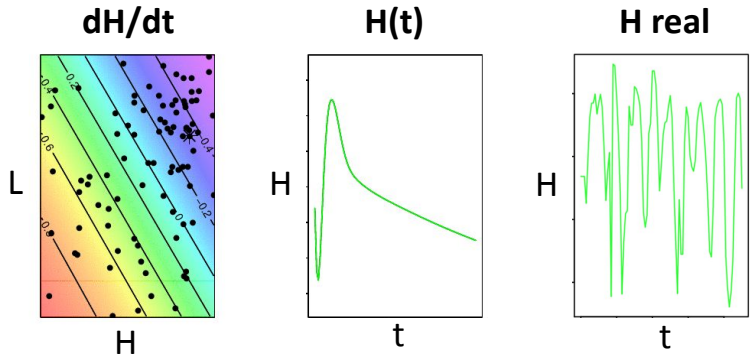


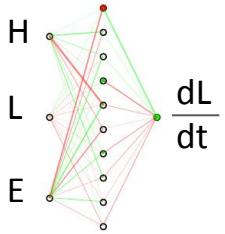


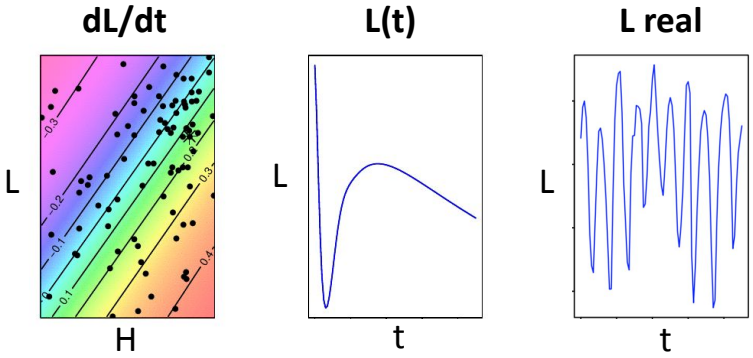


Bonnaffé, Sheldon, & Coulson 2021

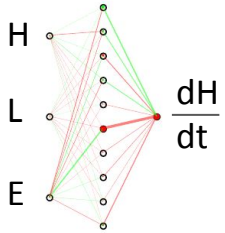


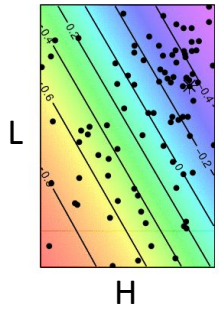


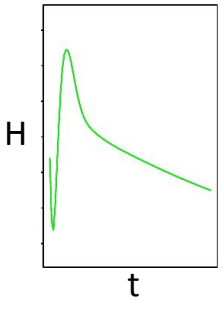


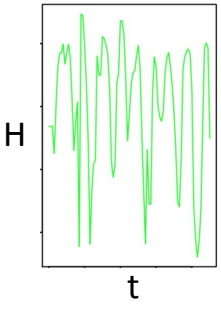


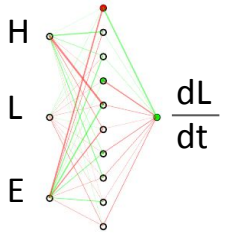
Bonnaffé, Sheldon, & Coulson 2021

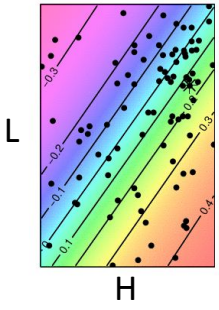


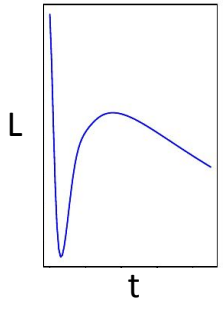


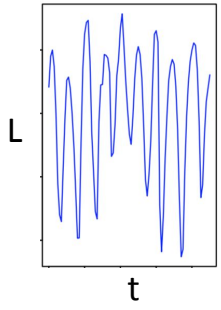












Unobserved states
H(t), L(t)

Dynamics
dH/dt, dL/dt

Effects/sensitivities
∂(dH/dt)/∂H ∂(dH/dt)/∂L
∂(dL/dt)/∂H ∂(dL/dt)/∂L

Contributions (Geber)
$C_{H \to L}$ = ∂(dL/dt)/∂H dH/dt

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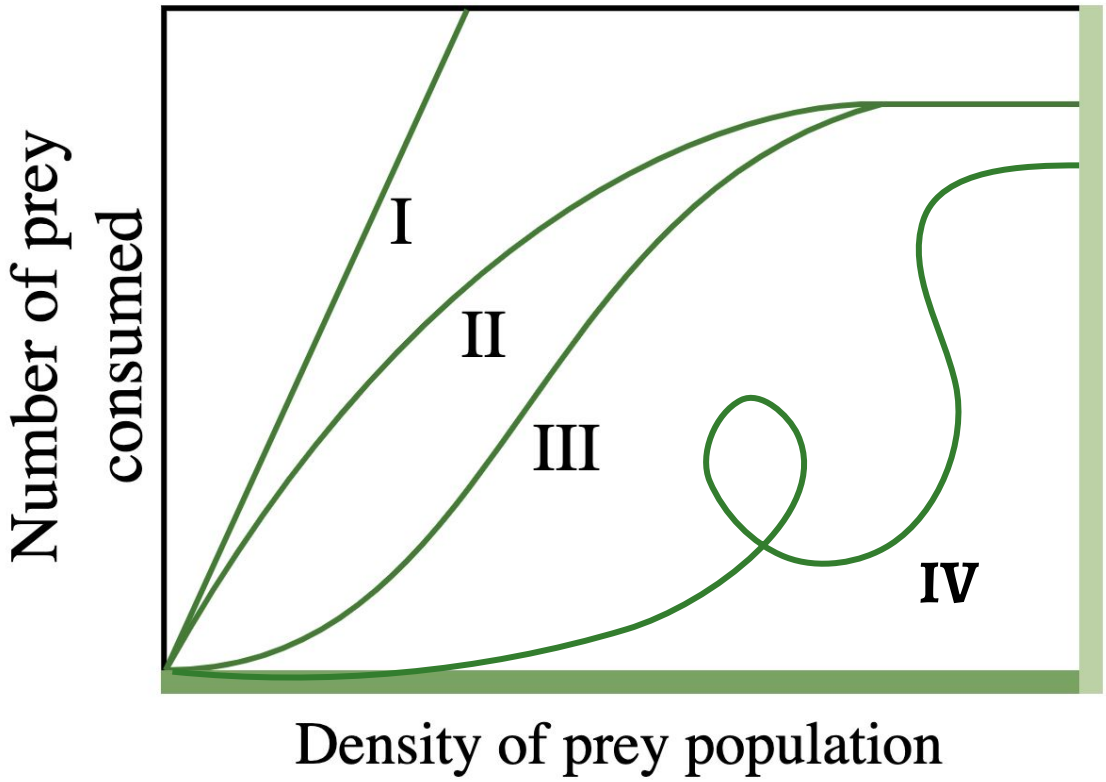
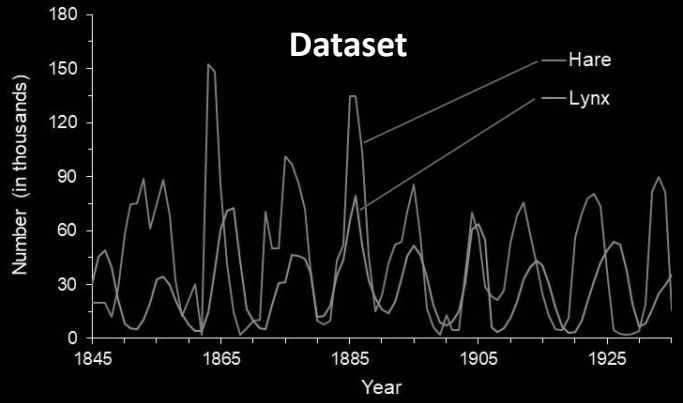

Number of prey consumed
I
II
III
IV
Density of prey population

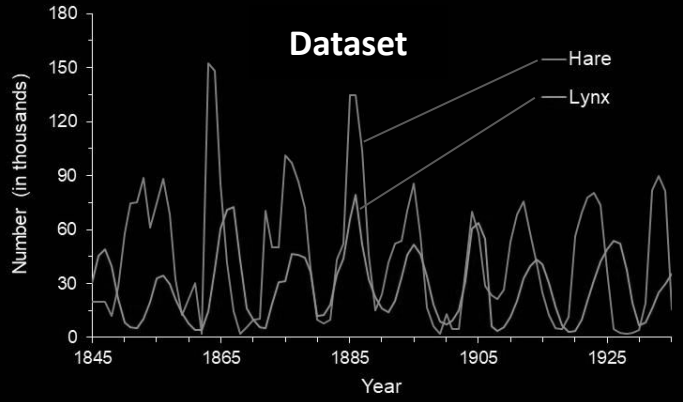
Practical Explore code
Reproduce results for hare-lynx system
Forecast & hindcast
Estimate prey-predator interactions

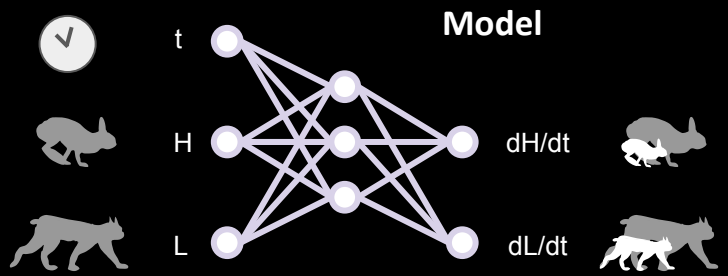
Assignment Explore dynamics of a tri-trophic system

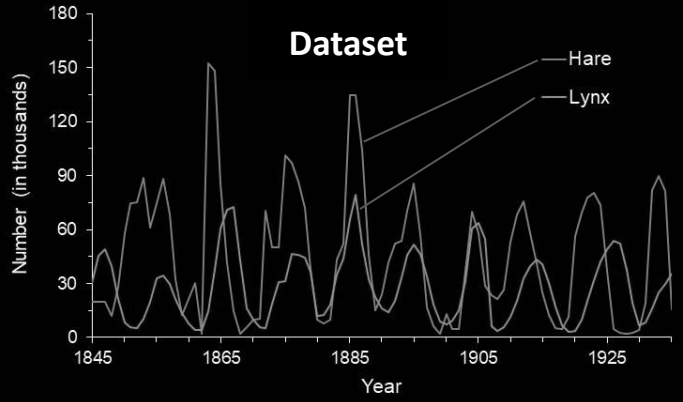
Practical Explore code
Reproduce results for hare-lynx system
Forecast & hindcast
Estimate prey-predator interactions

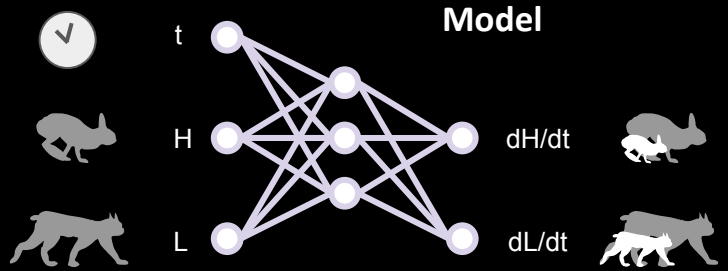
Assignment Explore dynamics of a tri-trophic system

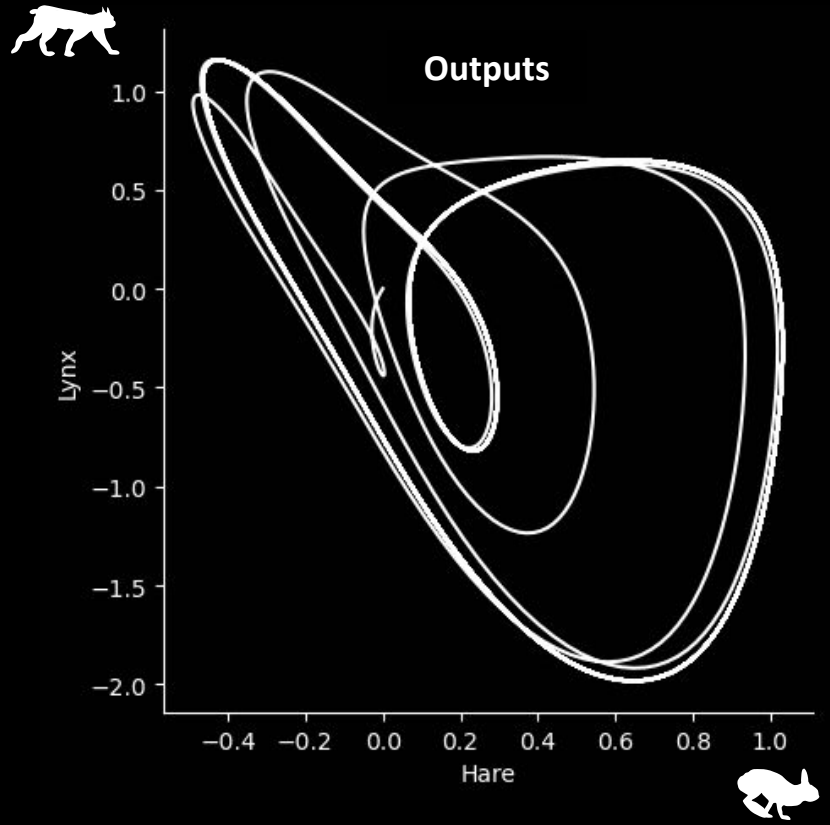

Dataset
Hare
Lynx
Number (in thousands)
180
150
120
90
60
30
0
1845
1865
1885
1905
1925
Year

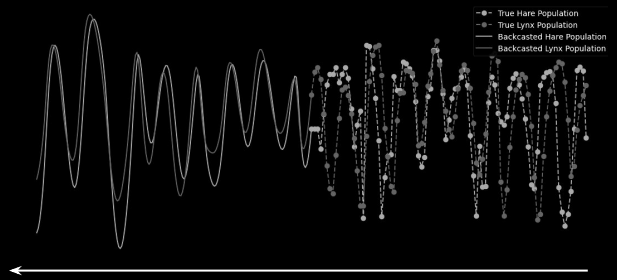

Dataset
Hare
Lynx
Number (in thousands)
180
150
120
90
60
30
0
1845
1865
1885
1905
1925
Year


Model
t
H
L
dH/dt
dL/dt


Dataset
Hare
Lynx
Number (in thousands)
180
150
120
90
60
30
0
1845
1865
1885
1905
1925
Year


Model
t
H
L
dH/dt
dL/dt

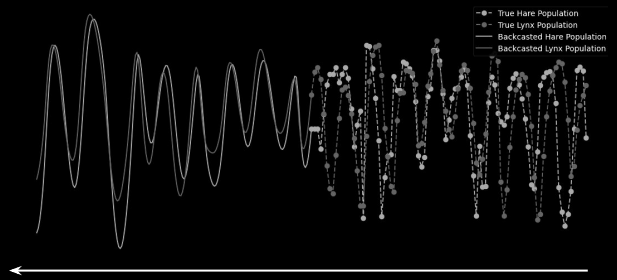

Outputs
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Lynx
−0.4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
Hare



Aim 1: Forecasting & backcasting

How long will the populations survive?

How was the system 50 years ago?



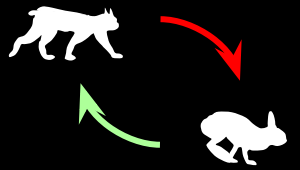
Aim 1: Forecasting & backcasting

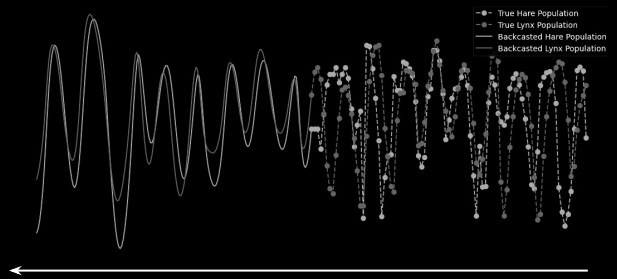
How long will the populations survive?

How was the system 50 years ago?

Aim 2: Understanding dynamics

How does the strength of the interaction between the prey and predator change?





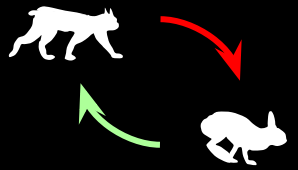
Aim 1: Forecasting & backcasting

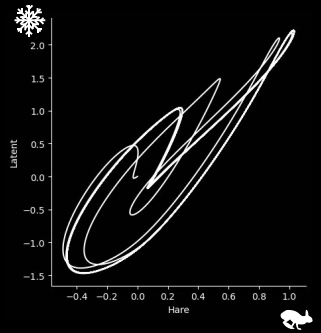
How long will the populations survive?

How was the system 50 years ago?

Aim 2: Understanding dynamics

How does the strength of the interaction between the prey and predator change?





Aim 3: Latent variables

Are unobserved forces at play?

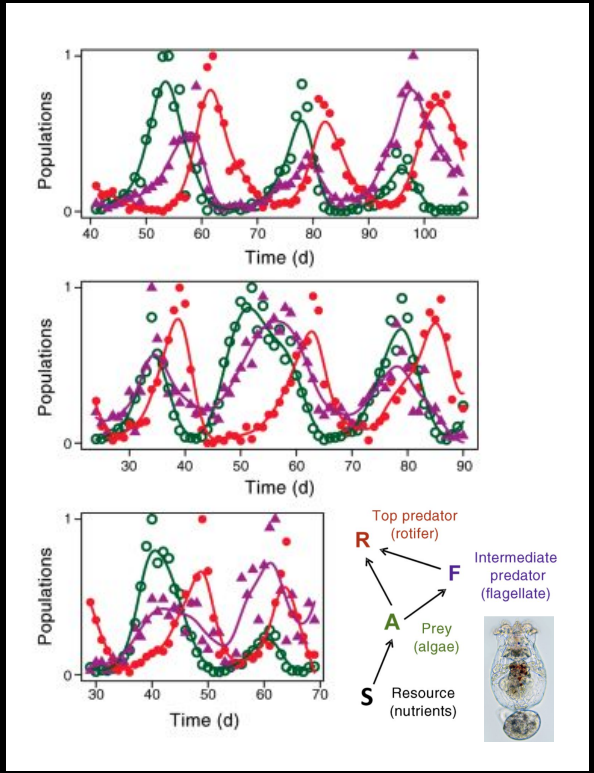
Can we account for them in forecasts?

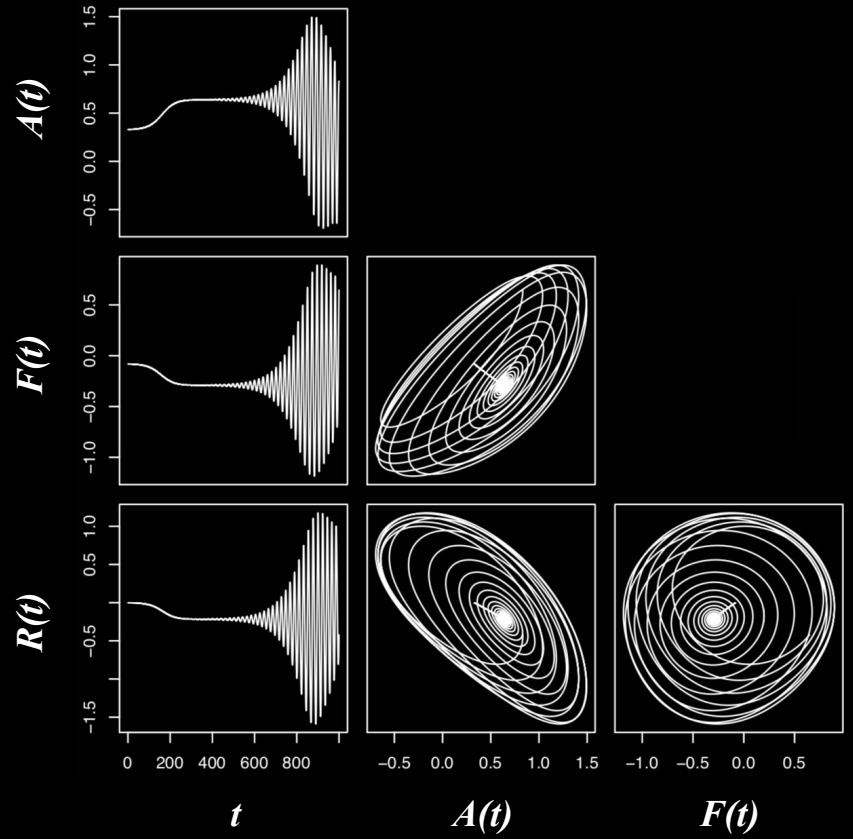
Practical

Explore code
Reproduce results for hare-lynx system
Forecast & hindcast
Estimate prey-predator interactions

Assignment

Explore dynamics of a tri-trophic system
References > Hiltunen-2013.pdf
data_rotifer_A.csv
data_rotifer_B.csv
data_rotifer_C.csv


Populations
Time (d)
Top predator
(rotifer)
R
F
Intermediate
predator
(flagellate)
A
Prey
(algae)
S
Resource
(nutrients)


A(t)
F(t)
R(t)
t
A(t)
F(t)