ECG signal generating function

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An Artificial ECG Signal Generating Function in MATLAB^{TM}

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Abstract

Data obtained from electrocardiogram (ECG) signals provides invaluable tools for diagnosing cardiac disorders. However, ECG signals recorded from electrocardiograph are usually corrupted by noise attributed to several factors. To help solve these problems, we develop a simple but inexpensive and easy-to-implement MATLAB^{TM} model that generates ECG signals and gives us mathematical control over the ECG signal. Our model fuses Mathematical functions in MATLAB^{TM} with physiological data.

Keywords: Electrograph, Electrogram, Electrocardiogram, signals, offline

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1 Introduction

ECG signals play a vital role in advanced diagnostic methods of various cardiovascular diseases. They provide crucial medical information on the overall health status of a patient. In addition to providing a reliable method for monitoring the electrical cardiac activity, basic aspects of a human physiology such as the Heart Rate Variability (HRV) can be determined from ECG signals [1]. A typical ECG signal shows the oscillations between cardiac contractions (systole) and relaxations (diastole) states as reflected in a heart rate (HR). Thus the ECG signal determines the number of heart beats per minute.

A number of important events characterize cardiac functions. Atrial and ventricular depolarization/re-polarization takes place for each heart beat. The cardiac cycle is associated with portions of the heart becoming positively charged, while the remaining parts become negatively charged interchangeably. This potential difference generated initiates the flow of current [2]. A typical ECG signal depicts a series of waveforms which occur in a repetitive order. The waveforms are initiated from the isometric line, from which a deflection indicates electrical activity. The principal features of an ECG signal, depicted by troughs and peaks, usually denoted by letters P, Q, R, S and T, are illustrated in Figure 1. One normal heart beat is represented by a set of three recognizable waveforms that start with the P-wave, followed by the QRS complex and ends with the T-wave. The relatively small P-wave is initiated by the depolarization of the atrial muscles and is related to their contraction. The large QRS-wave complex, made up of three waves, is caused by the depolarization of the ventricles and is connected to their contraction. Atrial re-polarization happens during the depolarization of the ventricles but its weak signal is undetected on an ECG. The T-wave is caused by currents flowing during the repolarization of the ventricles. A normal cardiac cycle of an individual at rest consisting of all waveforms (form $P-T$ waves) spans 0.8 seconds.

ECG signals are generated using electrocardiographs. Such signals are usually vitiated by several sources of noise which include (i) electrical interference from surrounding equipment (e.g. effect of the electrical mains supply), (ii) measurement (or electrode contact) noise, (iii) electromyographic (muscle contraction), (iv) movement artifacts, (v) baseline drift and respiratory artifacts and (vi) instrumentation noise (such as artifacts from the analogue to digital conversion process) [3]. One method of dealing with corrupt ECG signals is through the use of signal filtration systems (see for example) [4].

2 Generating Artificial ECG Signals Using Dynamical Functions

In order to eliminate this perennial problem of dealing with noise from tainted ECG signals, dynamic functions are usually developed for the generation of artificial synthetic ECG signals. Such functions are usually equation based. Of particular note is the of work McSharry and Clifford, that relies on three coupled differential equations [5] to generate ECG signals. We summarize their method below.

McSharry and Clifford captured the spectral characteristics of beat-to-beat RR intervals or RR tachogram, including both the oscillation in the RR tachogram resulting from parasympathetic activity, in synchrony with respiration (Respiratory Sinus Arrhythmia), [6] and the waves in arterial blood pressure (Mayer waves), using a bi-modal spectrum made up of the sum of two Gaussian functions. This is given by:

$$S(f) = \frac{\sigma_1^2}{\sqrt{2\pi c_1^2}} exp\left(\frac{(f-f_1)^2}{2c_1^2}\right) + \frac{\sigma_2^2}{\sqrt{2\pi c_2^2}} exp\left(\frac{(f-f_2)^2}{2c_2^2}\right) \tag{1}$$

where f_1 and f_2 are the two means, and c_1 and c_2 are the corresponding standard deviations. In spectral analysis of the RR tachogram, two critical frequency bands, usually referred to as the low-frequency (LF) band (0.04 to 0.15 Hz) and high-frequency (HF) band (0.15 to 0.4 Hz) [7] are considered. The power in the LF and HF bands are denoted by σ_1^2 and σ_2^2 respectively and the variance is represented by $\sigma^2 = \sigma_1^2 + \sigma_2^2$. Consequently, the LF/HF ratio is given by σ_1^2/σ_2^2

The function $S(f)$ gives the spectrum of a time series denoted by $T(t)$. In order to obtain $T(t)$, the inverse Fourier transform is applied on a sequence of complex numbers whose amplitudes are given by $\sqrt{(S(f))}$, such that the phases are randomly distributed between the interval 0 and 2π. Next, an appropriate scaling constant is chosen to multiply the resulting time series and then an offset value is added. This makes it possible to assign any required mean and variance to the resulting time series $T(t)$ based on the initial series. It also specifies different realizations of the random phases by simply changing the seed of the random number generator. The resulting series will inherit the same temporal and spectral properties of the original.

To complete the process, a dynamical model is then used to obtain the quasi-periodicity property of the ECG signal. This is guaranteed by making sure that the model has an attracting limit cycle. One complete revolution around the limit cycle in the $x-y$ plane mimics a heart-beat. The $PQRST$ peaks and troughs of the ECG signal are captured by a series of exponential functions inherent in the model, while the $PQRST$ extrema of the signal are specified by five angles correspondingly denoted by $\theta_p, \theta_q, \theta_r, \theta_s$, and θ_t. The mathematical model is represented by the following system of differential equations. See figure 1 for an example of a ECG signal.

FIGURE 2 NEAR HERE

$$\begin{aligned} \dot{x} &= \alpha x - \omega y \\ \dot{y} &= \alpha y + \omega x \\ \dot{z} &= -\sum_{i\in\{P,Q,R,S,T\}} \alpha_i \Delta\theta_i exp(-\Delta\theta_i^2/2b_i^2) - (z - z_0) \end{aligned} \tag{2}$$

where $\alpha = 1 - \sqrt{x^2 - y^2}, \Delta\theta_i = (\theta - \theta_i)mod(2\pi), \theta = atan2(y, x)$ is the four quadrant arctangents of the elements of x and y, ranging over $[-\pi, \pi]$ and ω is the angular frequency of the trajectory in its motion around the limit cycle and is related to the heart beat rate as $2\pi f$ [9]. The coefficients α_i control the magnitude of peaks while the b_i's defines the width of each peak. There is the possibility of the introduction of baseline wandering when the baseline value z_0 in (2) is coupled to the respiratory frequency f_2 by

$$z_0(t) = Asin(2\pi f_2 t) \tag{3}$$

In this case, $A = 0.15mV$. The system is then solved numerically, yielding the output synthetic ECG signal $s(t)$, which is the vertical component of the three-dimensional ordinary differential equations in equation (2) such that $s(t) = z(t)$. But for the use of techniques to filter and remove such noise in raw ECG signals, diagnosis from raw ECGs would have been medically inaccurate.

Sameni and Shamsollahi for example, applied the Extended Kalman filter [4, 8] to the dynamic model given in (2) in order to rid the output of the inherent noise. Their method consist of first linearizing (2) and then defining the filter by defining the actual state vector $\underline{X}_k = [x_k \;\; y_k \;\; z_k]^T$, where y_k is the driving function and z_k is the process noise. Then

$$s_k = [0 \;\; 0 \;\; 1]\underline{X}_k + v_k \tag{4}$$

where s_k and v_k are measurement vector and measurement noise respectively, and $R_k = E\{\underline{v}_k \underline{X}_k^T\}$, where R_k is the measurement noise covariance. This filter is designed to eliminate simple additive Gaussian noise and not the more complex nonlinear noise.

In this paper, we develop an easy-to-use MATLAB function capable of generating a 10 seconds artificial Electrocardiogram (ECG) by entering the heart rate (HR) and the peak voltage. This function utilizes several inbuilt MATLAB functions to achieve its goal.

3 Generating the Function Synecg in MATLAB

The aim of this function is to produce different ECG waveforms using MATLAB. The use of the function has many advantages in the generation of ECG waveforms. Firstly, time and money are saved. Secondly, the difficulty of taking real ECG signals with invasive and noninvasive methods is curtailed. Thirdly, the function enhances learning experience, that is to study and analyze abnormal and normal waveforms without using heavy medical equipments. Finally, the function is geared towards noise reduction. It also gives us mathematical control over ECG signals. In order to achieve this, we capture the temporal dynamics of ECG signal using a computer-based model. MATLAB^{TM} is used to develop and test "synecg", the artificial ECG generating function. The five inbuilt MATLAB^{TM} functions used to develop "Synecg" are "sgolayfilt", "kron" "ones", "round", and "linspace". These functions are described briefly below. Function $sgolayfilt()$ is known as the Savitzky- Golay Filter. $y = sgolayfilt(x, k, f)$ applies a Savitzky-Golay FIR smoothing filter to the data in vector x . If x is a matrix, $sgolayfilt$ operates on each column. The polynomial order k must be less than the frame size, f, which must be odd. If $k = f - 1$, the filter produces no smoothing [3]. Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least-squares smoothing filters) are typically used to "smooth out" a noisy signal whose frequency span (without noise) is large. It also helps preserve the peaks and valleys of ECG signals better than a standard FIR filter.

The second function, Function $kron()$ is the Kronecker Tensor Product. For example, $K = kron(X, Y)$ returns the Kronecker tensor product of X and Y. The result is a large array formed by taking all possible products between the elements of X and those of Y. If X is m-by-n and Y is p-by-q, then $kron(X, Y)$ is $m * p$-by-$n * q$. For example, if X is 2-by-3, then

$$\begin{aligned} kron(X, Y) &= [X(1,1) * Y X(1,2) * Y X(1,3) * Y X(2,1) \\ &* \quad Y X(2,2) * Y X(2,3) * Y] \end{aligned} \tag{5}$$

Function $ones()$ is used to generate a matrix of any dimension with ones (1) as entries. The command $ones(m, n)$ generates an m by n dimensional matrix with ones as entries.

The function $round()$ acts on single numbers or numbers in a matrix to round a number or numbers in a matrix of any decimal points to a whole number. The command round(2.43345534) will result in the answer 2. Likewise if M is a two by two matrix with $M = [2.323 \;\; 5.2344; -1.234 \;\; 8.4353]$ then $round(M)$ will result in [2 5;-1 8]. Function $linspace()$ is the inbuilt MATLAB^{TM} function $linspace(a, b, n)$, which generates a one-dimensional array of n evenly spaced numbers in the interval $[a \;\; b]$.

We demonstrate the use of our synecg function to generate 10-seconds synthetic ECGs for a number of heart rates of a resting individual with a peak voltage of 1.2mV. Figure 1 illustrates how the function is summoned in the MATLAB command window.

The way by which our function differs from other typical ECG models is that we used just heart rate per minute and the desired voltage in millivolts to generate synthetic electrocardiograms. The method used and other necessary descriptions are included in our MATLAB function in the next section.

4 Synecg Function

This function is able to generate synthetic electrocardiograms for 10 seconds. We just input heart rate (per minute) value and the desired peak voltage in millivolts.

```
function synecg
# DEVELOPING SYNTHETIC ECG IN MATLAB
 disp('This function is able to generate synthetic electrocardiograms')
 disp('for 10 seconds. Input your heart rate (per minute) and the')
 disp('desired peak voltage in millivolts.')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 h=input('Input your heart rate (in beats per minute)=');
 p=input('Input desired Peak voltage(in mV)=');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
 t1=10;
# t1=10 seconds is preferred to enable us have clearer RR tachograms.
# indicates the duration
            x1 = p*ecg(6000).';
            # denoising ecg signals with savitsky-golay filter for us to
            # get noiseless or ideal ECG
            y1 = sgolayfilt(kron(ones(1,19),x1),0,25);
             m = round(h/6);
            disp('Number of heart beats per 10 seconds=')
            m
            if h <= 105;
                    if m==5;
                        n1=32000;
                        elseif m==6;
                        n1=37000;
                        elseif m==7;
                        n1=43000;
                        elseif m==8;
```

```
                    n1=49000;
                    elseif m==9;
                    n1=55000;
                    elseif m==10;
                    n1=60000;
                    elseif m==11;
                    n1=66000;
                    elseif m==12;
                    n1=75000;
                    elseif m==13;
                    n1=80000;
                    elseif m==14;
                    n1=85100;
                    elseif m==15;

                    n1=92000;
                    elseif m==16;
                    n1=98000;
                    else m==17;
                    n1=100000;
                end
        else
          disp('Heart Rate input
exceeds limit')
        end
n = 1:n1;
del = round(6000*rand(1));
mhb = y1(n + del); t =
linspace(0,t1,n1);
colordef white; plot(t,mhb, 'g');
q=p+0.5;
axis([0 t1 -q q]); grid;
xlabel('Time [sec]'); ylabel('Voltage
[mV]');
title('Artificial Electrocardiogram');
legend('Isoelectric line');
end
```

5 Results

We use the synecg to generate 10-seconds synthetic ECGs for a number of heart rates of a resting individual with a peak voltage of 1.2mV. Figure 1.2

illustrates how the function is summoned in the MATLAB command window; Figures 2 through 4 show the artificial ECGs generated for 60bpm, 80bpm and 95bpm heart rates at a desired peak voltage of 1.2mV. We note that heart beats in a regular rhythm is usually between 60 and 100, that is when the signal-to-noise ratio is usually quite good in a person at rest. Basically, the ECG is a piecewise continuous graph of potential difference (in mV) against time (in seconds). The Synthetic ECG generated for 95bpm displays approximately 17 cardiac cycles in 10 seconds. The results are shown below:

F

6 Discussion

With the synecg, we have been able to generate a 10-seconds artificial ECG for some selected heart rates. The 60pbm generated 10-seconds ECG displays an approximate 10 cardiac cycles. As heart rate increases, the number of approximate cardiac cycles in the 10-seconds ECG also increases. Variation in the heart rates time intervals can be monitored with this increase in heart rate. A careful observation of the 60bpm ECG generated by synecg reveals that one cardiac cycle takes 0.8 seconds as human physiology suggests. We observe from the generated ECGs that as heart rate increases; the time taken for a cardiac cycle reduces. Similar observations were made for 80bpm and 95bpm. The various waveforms which constitute a cardiac cycle vary in length as heart rates change from time to time for the resting individual.

7 Conclusion

With the synecg function, synthetic ECG signals of various heart rates can be generated at a desired peak voltage for off-line study of the electrical activity of the heart. Our function made use of the function "sgolay" which is specifically geared towards ECG noise reduction and uses Savitzy-Golay polynomial filtering which is well-suited to smoothing of ECG data and preserve the peaks and valleys of the ECG signals better than a standard FIR filter. We have been able to show that our function generate synthetic ECG for 10 seconds by inputting heart rate (per minute) and the desired peak in millivolts. We propose that our method has more economic advantage than other methods since inputs to obtain an ECG signal is just the heart beat rate. The future prospects of this function are many. Synecg can be used to further develop a complete dynamic signal monitoring software for the electrical activity of the heart. This is an example of how Mathematical principles can be used in the dynamic modeling of biological rhythms of the human body as in the field of Biomathematics. The heart rate input are as follows; 60bpm, 80pbm and 95bpm. In all cases, we observe the number of cardiac cycles in the time duration.

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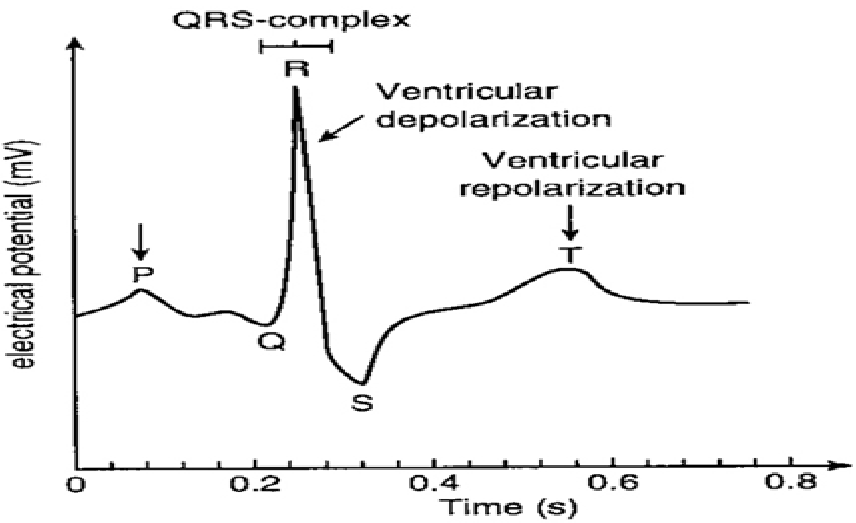

Figure 1: A single cycle of a typical ECG with the important points labeled, i.e. P, Q, R, S and T. We note the duration of the cardiac cycle; 0.8 seconds.

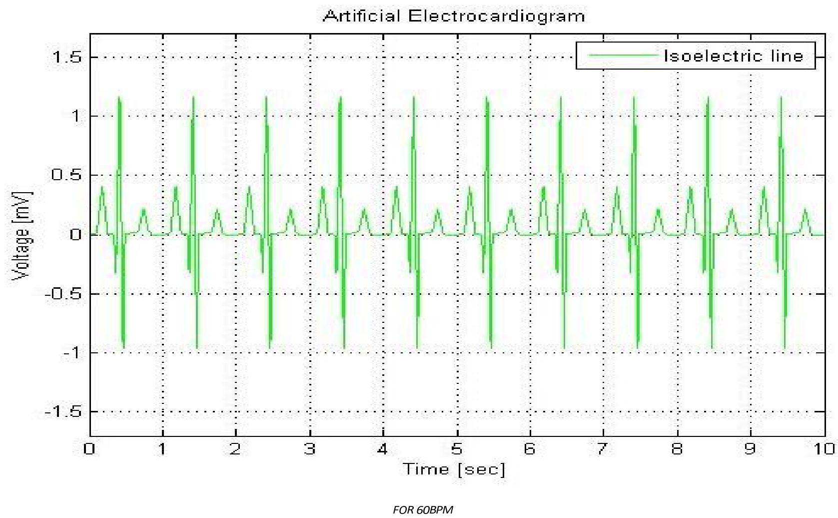

Figure 2: Synthetic ECG signal generated from SYNECG using 60bpm. This displays approximately 10 cardiac cycles in ten seconds.

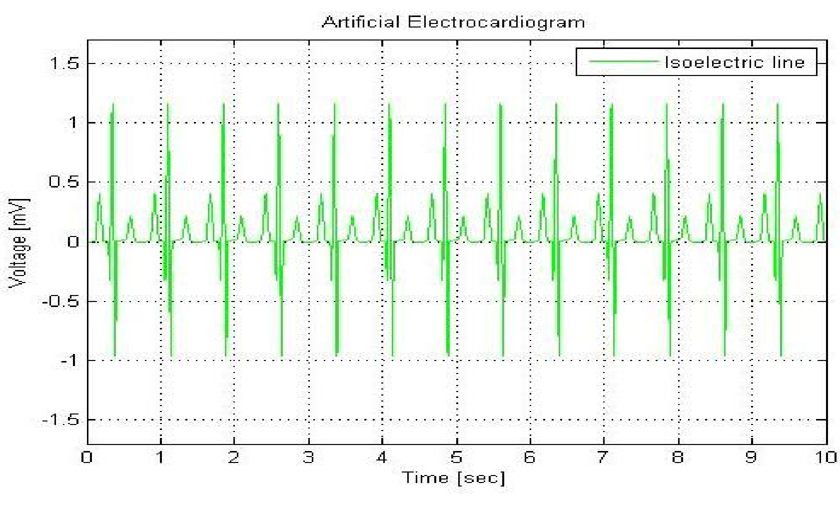

Figure 3: Synthetic ECG signal generated from SYNECG for 80bpm. This displays approximately 13 cardiac cycles in ten seconds.

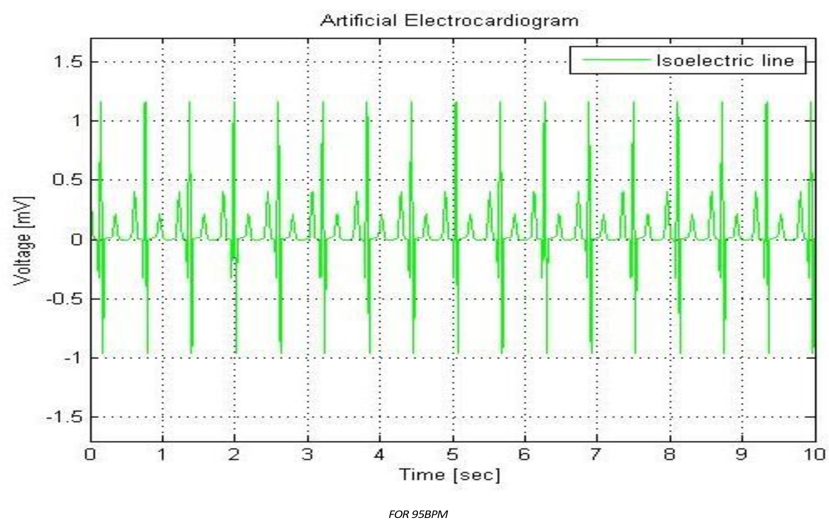

Figure 4: Synthetic ECG signal generated from SYNECG using 95bpm. The graph displays approximately 17 cardiac cycles in ten seconds.