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Anticipatory freight selection in intermodal long-haul round-trips

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ABSTRACT

We consider the planning problem faced by Logistic Service Providers (LSPs) transporting freights periodically, using long-haul round-trips. In each round-trip, freights are delivered and picked up at different locations within one region. Freights have time-windows and become known gradually over time. Using probabilistic knowledge about future freights, the LSP's objective is to minimize costs over a multi-period horizon. We propose a look-ahead planning method using Approximate Dynamic Programming. Experiments show that our approach reduces costs up to 25.5% compared to a single-period optimization approach. We provide managerial insights for several intermodal long-haul round-trips settings and provide directions for further research.

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1. Introduction

In a world with increasing trade and environmental consciousness, Logistic Service Providers (LSPs) are looking for better ways of organizing their long-haul transportation processes. Nowadays, LSPs aim towards network efficiency while maximizing profitability. This aim brings various challenges, one of which we study in this paper. We investigate the challenge faced by a Dutch LSP that transports containers from the Eastern part of the country to the Port of Rotterdam, and vice versa, in daily long-haul round-trips. Each day, a barge transports containers from a single inland terminal to different deep-sea terminals within the port. While delivering containers, the same barge picks up containers from the same, and other terminals, and transports them back to the inland terminal where it started. Alternatively, the LSP has trucks to transport containers. The challenge consists on how to assign the new containers that arrive for both parts of the round-trip, either to the barge or to trucks, to achieve the best network performance over time.

Ideally, the barge would visit as few terminals in the port as possible and trucks would be seldom used. However, the variability in the amount and type of containers that arrive each day makes the ideal situation hard to achieve. Each day, the LSP must choose which containers to consolidate and which to postpone, in order for its operations to be as close to ideal over time. For example, postponing the transport of a container to, or from, a given terminal today can reduce the number of terminals visited today without increasing the number of terminals visited tomorrow. Also, transporting a container that has a long time-window today can reduce the number of terminals that need to be visited tomorrow. The proper balance of consolidation and postponement in each round-trip will result in a better performance over a period of time.

In general terms, we study the decision problem of selecting freights for transportation in long-haul round-trips, periodically. In every period, a single round-trip is performed. In each round trip, freights are transported (i) from a single origin to

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multiple locations within a far away region and (ii) from locations in that region back to the origin, using a high capacity vehicle. The region is far away from the origin, but locations within the region are close to each other. As a result, the long-haul is the same in every round-trip and every period, and differences in costs arise due to the locations visited in the round-trip corresponding to each period and the use of an alternative transportation mode. The alternative transportation mode is more expensive than the high-capacity vehicle per freight. New freights, with different characteristics, arrive each period. Each freight has a given destination, a release-day, and a due-day. Although the number of freights, and their characteristics, vary from day to day, there is information about their probability distribution. The objective of the decisions is to reduce the total costs over a multi-period horizon (i.e., sum of transportation and handling costs over all modes and over all days) while transporting all freights.

Decisions that minimize the costs over a multi-period horizon are complex for three reasons. First, the freights that arrive in each period are uncertain. The uncertainty is not only on the number of freights that arrive, but also on their characteristics. Second, freights have different time-windows, which restrict the periods in which they can be consolidated and to which they can be postponed. Third, the cost advantage of consolidating the maximum number of freights in the high capacity vehicle, can be conflicting with the objective of minimizing costs over a multi-period horizon. To overcome these challenges, we model the optimization problem as a Markov Decision Process (MDP), and propose an Approximate Dynamic Programming (ADP) algorithm to solve it.

Our goal in this paper is twofold: (i) to model the stochastic and time dependent nature of the problem and design a solution approach that is applicable to solve realistic instances in reasonable time and (ii) to provide insight into the effect of various problem characteristics on the anticipatory freight selection decisions. With anticipatory, we mean making decisions today in anticipation of what might happen tomorrow. More specifically, we aim to answer the following two research questions: (i) how to design a proper look-ahead decision approach, i.e., a decision approach that incorporates information about future costs in current-day decisions and (ii) what performance can be expected from this look-ahead approach, with respect to costs savings under different stochastic freight characteristics, such as time-windows and destinations.

The remainder of this paper is structured as follows. In Section 2, we briefly review the relevant literature and specify our contribution to it. In Section 3, we introduce the notation and formulate the problem as an MDP. In Section 4, we present the ADP solution algorithm. In Section 5, we evaluate various designs for the ADP algorithm, and provide a comparison with optimal and heuristic solution approaches. Finally, we conclude this paper in Section 6 with insights about modeling and solving anticipatory freight selection problems in intermodal long-haul round trips, and provide directions for further research.

2. Literature review

The literature on freight consolidation in intermodal transportation networks is vast. In this brief review of it, we focus on two problem classes: (i) problems concerning assignment of freights to modes in an intermodal network and (ii) problems concerning anticipatory and dynamic selection of loads in transportation. In the first class, we summarize the key points and shortcomings of models and solution approaches proposed for Dynamic Service Network Design (DSND). In the second class, we provide examples on how the dynamic and stochastic nature of demand in transportation has been captured in routing and transportation problems, and what kind of solutions have been proposed. For an extensive review on research about the first problem class, we refer the reader to [Crainic and Kim \(2007\)](#) and [SteadieSeifi et al. \(2014\)](#); and for the second class, to [Pillac et al. \(2013\)](#) and [Powell et al. \(2007\)](#).

Decision problems in DSND involve the choice of transportation services for freight, over a multi-period horizon, where at least one problem characteristic varies over time ([SteadieSeifi et al., 2014](#)). However, two of the shortcomings in most DSND studies are that: (i) they do not incorporate time dependencies such as time-windows and information about pre-announced orders ([Crainic and Kim, 2007](#)) and (ii) they assume deterministic demand ([SteadieSeifi et al., 2014](#)). Furthermore, it seems that studies that tackle these shortcomings do so one at a time. For example, studies that model time dependencies, such as [Andersen et al. \(2009b\)](#), and consolidation opportunities, such as [Moccia et al. \(2011\)](#), assume deterministic demand. Recently, optimization studies that model multiple time dependencies in intermodal freight transportation networks, such as [Li et al. \(2015\)](#) and [Nabais et al. \(2015\)](#), use approaches based on receding horizons and model predictive control to take advantage of information that becomes known over time. Although these two studies do not explicitly incorporate probability distributions to capture uncertainty, they establish the benefits of including dynamic information in optimization models. Research that models uncertainty in the demand, such as [Hoff et al. \(2010\)](#), is usually developed for single mode. Furthermore, models that incorporate random variables, such as [Lium et al. \(2009\)](#), yield one initial plan that is robust to all realizations of the random variables. Only a few of these models, such as [Bai et al. \(2014\)](#) and [Lo et al. \(2013\)](#), include both planning and re-planning of a single transportation mode, in a two-stage approach.

One of the reasons why shortcomings have been tackled one at a time lies in the solution approaches used. Graph theory and meta-heuristics, which have been often proposed to solve DSND problems ([SteadieSeifi et al., 2014](#); [Wieberneit, 2008](#)), are less suitable for dealing with time-dependencies and stochastic demands. To deal with time-dependencies, mathematical programming techniques such as cycle-based variables ([Andersen et al., 2009a](#)), branch-and-price ([Andersen et al., 2011](#)), digraphs formulations ([Moccia et al., 2011](#)), and decompositions ([Ghane-Ezabadi and Vergara, 2016](#)) have been used. However, these techniques are computationally expensive. Consequently, meta-heuristics, such as those based on Tabu Search ([Crainic et al., 2000](#); [Verma et al., 2012](#)), have been used for larger problems ([SteadieSeifi et al., 2014](#)). Integrating

stochasticity in these techniques and heuristics requires additional designs, such as stochastic scenarios (Hoff et al., 2010), or two-stage stochastic approaches (Bai et al., 2014; Lo et al., 2013). Increasingly, the potential gains of integrating stochasticity are recognized in practice (Lium et al., 2009) and in theory (Zuidwijk and Veenstra, 2015).

In contrast to the first problem class, the second class concerning dynamic and stochastic freight transportation has been studied extensively for *single mode* routing (Pillac et al., 2013; Powell et al., 2007). Although our problem contains *multiple modes*, research done in this second class provides valuable insights. For example, knowing orders one or two days in advance has been shown to improve the performance of trucking companies (Zolfagharinia and Haughton, 2014). To model stochastic demand that is revealed dynamically over time, two strategies have been commonly applied: (i) sampling strategies and (ii) stochastic modeling (Pillac et al., 2013). Both strategies yield solutions that anticipate on the realization of the stochastic variables and that perform better than non-anticipatory approaches. An example that combines all of the insights above is Amazon's anticipatory package shipping (Spiegel et al., 2013). This patent describes the combination of information about existing and historical orders with forecasts of future orders, to transport pre-assembled packages to intermediate warehouses and even to trucks for the so-called "same day" deliveries. In our case, anticipatory decisions include two alternatives, either consolidating or postponing freight for better performance in the future.

The previous paragraph exemplifies the trend of using dynamic and probabilistic information in logistics and transportation decisions. The need for further research that includes probabilistic knowledge in the planning of dynamic transportation problems has been outlined (Corman and Meng, 2015). However, using such information, under the sampling and stochastic modeling strategies, comes with various difficulties. Sampling methods come with some form of bias and are heuristic in nature, such as the Indifference Zone Selection approach used by Ghiani et al. (2009). Stochastic modeling requires analytical models of the evolution of the system and its variability, which are usually non-scalable to real-life instances, such as the Markov Decision Process model used by Novoa and Storer (2009). To overcome the difficulties of each strategy, several techniques have been proposed (Pillac et al., 2013). To reduce the bias of sampling methods, Multiple Scenario Approaches with algorithms based on consensus, expectation, or regret of the probabilistic knowledge, have shown significant benefits (Bent and Hentenryck, 2004). To reduce the dimensionality issues of stochastic modeling, Approximate Dynamic Programming based on roll-out procedures and value function approximation has been used (Novoa and Storer, 2009; Simao et al., 2009).

Summarizing, research about intermodal and stochastic freight transportation has had different perspectives. Within DSND, there has been little research about large stochastic multi-period problems (Lium et al., 2009; SteadieSeifi et al., 2014; Wieberneit, 2008). Within dynamic and stochastic freight transportation, research about multiple modes and round-trips has been studied less in comparison to a single mode (Berbeglia et al., 2010; Pillac et al., 2013). Our paper deals with multi-period stochastic cost minimization through anticipatory freight selection in intermodal transportation. Our solution algorithm is categorized as *anticipatory* because it incorporates information about the future through a simulation of future demand realizations (Berbeglia et al., 2010; Ghiani et al., 2009; Ghiani et al., 2012; Thomas and White, 2004). For these reasons, we believe our work has four contributions to the existing literature. First, we propose an MDP model that includes stochastic freight demand and its characteristics for an intermodal network, handles complex time-dependencies, and measures performance over a multi-period horizon. Second, we propose an ADP algorithm to solve the model for large problem instances. Third, we provide methodological insights on the design process of an ADP algorithm for our problem. Fourth, we provide managerial insights on the consolidation and postponement decisions for several intermodal network settings and initial conditions.

3. Mathematical model

In this section, we formulate a model of the optimization problem using MDP theory. First, we introduce the notation for the problem characteristics mentioned in Section 1. Next, we formulate the stages, states, decisions, transitions, and the optimality equations of the MDP model. Finally, we discuss the dimensionality issues of this model.

3.1. Notation

We consider a multi-period horizon $\mathcal{T} = \{0, 1, 2, \dots, T^{\max} - 1\}$. At each period $t \in \mathcal{T}$, one high-capacity vehicle performs a round-trip, traveling from a single origin to a group of locations $\mathcal{D}' \subseteq \mathcal{D}$ within a region \mathcal{D} , and back. Freights transported by this vehicle are categorized in two types: (i) *delivery* and (ii) *pickup* freights. Delivery freights are those transported from the origin to a location $d \in \mathcal{D}$ and pickup freights are those transported from a location $d \in \mathcal{D}$ back to the origin. Since only one round-trip is planned each period, a total of T^{\max} consecutive round-trips are considered. Each period, the planner selects which of the released freights, of both types, to consolidate in that round-trip. For simplicity, we refer to a period as a "day", and to a delivery or pickup location as a "destination".

Each freight must be transported within its own time-window. Time-windows are characterized by a release-day $r \in \mathcal{R} = \{0, 1, 2, \dots, R^{\max}\}$ and a time-window length of $k \in \mathcal{K} = \{0, 1, 2, \dots, K^{\max}\}$ days. For modeling purposes, the release-day is relative to the current-day and the time-window length is relative to the release-day. For example, a freight that has $r = 1$ and $k = 0$ today will be released tomorrow and must also be transported tomorrow. Note that r is the number of days in advance that the LSP knows about a freight before it can be transported. Also note that k is the number of days within which the LSP must transport a freight, once it has been released.

Although freights are known only after they arrive, the LSP has probabilistic knowledge about them in the form of eight probability distributions. In between two consecutive days, $f \in \mathcal{F}$ delivery freights and $g \in \mathcal{G}$ pickup freights arrive with probability p_f^D and p_g^G , respectively. A freight has destination $d \in \mathcal{D}$ with probability $p_d^{D,F}$ in case of delivery, and $p_d^{D,G}$ in case of pickup. A freight has release-day $r \in \mathcal{R}$ with probability $p_r^{R,F}$ in case of delivery, and $p_r^{R,G}$ in case of pickup. A freight has time-window length $k \in \mathcal{K}$ with probability $p_k^{K,F}$ in case of delivery, and $p_k^{K,G}$ in case of pickup.

In each period, two transportation modes are available. First, there is one high-capacity vehicle doing the round-trip, with capacity of Q delivery freights and Q pickup freights. The costs $C_{\mathcal{D}'}$ of this vehicle depend on the group of destinations it visits $\mathcal{D}' \subseteq \mathcal{D}$. In addition to $C_{\mathcal{D}'}$, there is a cost B_d per freight with destination d consolidated in this vehicle. Second, we assume there is an unlimited number of alternative vehicles (e.g., trucks) for urgent freights, i.e., freights whose due-day is immediate, at a cost of A_d per freight to or from destination d . The restriction of using alternative vehicles only for urgent freights does not impact the decision making process since there are no holding costs and transportation costs do not change over time. We introduce this restriction to reduce the size of the decision space, and thus the computational complexity of the model. We do not consider holding (i.e., inventory costs) since our focus is on the long-haul round-trip decisions, not on the pre- and end-haulage operations, and the time-window lengths are a tighter restriction on the postponement decisions than the physical space.

3.2. Formulation

Each day corresponds to a *stage* in the MDP. Thus, stages are discrete, consecutive, and denoted by t . At each stage t , there are $F_{t,d,r,k}$ delivery freights and $G_{t,d,r,k}$ pickup freights with destination d , release-day r , and time-window length k . The *state* of the system \mathbf{S}_t consists of all freight variables at stage t , as seen in (1). We denote the state space of the system by \mathcal{S} , i.e., $\mathbf{S}_t \in \mathcal{S}$.

$$\mathbf{S}_t = [(F_{t,d,r,k}, G_{t,d,r,k})]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \quad (1)$$

At each stage t , the decision consists of which delivery and pickup freights from \mathbf{S}_t to consolidate in the high-capacity vehicle. This decision is restricted by the release-day of freights and by the capacity of the vehicle. We use the non-negative integer variables $x_{t,d,k}^F$ and $x_{t,d,k}^G$ to represent the number of released freights with destination d and time-window length k consolidated, for delivery and pickup freights respectively. The *decision* \mathbf{x}_t consists of all decision variables at stage t as seen in (2a), subject to constraints 2b,2c,2d,2e,2f, which define the feasible decision space \mathcal{X}_t .

$$\mathbf{x}_t = [(x_{t,d,k}^F, x_{t,d,k}^G)]_{\forall d \in \mathcal{D}, k \in \mathcal{K}} \quad (2a)$$

s.t.

$$0 \leq x_{t,d,k}^F \leq F_{t,d,0,k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (2b)$$

$$0 \leq x_{t,d,k}^G \leq G_{t,d,0,k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (2c)$$

$$\sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} x_{t,d,k}^F \leq Q, \quad (2d)$$

$$\sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} x_{t,d,k}^G \leq Q, \quad (2e)$$

$$x_{t,d,k}^F, x_{t,d,k}^G \in \mathbb{Z}^+ \cup \{0\} \quad (2f)$$

The costs of a decision depend on the destinations visited with the high-capacity vehicle and the use of the alternative transportation mode. We define $y_{t,d} \in \{0, 1\}$ as the binary variable that gets a value of 1 if destination d is visited by the high-capacity vehicle at stage t and 0 otherwise. We define $z_{t,d}$ as the variable representing the number of freights to destination d that were transported with the alternative mode. These variables depend on the state and decision variables, as seen in (3b) and (3c). Using these variables, the costs at stage t can be defined as a function of \mathbf{x}_t and \mathbf{S}_t , as shown in (3a).

$$C(\mathbf{S}_t, \mathbf{x}_t) = \sum_{\mathcal{D}' \subseteq \mathcal{D}} \left(C_{\mathcal{D}'} \cdot \prod_{d' \in \mathcal{D}'} y_{t,d'} \cdot \prod_{d' \in \mathcal{D} \setminus \mathcal{D}'} (1 - y_{t,d'}) \right) + \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} (B_d \cdot (x_{t,d,k}^F + x_{t,d,k}^G)) + \sum_{d \in \mathcal{D}} (A_d \cdot z_{t,d}) \quad (3a)$$

where

$$y_{t,d} = \begin{cases} 1, & \text{if } \sum_{k \in \mathcal{K}} (x_{t,d,k}^F + x_{t,d,k}^G) > 0 \\ 0, & \text{otherwise} \end{cases}, \quad \forall d \in \mathcal{D} \quad (3b)$$

$$z_{t,d} = F_{t,d,0,0} - x_{t,d,0}^F + G_{t,d,0,0} - x_{t,d,0}^G, \quad \forall d \in \mathcal{D} \quad (3c)$$

The objective is to minimize the costs over the entire planning horizon, i.e., the sum of (3a) over all $t \in \mathcal{T}$. However, there is uncertainty in the arrival of freights within this horizon, and thus in the states. Consequently, the formal objective is to minimize the expected costs over the horizon. Since for every possible state there is an optimal decision, the output has

to be a group of decisions rather than a single one. We define a policy $\pi \in \Pi$ as a function that maps each state $\mathbf{S}_t \in \mathcal{S}$ to a decision $\mathbf{x}_t^\pi \in \mathcal{X}_t$, for every $t \in \mathcal{T}$. The goal is to find the policy $\pi^* \in \Pi$ that minimizes the expected costs over the planning horizon, given an initial state \mathbf{S}_0 , i.e., initial conditions, as seen in (4).

$$\min_{\pi \in \Pi} \mathbb{E} \left\{ \sum_{t \in \mathcal{T}} C(\mathbf{S}_t, \mathbf{x}_t^\pi) \mid \mathbf{S}_0 \right\} \quad (4)$$

Using Bellman's principle of optimality, the optimal costs can be computed through a set of recursive equations. These recursive equations are expressed in terms of the current-stage and the expected next-stage costs, as seen in (5). Before solving these equations, we need to define how the system evolves over time, or in other words, the transition from \mathbf{S}_t to \mathbf{S}_{t+1} .

$$V_t(\mathbf{S}_t) = \min_{\mathbf{x}_t \in \mathcal{X}_t} (C(\mathbf{S}_t, \mathbf{x}_t) + \mathbb{E}\{V_{t+1}(\mathbf{S}_{t+1})\}), \quad \forall \mathbf{S}_t \in \mathcal{S} \quad (5)$$

The transition from \mathbf{S}_t to \mathbf{S}_{t+1} is influenced by \mathbf{x}_t and by the freights that arrive after this decision. Remind that arriving freights, and their characteristics, are stochastic and characterized by eight probability distributions. To model this arrival process, we introduce the variables $\tilde{F}_{t,d,r,k}$ and $\tilde{G}_{t,d,r,k}$ to represent the delivery and pickup freights, respectively, that arrived between stages $t-1$ and t having destination d , release-day r , and time-window length k . Note that we defined these variables with respect to stages $t-1$ and t , such that at t all information is known. The *exogenous information* \mathbf{W}_t at stage t consists of all the “new information”, represented by $\tilde{F}_{t,d,r,k}$ and $\tilde{G}_{t,d,r,k}$, as seen in (6).

$$\mathbf{W}_t = \left[\left(\tilde{F}_{t,d,r,k}, \tilde{G}_{t,d,r,k} \right) \right]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \quad (6)$$

A state \mathbf{S}_t at stage t occurs as the result of the state of the previous stage \mathbf{S}_{t-1} , the decision of the previous stage \mathbf{x}_{t-1} , and the exogenous information \mathbf{W}_t that became known between the stages. Remind that, to model the time-windows of freights, release-days r are indexed relative to t and time-window lengths k are indexed relative to r . Naturally, once a freight has been released, the time-window length must be decreased by one every day that passes until the freight is transported. All of these factors, and index relations, are used to capture the transition of the system. We represent them using the *transition function* S^M shown in (7a), which works as follows. The transition of delivery ($F_{t,d,r,k}$) and pickup ($G_{t,d,r,k}$) freights with destination d , release-day r , and time-window length k , from \mathbf{S}_{t-1} to \mathbf{S}_t , is defined having four considerations. First, freights that are released at day t and have a time-window length $k < K^{\max}$, are the result of freights from the previous day $t-1$ that were already released, had a time-window length $k+1$, and were not consolidated in the previous round-trip; in addition to freights from the previous day $t-1$ that had a next day release and the same time-window length k , and the freights that arrived between the previous and the current-day with release-day 0 and time-window length k , as seen in (7b) and (7c). Second, freights that are already released at day t and have a time-window length $k = K^{\max}$ are the result of freights from the previous day $t-1$ that had a next day release and the same time-window length K^{\max} , in addition to the freights that arrived between the previous and the current-day with the same characteristics, as seen in (7d) and (7e). Third, freights that are not released at day t , do not have the maximum release-day, i.e., $0 < r < R^{\max}$, and have time-window length k , are the result of freights from the previous day $t-1$ that had a release-day $r+1$ and a time-window length k , in addition to the freights that arrived between the previous and the current-day with the same characteristics, as shown in (7f) and (7g). Fourth, freights that are not released at day t , have the maximum release-day R^{\max} , and have time-window length k , are the result only of the freights that arrived between the previous and the current-day with release-day R^{\max} and time-window length k , as seen in (7h) and (7i).

$$\mathbf{S}_t = S^M(\mathbf{S}_{t-1}, \mathbf{x}_{t-1}, \mathbf{W}_t), \quad \forall t \in \mathcal{T} | t > 0 \quad (7a)$$

where

$$F_{t,d,0,k} = F_{t-1,d,0,k+1} - \mathbf{x}_{t-1,d,k+1}^F + F_{t-1,d,1,k} + \tilde{F}_{t,d,0,k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \setminus K^{\max} \quad (7b)$$

$$G_{t,d,0,k} = G_{t-1,d,0,k+1} - \mathbf{x}_{t-1,d,k+1}^G + G_{t-1,d,1,k} + \tilde{G}_{t,d,0,k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \setminus K^{\max} \quad (7c)$$

$$F_{t,d,0,K^{\max}} = F_{t-1,d,1,K^{\max}} + \tilde{F}_{t,d,0,K^{\max}}, \quad \forall d \in \mathcal{D} \quad (7d)$$

$$G_{t,d,0,K^{\max}} = G_{t-1,d,1,K^{\max}} + \tilde{G}_{t,d,0,K^{\max}}, \quad \forall d \in \mathcal{D} \quad (7e)$$

$$F_{t,d,r,k} = F_{t-1,d,r+1,k} + \tilde{F}_{t,d,r,k}, \quad \forall d \in \mathcal{D}, r \in \mathcal{R} \setminus \{0, R^{\max}\}, k \in \mathcal{K} \quad (7f)$$

$$G_{t,d,r,k} = G_{t-1,d,r+1,k} + \tilde{G}_{t,d,r,k}, \quad \forall d \in \mathcal{D}, r \in \mathcal{R} \setminus \{0, R^{\max}\}, k \in \mathcal{K} \quad (7g)$$

$$F_{t,d,R^{\max},k} = \tilde{F}_{t,d,R^{\max},k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (7h)$$

$$G_{t,d,R^{\max},k} = \tilde{G}_{t,d,R^{\max},k}, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (7i)$$

Using the transition function S^M , we can rewrite (5) in terms of the \mathbf{W}_{t+1} as shown in (8). In (8), the only stochastic variable at stage t is \mathbf{W}_t . Note that the amount of realizations of the exogenous information \mathbf{W}_t is discrete and finite. We denote the set of all these realizations with Ω , i.e., $\mathbf{W}_t \in \Omega$, $\forall t \in \mathcal{T}$. For each realization $\omega \in \Omega$, there is an associated probability p_ω^Ω . Using these probabilities, the expectation in (8) can be rewritten, as seen in (9).

$$V_t(\mathbf{S}_t) = \min_{\mathbf{x}_t \in \mathcal{X}_t} \left(C(\mathbf{S}_t, \mathbf{x}_t) + \mathbb{E} \left\{ V_{t+1} \left(S^M(\mathbf{S}_t, \mathbf{x}_t, \mathbf{W}_{t+1}) \right) \right\} \right), \quad \forall \mathbf{S}_t \in \mathcal{S} \quad (8)$$

$$V_t(\mathbf{S}_t) = \min_{\mathbf{x}_t \in \mathcal{X}_t} \left(C(\mathbf{S}_t, \mathbf{x}_t) + \sum_{\omega \in \Omega} \left(p_\omega^\Omega \cdot V_{t+1} \left(S^M(\mathbf{S}_t, \mathbf{x}_t, \omega) \right) \right) \right), \quad \forall \mathbf{S}_t \in \mathcal{S} \quad (9)$$

The probability p_ω^Ω depends on the realization $\omega \in \Omega$ in three ways. First, it depends on the total number of delivery and pickup freights arriving, denoted by f and g respectively. Second, it depends on the probability that $\tilde{F}_{d,r,k}^\omega$ delivery freights and $\tilde{G}_{d,r,k}^\omega$ pickup freights will have destination d , release-day r and time-window length k . Third, it depends on a multinomial coefficient β (Riordan, 2002) that counts the ways of assigning the total number of arriving delivery freights f and pickup freights g to each variable $\tilde{F}_{d,r,k}^\omega$ and $\tilde{G}_{d,r,k}^\omega$, respectively. This coefficient is necessary since the order in which freights arrive does not matter and freight characteristics are allowed to “repeat”. With these three aspects, the probability p_ω^Ω can be computed using (10a).

$$p_\omega^\Omega = \beta \cdot p_f^F p_g^G \cdot \prod_{d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \left(\left(p_d^D p_r^R p_k^K \right)^{\tilde{F}_{d,r,k}^\omega} \left(p_d^D p_r^R p_k^K \right)^{\tilde{G}_{d,r,k}^\omega} \right) \quad (10a)$$

where

$$\omega = \left[\left(\tilde{F}_{d,r,k}^\omega, \tilde{G}_{d,r,k}^\omega \right) \right]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \quad (10b)$$

$$f = \sum_{d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \tilde{F}_{d,r,k}^\omega, \quad g = \sum_{d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \tilde{G}_{d,r,k}^\omega \quad (10c)$$

$$\beta = \frac{f!}{\prod_{d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} (\tilde{F}_{d,r,k}^\omega!)} \cdot \frac{g!}{\prod_{d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} (\tilde{G}_{d,r,k}^\omega!)} \quad (10d)$$

The optimal costs and the optimal policy can be found solving (9) using dynamic programming and the probabilities from (10a). However, the computational effort increases faster than exponential with increasing size of the problem instance. In the following, we elaborate on the dimensionality issues of our model.

3.3. Dimensionality issues

The optimality equations in (9) suffer from what Powell (2007) calls “three curses of dimensionality”. The first curse comes from the set of all possible realizations of the exogenous information Ω . For each possible decision $\mathbf{x}_t \in \mathcal{X}$, the calculation of the expectation requires the next-stage value of $|\Omega|$ states. The second issue comes from evaluating all possible decisions. At each state, finding the decision that minimizes the sum of the current-day and expected future costs involves the evaluation of all possible combinations of the freights that are released. The third, and most difficult of all dimensionality issues, comes from the set of all possible states \mathcal{S} . In our model, the number of possible states increases faster than exponential with increasing domain of destinations, release-days, and time-window lengths. In addition, the number of release-days $|\mathcal{R}|$ determines the extent to which freights can “accumulate”. To provide the reader with a measurable idea on these issues, we elaborate on the first curse of dimensionality in Appendix A. Due to these dimensionality issues, an exact solution to (9) using backward dynamic programming, for example, is only feasible in small problem instances. Nevertheless, the model provides the foundation for our Approximate Dynamic Programming (ADP) approach, which can be applied to realistic instance sizes.

4. Solution algorithm

We propose a solution algorithm based on Approximate Dynamic Programming (ADP). ADP is a framework that contains several methods for tackling the curses of dimensionality in an MDP. The general idea of ADP is to modify Bellman’s equations with a series of components and algorithmic manipulations in order to approximate their solution, and thus the optimal policy. In this section, we elaborate on the components and algorithmic manipulations we use, as shown in Algorithm 1. First, we introduce the concepts of *post-decision state* and *forward dynamic programming*, which tackle the first and third dimensionality issue mentioned in Section 3.3. Second, we introduce the concept of *basis functions* as an approximation of the value of the post-decision states. Finally, we describe a way of tackling the second dimensionality issue of finding the optimal decision for a single stage.

Algorithm 1. Approximate dynamic programming solution algorithm.

```

1: Initialize  $\bar{V}_t^0, \forall t \in \mathcal{T}$ 
2:  $n := 1$ 
3: while  $n \leq N$  do
4:    $\mathbf{S}_0^n := \mathbf{S}_0$ 
5:   for  $t = 0$  to  $T^{\max} - 1$  do
6:      $\hat{v}_t^n := \min_{\mathbf{x}_t^n} (C(\mathbf{S}_t^n, \mathbf{x}_t^n) + \bar{V}_t^{n-1}(S^{M,x}(\mathbf{S}_t^n, \mathbf{x}_t^n)))$ 
7:      $\mathbf{x}_t^{n*} := \arg \min_{\mathbf{x}_t^n} (C(\mathbf{S}_t^n, \mathbf{x}_t^n) + \bar{V}_t^{n-1}(S^{M,x}(\mathbf{S}_t^n, \mathbf{x}_t^n)))$ 
8:      $\mathbf{S}_t^{n,x*} := S^{M,x}(\mathbf{S}_t^n, \mathbf{x}_t^{n*})$ 
9:      $\mathbf{W}_{t+1}^n := \text{RandomFrom}(\Omega)$ 
10:     $\mathbf{S}_{t+1}^n := S^M(\mathbf{S}_t^n, \mathbf{x}_t^{n*}, \mathbf{W}_{t+1}^n)$ 
11:   end for
12:   for  $t = T^{\max} - 1$  to  $1$  do
13:      $\bar{V}_{t-1}^n(\mathbf{S}_{t-1}^{n,x*}) := U^V(\bar{V}_{t-1}^{n-1}(\mathbf{S}_{t-1}^{n,x*}), \mathbf{S}_{t-1}^{n,x*}, \hat{v}_t^n)$ 
14:   end for
15:    $n := n + 1$ 
16: end while
17: return  $[\bar{V}_t^n]_{\forall t \in \mathcal{T}}$ 

```

4.1. Post-decision state and forward dynamic programming

To tackle the large set of realizations of the exogenous information Ω , we introduce two new components into the model: (i) a post-decision state $\mathbf{S}_t^{n,x}$, and (ii) an approximated next-stage cost $\bar{V}_t^n(\mathbf{S}_t^{n,x})$. The post-decision state is the state of the system directly after a decision \mathbf{x}_t^n has been made but before the exogenous information \mathbf{W}_t^n becomes known, at iteration $n = 1, 2, \dots, N$ of the algorithm. The approximated next-stage cost $\bar{V}_t^n(\mathbf{S}_t^{n,x})$ serves as an estimated measurement for the next-stage costs (i.e., $\bar{V}_t^n(\mathbf{S}_t^{n,x}) \approx \mathbb{E}\{V_{t+1}(\mathbf{S}_{t+1}^n)\}$). We elaborate on this measurement later on. For now, we focus on the post-decision state. In a similar way to the freight variables of a state, the post-decision freight variables $F_{t,d,r,k}^{n,x}$ and $G_{t,d,r,k}^{n,x}$ form the post-decision state $\mathbf{S}_t^{n,x}$, as seen in (11). Note that these components are all indexed with a superscript n , which denotes the iteration n they correspond to.

$$\mathbf{S}_t^{n,x} = \left[\left(F_{t,d,r,k}^{n,x}, G_{t,d,r,k}^{n,x} \right) \right]_{\forall d \in \mathcal{D}, r \in \mathcal{R}, k \in \mathcal{K}} \quad (11)$$

To define a post-decision state $\mathbf{S}_t^{n,x}$, we define a function $S^{M,x}$ that relates the post-decision freight variables $\mathbf{S}_t^{n,x}$ with the state \mathbf{S}_t^n and decision \mathbf{x}_t^n , as shown in (12a). The workings of this function are similar to the transition function S^M defined in (7a), leaving out the exogenous information \mathbf{W}_{t+1}^n .

$$\mathbf{S}_t^{n,x} = S^{M,x}(\mathbf{S}_t^n, \mathbf{x}_t^n), \quad \forall t \in \mathcal{T} \quad (12a)$$

where

$$F_{t,d,0,k}^{n,x} = F_{t,d,0,k+1}^n - x_{t,d,k+1}^n + F_{t,d,1,k}^n, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \setminus K^{\max} \quad (12b)$$

$$G_{t,d,0,k}^{n,x} = G_{t,d,0,k+1}^n - x_{t,d,k+1}^n + G_{t,d,1,k}^n, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \setminus K^{\max} \quad (12c)$$

$$F_{t,d,0,K^{\max}}^{n,x} = F_{t,d,1,K^{\max}}^n, \quad \forall d \in \mathcal{D} \quad (12d)$$

$$G_{t,d,0,K^{\max}}^{n,x} = G_{t,d,1,K^{\max}}^n, \quad \forall d \in \mathcal{D} \quad (12e)$$

$$F_{t,d,r,k}^{n,x} = F_{t,d,r+1,k}^n, \quad \forall d \in \mathcal{D}, r \in \mathcal{R} \setminus \{1, R^{\max}\}, k \in \mathcal{K} \quad (12f)$$

$$G_{t,d,r,k}^{n,x} = G_{t,d,r+1,k}^n, \quad \forall d \in \mathcal{D}, r \in \mathcal{R} \setminus \{1, R^{\max}\}, k \in \mathcal{K} \quad (12g)$$

To tackle the large state space \mathcal{S} , we use the algorithmic manipulation of “forward dynamic programming”. In contrast to backward dynamic programming, forward dynamic programming starts at the first stage and, at each stage, solves an “optimality” equation for only one state, as seen in (13). This equation follows the same reasoning as the Bellman’s equation from (5), with two differences: (i) the next-stage costs are approximated and (ii) each feasible decision \mathbf{x}_t^n has only one corresponding post-decision state.

$$\hat{v}_t^n = \min_{\mathbf{x}_t^n \in \mathcal{X}_t^n} (C(\mathbf{S}_t^n, \mathbf{x}_t^n) + \bar{V}_t^{n-1}(\mathbf{S}_t^{n,x})) = \min_{\mathbf{x}_t^n \in \mathcal{X}_t^n} (C(\mathbf{S}_t^n, \mathbf{x}_t^n) + \bar{V}_t^{n-1}(S^{M,x}(\mathbf{S}_t^n, \mathbf{x}_t^n))) \quad (13)$$

To advance in time, a random realization \mathbf{W}_{t+1}^n from the set of exogenous information Ω is obtained via a Monte Carlo simulation. Subsequently, the transition function S^M of the MDP model, defined in (7a), is used. Although the simulation step introduces variability in the algorithm, it also represents the stochastic nature of the problem and can be used to improve the approximation iteratively. After processing the entire horizon, the approximated next-stage cost $\bar{V}_t^n(\mathbf{S}_t^{n,x})$ is updated retrospectively. In the following section, we explain how the approximation works and how it is updated in every iteration.

4.2. Basis functions and non-stationary least-squares update function

The approximated next-stage cost $\bar{V}_t^n(\mathbf{S}_t^{n,x})$ represents the future costs after stage t , estimated at iteration n . To build a good approximation, we use a “basis functions” approach. Basis functions are quantitative characteristics, or features, of a post-decision state, which explain, to some extent, the next-stage costs. Examples of basis functions are the sum of all freights in a post-decision state, the number of destinations of the released-freights, etc. We denote a basis function as $\phi_a(\mathbf{S}_t^{n,x})$, where a belongs to the set of basis functions or features \mathcal{A} . The approximated next-stage cost of a post-decision stage $\bar{V}_t^n(\mathbf{S}_t^{n,x})$ is a weighted sum of all basis functions, as shown in (14), where $\theta_a \in \mathbb{R}$ is the weight of each basis function $a \in \mathcal{A}$.

$$\bar{V}_t^n(\mathbf{S}_t^{n,x}) = \sum_{a \in \mathcal{A}} (\theta_a \cdot \phi_a(\mathbf{S}_t^{n,x})) \quad (14)$$

Naturally, with each iteration there are more observations of the stochastic characteristics of the problem. Since in every iteration and every stage, a Monte Carlo simulation of the exogenous information is performed, the costs of the newly seen state can be used to improve our knowledge of the next-stage costs. However, due to the time nature of the post-decision state, costs have to be updated retrospectively. We define a function U^V to denote the process that updates the approximated costs $\bar{V}_{t-1}^n(\mathbf{S}_{t-1}^{n,x})$ at iteration n , using (i) the approximated costs, from the previous iteration, of the post-decision state of the current iteration and of the previous stage $\bar{V}_{t-1}^{n-1}(\mathbf{S}_{t-1}^{n,x})$, (ii) the post-decision state of the current iteration and the previous stage itself ($\mathbf{S}_{t-1}^{n,x}$), and (iii) the solution to the forward optimality equation \hat{v}_t^n corresponding to the current iteration and of the current stage, as seen in (15).

$$\bar{V}_{t-1}^n(\mathbf{S}_{t-1}^{n,x}) = U^V(\bar{V}_{t-1}^{n-1}(\mathbf{S}_{t-1}^{n,x}), \mathbf{S}_{t-1}^{n,x}, \hat{v}_t^n) \quad (15)$$

The logic behind the retrospective update is as follows. At stage t of iteration n , the system has moved from \mathbf{S}_{t-1}^n to \mathbf{S}_t^n . The optimal costs \hat{v}_t^n of \mathbf{S}_t^n are the “realized” next-stage costs of the previous-stage post-decision state that the algorithm estimated in the previous iteration. In other words, the approximated next-stage cost that was calculated at the $t-1$, using the previous iteration $n-1$ estimate, has now been observed at t in iteration n .

Since we use a basis functions approach for the approximation, the update function U^V only modifies the weights θ_a of each basis function $a \in \mathcal{A}$. A suitable updating mechanism using sequential observations is the non-stationary least square method (Powell, 2007), as seen in (16). This method updates each weight based on the observed error $(\bar{V}_{t-1}^{n-1}(\mathbf{S}_{t-1}^{n,x}) - \hat{v}_t^n)$ and the value of its corresponding basis function $\phi_a(\mathbf{S}_t^{n,x})$. The matrix $(G^n)^{-1}$ makes sure all weights are updated with a magnitude that minimizes the squared errors in the non-stationary data, comparable to linear regression models. Solving (16) for all basis functions $a \in \mathcal{A}$ takes places in a “double-pass” fashion in lines 12 to 14, which further reduces the approximation bias. For more information on the double-pass logic, see Powell (2007).

$$\theta_a^n = \theta_a^{n-1} - (G^n)^{-1} \phi_a(\mathbf{S}_t^{n,x}) (\bar{V}_{t-1}^{n-1}(\mathbf{S}_{t-1}^{n,x}) - \hat{v}_t^n) \quad (16)$$

Finally, the decision space \mathcal{X}_t is relatively small for our test problem instances. Nevertheless, for larger problem instances, enumerating all possible decisions can be computationally difficult. To tackle this second dimensionality issue of finding the optimal decision for a single stage, we formulate a Mixed-Integer Linear Program (MILP) for the single-stage decision problem, as seen in Appendix B.

5. Numerical experiments

In this section, we analyze the performance of our approach under various instances (i.e., problem settings). Instances reflect the network characteristics of the Dutch LSP, with modifications to serve our analysis. The analysis of our ADP approach consists of two phases: (i) basis function design and comparison to the optimal solution and (ii) comparison to a benchmark heuristic. We use these two phases to test the theoretical and the practical relevance of our approach, respectively. The section is divided as follows. First, we present our experimental setup and the specific goals of each experiment. Second, we analyze the results of our experiments from the two phases and answer our research questions. Finally, we summarize the principal findings and provide a discussion on the benefits and the shortcomings of our approach, as well directions for further research.

5.1. Experimental setup

In the first phase of our analysis, we study the optimality gap of our ADP approach in combination with three different sets of basis functions. The optimal solution of the MDP model is obtained using dynamic programming, i.e., solving the Bellman equations in (9) for all states and all stages, working backwards from the last stage. Due to the dimensionality issues of the MDP model, this is only possible for small instances of the problem. We create six small-sized instances: I_1^S and I_6^S , which are considerably smaller than the network of the Dutch LSP, in terms of number of freights and destinations, but have similar relations of probability distributions. The probability distributions for these six instances are shown in Table 1. Furthermore, we consider a time horizon of a working week $T^{\max} = 5$ and a vehicle capacity of $Q = 2$ for all six instances. The ranges for the costs are the same for all six instances, and are defined as follows: $C_{D'} = [250, 1000]$, $B_d = [50, 100]$, and $A_d = [500, 1000]$. Remind that $C_{D'}$ is the cost of visiting a subset of destinations with the high-capacity mode (i.e., transportation cost), B_d is the cost per freight consolidated in the high-capacity mode, to a given destination (i.e., handling, loading, unloading cost), and A_d is the cost per freight transported using the alternative mode, to a given destination (i.e., transportation and handling cost). This cost setup is based upon three cost considerations. First, there are costs only if the high-capacity vehicle departs or the alternative transport mode is used. Second, the high-capacity vehicle costs depend predominantly on the subset of destinations visited. This means that the costs for transporting an additional container to (or from) a destination already scheduled to be visited are small compared to visiting an additional destination. Third, using the alternative transport mode, for the same number of freights that a high-capacity vehicle can carry, is more expensive. This reflects the economies of scale achieved through consolidation in high-capacity vehicles. In addition to the cost setup above, we do experiments with $B_d = 0$ to be comparable with our preliminary study in Pérez Rivera and Mes (2015), where we only formulate the first half of the round-trip (what we call a single trip). The comparison shows the design challenges of ADP between two similar and associated problems.

The reasoning behind varying the probability distributions in Instances I_1^S to I_6^S is to test the optimality gap in different network characteristics. For example, I_1^S represents a “balanced” network, where the probability distributions of the random variables in the delivery freight are the same as the ones in the pickup freights, whereas I_2^S represents an “unbalanced” network where these distributions differs. Furthermore, Instances I_3^S and I_4^S represent balanced networks with the freights having a short time-window and a long time-window, respectively. The shorter the time-window, the less possibilities to postpone freights. Finally, I_5^S and I_6^S follow the same idea of testing opposite time-window distributions, but for unbalanced networks.

Defining a set of basis functions, or features of a post-decision state, that properly capture future costs within an ADP algorithm is both a science and an art. With scientific approaches such as factor analysis and regression analysis, one can conjecture how “good” a feature is. However, designing features requires creativity about the causes of costs. We build three different sets of features based on a common “job” description used in transportation: *MustGo*, *MayGo*, and *Future* jobs. In our case, *MustGo* freights are those released freights whose due-day is immediate. *MayGo* freights are those released freights whose due-day is not immediate. *Future* freights are those that have not yet been released. We use the *MustGo*, *MayGo* and *Future* adjectives in destinations as well, with an analogous meaning. In Table 2 we show the three sets of features, which we name Value Function Approximation (VFA) 1, 2, and 3. All feature types in this table are related to the freights of a post-decision state. The symbol “•” denotes a VFA set containing the corresponding feature type. All feature types are numerical, and either indicate (i.e., 1 if yes, 0 if no), count (1,2,...), number (add), or multiply (i.e., product between two numbers) the different type of freights and destinations. Between parentheses we show the number of basis functions of

Table 1
Input parameters of the numerical experiments for the first phase.

Parameter	Instance					
	I_1^S	I_2^S	I_3^S	I_4^S	I_5^S	I_6^S
\mathcal{D}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}	{1, 2, 3}
p_k^{DF}	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$	$\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$
p_d^{DG}	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{8}{10}, \frac{1}{10}\}$
\mathcal{R}	{0}	{0}	{0}	{0}	{0}	{0}
p_r^{RF}	{1}	{1}	{1}	{1}	{1}	{1}
p_r^{RG}	{1}	{1}	{1}	{1}	{1}	{1}
\mathcal{K}	{0, 1, 2}	{0, 1, 2}	{0, 1, 2}	{0, 1, 2}	{0, 1, 2}	{0, 1, 2}
p_k^{KF}	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
p_k^{KG}	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{1}{10}, \frac{5}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
$\mathcal{F} = \mathcal{G}$	{1}	{1}	{1}	{1}	{1}	{1}
p_f^F	{1}	{1}	{1}	{1}	{1}	{1}
p_f^G	{1}	{1}	{1}	{1}	{1}	{1}

Table 2

Various sets of features (basis functions of a post-decision state).

Feature type	VFA 1	VFA 2	VFA 3
All post-decision state variables (18)	•	•	•
All post-decision state variables squared (18)	•	–	–
Count of MustGo destinations (1)	•	•	•
Number of MustGo freights (1)	•	•	•
Product of MustGo destinations and MustGo freights (1)	•	–	–
Count of MayGo destinations (1)	•	•	•
Number of MayGo freights (1)	•	•	•
Product of MayGo destinations and MayGo freights (1)	•	–	–
Count of Future destinations (1)	•	•	•
Number of Future freights (1)	•	•	•
Product of Future destinations and Future freights (1)	•	–	–
Indicator MustGo freights per destination (3)	–	•	–
Indicator MayGo freights per destination (3)	–	•	–
Indicator Future freights per destination (3)	–	•	–
Number of all freights (1)	•	•	•
Constant (1)	•	•	•

each feature type. For example, there is one post-decision state variable per destination, per time-window length, both for the delivery and the pickup, resulting in a total of $3 \times 3 \times 2 = 18$ post-decision state variables.

To measure the optimality gap of the ADP approach in combination with each VFA, in each small instance, we use a two-step methodology. First, we apply the ADP algorithm to each of the 19,321 states of the instance. The algorithm settings used are those recommended in the literature (Pérez Rivera and Mes, 2015; Powell, 2007). In our experiments, 500 iterations are enough for the algorithm to converge to a constant estimate of the costs of each initial state, within a reasonable computational time. A larger number of iterations would not guarantee a better performance since the state space is too large to measure it in its entirety within reasonable time, and the performance is also influenced by the chosen approximation function (i.e., set of features) and the updating mechanism. Second, for each state, we simulate the use of the policy resulting from the ADP algorithm on 500 replications of the time horizon. We store the average costs of this simulation, for each state, and calculate the percentage difference between these average performance costs and the optimal expected costs. Finally, the optimality gap is measured as the average percentage difference over all states, for each VFA and each small instance.

In the second phase of our analysis, we compare the ADP approach, in combination with the best set of basis functions from the first phase, against a benchmark policy. This benchmark policy consists of solving the single-stage decision problem to optimality, and in case of multiple optimal decisions, the one with the most transported freights is chosen. In other words, this heuristic evaluates all released freights, and assigns them to the high-capacity vehicle or the alternative mode such that minimum costs are achieved that day. Once the decision with minimum costs is found, the heuristic tries to fill in the high-capacity vehicle with freights that do not increase the costs (e.g., freights to the same destinations of the selected ones). Similar to the first phase, we assess the differences between the two planning methods in different instances representing different networks. The parameters in these instances have similar values and probability distributions as the Dutch LSP characteristics. For sensitivity analysis, we modify the probability distributions per instance to represent different companies. The probability distributions for these large instances, named I_1^L to I_6^L , are shown in Table 3. The time horizon is

Table 3

Input parameters of the numerical experiments for the second phase.

Parameter	Instance					
	I_1^L	I_2^L	I_3^L	I_4^L	I_5^L	I_6^L
\mathcal{D}	$\{1, 2, \dots, 12\}$	$\{1, 2, \dots, 12\}$	$\{1, 2, \dots, 12\}$	$\{1, 2, \dots, 12\}$	$\{1, 2, \dots, 12\}$	$\{1, 2, \dots, 12\}$
$p_{d_i}^{DF}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$
$p_{d_i}^{DG}$	$\approx \frac{6^d}{d!} e^{-6}$	$\frac{1}{12}, \forall d \in \mathcal{D}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$	$\approx \frac{6^d}{d!} e^{-6}$
\mathcal{R}	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
p_r^{RF}	$\{\frac{3}{10}, \frac{3}{10}, \frac{4}{10}\}$	$\{\frac{3}{10}, \frac{3}{10}, \frac{4}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
p_r^{RG}	$\{\frac{3}{10}, \frac{3}{10}, \frac{4}{10}\}$	$\{\frac{3}{10}, \frac{3}{10}, \frac{4}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
\mathcal{K}	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1, 2\}$
p_k^{KF}	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
p_k^{KG}	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{2}{10}, \frac{3}{10}, \frac{5}{10}\}$	$\{\frac{8}{10}, \frac{1}{10}, \frac{1}{10}\}$	$\{\frac{1}{10}, \frac{1}{10}, \frac{8}{10}\}$
$\mathcal{F} = \mathcal{G}$	$\{1, 2, \dots, 10\}$	$\{1, 2, \dots, 10\}$	$\{1, 2, \dots, 10\}$	$\{1, 2, \dots, 10\}$	$\{1, 2, \dots, 10\}$	$\{1, 2, \dots, 10\}$
p_f^F	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$
p_f^G	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-2}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$	$\approx \frac{4^f}{f!} e^{-4}$

$T^{\max} = 5$ and the vehicle capacity is $Q = 10$ for all six instances. Although 10 freights seem small compared to the average capacity of a barge of 48 containers, we note that freights can represent orders with multiple containers for a barge (in the case of the Dutch LSP, freight orders usually have at least 4 containers). The ranges for the costs are the same for all six instances, and are defined as follows: $C_p = [250, 2150]$, $B_d = [50, 150]$, and $A_d = [300, 800]$. The cost setup is based upon the same three considerations as in the first phase.

Instances I_1^L to I_6^L differ with respect to the input probability distributions. Similar to the first phase, the rationale behind this difference is to test three characteristics of a network: (i) the “balance” of delivery and pickup freight destinations, (ii) the number of pre-announced orders, i.e., freights whose release day is in the future, and (iii) the “urgency” of freight, whether they must be carried the same day they are announced or can be carried some days later. In this rationale, I_1^L represents a totally balanced network, while I_2^L represents a totally unbalanced network. Instances I_3^L through I_6^L represent balanced networks with different time-window characteristics. Freights are mostly released “immediately” in I_3^L , whereas in I_4^L they are mostly released “in the future”. In I_5^L , freights are mostly urgent, i.e., immediate release and due-day, whereas in I_6^L freights have future release and due day.

In a similar way to the first phase, we use a two-step methodology to measure differences in performance between the ADP algorithm and the benchmark heuristic. However, instead of doing the steps for all states, we do it for a subset of states based on a categorization of states according to two criteria: (i) number of released freights and (ii) number of destinations. Each criteria is divided into three levels, for a total of 9 categories, as seen in Appendix D. The two steps to measure performance are then applied to one state per category, for each instance. The state is chosen randomly among those states close to the center of the cluster of states from the given category, in terms of the two aforementioned criteria for each category. In the first step, we apply the ADP algorithm to each of these states, using the best VFA and the settings from the first phase. In the second step, we simulate the use of the policy resulting from the ADP algorithm and compare this with the benchmark heuristic. We use common random numbers to allow a pairwise analysis of differences among the two planning methods.

5.2. Results

We now present the results of the two phases of our experiments separately. In this section, ADP performance refers to the result of the two-step methodology, i.e., the result of a simulation of 500 replications using the weights of the basis functions learned by the ADP algorithm in 500 iterations.

5.2.1. Experiments of Phase I

In the first phase, we test the performance of each set of features (which we call VFAs) from Table 2, over the entire state space. We show the optimality gap as a percentage and the Mean Square Error (MSE), which is a measure for the spread of the error among all states of an instance, in Table 4. In Table 5, we show the average costs for each instance, over all states, for the MDP model (i.e., optimal costs) and for the three VFAs, and their computational times, in seconds on a personal computer. The time for the VFAs corresponds to the average time it takes the ADP algorithm to run the 500 iterations and learn the weights in (14). Remind that ADP starts with an initial state but generalizes the future costs for all possible states within the horizon. In the following paragraph we discuss these results in detail.

Although the three VFAs have a similar performance, there are some small differences that the designer can consider when choosing the best. With respect to optimality gap, there are no significant differences among the average optimality gaps of the VFAs. VFA 1 ranges between 1.5% and 7.1% away from optimal, VFA 2 between 1.5% and 7.0%, and VFA 3 between 1.3% and 6.5%. With respect to the spread of the error within an instance, differences among the VFAs are more noticeable. For example, in Instance I_2^L , both VFA 1 and VFA 2 have the same average optimality gap, but VFA 2 has a lower MSE indicating a smaller spread in error across states of the same instance. We reflect upon the difference in optimality gap among states of the same instance in Section 5.3. Finally, the most noticeable difference among the VFAs arise in computational time. Although all VFAs are significantly faster than the MDP (3 s against 830 s on average), VFA 3 is the fastest and VFA 2 the slowest. Note that 830 s is still reasonable for daily decision making, however, for realistic instances (see Section 5.2.2) the MDP cannot be solved within a day.

Table 4
Optimality gap of the different VFAs in instances I_1^L to I_6^L .

Instance	VFA1		VFA2		VFA3	
	Avg. (%)	MSE	Avg. (%)	MSE	Avg. (%)	MSE
I_1^L	4.7	34,203	4.1	27,253	4.0	25,213
I_2^L	6.2	57,065	6.2	54,110	5.9	51,032
I_3^L	2.1	16,196	1.9	14,409	2.0	14,824
I_4^L	3.3	23,622	3.7	24,878	3.5	24,148
I_5^L	1.5	11,895	1.5	10,000	1.3	9405
I_6^L	7.1	66,958	7.0	67,880	6.5	60,010

Table 5Costs and times of the MDP model and the different VFAs in instances I_1^S to I_6^S .

Instance	MDP		VFA1		VFA2		VFA3	
	Avg.	Time	Avg.	Time	Avg.	Time	Avg.	Time
I_1^S	3245	823.1	3396	3.1	3376	4.7	3372	2.5
I_2^S	3447	820.3	3657	3.0	3657	4.7	3649	2.4
I_3^S	4228	795.7	4317	3.0	4309	4.8	4312	2.4
I_4^S	2921	843.2	3017	2.4	3028	4.7	3024	1.9
I_5^S	4405	850.1	4469	2.1	4469	4.6	4462	1.6
I_6^S	2975	843.2	3183	2.4	3179	4.9	3166	1.7

Focusing on differences among instances, we observe that when freights have longer time-windows, the optimality gap of the VFAs is lower than when they have shorter time-windows. This result indicates that the more freights can be postponed in a network, the better our ADP algorithm can identify the optimal policy. The lowest optimality gap is achieved in Instance I_5^S (1.4% on average). This result indicates that a network that has many and complex possibilities for postponing freights, such as the “unbalanced” Instance I_5^S with long time-windows, has more potential for our algorithm to identify the optimal policy. We also observe that in terms of the optimality gap and MSE, Instance I_6^S was the most difficult for all VFAs. We elaborate on the difficulties of the ADP performance under different problem settings (instances) in Section 5.3.

All the previous experiments were done in a simulation of all states in the state space. However, statistical inference can be used to conjecture about the potential of different sets of features. For example, due to the linear relation between basis functions and estimated future costs (in the value function approximation of the ADP algorithm), the coefficient of determination R^2 of a linear regression is a measure that could be used to infer which is the best set of features. Computing this measure is faster than simulating the use of each set of features for all states. However, its results can mislead the designer on what performance to expect from a given set of features. We use R^2 to show the difficulties in establishing the potential of basis functions (i.e., sets of features of a state) in the design of the ADP algorithm.

To exemplify the use of R^2 and compare our results to the ones from our preliminary study, we use the six instances defined in Table 1 with the cost $B_d = 0$ and show six additional instances denoted by I_n^S -st in Table 6. These instances correspond to the single delivery trip setting of Instances I_1^S to I_6^S , as considered in Pérez Rivera and Mes (2015). The results of the optimality gap experiments, and the linear regression, are shown in Table 6. We observe that all single trip (-st) instances have a higher R^2 than the round-trip ones, but half of them have worse optimality gap. This result shows that, although applying a statistical test to a simplified version of the problem can be easier computationally wise, it can also lead to a different choice of “best” set of features. Furthermore, we observe that a higher coefficient of determination does not necessarily mean a lower optimality gap, especially in the round-trip instances. This result indicates that more basis functions to capture costs does not necessarily lead to a better approximation of the optimal policy.

In addition, we can analyze the sensitivity of ADP to the cost setting $B_d > 0$ and $B_d = 0$ by comparing Tables 4 and 6. We observe that all instances in Table 4 have a lower optimality gap and MSE than in Table 6. In other words, ADP is closer to optimal in the cost setting $B_d > 0$. A reason for this is the fact that there are larger differences in costs among similar states, making it easier for ADP to identify which post-decision states are more valuable to go to. We elaborate more on the challenges of ADP design in Section 5.3.

Table 6Performance of the different VFAs in instances I_1^S to I_6^S with $B_d = 0$.

Instance	R^2			Optimality gap entire state space					
	VFA1	VFA2	VFA3	VFA1		VFA2		VFA3	
				Avg. (%)	MSE	Avg. (%)	MSE	Avg. (%)	MSE
I_1^S	0.632	0.686	0.554	5.6	24,304	5.9	26,306	5.6	23,405
I_1^S -st	0.890	0.892	0.890	2.1	7820	2.1	7503	2.1	7373
I_2^S	0.636	0.684	0.555	6.6	35,144	7.7	44,680	6.8	35,260
I_2^S -st	0.895	0.895	0.895	10.1	147,860	10.9	172,340	11.0	175,301
I_3^S	0.661	0.709	0.589	3.3	19,368	3.4	19,924	3.3	19,258
I_3^S -st	0.942	0.919	0.916	4.6	58,803	4.5	58,250	4.5	62,550
I_4^S	0.637	0.698	0.563	5.3	21,043	5.5	20,765	5.5	20,407
I_4^S -st	0.922	0.877	0.875	3.5	19,902	2.3	5810	2.3	5869
I_5^S	0.640	0.683	0.558	2.8	15,030	2.8	14,735	3.0	16,018
I_5^S -st	0.944	0.915	0.915	3.5	38,799	3.3	34,985	3.3	34,547
I_6^S	0.639	0.691	0.561	9.9	59,733	11.4	78,208	10.4	67,544
I_6^S -st	0.922	0.876	0.874	4.8	30,960	4.4	28,329	4.1	26,624

Overall, there are several ways to measure performance among different VFAs. Naturally, to select the “best” VFA, several design characteristics must be considered, among which the optimality gap, error spread, and computational time. We select VFA 3 for two reasons: (i) it only contains linear features, which make it directly applicable to be used in the MILP for the single-stage decision and (ii) it contains the least number of features, improving the computational time of the updating function within the ADP algorithm. With this third set of basis functions, we continue the experiments of the second phase.

5.2.2. Experiments of Phase II

In the second phase, we show the performance of the ADP algorithm on the large instances I_1^L through I_6^L , using VFA 3. We measure performance in these experiments as the pairwise difference between the simulated costs of using the policy resulting from the ADP algorithm and the simulated costs of using the benchmark heuristic mentioned in Section 5.1. By pairwise difference we mean the use of common random numbers in the replications of the simulations, to rule out variability in the network as a cause for the differences. The differences, as a percentage, for each category of each instance, are shown in Table 7. A negative percentage in this table can be interpreted as how much lower costs the ADP approach achieves compared to the heuristic.

On average, the ADP approach achieves 6.6% lower costs than the heuristic over all categories and instances, as seen in Table 7. To begin our analysis, we focus on the different categories (rows in Table 7). The savings range from 2.8% to 11.2%. However, when we look into the performance of a category in the various instances, two observations stand out. First, in C4, the ADP approach achieves the largest cost reduction of all categories in all instances (25.5% in I_2^L), as well as the largest average cost reduction among all instances (11.2%). Second, in C9, the ADP approach achieves the worst performance of all categories in all instances (4.7% higher costs than the heuristic in I_5^L), as well as the smallest average cost reduction among all instances (2.8%). In these experiments we show that, although reductions are achieved in all categories on average over the test instances, a careful analysis must be done for some of the categories of the states that the system can be in. We elaborate more on this difference in performance among categories in Section 5.3.

Focusing on the instances (columns in Table 7), we observe that, on average, cost reductions larger than 5.9% are achieved in all instances except I_5^L . Furthermore, we observe that in I_5^L , the ADP approach generates higher costs than the heuristic in two categories (C3 and C9). In I_1^L and I_2^L , the ADP policy generates higher costs than the heuristic in only one category (C6 and C9, respectively). To study more into detail the pairwise differences, we calculate the confidence intervals for all differences, as seen in Appendix E. In most instances and categories, significant cost reductions are achieved. In addition to the two categories of I_5^L that had no savings on average (C3 and C9), we have two categories where the savings are not significant (C1 and C2). We further elaborate on why the ADP policy under-performs in these cases, when compared to the others, in Section 5.3.

5.3. Discussion

Before applying our approach in practice, two aspects of our model should be brought to attention. First, all random variables in our model are defined as empirical distributions. These distributions can be derived from historical data. Furthermore, the “finite” nature of our model is in line with the constraints that hold in practice, such as the size of the container yard, maximum number of days a leased container can be used, etc. The finite time-window length means there is a maximum number of days that freight can be postponed. Second, the costs of a long-haul vehicle are modeled to depend on the subset of destinations visited rather than the route in which they are visited. The motivation for this choice comes from the destination characteristics of our problem: the Port of Rotterdam contains a limited number of container terminals. Thus, the optimal route for every combination of destinations can be calculated beforehand and used once the freights are selected. In addition, differences in waiting time, CO₂ emissions, or other indicators of terminals, can be easily incorporated with the full definition of all subsets of destinations. In case there are many destinations in the region of the round-trip, these costs can be troubling to compute. To overcome this limitation in large networks, routing decisions should be incorporated

Table 7

Average cost difference between the ADP policy and the competing policy.

Category	I_1^L (%)	I_2^L (%)	I_3^L (%)	I_4^L (%)	I_5^L (%)	I_6^L (%)	Average (%)
C1	−5.9	−8.6	−9.4	−5.5	−0.6	−5.2	−5.9
C2	−9.1	−12.3	−4.0	−2.7	−0.6	−11.0	−6.6
C3	−1.9	−6.7	−8.2	−3.1	1.1	−7.2	−4.3
C4	−14.9	−25.5	−5.2	−11.8	−1.5	−8.0	−11.2
C5	−15.1	−1.5	−9.7	−25.9	−0.4	−9.7	−10.4
C6	1.3	−4.5	−3.8	−10.6	−2.0	−7.8	−4.6
C7	−4.4	−3.7	−24.2	−0.1	−11.0	−7.3	−8.4
C8	−2.3	−16.7	−2.1	−7.1	−0.6	−3.3	−5.3
C9	−0.9	2.3	−4.4	−11.0	4.7	−7.6	−2.8
Average	−5.9	−8.6	−7.9	−8.6	−1.2	−7.5	−6.6

into the problem. These decisions provide a new research opportunity on finding the balance between optimal routing and optimal postponing in large, stochastic, intermodal networks over a multi-period horizon.

After defining the input parameters for implementation in practice, there are several challenges in constructing a close-to-optimal ADP algorithm as shown in our experiments. Next to the challenge of designing “features” of a state, there is a challenge in estimating their accuracy. The coefficient of determination R^2 is one of many statistical measures one could use to conjecture the performance of a design in a fast way. However, as shown in our experiments, the results of such a statistical measure are not generalizable to judge the performance of a set of features for different network settings. Another option for estimating a design's accuracy is to test the designs using simulation of smaller problem instances that can be solved to optimality. Although this method allows for a deeper analysis of the differences among ADP designs with respect to the optimal solution, it is possible that not all significant interactions among the stochastic variables are present in these small instances. Without those relations in the accuracy tests, results can differ in larger instances, which are common in practice.

To illustrate that a smaller problem may not capture all relations of the larger problem, we compare the performance of ADP in Instances I_2^S and I_2^L . These instances have similar probability distributions but differ in size. We use the categorization procedure from [Appendix D](#), which was also used for I_2^L , and calculate descriptive statistics about the optimality gap of the ADP approach in Instance I_2^S . The full results are shown in [Appendix F](#), but the main conclusion is that the differences in ADP performance among the different categories is much larger in the large instance I_2^L than in the small instance I_2^S . Although this analysis is limited by our categorization of states, it supports the need for further research on the performance of ADP in different states of the same problem instance. For example, if properties of states with respect to ADP performance are identified, hybrid ADP designs that use different VFAs for different categories of states could be used when approximating the value function. The results of our second phase of experiments suggest some of these properties, which we elaborate upon in the following paragraph.

The results of the second phase of experiments showed that ADP performs better in some categories of states than others (see [Table 7](#)). States in these experiments represent initial conditions. As expected with the finite nature of our problem, some initial conditions, or categories of states, have a large impact on the performance of ADP over the entire horizon. In C4, which has a medium number of freights and a low number of destinations, the ADP approach performs the best. In C9, which has a large number of freights and a large number of destinations, the ADP approach performs the worst. These results are to be expected when considering the possible decisions in each of these two categories, with respect to the capacity of the round-trip vehicle. In C4, which has a medium number of freights, it is possible to send a full barge, or a less than full barge. Naturally, these two actions differ in current-day costs, but also differ in future costs due to future consolidation possibilities. The ADP approach seems to capture better the future consolidation possibilities and therefore performs better over the entire horizon. In C9, which has a large number of freights, the alternative transport mode needs to be used frequently and postponement makes no sense. In this case, the daily costs will be large anyways, meaning that optimizing for each day individually coincides with the optimal policy. In other words, ADP and the heuristic have a similar performance. Investigating this, and other factors that influence the savings of ADP will lead to better designs.

In broader view of performance, we also observed that the ADP approach performs well in 7 out of the 8 instances tested in the second phase of experiments. Instances correspond to different problem settings, and could represent different transportation companies. In I_2^L and I_4^L , the ADP approach performs at least 8% better on average than the heuristic. I_2^L represents an “unbalanced” network, in terms of number of freights and destination probabilities, and I_4^L a network with many pre-announced orders. It seems that, when there are complex relations in the demand and when the planner knows about orders in advance, the ADP approach particularly pays off. In I_5^L , there is almost no difference between the ADP approach and the heuristic. I_5^L represents an “urgent” network in which freights are mostly due on the day they are released, and they are mostly released the same day their information becomes known. In line with the observations from the last paragraph, in I_5^L there is less freedom on the decisions, and consequently lower benefits of using ADP in comparison to the heuristic. This analysis shows that we can establish up front whether ADP works better than the heuristic for a given problem setting. As [Powell \(2007\)](#) points out, ADP performs well if the problem benefits from looking into the future.

A natural limitation of single-period optimization is its tendency to postpone the transportation of freights. If there would be additional terminal constraints (e.g., operating schedules, fluctuating handling capacity, etc.), the postponement decisions will be more restricted and the performance of the ADP would get closer to that of the myopic heuristic. In addition, postponement decisions may be less desirable when there are cost incentives to consolidate as much as possible. For example, in a problem setting where there are no costs per container, i.e., $B_d = 0$, and all other settings remain the same, the heuristic (i.e., single-period optimization) sends as many freights as possible to the terminals it must visit in the high-capacity vehicle, since they do not add any costs. In a way, this is a single-period optimization method that “looks-ahead” by transporting freights that can be transported for free today instead of tomorrow. In a small sensitivity analysis using this cost setting and our large instances, the ADP approach achieves average savings of 3.1% instead of the 6.6% of the second phase experiments ([Table 7](#)). Although our sensitivity analysis is limited to a single cost parameter, it indicates how costs influence which decision method works best under stochastic multi-period problems. Further research is necessary with respect to both, the analysis of different cost structures, and the comparison over different look-ahead decision methods for other cost structures.

6. Conclusions

In this paper, we provided an MDP model for the anticipatory freight selection problem in intermodal long-haul round-trips, and an ADP algorithm to solve it. With our approach, we studied the tradeoff between selecting freights for today's round-trip and postponing them for future trips, under stochastic demand and various time-dependencies, for a multi-period horizon in realistic problem instances. Furthermore, we tested how decisions that take into account probabilistic knowledge of freights and their characteristics compare to those that do not. We analyzed the optimality gap of our ADP approach and compared its cost-reduction capabilities to a benchmark heuristic.

With respect to the optimality gap, we provided methodological insights on designing a close-to-optimal ADP algorithm and overcoming the challenges this brings. First, we showed that a careful analysis is required to measure the accuracy of different ADP designs. Specifically, we showed that additional mechanisms besides statistical inference and simulation are needed to translate results from small problem instances to larger instances. Second, we showed that ADP performs better in some states and problem instances than others. These differences in performance suggested problem characteristics in which ADP particularly pays off, such as the freedom in the decisions and availability of pre-announced orders.

With respect to cost-reduction capabilities, we showed that ADP results in significant savings compared to single-period optimization (between 5.9% and 8.6% on average on 7 out of the 8 instances tested). Moreover, we pointed out under which network settings ADP performs best. As a managerial insight, companies with unbalanced demand and a large number of pre-announced orders can expect the largest savings when using ADP instead of single-period optimization.

Finally, we identified two areas for further research. The first deals with differences in performance of our ADP approach across different initial states of the same problem instance. Investigating the causes of these differences can lead to improved ADP designs. The second deals with the benefits that can be expected from the ADP approach in comparison to other benchmark heuristics. Research on which look-ahead approach is the most appropriate, considering additional criteria such as ease of implementation, can accelerate the adoption of look-ahead approaches in practice. Exploring these two areas will increase the value of decision methods that use probabilistic knowledge in transportation and expand the knowledge on anticipatory decision-making in logistics.

Appendix A. Number of realizations of the exogenous information

Each realization $\omega \in \Omega$ is basically a combination of the values that the exogenous information variables $\tilde{F}_{t,d,r,k}$ and $\tilde{G}_{t,d,r,k}$ can have. Since both $\tilde{F}_{t,d,r,k}$ and $\tilde{G}_{t,d,r,k}$ can have a value greater than one due to multiple freights having the same characteristics, the number of possible realizations of exogenous information $|\Omega|$ depends not only on the number of different characteristics, but also on the number of freights that can have the same characteristics, as seen in (A.1).

$$|\Omega| = \sum_{n=0}^{|\mathcal{F}|} \binom{|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| + n - 1}{n} \cdot \sum_{n=0}^{|\mathcal{G}|} \binom{|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| + n - 1}{n} = \sum_{n=0}^{|\mathcal{F}|} \frac{(|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| + n - 1)!}{n!(|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| - 1)!} \cdot \sum_{n=0}^{|\mathcal{G}|} \frac{(|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| + n - 1)!}{n!(|\mathcal{D}| \cdot |\mathcal{R}| \cdot |\mathcal{K}| - 1)!} \quad (\text{A.1})$$

In a similar way to Ω , the state space \mathcal{S} grows faster than exponential with an increasing number of possible freight characteristics. The set of possible decisions \mathcal{X} grows fast as well, but the optimal action can be obtained (for realistic problems and in reasonable time) through an Integer Linear Program (ILP), as we show later on. Due to these dimensionality issues, an exact solution is only feasible in small problem instances.

Appendix B. Single-stage decision problem

The single-stage decision problem can be formulated as a MILP, as seen in (B.1a)–(B.1w).

$$\min C(\mathbf{S}_t^n, \mathbf{x}_t^n) = \sum_{\mathcal{D}' \subseteq \mathcal{D}} (C_{\mathcal{D}'} \cdot w_{t,\mathcal{D}'}) + \sum_{d \in \mathcal{D}} (A_d \cdot z_{t,d}) + \sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} (B_d \cdot (x_{t,d,k}^F + x_{t,d,k}^G)) + \sum_{a \in \mathcal{A}} (\theta_a \cdot \phi_a(\mathbf{S}_t^{n,x})) \quad (\text{B.1a})$$

s.t.

$$\sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} x_{t,d,k}^F \leq Q \quad (\text{B.1b})$$

$$\sum_{d \in \mathcal{D}} \sum_{k \in \mathcal{K}} x_{t,d,k}^G \leq Q \quad (\text{B.1c})$$

$$x_{t,d,0}^F + x_{t,d,0}^G + z_{t,d} = F_{t,d,0,0}^n + G_{t,d,0,0}^n, \quad \forall d \in \mathcal{D} \quad (\text{B.1d})$$

$$\sum_{k \in \mathcal{K}} x_{t,d,k}^F - \sum_{k \in \mathcal{K}} (F_{t,d,0,k}) \cdot y_d \leq 0, \quad \forall d \in \mathcal{D} \quad (\text{B.1e})$$

$$\sum_{k \in \mathcal{K}} x_{t,d,k}^G - \sum_{k \in \mathcal{K}} (G_{t,d,0,k}) \cdot y_d \leq 0, \quad \forall d \in \mathcal{D} \quad (\text{B.1f})$$

$$x_{t,d,k+1}^F + F_{t+1,d,0,k}^{n,x} = F_{t,d,0,k+1}^n + F_{t,d,1,k}^n, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} | k < K^{max} \quad (\text{B.1g})$$

$$x_{t,d,k+1}^G + G_{t+1,d,0,k}^{n,x} = G_{t,d,0,k+1}^n + G_{t,d,1,k}^n, \quad \forall d \in \mathcal{D}, k \in \mathcal{K} | k < K^{max} \quad (\text{B.1h})$$

$$F_{t+1,d,0,K^{max}}^{n,x} = F_{t,d,1,K^{max}}^n \quad \forall d \in \mathcal{D} \quad (\text{B.1i})$$

$$G_{t+1,d,0,K^{max}}^{n,x} = G_{t,d,1,K^{max}}^n \quad \forall d \in \mathcal{D} \quad (\text{B.1j})$$

$$F_{t+1,d,r,k}^{n,x} = F_{t,d,r+1,k}^n \quad \forall d \in \mathcal{D}, k \in \mathcal{K}, r \in \mathcal{R} | r < R^{max} \quad (\text{B.1k})$$

$$G_{t+1,d,r,k}^{n,x} = G_{t,d,r+1,k}^n \quad \forall d \in \mathcal{D}, k \in \mathcal{K}, r \in \mathcal{R} | r < R^{max} \quad (\text{B.1l})$$

$$w_{t,\mathcal{D}'} - y_{t,d'} \leq 0, \quad \forall \mathcal{D}' \subseteq \mathcal{D}, d' \in \mathcal{D}' \quad (\text{B.1m})$$

$$w_{t,\mathcal{D}'} + y_{t,d'} \leq 1, \quad \forall \mathcal{D}' \subseteq \mathcal{D}, d' \in \mathcal{D} \setminus \mathcal{D}' \quad (\text{B.1n})$$

$$w_{t,\mathcal{D}'} - \sum_{d' \in \mathcal{D}'} y_{t,d'} + \sum_{d'' \in \mathcal{D}' \setminus \mathcal{D}} y_{t,d''} \geq 1 - |\mathcal{D}'|, \quad \forall \mathcal{D}' \subseteq \mathcal{D} \quad (\text{B.1o})$$

$$\sum_{\mathcal{D}' \subseteq \mathcal{D}} w_{t,\mathcal{D}'} = 1 \quad (\text{B.1p})$$

$$x_{t,d,k}^F \in \mathbb{Z} \cap [0, F_{t,d,0,k}^n], \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (\text{B.1q})$$

$$x_{t,d,k}^G \in \mathbb{Z} \cap [0, F_{t,d,0,k}^n], \quad \forall d \in \mathcal{D}, k \in \mathcal{K} \quad (\text{B.1r})$$

$$y_{t,d} \in \{0, 1\}, \quad \forall d \in \mathcal{D} \quad (\text{B.1s})$$

$$z_{t,d} \in [0, F_{t,d,0,0}^n + G_{t,d,0,0}^n], \quad \forall d \in \mathcal{D} \quad (\text{B.1t})$$

$$w_{t,\mathcal{D}'} \in [0, 1], \quad \forall \mathcal{D}' \subseteq \mathcal{D} \quad (\text{B.1u})$$

$$F_{t+1,d,0,k}^{n,x} \in [0, F_{t,d,0,k+1}^n + F_{t,d,1,k}^n], \quad \forall d \in \mathcal{D}, k \in \mathcal{K} | k < K^{max} \quad (\text{B.1v})$$

$$G_{t+1,d,0,k}^{n,x} \in [0, G_{t,d,0,k+1}^n + G_{t,d,1,k}^n], \quad \forall d \in \mathcal{D}, k \in \mathcal{K} | k < K^{max} \quad (\text{B.1w})$$

The objective in (B.1a) is to minimize the sum of (i) a linearized version of the current-day costs of a decision, as shown in (3a), and (ii) the approximated next-stage cost. Constraints (B.1b)–(B.1f) define the feasible decision space and the auxiliary variables. Constraints (B.1g)–(B.1l) define the post-decision freight variables. Constraints (B.1m)–(B.1p) linearize costs through the use of a binary variable $w_{t,\mathcal{D}'}$ that gets a value of 1 if the subset of destinations \mathcal{D}' is visited with the long-haul vehicle and 0 otherwise. Constraints (B.1q)–(B.1w) define the domain of all variables in the MILP. Note that not all post-decision freight variables are included in the model, only the ones that are affected by the decision. Furthermore, the basis functions $\phi_a(\mathbf{S}_t^{n,x})$ for all $a \in \mathcal{A}$ are assumed to be linear in the decision variables of the MILP model. For examples on how to incorporate basis functions in the MILP, see Appendix C.

Appendix C. Basis functions in the MILP

To model the various sets of features seen in Table 2 of the main text, we introduce two additional variables into the MILP shown in (B.1a)–(B.1w). The binary variable $u_{t,d}$ gets a value of 1 if destination d has any MustGo freight, and 0 otherwise. The binary variable $v_{t,d}$ gets a value of 1 if destination d has any MayGo freight, and 0 otherwise. To define these variables within the MILP, we introduce constraints (C.1b)–(C.1g). To account for the future costs (i.e., weights from the ADP algorithm), we need to construct weights for the MILP decision variables depending on the set of features \mathcal{A} used, as shown in (C.1a). For example, in VFA 2, we have the feature “Indicator of MustGo freights per destination”, whereas in VFA 3 we do not have this feature. In both VFA 2 and VFA 3 we have the feature “Count of MustGo destinations”. When using VFA 2, the weight $\theta_d^{u,\mathcal{A}}$ in the MILP will be the sum of weights corresponding to the feature a “Indicator of MustGo freights per destination” and the feature a' “Count of MustGo destinations” (i.e., $\theta_d^{u,\mathcal{A}} = \theta_a + \theta_{a'}$). When using VFA 3, the weight $\theta_d^{u,\mathcal{A}} = \theta_{a'}$, since this set of features only has the feature a' “Count of MustGo destinations”, related to the variable $u_{t,d}$. At the end of (C.1a), we add the constant δ_t^n , which takes into account the total costs of features that are not dependent on the MILP variables, but only on the state itself (i.e., “Number of future freights”, “Constant”, etc.).

$$\sum_{a \in \mathcal{A}} (\theta_a \cdot \phi_a(\mathbf{S}_t^{n,x})) = [\theta_{d,k}^{F,\mathcal{A}} \cdot F_{t+1,d,0,k}^{n,x}]_{\forall d \in \mathcal{D}, k \in \mathcal{K}} + [\theta_{d,k}^{G,\mathcal{A}} \cdot G_{t+1,d,0,k}^{n,x}]_{\forall d \in \mathcal{D}, k \in \mathcal{K}} + [\theta_d^{u,\mathcal{A}} \cdot u_{t,d}]_{\forall d \in \mathcal{D}} + [\theta_d^{v,\mathcal{A}} \cdot v_{t,d}]_{\forall d \in \mathcal{D}} + \delta_t^n \quad (\text{C.1a})$$

$$F_{t+1,d,0,0}^{n,x} - (F_{t,d,0,1}^n + F_{t,d,1,0}^n) \cdot u_{t,d} \leq 0, \quad \forall d \in \mathcal{D} \quad (\text{C.1b})$$

$$G_{t+1,d,0,0}^{n,x} - (G_{t,d,0,1}^n + G_{t,d,1,0}^n) \cdot u_{t,d} \leq 0, \quad \forall d \in \mathcal{D} \quad (\text{C.1c})$$

$$\sum_{k \in \mathcal{K} | k > 0} F_{t+1,d,0,k}^{n,x} - \left(\sum_{k \in \mathcal{K} | k < K^{max}} (F_{t,d,0,k+1}^n) + \sum_{k \in \mathcal{K}} (F_{t,d,1,k+1}^n) \right) \cdot v_{t,d} \leq 0 \quad \forall d \in \mathcal{D} \quad (\text{C.1d})$$

$$\sum_{k \in \mathcal{K} | k > 0} G_{t+1,d,0,k}^{n,x} - \left(\sum_{k \in \mathcal{K} \setminus \mathcal{K}^{max}} (G_{t,d,0,k+1}^n) + \sum_{k \in \mathcal{K}} (G_{t,d,1,k+1}^n) \right) \cdot v_{t,d} \leq 0 \quad \forall d \in \mathcal{D} \quad (\text{C.1e})$$

$$u_{t,d} \in \{0, 1\}, \quad \forall d \in \mathcal{D} \quad (\text{C.1f})$$

$$v_{t,d} \in \{0, 1\}, \quad \forall d \in \mathcal{D} \quad (\text{C.1g})$$

Appendix D. Categorization of states in normal-sized experiments

Each instance of the second phase of experiments has much more than $7 \cdot 10^{27}$ states. For this reason, we cannot evaluate the performance for all initial states as we did in the first phase. However, choosing a subset of the entire state space has two complications: (i) results hold for the chosen sample of states but not necessarily to the entire state space and (ii) states have different characteristics that influence the performance of the ADP algorithm. To measure different regions of the state space in a representative way, we build a subset of states as follows. First, we generate a random sample of 10,000 states for each network I_1^L to I_6^L , which are “commonly encountered” in such a network. To achieve a commonly encountered state, we simulate the network using the benchmark heuristic. We begin with a random initial state, and each day within the horizon of a week, we simulate the decisions and arrival of new freights. After the simulated week, we stop the simulation and store the state in which the system is at that moment. Naturally, the resulting sample contains various types of states representing

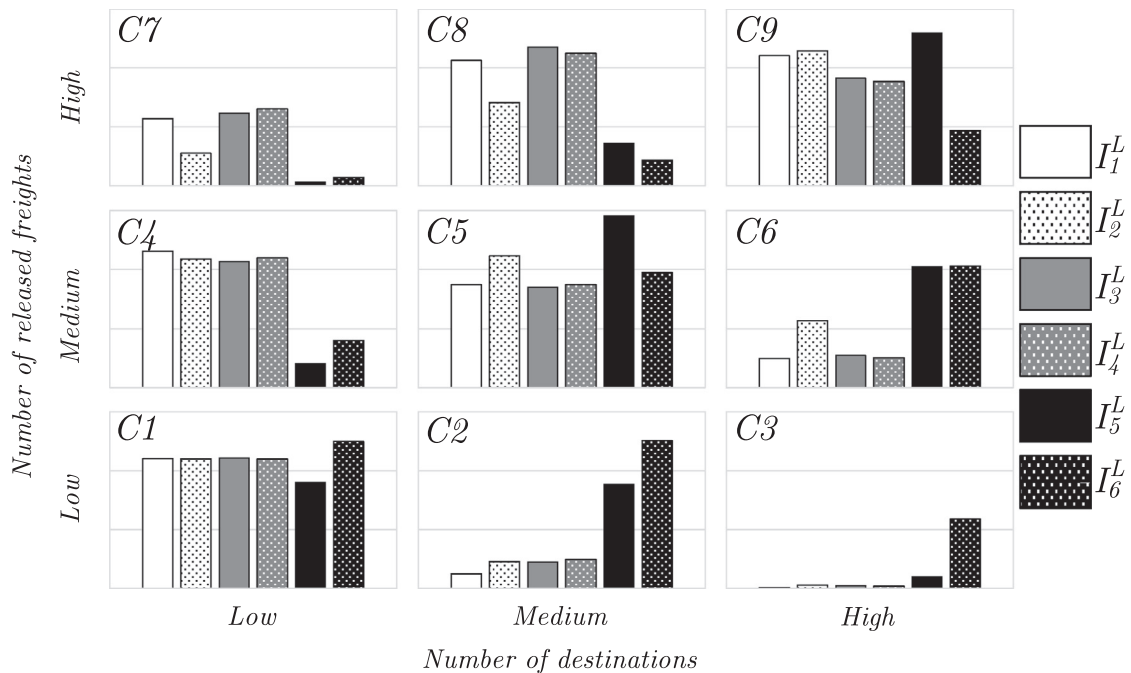


Fig. D.1. Categorization of the 10,000 sampled states of the Instances I_1^L to I_6^L .

Table D.8

Categorization of states.

Category	Freights		Destinations		I_1^L		I_2^L		I_3^L		I_4^L		I_5^L		I_6^L	
	(F)	(D)	(F)	(D)	F	D	F	D	F	D	F	D	F	D	F	D
C1	Low	Low	[0,13]	[0,4]	[0,11]	[0,4]	[0,14]	[0,4]	[0,14]	[0,4]	[0,8]	[0,2]	[0,22]	[0,4]		
C2	Low	Medium	[0,13]	[4,5]	[0,11]	[4,5]	[0,14]	[4,5]	[0,14]	[4,5]	[0,8]	[2,3]	[0,22]	[4,5]		
C3	Low	High	[0,13]	[5,9]	[0,11]	[5,10]	[0,14]	[5,8]	[0,14]	[5,9]	[0,8]	[3,7]	[0,22]	[5,9]		
C4	Medium	Low	[13,20]	[0,4]	[11,18]	[0,4]	[14,20]	[0,4]	[14,20]	[0,4]	[8,14]	[0,2]	[22,30]	[0,4]		
C5	Medium	Medium	[13,20]	[4,5]	[11,18]	[4,5]	[14,20]	[4,5]	[14,20]	[4,5]	[8,14]	[2,3]	[22,30]	[4,5]		
C6	Medium	High	[13,20]	[5,9]	[11,18]	[5,10]	[14,20]	[5,8]	[14,20]	[5,9]	[8,14]	[3,7]	[22,30]	[5,9]		
C7	High	Low	[20,57]	[0,4]	[18,45]	[0,4]	[20,52]	[0,4]	[20,47]	[0,4]	[14,37]	[0,2]	[30,54]	[0,4]		
C8	High	Medium	[20,57]	[4,5]	[18,45]	[4,5]	[20,52]	[4,5]	[20,47]	[4,5]	[14,37]	[2,3]	[30,54]	[4,5]		
C9	High	High	[20,57]	[5,9]	[18,45]	[5,10]	[20,52]	[5,8]	[20,47]	[5,9]	[14,37]	[3,7]	[30,54]	[5,9]		

Table E.9

Confidence intervals of the difference between the benchmark heuristic and the ADP algorithm (VFA 3).

State	I_1^L	I_2^L	I_3^L	I_4^L	I_5^L	I_6^L
C1	[−7.0%,−4.8%]	[−9.6%,−7.5%]	[−10.3%,−8.4%]	[−6.1%,−4.9%]	[−1.3%,0.0%]	[−5.9%,−4.5%]
C2	[−9.7%,−8.4%]	[−13.1%,−11.6%]	[−4.8%,−3.3%]	[−3.6%,−1.8%]	[−1.2%,0.1%]	[−11.6%,−10.4%]
C3	[−2.7%,−1.2%]	[−7.2%,−6.1%]	[−9.1%,−7.4%]	[−3.8%,−2.4%]	[0.5%,1.7%]	[−7.7%,−6.7%]
C4	[−16.0%,−13.8%]	[−26.5%,−24.6%]	[−6.2%,−4.1%]	[−12.5%,−11.2%]	[−2.2%,−0.7%]	[−8.4%,−7.6%]
C5	[−15.9%,−14.3%]	[−2.0%,−0.9%]	[−10.5%,−8.8%]	[−26.5%,−25.3%]	[−1.0%,0.1%]	[−10.3%,−9.2%]
C6	[0.5%,2.1%]	[−5.1%,−3.9%]	[−4.5%,−3.1%]	[−11.1%,−10.0%]	[−2.6%,−1.4%]	[−8.2%,−7.3%]
C7	[−4.7%,−4.0%]	[−4.3%,−3.0%]	[−25.0%,−23.5%]	[−0.6%,0.4%]	[−12.2%,−9.8%]	[−7.9%,−6.8%]
C8	[−2.9%,−1.7%]	[−17.1%,−16.3%]	[−2.5%,−1.6%]	[−7.5%,−6.7%]	[−0.9%,−0.2%]	[−3.7%,−2.9%]
C9	[−1.5%,−0.3%]	[1.8%,2.8%]	[−5.4%,−3.5%]	[−11.4%,−10.7%]	[3.9%,5.4%]	[−7.9%,−7.2%]

Table F.10Statistics about optimality gap in Instance I_2^S .

Category	Freights	Destinations	Min (%)	Max (%)	Average (%)	95th Percentile (%)	States
C1	Low	Low	2.8	18.2	7.2	9.6	282
C2	Low	Medium	2.5	39.8	7.5	15.5	2244
C3	Low	High	1.9	31.5	7.5	17.9	1416
C4	Medium	Low	2.8	26.4	6.5	10.2	150
C5	Medium	Medium	2.0	36.2	6.9	16.0	2940
C6	Medium	High	1.8	33.4	6.9	15.8	4008
C7	High	Low	2.1	38.3	5.8	8.5	75
C8	High	Medium	1.8	41.3	6.7	17.5	2550
C9	High	High	1.5	36.5	6.4	15.6	5656

various regions of the entire state space. To categorize these regions, we use two state-characteristics that are of great importance in practice: the number of released freights, and the number of different destinations of the released freights. We define three levels in each characteristic: Low, Medium, and High, and categorize the 10,000 states of each instance into nine different categories C1–C9, as seen in Fig. D.1. Since the distribution of state characteristics differ per instance, we define the boundary of the levels individually for each instance as seen in Table D.8. These boundaries are computed with the objective of minimizing the difference between the category with the largest number of states and the one with the lowest number of states.

Appendix E. Confidence intervals of the performance in normal-sized experiments

See Table E.9.

Appendix F. Differences in optimality gap among categories of a small instance

See Table F.10.

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