

Preservation

Part 1

Assume:

$$\vdash H : \star$$

$$H \vdash F : \sigma$$

$$H; a \vdash F \text{ ok}$$

$$H, F \rightarrow H', F'$$

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Show:

$$\vdash H' : \star$$

$$H' \vdash F' : \sigma$$

$$H'; a \vdash F' \text{ ok}$$

Part 2

$$\vdash H : \star$$

$$H \vdash FS$$

$$H; a \vdash FS \text{ ok}$$

$$H, FS \rightarrow H', FS'$$

Shown in LaCava

$$\vdash H' : \star$$

$$H' \vdash FS'$$

$$H'; b \vdash FS' \text{ ok}$$

Part 3: Taskset reduction

Assume:

Show:

Cases of \leadsto

- | | | |
|--|--|-------------|
| 1) $\vdash H : *$ | $\vdash H' : *$ $\checkmark \longrightarrow$ Trivial as $H = H'$ | E-ASYNC |
| 2) $H \vdash TS$ | <u>I</u> $H' \vdash TS'$ | E-FINISH1 |
| 3) $H \vdash TS \text{ ok}$ | <u>II</u> $H' \vdash TS' \text{ ok}$ | E-FINISH2 |
| 4) $H, \{T\} \cup TS \leadsto H', TS'$ | | E-TASK-DONE |

E-Async

$$L' = [x \rightarrow 0] \quad P = \emptyset \quad \Gamma' = x : C$$

E-Box

E-CAPTURE

E-SWAP

$$\Gamma; \text{ocap} \vdash t : \tau \quad \text{by } 2, T\text{-ASYNC}$$

$$\vdash H \vdash \Gamma; L' \quad \text{by } WF\text{-ENV?}$$

$$\vdash \vdash \Gamma; L; P \quad \text{by } WF\text{-PERM}$$

$$\leadsto H \vdash \{T_1\}$$

$$\Gamma'' = \Gamma \setminus \text{Perm}[Q]$$

$$\Gamma''; a \vdash s : \sigma \quad \text{by } 2, T\text{-ASYNC}$$

$$\vdash H \vdash \Gamma''; L \quad \text{by } 2, \Gamma'' \subseteq \Gamma, WF\text{-ENV?}$$

$$\vdash \vdash \Gamma''; L; P \setminus \{p\} \quad \text{by } WF\text{-PERM?}$$

$$\leadsto H \vdash \{T_2\}$$

$$\leadsto H \vdash \{T_1, T_2\} \cup TS \quad \text{I}$$

I $H' \vdash TS'$ checks for coherency between types and heap+variable bindings. The other rules are likely easy to prove and any problems require only minor changes to the typing rules.

II $H' \vdash TS' \text{ ok}$

$H \vdash \{T_1, T_2\} + TS \text{ ok}$

- $f < f'$ trivial, no FINISH gets added

- No two tasks await same id. Trivial

Two cases:

- Async was checked with ocap

-> $T_1 + T_2$ are also checked with ocap

-> TS' ok

- Async wasn't checked with ocap

-> T_1 is ocap, T_2 not

-> TS' ok because we still have only one non-ocap active task

E-FINISH1

$f < f'$ by definition of f' fresh

No two tasks await same id by def of fresh

Current task becomes inactive, awaits new task

Currently ocap \rightarrow new also ocap \rightarrow no problem

Currently not ocap \rightarrow current becomes inactive, new also not ocap
 \rightarrow no problem

E-FINISH2

$f < f'$ trivial

No two tasks await same id trivial

if task was not ocap \rightarrow no non-ocap active tasks exists (only it and its direct ancestors are allowed to be non-ocap. I.e. if no active tasks exist, only this inactive task does) \rightarrow This task becomes the new non-ocap active task

E-TASK-DONE

$f < f'$ trivial

No two tasks await same id trivial

Only one non-ocap active task:

If task was non-ocap

\rightarrow No active non-ocap task exists

\rightarrow Its parent might become next (see E-FINISH2), otherwise everything is ocap

E-BOX/CAPTURE/SWAP

$f < f'$ trivial

No two tasks await same id trivial

only one non-ocap active task trivial. Removing tasks does not invalidate any invariant