Progress

Assume:

(1) $\vdash H : *$

(2) $H \vdash TS$?

(3) $\forall (f, FS) \in TS.$
$H; a \vdash FS$ ok

Show:

(I) $H, TS \rightsquigarrow H', TS'$

(II) $\vee$ $TS = \emptyset$

(III) $\vee$ $FS = F \circ FS'$ where something null

(4) $\exists (\_, FS) \in TS, f.\ FS = \varepsilon \vee FS = F \circ FS' \wedge (F \neq \langle \text{FINISH } f \rangle \vee F = \langle \text{FINISH } f \rangle \wedge \nexists (f, \_) \in TS)$ by Reducable Taskset

ID-Ordering

TS finite

$TS \neq \emptyset$

Case distinction on TS:

- $\emptyset$: Done by (II)
- $T \uplus TS'$: Obtain Task $T = (f, FS)$ by (4), $TS \neq \emptyset$

  Case distinction on FS:

  - ε: $H, T \uplus TS' \rightsquigarrow H, TS'$ by E-TASK-DONE
  - $\langle \text{FINISH } f' \rangle \circ FS'$: $H, \{(f, \langle \text{FINISH } f' \rangle \circ FS')\} \uplus TS \rightsquigarrow H, \{(f, FS')\} \uplus TS$ by E-FINISH2
  - $\langle L, u, P \rangle^{l} \circ FS'$:

    Induction on u:

    See following pages

$F = \langle L, \text{let } x = \text{task}(b')\{x \Rightarrow t\} \text{ in } s, P \rangle^{l}$

- (5) $H \vdash \langle L, u, P \rangle^{l}$ by (2), T-FS-A, T-FS-NA
  $H \vdash \Gamma; L$ by prev, T-FRAME1
  $H \vdash \Gamma; L; b'$ by prev, WF-ENV, (6)

  $\Gamma; a \vdash \text{let } x = \text{task}(b')\{x \Rightarrow t\} \text{ in } s : \sigma$ by (5), T-Frame 1
  $\Gamma; a \vdash \text{task}(b')\{x \Rightarrow t\} : C$ by ", T-Let
- (7) $\Gamma; a \vdash b' : Q \triangleright \text{Box}[C]$ by ", T-Task
- (6) $b' \in dom(\Gamma)$ by ", T-Var
- (8) $L(b') = null \lor L(b') = b(o, p)$ by ", WF-Var, (7)
  ↯ typeof(H, o) <: C missing

Cases of $L(b')$ by (8)

- null : Case III: Stuck
- $b(o, p)$: E-Task

  $H, \{T\} \uplus TS \rightsquigarrow H, \{(f, \langle L[x \mapsto \text{task}(b(o,p), t)], s, P \rangle^{l} \circ FS')\} \uplus TS'$

$F = \langle L, \text{async}(y)\{s\}, \{p\} \uplus P \rangle'$

Show:

IV $L(y) = \text{task}(b(o,p),t)$

- (5) $H \vdash F$ by (2), T-FS-A, T-FS-NA
  $H \vdash \Gamma; L$ by prev, T-FRAME1
  (7) $\forall x \in dom(\Gamma).\ H \vdash \Gamma; L; x$

- (6) $\Gamma; a \vdash \text{async}(y)\{s\} : \sigma$ by (5), T-FRAME1
  (9) $\Gamma; a \vdash y : Q \triangleright Task[C]$ by prev, T-ASYNC
  (10) $y \in dom(\Gamma)$ by prev, T-VAR ← How do we know that $\Gamma(y) = Q \triangleright Task[C]$?
  (8) $H \vdash \Gamma; L; y$ by prev, (7)
  $Perm[Q] \in \Gamma$ by (6), T-ASYNC

  Is this covered by WFness?

- $L(y) = null \lor L(y) = \text{task}(b(o,p),t) \land \Gamma(y) = Q \triangleright Task[C] \land typeof(H,o) <: C$ by WF-VAR, (8)

  $L(y) = null \lor L(y) = \text{task}(b(o,p),t) \land typeof(H,o) <: C$

  $L(y) = null \lor L(y) = \text{task}(b(o,p),t)$

  Shows IV or III (null)

V $p \in P$

$\Gamma(y) = Q \triangleright Task[C] \land L(y) = \text{task}(b(o,p),t) \land Perm[Q] \in \Gamma \Rightarrow p \in P$ by WF-PERM, $\underset{?}{\underline{\gamma(Q) \in P}}$

$p \in P$ by IV, (10), (9) + red comment by (9)

$F = \langle L_1 \text{ let } x = \text{finish } \{t\} \text{ in } s \text{ , } P \rangle^{c}$

Show: $\emptyset$

Reducable Taskset

Assume

ID-ordering:

(1) $\forall (f, F \circ FS) \in TS.\ F = \langle \text{FINISH } f' \rangle \rightarrow f < f'$

ID-uniqueness:

$\forall T, T' \in TS.\ T \neq T' \wedge T = (\_, \langle \text{FINISH } f \rangle \circ \_) \wedge T' = (\_, \langle \text{FINISH } f' \rangle \circ \_) \rightarrow f \neq f'$

(2) $TS \neq \emptyset \wedge TS$ finite

Show: $\exists (\_, F \circ FS) \in TS, f.\ F = \langle \text{FINISH } f \rangle \wedge \nexists (f, FS') \in TS$
$\vee F \neq \langle \text{FINISH } f \rangle$

Proof by contradiction:

Assume $\forall (\_, F \circ FS) \in TS.\ \exists f.\ F = \langle \text{FINISH } f \rangle \wedge \exists (f, FS') \in TS$

$\forall T \in TS.\ \exists T' \in TS, f, g.\ (f, -) = T \wedge (g, -) = T' \wedge f < g$ by ", (1)

↯ Contradiction: (1), (2) implies that a maximal element exist in TS