Reducable Taskset

Assume

H, TS, WS ok

(1) $\forall ((\ell,\_,\_),\ell') \in WS.\ \ell < \ell'$

(2) $WS \neq \emptyset$

Show: $TS = \emptyset \Rightarrow \exists (T,\ell) \in WS.$

$H, \emptyset, \{(T,\ell)\} \uplus WS \rightsquigarrow H, \{T\}, WS$

Proof by contradiction:

Assume $TS = \emptyset \land \nexists (T,\ell) \in WS.\ \nexists (\ell,\_,\_) \in TS \land \nexists ((\ell,\_,\_),\_) \in WS$

$\Rightarrow \forall (T,\ell) \in WS.\ \cancel{\exists (\ell,\_,\_) \in TS} \lor \exists ((\ell,\_,\_),\_) \in WS$

$TS = \emptyset$

$\Rightarrow \forall (T,\ell) \in WS.\ \exists ((\ell,\_,\_),\_) \in WS$

$\lightning$ Contradiction: (1),(2) implies that a maximal element exist in TS

More rigorously: Start with element, take a step to its blocker, repeat.