MAT 1440 Exam 1 will cover Sections 4.1, 4.2, 4.3, and 4.4
Please show all work and clearly state answers as demonstrated in class.
You will turn in this exam review on the day of the exam.
This is not a practice exam. I have selected example problems from the textbook that are representative of the types of problems that we encountered in this unit. In addition to completing this review, you should review examples from notes, practice problems from classwork, and homework problems from Connect Math to prepare for the exam.
I will post my work and answers. Please check and correct your work before taking the exam and before turning in your review.

1. Complete the table.

θ in degrees	θ in radians	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$0°$	0	0	1	0	undefined	1	undefined
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$90°$	$\frac{\pi}{2}$	1	0	undefined	1	undefined	0

Section 4.1

Conversions

2. Convert $96°28'$ to decimal degrees. Round to 2 decimal places.

$$96°28' = 96° + 28\,\cancel{\text{min}} \cdot \frac{1°}{60\,\cancel{\text{min}}}$$

$$\approx 96.47°$$

3. Convert $225.24°$ to DMS form. Round to the nearest second if necessary.

$$225.24° = 225° + 0.24° \cdot \frac{60\text{ min}}{1°}$$

$$= 225° + 14.4\text{ min}$$

$$= 225°\,14' + 0.4\text{ min} \cdot \frac{60\text{ sec}}{1\text{ min}}$$

$$= 225°\,14'\,24''$$

4. Convert $124°$ to radians. Give the answer in exact form in terms of π.

$$124° = \overset{31}{\cancel{124°}} \cdot \frac{\pi}{\underset{45}{\cancel{180°}}}$$

$$= \frac{31\pi}{45}$$

5. Convert $\frac{5\pi}{8}$ radians to decimal degrees. Round to 1 decimal place if necessary.

$$\frac{5\pi}{8} = \frac{5\cancel{\pi}}{8} \cdot \frac{180°}{\cancel{\pi}}$$

$$= 112.5°$$

Coterminal Angles

6. Final an angle between $0°$ and $360°$ that is coterminal to $745°$.

$$745° - 2(360°) = 745° - 720°$$
$$= 25°$$

7. Find an angle between 0 and 2π that is coterminal to $-\frac{19\pi}{6}$

$$-\frac{19\pi}{6} + 4\pi = -\frac{19\pi}{6} + \frac{24\pi}{6}$$
$$= \frac{5\pi}{6}$$

Arc Length, Area of a Sector, Angular and Linear Speed

8. A unicycle with a wheel diameter of 24 in. moves through an angle of $140°$. What distance does a point on the edge of the wheel move? Round the answer to the nearest tenth of an inch.

$r = 12\text{ in}$

$\theta = \cancel{140}^{7}° \cdot \frac{\pi}{\cancel{180°}_{9}}$

$\theta = \frac{7\pi}{9}$

$S = r\theta$

$S = \cancel{12}^{4}\text{ in} \cdot \frac{7\pi}{\cancel{9}_{3}}$

$= \frac{(4\text{ in})(7\pi)}{3}$

$\approx 29.3\text{ in}$

A point on the edge of the wheel moves about 29.3 inches.

9. A pulley is 1.5 ft in diameter. Find the distance the load will move if the pulley is rotated $750°$. Find the exact distance in terms of π and then round the answer to the nearest tenth of a foot.

$r = \frac{1.5\text{ ft}}{2} = 0.75\text{ ft} = \frac{3}{4}\text{ ft}$

$\theta = \cancel{750}^{25}° \cdot \frac{\pi}{\cancel{180°}_{6}} = \frac{25\pi}{6}$

$S = r\theta$

$S = (0.75\text{ ft})\left(\frac{25\pi}{6}\right)$

$S = 3.125\pi\text{ ft}$

$S \approx 9.8\text{ ft}$

using $\frac{3}{4}$ ft →

$S = r\theta$

$S = \left(\frac{\cancel{3}^{1}}{4}\text{ ft}\right)\left(\frac{25\pi}{\cancel{6}_{2}}\right)$

$S = \frac{25\pi}{8}\text{ ft}$

$S \approx 9.8\text{ ft}$

180 | 18 | 6

10 | 750 | 75 | 25 | 3

GCF 15 30

10. A spinning disk has radius of 10 in. and rotates at 2800 revolutions per minute. For a point at the edge of the disk,
 a) Find the exact value of the angular speed. → $\omega = \frac{\theta}{t}$

$$\omega = \frac{2800\,\cancel{\text{rev}}}{1\text{ min}} \cdot \frac{2\pi}{1\,\cancel{\text{rev}}} = 5600\pi/\text{min}$$

 b) Find the linear speed. Round the answer to the nearest inch per minute.

$v = r\omega$ →

$$v = (10\text{ in})(5600\pi/\text{min})$$
$$v = 56000\pi\text{ in/min}$$
$$v \approx 175{,}929\text{ in/min}$$

11. A round pie 10 in. in diameter is cut into a slice with a $30°$ angle. Find the exact area of the slice, then round the result to the nearest tenth of a square inch.

$r = 5\text{ in}$

$\theta = 30° \cdot \frac{\pi}{180°}$

$\theta = \frac{\pi}{6}$

$A = \frac{1}{2}r^2\theta$

$A = \left(\frac{1}{2}\right)(5\text{ in})^2\left(\frac{\pi}{6}\right)$

$= \left(\frac{1}{2}\right)(25\text{ in}^2)\left(\frac{\pi}{6}\right)$

$= \frac{25\pi}{12}\text{ in}^2$

$\approx 6.5\text{ in}^2$

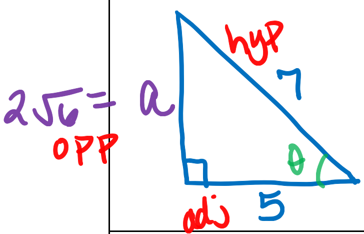
Exact Area: $\frac{25\pi}{12}\text{ in}^2$

Approximate Area: 6.5 in^2

Section 4.3

Using Right Triangles to Find Function Values

12. If $\cos\theta = \frac{5}{7}$, find $\csc\theta$ and $\tan\theta$.



$a^2 + b^2 = c^2$

$a^2 + (5)^2 = (7)^2$

$a^2 = 49 - 25$

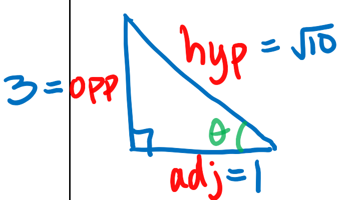
$a = \sqrt{24}$

$a = 2\sqrt{6}$

$\csc\theta = \frac{hyp}{opp} = \frac{7}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{7\sqrt{6}}{12}$

$\tan\theta = \frac{opp}{adj} = \frac{2\sqrt{6}}{5}$

13. If $\tan\theta = 3$, find $\sin\theta$ and $\sec\theta$.



$\tan\theta = \frac{3}{1}$ ← opp, ← adj

$c^2 = (3)^2 + (1)^2$

$c^2 = 9 + 1$

$c = \sqrt{10}$

$\sin\theta = \frac{opp}{hyp} = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$

$\sec\theta = \frac{hyp}{adj} = \frac{\sqrt{10}}{1} = \sqrt{10}$

Using Trigonometric Identities

14. Given $\sin\theta = \frac{99}{101}$ and $\cos\theta = \frac{20}{101}$, use the reciprocal and quotient identities to find the values of the other trigonometric functions of θ.

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{99}{101}}{\frac{20}{101}}$

$= \frac{99}{101} \cdot \frac{101}{20}$

$= \frac{99}{20}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{20}{99}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{101}{99}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{101}{20}$

15. Given $\cos\theta = \frac{40}{41}$. Use an appropriate Pythagorean identity to the value of $\sin\theta$ for an acute angle θ.

QI so $\sin\theta > 0$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta + \left(\frac{40}{41}\right)^2 = 1$

$\sin^2\theta = 1 - \frac{1600}{1681}$

$\sin^2\theta = \frac{1681}{1681} - \frac{1600}{1681}$

$\sin\theta = \sqrt{\frac{81}{1681}}$

$\sin\theta = \frac{9}{41}$

16. Given $\tan\frac{\pi}{12} = 2 - \sqrt{3}$. Find a cofunction of another angle with the same function value.

$\tan\frac{\pi}{12} = \cot\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$

$= \cot\frac{5\pi}{12}$

$\tan\frac{\pi}{12} = \cot\frac{5\pi}{12} = 2 - \sqrt{3}$

17. Given that $\cos x \approx 0.6691$, approximate $\sin x$ and $\sin\left(\frac{\pi}{2} - x\right)$. Round to four decimal places.

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x + (0.6691)^2 = 1$

$\sin^2 x = 1 - (0.6691)^2$

$\sin x = \sqrt{1 - (0.6691)^2}$

$\sin x \approx 0.7432$

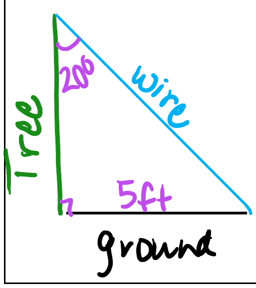
$\sin\left(\frac{\pi}{2} - x\right) = \cos x \approx 0.6691$

$\sin x \approx 0.7432$

$\sin\left(\frac{\pi}{2} - x\right) \approx 0.6691$

Application of Right Triangle Trigonometry

18. A newly planted tree is anchored by a covered wire running from the top of the tree to a post in the ground 5 ft from the base of the tree. If the angle between the wire and the top of the tree is $20°$, what is the length of the wire? Round to the nearest foot.



Let x = length of wire

$\sin 20° = \frac{5}{x}$

$\frac{x \cdot \sin 20°}{\sin 20°} = \frac{5}{\sin 20°}$

$x \approx 15$

The length of the wire is about 15 ft.

Section 4.2

Using the Unit Circle

19. The real number t corresponds to the point $P\left(-\frac{3\sqrt{5}}{7}, \frac{2}{7}\right)$ on the unit circle. Evaluate the six trigonometric functions of t.

$\sin t = y = \frac{2}{7}$

$\cos t = x = -\frac{3\sqrt{5}}{7}$

$\tan t = \frac{y}{x} = \frac{2}{\cancel{7}} \cdot \frac{-\cancel{7}}{3\sqrt{5}}$

$= \frac{-2}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= -\frac{2\sqrt{5}}{15}$

$\csc t = \frac{7}{2}$

$\sec t = -\frac{7}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{-7\sqrt{5}}{15}$

$\cot t = \frac{-3\sqrt{5}}{\cancel{7}} \cdot \frac{\cancel{7}}{2} = \frac{-3\sqrt{5}}{2}$

Identify the ordered pair on the unit circle corresponding to each real number t.

20. $t = \frac{2\pi}{3}$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$	21. $t = \frac{5\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$	22. $t = \frac{11\pi}{6}$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Use the unit circle and the period of the function to evaluate the function or state that the function is undefined at the given value.

23. $\cos\frac{15\pi}{2} = \cos\left(\frac{15\pi}{2} - 3(2\pi)\right) = \cos\left(\frac{15\pi}{2} - \frac{12\pi}{2}\right) = \cos\frac{3\pi}{2} = 0$ $(0,-1)$	24. $\cot 540° = \cot(540° - 360°) = \cot 180°$ undefined $(-1,0)$	25. $\csc(-240°) = \csc(-240° + 360°) = \csc(120°) = \frac{2\sqrt{3}}{3}$ $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Use the even-odd and periodic properties of the trigonometric functions to simplify.

26. $\tan(\pi - t) \cdot \cos(2\pi - t)$ $= \tan(-t+\pi) \cdot \cos(-t+2\pi)$ $= \tan(-t) \cdot \cos(-t)$ $= -\tan t \cdot \cos t$ $= -\frac{\sin t}{\cancel{\cos t}} \cdot \cancel{\cos t}$ $= -\sin t$	27. $2\cot(-\theta) - \cot(-\theta + \pi)$ $= -2\cot(\theta) - \cot(-\theta)$ $= -2\cot\theta + \cot\theta$ $= -\cot\theta$

Section 4.4

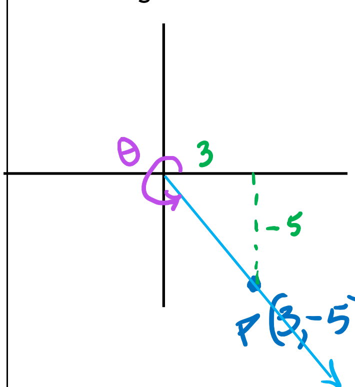
Finding Reference Angles

Find the reference angle for the given angle.

28. $-\frac{10\pi}{3} = \theta$ Coterminal: $-\frac{10\pi}{3} + \frac{6\pi}{3} = -\frac{4\pi}{3}$ $-\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{2\pi}{3}$ $\frac{\pi}{3} = \theta'$ $\theta' = \frac{\pi}{3}$	29. $\frac{5\pi}{6} = \theta$ $\theta' = \pi - \theta$ $\theta' = \pi - \frac{5\pi}{6}$ $= \frac{\pi}{6}$	30. $260° = \theta$ $\theta' = \theta - 180°$ $\theta' = 260° - 180°$ $= 80°$
31. $\frac{7\pi}{4} = \theta$ $\theta' = 2\pi - \theta$ $\theta' = \frac{8\pi}{4} - \frac{7\pi}{4}$ $= \frac{\pi}{4}$	32. $-200°$ Coterminal: $-200° + 360° = 160°$ $\theta' = 180° - 160° = 20°$	33. $750°$ Coterminal: $750° - 720° = 30°$ $\theta' = 30°$

Evaluating Trigonometric Functions

34. Let $P(3,-5)$ be a point on the terminal side of angle θ drawn in standard position. Find the values of the six trigonometric functions of θ.



$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(3)^2 + (-5)^2}$

$r = \sqrt{9+25}$

$r = \sqrt{34}$

$x = 3$

$y = -5$

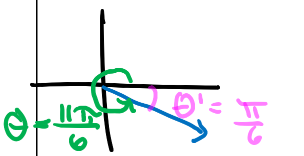
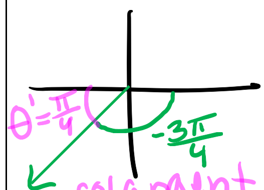
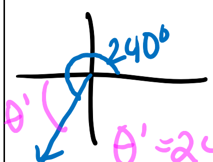
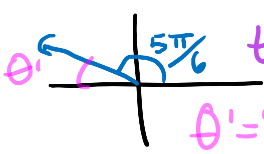
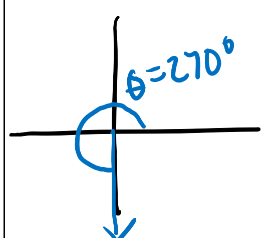
$\sin\theta = \frac{-5}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$ $\qquad \csc\theta = -\frac{\sqrt{34}}{5}$

$\cos\theta = \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$ $\qquad \sec\theta = \frac{\sqrt{34}}{3}$

$\tan\theta = -\frac{5}{3}$ $\qquad \cot\theta = -\frac{3}{5}$

$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

Use reference angles to find the exact value.

35. $\cos\frac{11\pi}{6}$ $= \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$  $\theta' = 2\pi - \frac{11\pi}{6} = \frac{\pi}{6}$	36. $\sin\left(-\frac{5\pi}{3}\right)$ $= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ Coterminal: $-\frac{5\pi}{3} + 2\pi = \frac{\pi}{3}$
37. $\tan\frac{\pi}{2}$ is undefined  	38. $\cot\left(-\frac{3\pi}{4}\right)$ $= \cot\frac{\pi}{4} = 1$  cotangent is positive in Q III
39. $\csc(-120°)$ $= -\csc 60° = -\frac{2\sqrt{3}}{3}$ Coterminal: $-120° + 360° = 240°$ $\csc\theta$ is negative in Q III  $\theta' = 240° - 180° = 60°$	40. $\sec 240°$ $= -\sec 60° = -2$ $\sec\theta$ is neg. in Q III
41. $\tan\frac{17\pi}{6}$ $= -\tan\frac{\pi}{6} = -\frac{\sqrt{3}}{3}$ Coterminal: $\frac{17\pi}{6} - \frac{12\pi}{6} = \frac{5\pi}{6}$  $\tan\theta$ is neg. in Q II $\theta' = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$	42. $\sec 270°$ is undefined 
43. Given $\tan\theta = -\frac{2}{3}$ and $\sin\theta > 0$, find $\sec\theta$. since $\tan\theta < 0$ & $\sin\theta > 0$, θ in Q II this means $\sec\theta < 0$ $\tan\theta = -\frac{2}{3}$ ← $y = 2$, ← $x = -3$ $r = \sqrt{(-3)^2 + (2)^2}$ $r = \sqrt{13}$ $\sec\theta = -\frac{\sqrt{13}}{3}$	44. Given $\sin\theta = -\frac{60}{61}$ and θ in Quadrant III, find $\cot\theta$. ↑ positive $\sin\theta = \frac{-60}{61}$ ← y, ← r $x^2 + y^2 = r^2$ $x^2 + (-60)^2 = (61)^2$ $x^2 = 3721 - 3600$ $x^2 = 121$ $x = \pm 11$ b/c Q III $\cot\theta = \frac{11}{60}$

$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$