5 a) $$\sum_{x=0}^{N-1} e^{-2\pi i k x/N} = \sum_{x=0}^{N-1} \left(e^{-2\pi i k/N}\right)^x$$

$$= \sum_{x=0}^{N-1} \alpha^x = S$$

$$\Rightarrow \alpha S = \sum_{x=1}^{N} \alpha^x$$

$$\Rightarrow S - \alpha S = 1 - \alpha^N$$

$$\boxed{\Rightarrow S = \frac{1-\alpha^N}{1-\alpha} = \frac{1-e^{-2\pi i k}}{1-e^{-2\pi i k/N}}}$$

b) as $k \to 0$:

$$\sum_{x=0}^{N-1} e^0 = N$$

for integer k not a multiple of N:

$$\frac{1-e^{-2\pi i k}}{1-e^{-2\pi i k/N}} = \frac{1-\cos(2\pi k)+i\sin(2\pi k)}{1-\cos(2\pi k/N)+i\sin(2\pi k/N)}$$

$$= \frac{1-1+0}{\text{"} \qquad \text{"}} = 0$$

c) $F(k) = \sum_{x=0}^{N-1} \frac{(e^{i\theta} - e^{-i\theta})}{2i} \cdot e^{-2\pi i k x/N}$

where $\theta = 2\pi l x/N$

$\Rightarrow F(k) = \frac{1}{2i} \sum_{x=0}^{N-1} \left[e^{2\pi x/N(l-k)} - e^{-2\pi x/N(l+k)} \right]$

where l is a constant