PHYS 512

William Frost - 260767513 - HW1

1. a) $\boxed{\text{we have 4 taylor expansions}}$

$$f(x+\delta) = f + f'\delta + \frac{1}{2}f''\delta^2 + \frac{1}{6}f'''\delta^3 + \dots$$

$$f(x-\delta) = f - f'\delta + \frac{1}{2}f''\delta^2 - \frac{1}{6}f'''\delta^3 + \dots$$

$$f(x+2\delta) = f + 2f'\delta + 2f''\delta^2 + \frac{4}{3}f'''\delta^3 + \dots$$

$$f(x-2\delta) = f - 2f'\delta + 2f''\delta^2 - \frac{4}{3}f'''\delta^3 + \dots$$

$\boxed{\text{We desire to use a linear combination of } f(x \pm \delta) \text{ and } f(x \pm 2\delta) \text{ to define } f'\delta}$

$$\Rightarrow f'\delta = c_1 f(x+\delta) + c_2 f(x-\delta) + c_3 f(x+2\delta) + c_4 f(x-2\delta)$$

$$= (c_1 + c_2 + c_3 + c_4) f + (c_1 - c_2 + 2c_3 - 2c_4) f'\delta$$

$$+ \left(\frac{1}{2}c_1 + \frac{1}{2}c_2 + 2c_3 + 2c_4\right) f''\delta^2 + \left(\frac{1}{6}c_1 - \frac{1}{6}c_2 + \frac{4}{3}c_3 - \frac{4}{3}c_4\right) f'''\delta$$

$\boxed{\text{We need:}}$

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 2 & -2 \\ 1/2 & 1/2 & 2 & 2 \\ 1/6 & -1/6 & \frac{4}{3} & -4/3 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Solving for c_i gives $c_1 = \frac{2}{3}$, $c_2 = \frac{-2}{3}$, $c_3 = \frac{-1}{12}$, $c_4 = \frac{1}{12}$

$$\boxed{\Rightarrow f' = \frac{\frac{2}{3}\left[f(x+\delta) - f(x-\delta)\right] + \frac{1}{12}\left[f(x-2\delta) - f(x+2\delta)\right]}{\delta}}$$

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I 6) The error in our estimation can be seen as

$$f' = f'_{true} + \frac{2}{3}\left[\frac{1}{24}\left(f^{(4)}\delta^3 - f^{(4)}\delta^3\right)\right] + \frac{1}{12}\left[\frac{8}{24}\left(f^{(4)}\delta^3 - f^{(4)}\delta^3\right)\right] \rightarrow = 0$$

$$+ \frac{2}{3}\left[\frac{2}{120} f^{(5)}\delta^4\right] + \frac{1}{12}\left[\frac{2\cdot 16}{120} f^{(5)}\delta^4\right] + g\epsilon f/\delta$$

$$= f'_{true} + \frac{1}{30} f^{(5)}\delta^4 + g\epsilon f/\delta$$

$$\Rightarrow \frac{dE}{d\delta} = \frac{4}{30} f^{(5)}\delta^3 - g\epsilon f/\delta^2 = 0$$

$$\Rightarrow \delta^5 = \frac{15}{2} g\epsilon f/f^{(5)}$$

$$\boxed{\Rightarrow \delta \approx \sqrt[5]{\frac{15}{2}\epsilon \frac{f}{f^{(5)}}}}$$

Therefore, for single precision $\delta \approx 1.5\cdot 10^{-8/5} \approx 10^{-2}$

double $\Longrightarrow$ $\delta \approx 1.5\cdot 10^{-16/5} \approx 10^{-3}$

For $\exp(0.01x)$:

Single: $\delta \approx 1.5\left(10^{-8}\cdot 10^{10}\right)^{1/5} = 1.5\cdot 10^{2/5} \approx 3.8$

double: $\delta \approx 1.5\left(10^{-16}\cdot 10^{10}\right)^{1/5} = 1.5\cdot 10^{-4/5} \approx 10^{-1}$