William Daniels AMS 326 exam two writeup

March 21, 2023

All of my source files can be found in this github repositiory:
https://github.com/William-J-Daniels/DanielsAms326.git

A detailed README on how the repositiory is used is there. The commit that represents the state of the repository upon submission of the assignment is the one at 6:05 PM on March 21 (I can't give the hash before I commit).

Problem One

The problem is to numerically approximate the Airy function $Ai(x)=\frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3+x}t\right)dt$ for various values of x to four decimals precision.

Algorithms

All of my integration implimentations inherit from the file class `Integrator`, defined in `Integrals/include/integrator.h` and `Integral/src/integrator.cpp`. Each integrator is threaded and uses the same interface, which consists of an evaluate, converge, and reset method, where evaluate evaluates the integral with a given number of subdivisions, converge attempts to reach a convergence driven by `Integrator::precision`, and reset sets all the internal values back to their initialized state.

Simpon

Simpson's method approximates an integral by subdividing a range and approximating the function as a parabola for that range. Since we know the integral of a parabola analytically, this enables us to construct a weighted sum that approximates the integral.

Midpoint

The midpoint rule approximates an integral as the sum of the area of some number of boxes whose heights are determined by the value of the integrand at the midpoint of the division.

Trapezoid

The trapezoid rule approximates an integral as the sum of the area of linear interpolations of the integrand at some number of subdivions.

Results

The following table details my results. I've added a row for the approximation from Maple since my results were not agreeing with the "I got" row. We must have different ages.

x	-3	-2	-1	0	1	2	3
Ai(x) given	-0.378814	0.227407	0.535561	0.355028	0.135292	0.034924	0.006591
Ai(x) Simp	-0.378105	0.226889	0.535421	0.355697	0.134701	0.0349051	0.0072014
Ai(x) Mid	-0.378105	0.226889	0.535421	0.355697	0.134701	0.0349051	0.0072014
Ai(x) Trap	-0.378105	0.226889	0.535421	0.355697	0.134701	0.0349051	0.0072014
Ai(x) Maple	-0.378105	0.226889	0.535421	0.355719	0.134701	0.0349051	0.0072011