William Daniels AMS 326 final exam writeup

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All of my source files can be found in this github repositiory:
https://github.com/William-J-Daniels/DanielsAms326.git

A detailed README on how the repositiory is used is there.

Problem One

Description

This is the Buffon's Needle problem, in which we simulate probabilites of dropping a needle onto a grid of wires without the needle touching the wire

Algorithms

We use a vector of x position, y position, and angle to describe the needle and assume that the position and angle are uniformly distributed. By the translational symmetry of the problem, we find that it is sufficient to simulate a needle dropping into a single square.

The sinulation consists of generating three random numbers to describe the pin drop, then using those parameters to determine if the pin touches the wire or not. This is done by checking if either extreame of the pin lies beyond the boundary of a box using simple right-triangles.

The entire implimentation is about 50 lines in `Final/examples/prob_one.cpp`.

Results

Here are the probabilites that a needle does not touch the wires for the specified grid-sizes, simulating one million needle drops for each size.

	d=1	d=2	d=3
Probabilities	0.350773	0.541173	0.83658

Remarks

The assumptions made were sound and since the pRNG used is a simple one based on fmod, the code executes very quickly. Roundoff error is all but neglegable here.

Problem Two

Description

In this problem we simulate the performance of an initial investment of one million dollars over the course of a year.

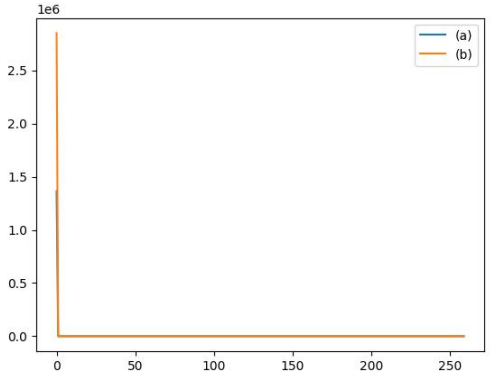
Algorithms

The change rate for each day is computed according to the given formula assiming an initial change rate of 0.0001. If the value of the investment increases, the result for the day is also multiplied by 0.9334, which is for the trader's cut. The data is written to the disk and plotted.

The implimentations are in `Final/examples/prob_two.cpp` and `Final/tools/F2.py`.

Results

I produced the following disapointing plot.



Remarks

This is obviously wrong. As far as I can tell, I was able to impliment the problem as I understood it, so I must have misunderstood the problem. I spent a significant time trying to fix it before realizing that I had spent too much.

Problem 3

Description

This problem is to solve the Airy BVP using the shooting method and the finite-differnce method.

Algorithms

Shooting method

One variation of the shooting method for solving BVPs is to create an array of initial conditions to test, and then apply bisection to that array. That is, when by the intermediate value theorem, when two array elements border the desired result, try using the mean of the initial conditions producing the array elements to get a more accurate solution. This is repeated until a convergence condition is met.

Finite element

I did not have time to impliment the methods...