Will Daniels; 112774725
February 10, 2023
Computational Astrophysics; AST 390
Homework 1

Contents

1 Second derivative

Consider the taylor expansions

$$f_{i+1} = f_i + \Delta x \left.\frac{df}{dx}\right|_i + \frac{1}{2}\Delta x^2 \left.\frac{d^2}{dx^2}\right|_i + \mathcal{O}\left(\Delta x^3\right) \tag{1}$$

$$f_{i-1} = f_i - \Delta x \left.\frac{df}{dx}\right|_i + \frac{1}{2}\Delta x^2 \left.\frac{d^2}{dx^2}\right|_i + \mathcal{O}\left(\Delta x^3\right) \tag{2}$$

To find the second order approximation of the first derivative, we subtracted the two expressions. Now we add them so the second derivative term doesn't cancel.

$$f_{i+1} + f_{i-1} = 2f_i + \Delta x^2 \frac{d^2 f}{dx^2} + \mathcal{O}\left(\Delta x^3\right) \tag{3}$$

$$\frac{d^2 f}{dx^2} = \frac{f_{i+1} - f_i + f_{i-1}}{\Delta x^2} + \mathcal{O}\left(\Delta x^3\right) \tag{4}$$

As expected, the algorithm converges quadraticly. Ten iterations are presented in table 1.

Table 1: Ten iterations of the algorithm. The initial step size is 0.5 and it reduces by a factor of two each time

Iteration	Error	Ratio
1	0.00206261	–
2	0.000518884	0.252
3	0.000129924	0.250
4	3.24936e-5	0.250
5	8.12423e-6	0.250
6	2.0311e-6	0.250
7	5.07778e-7	0.250
8	1.26944e-7	0.250
9	3.17375e-8	0.250
10	7.94152e-9	0.250

2 Roundoff error

Let's start by finding the equivalent expression with no floating point subtractions.

$$\sqrt{x^2+1}\left(\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}\right) \tag{5}$$

$$\frac{x^2+1-1+1}{\sqrt{x^2+1}+1} \tag{6}$$

$$\frac{x^2}{\sqrt{x^2+1}+1} \tag{7}$$

The results of using each form are given in table 2. Clearly the modified equation is more accurate.

Table 2: Comparison of equivalent forms at specified inputs

Evaluation point	Original	Equivalent
1.0e-6	2	1e-12
1.0e-7	2	1e-14
1.0e-8	2	1e-16