1.6 Rules of Inference

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Outline

- What is rule of inference
- Rules of inference for Propositional Logic
- Rules of inference for First-order Predicate Logic
- Using Rules of Inference

推理(Inference)

- 数理逻辑的主要任务是提供一套推理规则，利用推理规则从给定的前提出发，推导出一个结论来。
- 前提是一些已知的公式。
- 结论是从前提出发，应用推理规则推出的公式。
- 这推理过程称为演绎（Calculus）或形式证明。

关于重言蕴含⇒

- 定义：
  - A、C是两个公式，如果A→C是重言式（tautologies），则称A重言蕴含C，或称A能逻辑地推出 C，记作A ⇒ C
- 说明：→与⇒是有区别的
  - →是联结词，A→C仍然是公式，公式A→C，当且仅当A真C假时才为假
  - ⇒是公式间的关系符，描述了两个公式间的关系，只能说A⇒C式成立或不成立
  - 重言蕴含A⇒C要成立的充要条件是对一切赋值，如果使A为真，则C也必须为真（此时A→C 为永真式/重言式）。

重言蕴含⇒

- **定义：**
  - 设A_1、A_2、…、A_m，C是公式，如果($A_1 \wedge A_2 \wedge \ldots \wedge A_m$）→C是**重言式**，则称C是前提集合{$A_1$、$A_2$、…、$A_m$}的有效结论，或称由{$A_1$、$A_2$ 、…、A_m}逻辑地推论出C，或称{A_1、A_2、…、A_m} 重言蕴含C。
  - 由⇒的定义，可以记作（$A_1 \wedge A_2 \wedge \ldots \wedge A_m$）⇒ C。

Rules of Inference for PL

- Many of the tautologies are rules of inference. They have the form
  - $P_1 \wedge P_2 \wedge \ldots\ldots \wedge P_n \Rightarrow Q$
- where
  - P_i are called the *hypotheses or premises*（前提）
- and
  - Q is the *conclusion*（结论）.

***Q* *logically follows* from P_1 , P_2, …, P_n**

Rules of Inference

- As a rule of inference they take the symbolic form:

$$\begin{array}{l} P_1 \\ P_2 \\ \cdot \\ \cdot \\ P_n \\ \therefore Q \end{array}$$

- where $\therefore$ means 'therefore' or 'it follows that.'

Rules of Inference

- **Note:**
  - To "prove the theorem" means to show that the implication is a tautology.
  - not trying to show that Q (the conclusion) is true, but only that Q will be true if all the P_i are true.
  - The proof does not show that Q is true, but simply shows that Q has to be true if the P_i are all true.
  - mathematical proofs often begin with the statement "suppose that P_1 , P_2, . . . , and P_n are true" and conclude with the statement "therefore, Q is true ".

Rules of Inference

- **Arguments** based on tautologies represent universally correct methods of reasoning.
- Their ***validity*** depends only on the form of the statements involved and not on the truth values of the variables they contain. Such arguments are called ***rules of inference***.

- The various steps in a mathematical proof of a theorem must follow from the use of various rules of inference.
- **A mathematical proof of a theorem** must begin with the ***hypotheses***, proceed through various steps, each justified by some ***rule of inference***, and arrive at the ***conclusion***.

假言推理规则(MODUS PONENS)

- The tautology $P \wedge (P \rightarrow Q) \rightarrow Q$ becomes

$$\boldsymbol{P}$$
$$\boldsymbol{P \rightarrow Q}$$
$$\therefore \boldsymbol{Q}$$

- This means that whenever P is true and $P \rightarrow Q$ is true we can conclude logically that Q is true.

Famous rules of inference

- P
  $\therefore P \vee Q$ **Addition（析取引入规则）**

- $P \wedge Q$
  $\therefore P$ **Simplification（合取消去规则）**

- $\sim Q$
  $P \rightarrow Q$
  $\therefore \sim P$ **Modus Tollens （拒取式）**

- $P \rightarrow Q$
  $Q \rightarrow R$
  $\therefore P \rightarrow R$ **Hypothetical syllogism（假言三段论）**

Famous rules of inference

- $P \vee Q$
  $\sim P$
  $\therefore Q$ **Disjunctive syllogism （析取三段论）**

- P
  Q
  $\therefore P \wedge Q$ **Conjunction （合取引入规则）**

- $P \vee Q$
  $\sim P \vee R$
  $\therefore Q \vee R$ **Resolution（消解规则）**

- $(P \rightarrow Q) \wedge (R \rightarrow S)$
  $P \vee R$
  $\therefore Q \vee S$ **Constructive dilemma (二难推论)**

Formal Proofs

- To prove an argument is valid or the conclusion follows *logically* from the hypotheses:
  - Assume the hypotheses are true.
  - Use the **rules of inference** and **logical equivalences** to determine that the conclusion is true.

Formal Proofs

Example: From the single proposition $p \wedge (p \rightarrow q)$
Show that q is a conclusion.

Solution:

Step	**Reason**
1. $p \wedge (p \rightarrow q)$	Premise
2. p	Simplificaiton using (1)
3. $p \rightarrow q$	Simplificaiton using (1)
4. q	Modus Ponens using (2) and (3)

Formal Proofs

- **Example:** Consider the following logical argument:
  - *If horses fly or cows eat artichokes, then the mosquito is the national bird. If the mosquito is the national bird then peanut butter takes good on hot dogs. But peanut butter tastes terrible on hot dogs. Therefore, cows don't eat artichokes.*

- Assign propositional variables to the component propositions in the argument:
  - f Horses fly
  - a Cows eat artichokes
  - m The mosquito is the national bird
  - p Peanut butter tastes good on hot dogs

Formal Proofs

- Represent the formal argument using the propositions

  $1. (f \vee a) \to m$

  $2. m \to p$

  $3. \sim p$

  $\therefore \sim a$

- Use the hypotheses 1., 2., and 3. and the above rules of inference and any logical equivalences to construct the proof.

Formal Proofs

$(f \vee a) \to m,\ m \to p,\ \sim p$
$\therefore \sim a$

- **Proof:**

Assertion	**Reasons**
1.$(f \vee a) \to m$	Hypothesis 1.
2.$m \to p$	Hypothesis 2.
3.$(f \vee a) \to p$	steps 1 and 2 and ***hypothetical syll***.
4.$\sim p$	Hypothesis 3.
5.$\sim(f \vee a)$	steps 3 and 4 and ***modus tollens***
6.$\sim f \wedge \sim a$	step 5 and ***De Morgan's***
7.$\sim a$	step 7 and ***simplification***

- Q. E. D.

逻辑推理只关注推理过程的正确性
并不关注句子本身的含义是否正确

Formal Proofs

- **Example 6:**
- Suppose we have the following premises:
  - "It is not sunny and it is cold."
  - "We will swim only if it is sunny."
  - "If we do not swim, then we will canoe."
  - "If we canoe, then we will be home early."
- Given these premises, prove the theorem **"We will be home early"** using inference rules.

Formal Proofs

- Choose propositional variables:
  - p : "It is sunny."
  - q : "It is cold."
  - r : "We will swim."
  - s : "We will canoe."
  - t : "We will be home early."

- Translation into propositional logic:
  - Then, the premises can be written as:
    (1) $\neg p \wedge q$ (2) $r \rightarrow p$ (3) $\neg r \rightarrow s$ (4) $s \rightarrow t$
  - Conclusion: $\boldsymbol{t}$

Formal Proofs

(1) $\neg p \wedge q$ (2) $r \rightarrow p$ (3) $\neg r \rightarrow s$ (4) $s \rightarrow t$

Step	Reason
1. $\neg p \wedge q$	Premise #1.
2. $\neg p$	Simplification of 1.
3. $r \rightarrow p$	Premise #2.
4. $\neg r$	Modus tollens on 2,3.
5. $\neg r \rightarrow s$	Premise #3.
6. s	Modus ponens on 4,5.
7. $s \rightarrow t$	Premise #4.
8. t	Modus ponens on 6,7.

Q. E. D.

Formal Proofs-Excercise

- **Example 9:**
- Show that the hypotheses $(p \wedge q) \vee r$ and $r \to s$ imply the conclusion $p \vee s$.

Step	Reason
1. $(p \wedge q) \vee r$	Premise #1.
2. $(p \vee r) \wedge (q \vee r)$	Distributive of 1.
3. $p \vee r$	$\wedge$ - of 2.
4. $r \to s$	Premise #2.
5. $\neg r \vee s$	Implication of 4.
6. $p \vee s$	Resolution on 3,5.

Q. E. D.

Rules of Inference for Predicate Logic

- **A⇒B**
  - 读作：A推出B（A逻辑蕴含B）
  - 含义：A为真时, B也为真
- A⇒B 当且仅当 A→B是永真式
- 例如：$\forall xF(x) \Rightarrow \exists xF(x)$

- 来源：
  - 1. 命题逻辑推理规则的代换实例
  - 2. 一阶逻辑等值式生成的推理规则
  - 3. 一阶逻辑蕴含式生成的推理规则
  - 4. 一阶逻辑量词相关的推理规则

1.代换实例

- **命题逻辑推理规则的代换实例**

例如: 假言推理规则（Modus Ponens）:

$(A\rightarrow B)\wedge A\Rightarrow B$

代入 $A=F(a)$, $B=G(a)$,

得到 $(F(a)\rightarrow G(a))\wedge F(a)\Rightarrow G(a)$

即：

$$\frac{\begin{array}{l}F(a)\\F(a)\rightarrow G(a)\end{array}}{\therefore\ G(a)}$$

1.代换实例

- 命题逻辑推理规则的代换实例

假言推理	Modus Ponens
析取引入	Addition
合取消去	Simplification
拒取式	Modus Tollens
合取引入	Conjunction
假言三段论	Hypothetical syllogism
析取三段论	Disjunctive syllogism
消解规则	Resolution
构造性两难	Constructive dilemma

2. 一阶逻辑等值式生成的推理规则

- **一阶逻辑等值式生成的推理规则**

  即由 $A \Leftrightarrow B$ 可得 $A \Rightarrow B$ 和 $B \Rightarrow A$

- 如：量词分配等值式:

$$\forall x(A(x) \wedge B(x)) \Leftrightarrow \forall x A(x) \wedge \forall x B(x)$$

可得

$$\forall x(A(x) \wedge B(x)) \Rightarrow \forall x A(x) \wedge \forall x B(x)$$

$$\forall x A(x) \wedge \forall x B(x) \Rightarrow \forall x(A(x) \wedge B(x))$$

3. 一阶逻辑蕴含式生成的推理规则

- 一阶逻辑蕴含式生成的推理规则
  - $\forall xA(x) \vee \forall xB(x) \Rightarrow \forall x(A(x) \vee B(x))$

    $\exists x(A(x) \wedge B(x)) \Rightarrow \exists xA(x) \wedge \exists xB(x)$

    $\forall x(A(x) \rightarrow B(x)) \Rightarrow \forall xA(x) \rightarrow \forall xB(x)$

    $\exists x(A(x) \rightarrow B(x)) \Rightarrow \forall xA(x) \rightarrow \exists xB(x)$

    $\exists xA(x) \rightarrow \forall xB(x) \Rightarrow \forall x(A(x) \rightarrow B(x))$

4.一阶逻辑量词相关推理规则

- 与量词相关的推理规则
  - **UI：** universal instantiation
  - **UG：** universal generalization
  - **EI：** existential instantiation
  - **EG：** existential generalization

UI规则(UNIVERSAL INSTANTIATION)

- 表示为

$$\frac{\forall x A(x)}{\therefore A(y)} \quad \text{或} \quad \frac{\forall x A(x)}{\therefore A(c)}$$

  - 注意1: y是自由变项；c是个体常项（论域内）
  - 注意2: 被消去量词的辖域是整个公式

- 例如

(1) $\forall x(F(x)\rightarrow G(x))$ Premise

(2) $F(a)\rightarrow G(a)$ (1)UI

UG规则(UNIVERSAL GENERALIZATION)

- 表示为

$$\frac{A(y)}{\therefore\ \forall x A(x)}$$

  - 注意1：y是自由变项
  - 注意2：量词加在整个公式前面

- 例如

  (1) $F(y)\rightarrow G(y)$ Premise

  (2) $\forall x(F(x)\rightarrow G(x))$ (1)UG

EI规则(EXISTENTIAL INSTANTIATION)

- 表示为

$$\frac{\exists x A(x)}{\therefore\ A(c)}$$

- 注意1：c是**特定的满足A**的个体常项
- 注意2：被消去量词的辖域是整个公式

- 例如

(1)　$\exists x(F(x)\land G(x))$　　Premise

(2)　$F(a)\land G(a)$　　(1)EI

EG规则(Existential Generalization)

- 表示为

$$\frac{A(c)}{\therefore\ \exists x A(x)}$$

  - 注意1：c是个体常项
  - 注意2：量词加在整个公式前面

- 例如

(1)	$F(c)\wedge G(c)$	Premise
(2)	$\exists x(F(x)\wedge G(x))$	(1)EG

Formal Proofs

- 前提：$\forall x(F(x)\rightarrow G(x))$，$F(a)$

  结论：$G(a)$

  证明:

(1) $F(a)$	Premise
(2) $\forall x(F(x)\rightarrow G(x))$	Premise
(3) $F(a)\rightarrow G(a)$	(2)UI
(4) $G(a)$	(1)(3)Modus Ponens

Q. E. D.

Formal Proofs

- **前提:** $\forall x(F(x) \rightarrow G(x))$，$\exists x F(x)$

  结论: $\exists x G(x)$

  证明1:

(1) $\exists x F(x)$	Premise
(2) $F(c)$	(1) EI
(3) $\forall x(F(x) \rightarrow G(x))$	Premise
(4) $F(c) \rightarrow G(c)$	(3) UI
(5) $G(c)$	(2)(4) Modus Ponens
(6) $\exists x G(x)$	(5) EG

Q. E. D.

Formal Proofs

- 证明2:

(1) $\forall x(F(x)\rightarrow G(x))$	Premise
(2) $F(c)\rightarrow G(c)$	(2) UI
(3) $\exists xF(x)$	Premise
(4) $F(c)$	(3) EI
(5) $G(c)$	(2)(4) Modus Ponens
(6) $\exists xG(x)$	(5) EG

- **说明：这个证明是错的**. (3)(4)应当在(1)(2)前，(4)中的c是特定的， (2)中的c是任意的

“先EI，后UI”

Formal Proofs

- 前提: $\sim\exists x(F(x)\wedge H(x))$, $\forall x(G(x)\rightarrow H(x))$,

  结论: $\forall x(G(x)\rightarrow\sim F(x))$

- 证明:

(1)	$\sim\exists x(F(x)\wedge H(x))$	Premise
(2)	$\forall x(\sim F(x)\vee\sim H(x))$	(1)De Morgan's
(3)	$\forall x(H(x)\rightarrow\sim F(x))$	(2)Implication
(4)	$H(y)\rightarrow\sim F(y)$	(3) UI
(5)	$\forall x(G(x)\rightarrow H(x))$	Premise
(6)	$G(y)\rightarrow H(y)$	(5) UI
(7)	$G(y)\rightarrow\sim F(y)$	(4)(6)Hypo Syll.
(8)	$\forall x(G(x)\rightarrow\sim F(x))$	(7) UG

Q. E. D.

Using Rules of Inference

- **Returning to the Socrates Example**

$$\begin{array}{l} \forall x(Man(x) \rightarrow Mortal(x)) \\ Man(Socrates) \\ \hline \therefore \quad Mortal(Socrates) \end{array}$$

Step	**Reason**
1. $\forall x(Man(x) \rightarrow Mortal(x))$	Premise
2. $Man(Socrates) \rightarrow Mortal(Socrates)$	UI from (4)
3. $Man(Socrates)$	Premise
4. $Mortal(Socrates)$	MP from (2) and (3)

Using Rules of Inference

- **Example:**
  - *Every man has two legs.*
  - *John Smith is a man.*
  - *Therefore, John Smith has two legs.*
- **Define the predicates:**

  $M(x)$: x is a man

  $L(x)$: x has two legs

  J: John Smith, a member of the universe

Using Rules of Inference

- **The argument becomes**
  - $1. \forall x(M(x) \to L(x))$
  - $2. M(J)$
  - $\therefore L(J)$

- **The proof is**

$1. \forall x(M(x) \to L(x))$	Hypothesis 1
$2. M(J) \to L(J)$	step 1 and UI
$3. M(J)$	Hypothesis 2
$4. L(J)$	steps 2 and 3 and *modus ponens*

- Q. E. D.

Using Rules of Inference

- **Example 13:**
  - Use the rules of inference to construct a valid argument showing that the conclusion: "*Someone who passed the first exam has not read the book.*"

  follows from the premises:

  "*A student in this class has not read the book.*"

  "*Everyone in this class passed the first exam.*"

- **Predicates:**

  C(x): "x is in this class,"

  B(x): "x has read the book,"

  P(x): "x passed the first exam."

$$\begin{array}{l} \exists x(C(x) \wedge \neg B(x)) \\ \forall x(C(x) \rightarrow P(x)) \\ \hline \therefore \ \exists x(P(x) \wedge \neg B(x)) \end{array}$$

Using Rules of Inference

$$\exists x(C(x) \land \neg B(x))$$
$$\forall x(C(x) \to P(x))$$
$$\therefore\ \exists x(P(x) \land \neg B(x))$$

Valid Argument:

Step	**Reason**
1. $\exists x(C(x) \land \neg B(x))$	Premise
2. $C(a) \land \neg B(a)$	EI from (1)
3. $C(a)$	Simplification from (2)
4. $\forall x(C(x) \to P(x))$	Premise
5. $C(a) \to P(a)$	UI from (4)
6. $P(a)$	MP from (3) and (5)
7. $\neg B(a)$	Simplification from (2)
8. $P(a) \land \neg B(a)$	Conj from (6) and (7)
9. $\exists x(P(x) \land \neg B(x))$	EG from (8)

Q. E. D.

SUPPL: Construction of Proofs

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构造证明1—直接证明法

- **例1：构造下面推理的证明**
  - 若明天是周二或周三，我就有课。若有课，今天必备课。我今天没备课。所以，明天不是周二和周三。
- **解：**
  - 设 p：明天是星期二

    q：明天是星期三

    r：我有课

    s：我备课
  - **推理的形式结构为**

    前提：$(p \vee q) \rightarrow r$，$r \rightarrow s$，$\neg s$

    结论：$\neg p \wedge \neg q$

- **例1证明**

①	$r \rightarrow s$	Premise
②	$\neg s$	Premise
③	$\neg r$	①②M.Tollens
④	$(p \vee q) \rightarrow r$	Premise
⑤	$\neg(p \vee q)$	③④M.Tollens
⑥	$\neg p \wedge \neg q$	⑤De Morgan’s

构造证明2—附加前提证明法(CP规则)

- 欲证明

前提：$A_1, A_2, \ldots, A_k$

结论：$C \rightarrow B$

- 等价地证明

前提：$A_1, A_2, \ldots, A_k, C$

结论：B

- 理由：$(A_1 \wedge A_2 \wedge \ldots \wedge A_k) \rightarrow (C \rightarrow B)$

$$\Leftrightarrow \neg(A_1 \wedge A_2 \wedge \ldots \wedge A_k) \vee (\neg C \vee B)$$

$$\Leftrightarrow \neg A_1 \vee \neg A_2 \vee \ldots \vee \neg A_k \vee \neg C \vee B$$

$$\Leftrightarrow \neg(A_1 \wedge A_2 \wedge \ldots \wedge A_k \wedge C) \vee B$$

$$\Leftrightarrow (A_1 \wedge A_2 \wedge \ldots \wedge A_k \wedge C) \rightarrow B$$

构造证明2—附加前提证明法(CP规则)

- 例2：构造下面推理的证明
  - 2是素数或合数；若2是素数，则$\sqrt{2}$是无理数.
  - 若$\sqrt{2}$是无理数，则4不是素数.
  - 所以，如果4是素数，则2是合数.
- 解：用附加前提证明法构造证明
  - 设 p：2是素数，q：2是合数，
  - r：$\sqrt{2}$是无理数，s：4是素数
- 推理的形式结构
  - 前提：$p \vee q$, $p \rightarrow r$, $r \rightarrow \neg s$
  - 结论：$s \rightarrow q$

构造证明2—附加前提证明法(CP规则)

前提：$p\vee q$，$p\rightarrow r$，$r\rightarrow\neg s$
结论：$s\rightarrow q$

- 例2证明

①	s	Premise(C.P)
②	$p\rightarrow r$	Premise
③	$r\rightarrow\neg s$	Premise
④	$p\rightarrow\neg s$	②③Hypo. Syllogism
⑤	$\neg p$	①④Modus Tollens
⑥	$p\vee q$	Premise
⑦	q	⑤⑥Disjunctive Syllogism

请用直接证明法证明之

构造证明2—附加前提证明法(CP规则)

- **例3：设有下列情况，判断论证是否有效。**
  - 1) 如果我不去玩游戏，我会有充足的时间；
  - 2) 假如我有充足的时间，我就会认真复习英语；
  - 3) 如果我认真复习英语，我的英语考试就不会不及格。
  - 结论：我的英语考试没及格，所以我肯定去玩游戏了。
- **证明**
  - 将前提和结论符号化：
  - P：我去玩游戏。　Q：我有充足的时间。
  - R：我复习英语。 S：我的英语考试及格了。
  - 前提：$\sim P \to Q$，$Q \to R$，$R \to S$,
  - 结论：$\sim S \to P$ 。
  - 由于结论$\sim S \to P$是一个条件式,故可采用CP规则来证明。

构造证明2—附加前提证明法(CP规则)

~P→Q，Q→R，R→S，~S→P

- **例3证明：**

(1) Q→R	Premise
(2) R→S	Premise
(3) Q→S	(1)(2) Hypo. Syll.
(4) ~S	Premise（C.P）
(5) ~Q	(4) Modus Tollens
(6) ~P→Q	Premise
(7) P	(5)(6) Modus Tollens
(8) ~S→P	CP (4)(7)

所以结论有效。**请用直接证明法证明之。**

构造证明3—反证法

- 欲证明：

  前提：$A_1, A_2, \ldots, A_k$

  结论：B

- 方法：
  - 将$\neg B$加入前提，若推出矛盾，则得证结论正确

- 理由：$A_1 \wedge A_2 \wedge \ldots \wedge A_k \rightarrow B$

  $\Leftrightarrow \neg(A_1 \wedge A_2 \wedge \ldots \wedge A_k) \vee B$

  $\Leftrightarrow \neg A_1 \vee \neg A_2 \vee \ldots \vee \neg A_k \vee B$

  $\Leftrightarrow \neg(A_1 \wedge A_2 \wedge \ldots \wedge A_k \wedge \neg B)$

$(A_1 \wedge A_2 \wedge \ldots \wedge A_k \rightarrow B)$为重言式当且仅当括号内为矛盾式

构造证明3—反证法

- **例4：构造下面推理的证明**

前提：$\neg(p\wedge q)\vee r$，$r\rightarrow s$，$\neg s$，p

结论：$\neg q$

①	q	结论否定引入
②	p	前提引入
③	$p\wedge q$	合取引入
④	$\neg(p\wedge q)\vee r$	前提引入
⑤	r	③④析取三段论
⑥	$r\rightarrow s$	前提引入
⑦	s	⑤⑥假言推理
⑧	$\neg s$	前提引入
⑨	$\neg s\wedge s$	矛盾

①	q	结论否定引入
②	$r\rightarrow s$	前提引入
③	$\neg s$	前提引入
④	$\neg r$	②③拒取式
⑤	$\neg(p\wedge q)\vee r$	前提引入
⑥	$\neg(p\wedge q)$	④⑤析取三段论
⑦	$\neg p\vee\neg q$	⑥德摩根律
⑧	$\neg p$	①⑦析取三段论
⑨	p	前提引入
⑩	$\neg p\wedge p$	⑧⑨合取，矛盾

Examples

- 例5：
  - 乌鸦都不是白色的，北京鸭是白色的，
  - 因此，北京鸭不是乌鸦。
- 符号化：

  $F(x)$: x是乌鸦

  $G(x)$: x是北京鸭

  $H(x)$: x是白色的

  前提：$\forall x(F(x)\rightarrow\neg H(x))$，$\forall x(G(x)\rightarrow H(x))$

  结论：$\forall x(G(x)\rightarrow\neg F(x))$

Examples

$\forall x(F(x)\to\neg H(x))$， $\forall x(G(x)\to H(x))$

结论：$\forall x(G(x)\to\neg F(x))$

- 例5证明：

① $\forall x(F(x)\to\neg H(x))$	Premise
② $F(y)\to\neg H(y)$	①UI
③ $\forall x(G(x)\to H(x))$	Premise
④ $G(y)\to H(y)$	③UI
⑤ $\neg H(y)\to\neg G(y)$	④Contrapositive
⑥ $F(y)\to\neg G(y)$	②⑤Hypo. Syllogism
⑦ $G(y)\to\neg F(y)$	⑥Contrapositive
⑧ $\forall x(G(x)\to\neg F(x))$	⑦UG

Examples

- **例6：** 前提：$\exists xF(x)\rightarrow\forall xG(x)$

  结论：$\forall x(F(x)\rightarrow G(x))$

- **证明：** $\exists x\mathbf{A(x)} \rightarrow \forall x\mathbf{B(x)} \Rightarrow \forall x\ (\mathbf{A(x)} \rightarrow \mathbf{B(x)})$

  ① $\exists xF(x)\rightarrow\forall xG(x)$ Premise

  ② $\forall x\forall y(F(x)\rightarrow G(y))$ ①Rename， Scope Extension

  ③ $\forall x(F(x)\rightarrow G(z))$ ②UI

  ④ $F(z)\rightarrow G(z)$ ③UI

  ⑤ $\forall x(F(x)\rightarrow G(x))$ ④UG

- **说明1:** 不能对$\exists xF(x)\rightarrow\forall xG(x)$直接消去量词, 不是前束范式
- **说明2：** 对此题不能用附加前提证明法.

Examples

- 例7：前提：$\forall x(F(x)\rightarrow G(x))$

  结论：$\forall xF(x)\rightarrow\forall xG(x)$

- 证明： $\forall x\,(A(x)\rightarrow B(x))\Rightarrow\forall xA(x)\rightarrow\forall xB(x)$

  ① $\forall xF(x)$ Premise(C.P)

  ② $F(y)$ ①UI

  ③ $\forall x(F(x)\rightarrow G(x))$ Premise

  ④ $F(y)\rightarrow G(y)$ ③UI

  ⑤ $G(y)$ ②④Modus Ponens

  ⑥ $\forall xG(x)$ ⑤UG

- 说明：本题可以使用附加前提证明法

Excercise

- 例8：有些运动员信任每一位教练，但没有运动员信任任何无能的人，所以教练都不是无能的人。
- 证明：
  - 令P(x)：x是运动员，　D(x)：x是教练，

    S(x)：x是无能的人，G(x,y)：x信任y。
  - 原论证的前提和结论符号化为：

    前提：

    $$\exists x(P(x) \wedge \forall y\, (D(y) \rightarrow G(x,y)))$$

    $$\forall x(P(x) \rightarrow \forall y(S(y) \rightarrow \sim G(x,y)))$$

    结论：$\forall x(D(x) \rightarrow \sim S(x))$

Excercise

$\exists x(P(x) \land \forall y\ (D(y) \to G(x,y)))$
$\forall x(P(x) \to \forall y(S(y) \to \sim G(x,y)))$
结论：$\forall x(D(x) \to \sim S(x))$

例8证明：

(1) $\exists x(P(x) \land \forall y(D(y) \to G(x,y)))$	Premise
(2) $P(c) \land \forall y(D(y) \to G(c,y))$	(1),EI
(3) $P(c)$	(2), Simplification
(4) $\forall y(D(y) \to G(c,y))$	(2), Simplification
(5) $D(z) \to G(c,z)$	(4),UI
(6) $\forall x(P(x) \to \forall y(S(y) \to \sim G(x,y)))$	Premise
(7) $P(c) \to \forall y(S(y) \to \sim G(c,y))$	(6),UI
(8) $\forall y(S(y) \to \sim G(c,y))$	(3)(7), M.P.
(9) $S(z) \to \sim G(c,z)$	(8),UI
(10) $G(c,z) \to \sim S(z)$	(9),Contrapositive
(11) $D(z) \to \sim S(z)$	(5)(10),Hypo.Syllogism
(12) $\forall x(D(x) \to \sim S(x))$	(11),UG

Homework

- § 1.6
  - 4, 6, 12, 16, 20, 24

设计命题或谓词
给出前提和结论的形式化表示
推理过程要完整
给出规范的三列形式