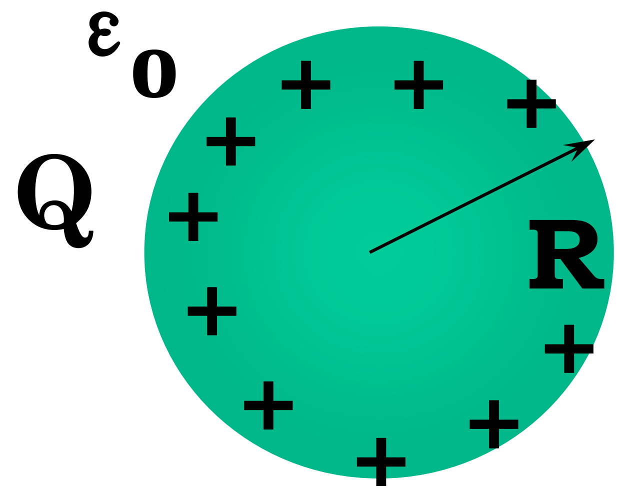
§6.4 电容器的电容

一、孤立导体的电容

电容 $$C = \frac{q}{V}$$

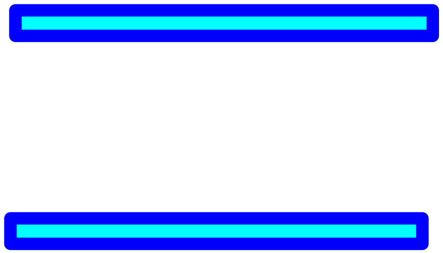
例：孤立导体球

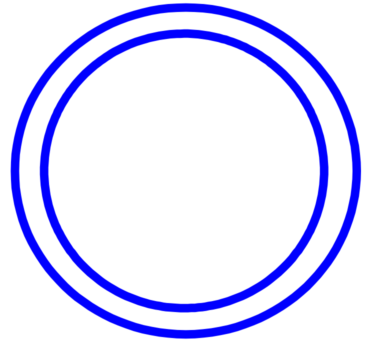


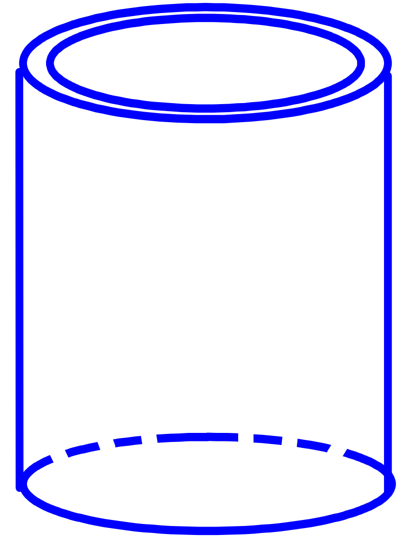
$$C = \frac{Q}{\dfrac{Q}{4\pi\varepsilon_0 R}} = 4\pi\varepsilon_0 R$$

单位：法拉(F)

二、电容器的电容







极板： 两块导体薄板

充电后，两板带**等量异号**的电荷

电容 $C=\dfrac{q}{V_1-V_2}=\dfrac{q}{U}$

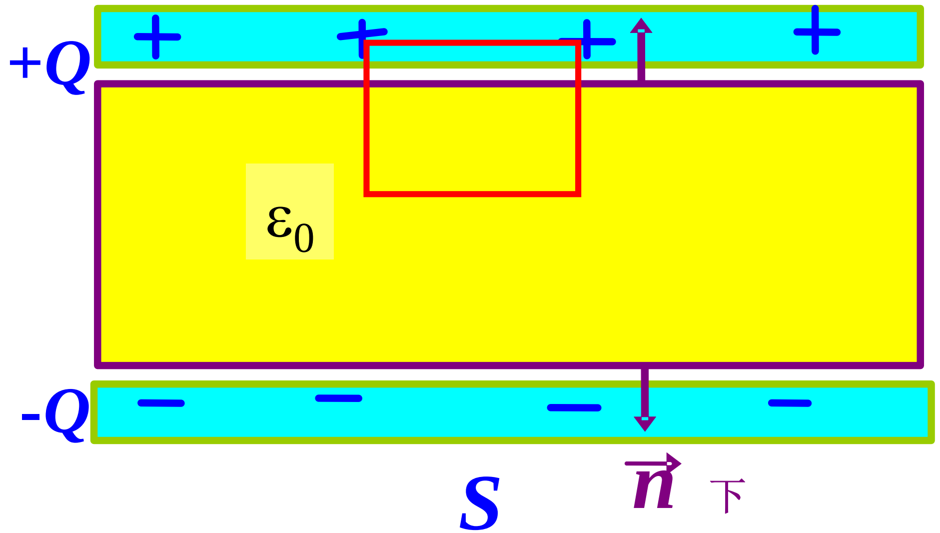
C与电容器极板的大小、形状、间距及充的电介质有关

1. 平行板电容器

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$ES_0 = \frac{\sigma_0}{\varepsilon_0}S_0$$

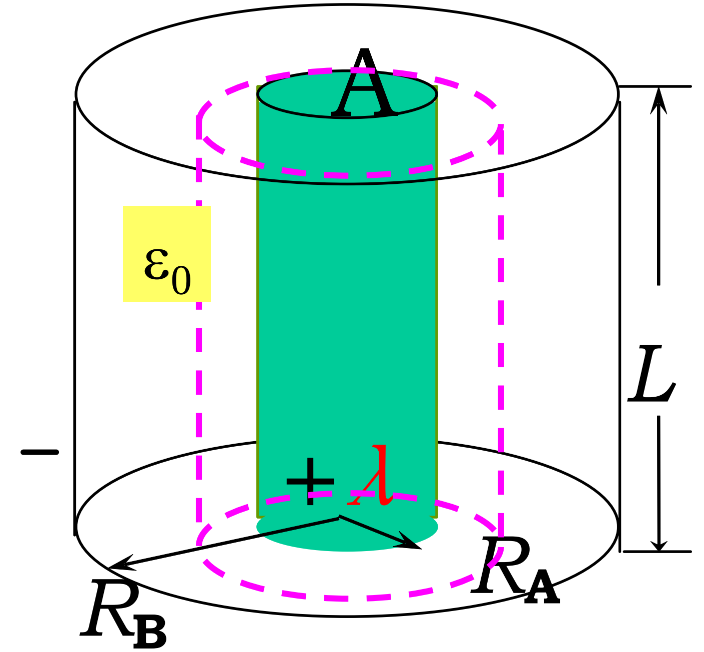
$$E = \frac{\sigma_0}{\varepsilon_0}$$



$$U = Ed$$

$$C = \frac{Q}{U} = \frac{\sigma_0 S}{Ed} = \frac{\varepsilon_0 S}{d}$$

2. 圆柱形电容器



$$\oint_S \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0}$$

$$E2\pi rL = \frac{\lambda L}{\varepsilon_0}$$

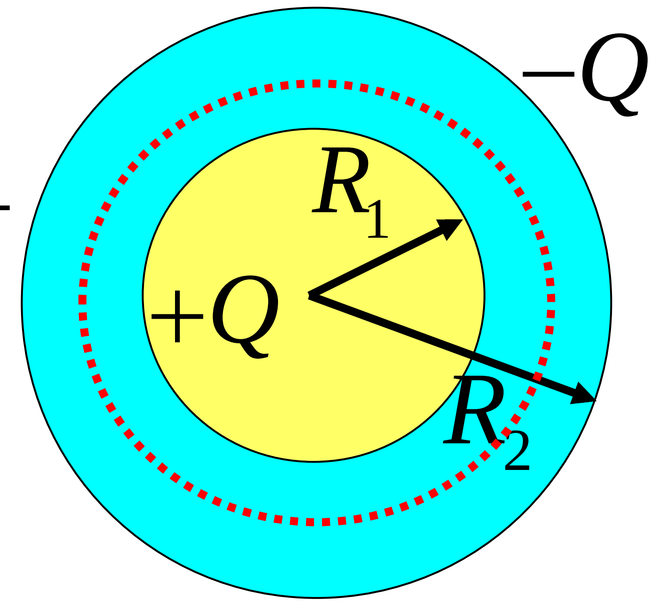
$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$U = \int_A^B \vec{E}\bullet d\vec{l} = \int_{R_A}^{R_B} \frac{\lambda}{2\pi\varepsilon_0 r} dr = \frac{\lambda}{2\pi\varepsilon_0} \ln\frac{R_B}{R_A}$$

$$C = \frac{Q}{U} = \frac{\lambda L}{U} = \frac{2\pi\varepsilon_0 L}{\ln\dfrac{R_B}{R_A}}$$

3. 球形电容器

$$\oint_S \vec{E}\cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2}$$



$$U = \int_{R_1}^{R_2} \vec{E}\bullet d\vec{l} = \frac{Q}{4\pi\varepsilon_0}\int_{R_1}^{R_2}\frac{dr}{r^2}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$C = \frac{Q}{U} = \frac{1}{\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

解题思路

$$U = \int_A^B \vec{E} \bullet d\vec{l}$$

$$\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\varepsilon_0} \Rightarrow \vec{E} \Rightarrow U \Rightarrow C$$

$$C = \frac{Q}{U}$$

三、电容器的串并联

1.串联：等效电容

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\cdots$$

·若仅有两个电容器串联

$$C=\frac{C_1C_2}{C_1+C_2}$$

2.并联：等效电容

$$C=C_1+C_2+C_3+\cdots$$

§6.5 静电场的能量

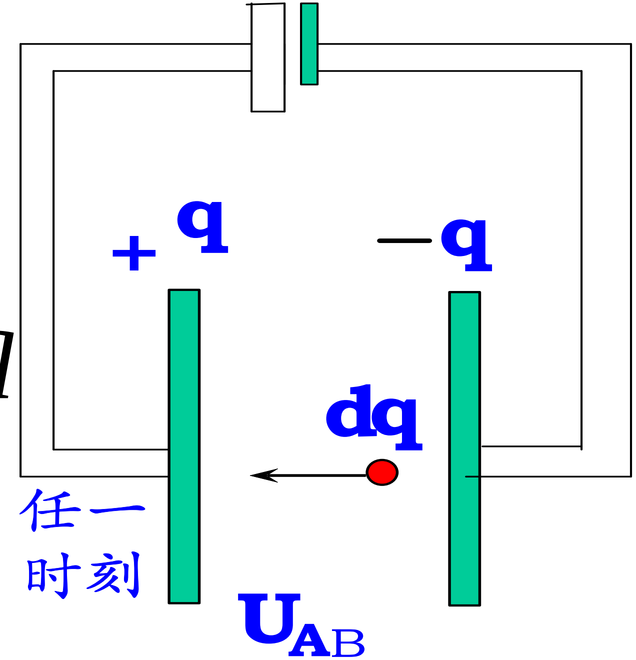
一、静电场能量和能量密度

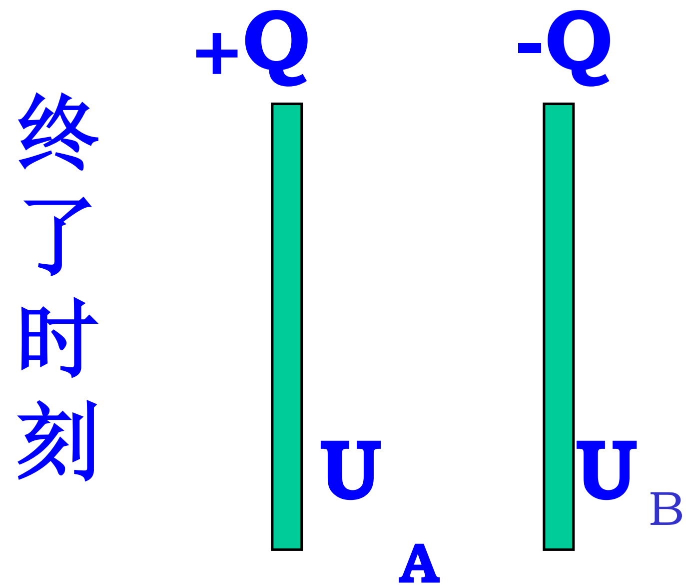
移动dq电荷外力作功

$$dA = dqEd = dqU_{AB} = \frac{q}{C}dq$$

由于 $U_{AB} = \dfrac{q}{C}$

所以 $A = \displaystyle\int_0^Q dq\frac{q}{C} = \frac{Q^2}{2C}$





外力作功转化为电场能量,于是有

$$W = A = \frac{Q^2}{2C} \Rightarrow W = \frac{1}{2}CU^2 = \frac{1}{2}QU$$

$$U = \frac{Q}{C}$$

因为 $C = \frac{\varepsilon_0 S}{d},\qquad U = Ed$

得到: $W = \frac{1}{2}\frac{\varepsilon_0 S}{d}(Ed)^2 = \frac{1}{2}\varepsilon_0 E^2 Sd = \frac{1}{2}\varepsilon_0 E^2 V$

单位体积内的
静电场能量称为能量密度

$$w_e = \frac{1}{2}\varepsilon_0 E^2$$

适用于任何带电体.

二、能量的计算

对于电容器

$$W=\frac{1}{2}CU^2=\frac{1}{2}QU=\frac{Q^2}{2C}$$

对于非电容器

$$W=\iiint_V w_e dV=\iiint_V \frac{1}{2}\varepsilon_0 E^2 dV$$

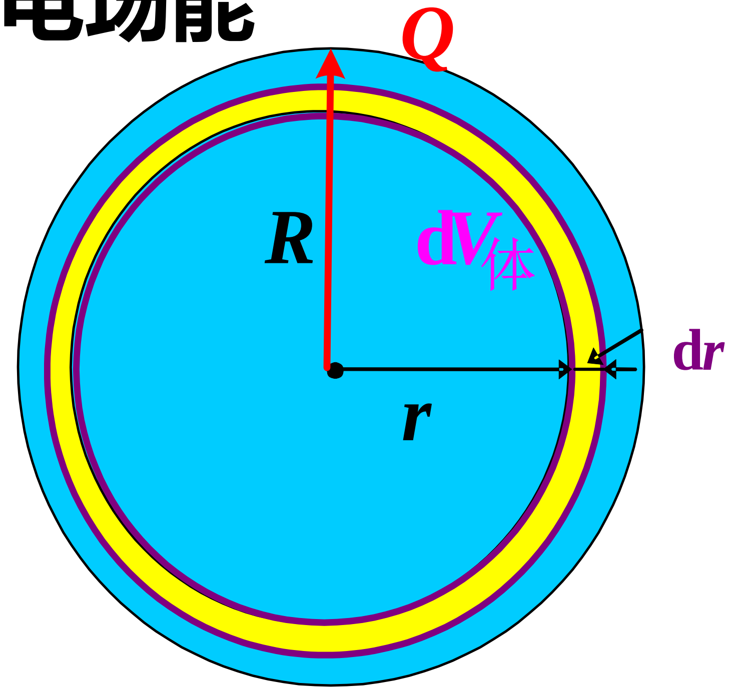
例1 求带电量为Q的导体球的电场能

$$E=\frac{Q}{4\pi\varepsilon_0 r^2}$$

$$W_e=\iiint\limits_{\text{球外空间}} w_e dV=\iiint\frac{1}{2}\varepsilon_0E^2dV$$

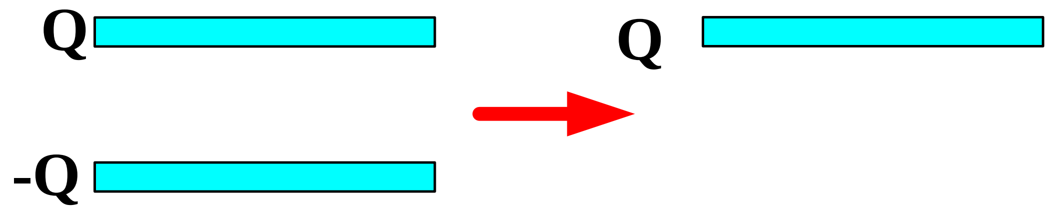
$$=\int_R^{\infty}\frac{Q^2}{32\pi^2\varepsilon_0 r^4}4\pi r^2dr$$

$$W_e=\frac{Q^2}{8\pi\varepsilon_0 R}$$

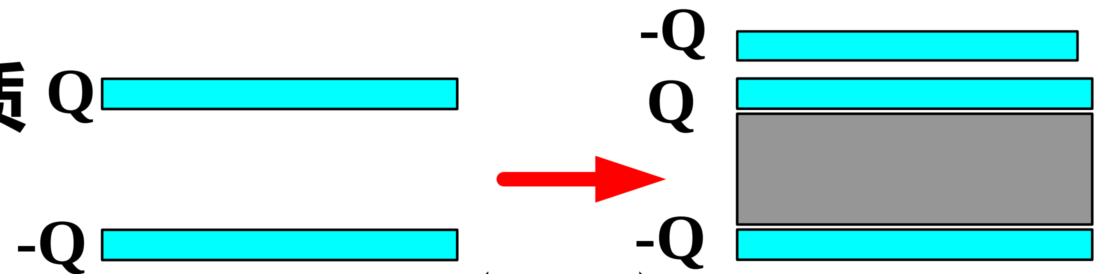


三、能量变化的计算

1、电容器结构情况的变化



2、电容器中电介质情况的变化



3、电容器连接情况的变化： 并联 ⟷ 串联

注意：

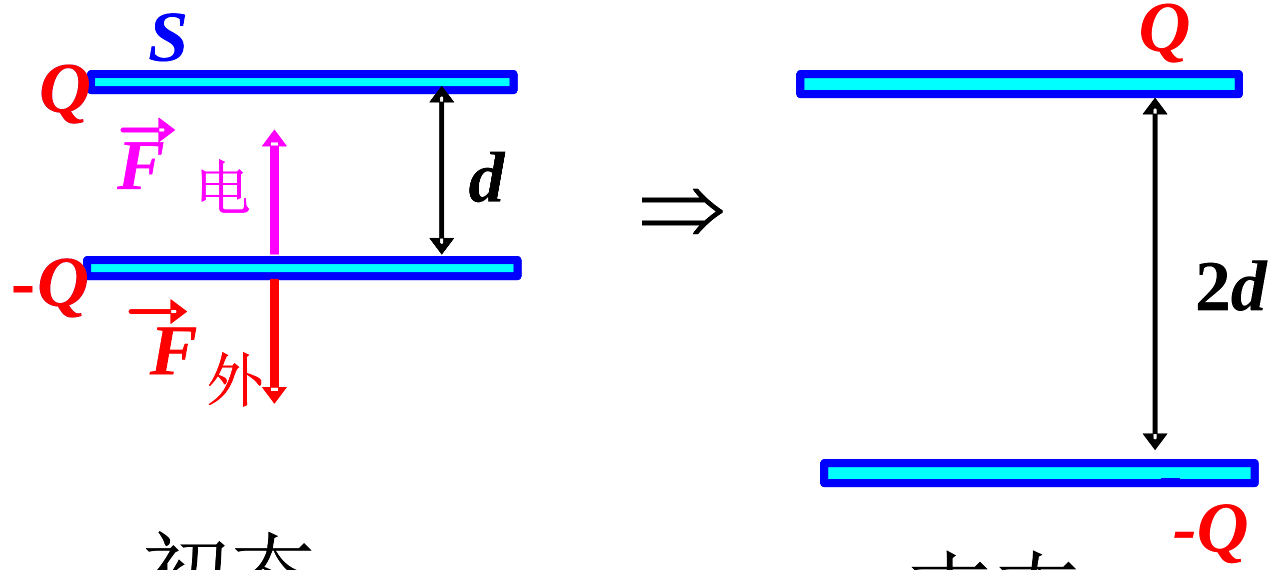
若电容器充电后和电源断开，**电容器的电量保持不变**

若电容器始终和电源相连，**电容器的电压保持不变**

例 带电Q的平板电容器板间距为d，现用力缓慢地拉动下极板，使板间距变为$2d$，

求(1)电容器能量的变化；

(2)外力所作的功。



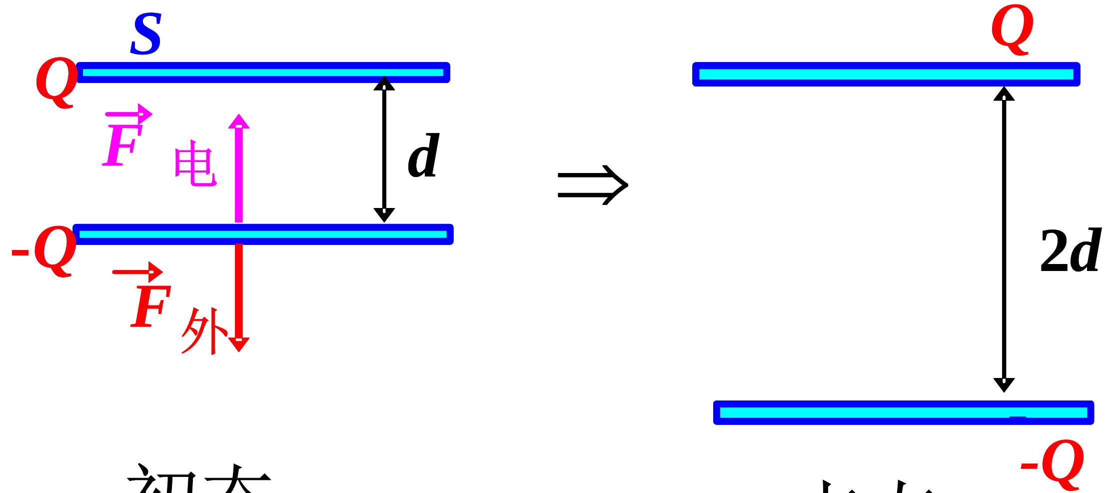
初态

末态

(1)电容器能量的变化

$$\Delta W = W_{末} - W_{初}$$

$$= \frac{Q^2}{2C_{末}} - \frac{Q^2}{2C_{初}}$$

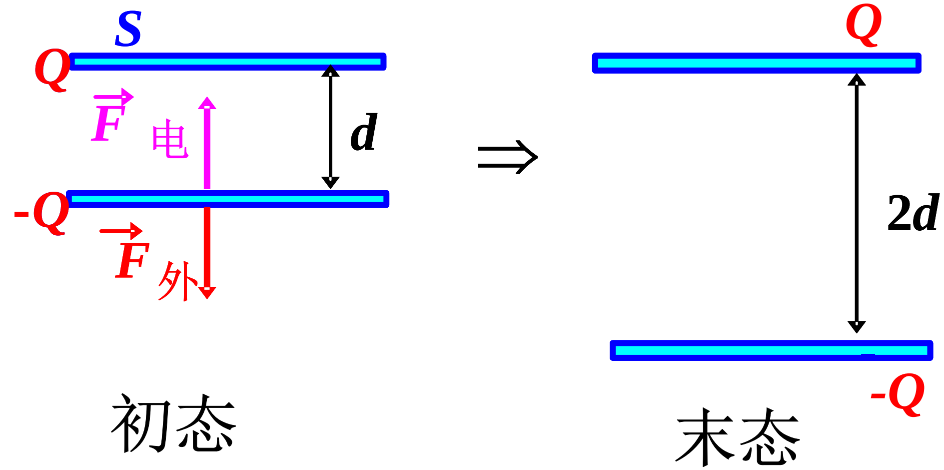


初态

末态

$$= \frac{Q^2}{2\left(\dfrac{\varepsilon_0 S}{2d}\right)} - \frac{Q^2}{2\left(\dfrac{\varepsilon_0 S}{d}\right)} = \frac{Q^2 d}{2\varepsilon_0 S}$$

(2)外力所作的功。



$$A = F_{外} d = F_{电} d$$

$$= Q E_{上} d$$

$$= Q\left(\frac{Q}{2\varepsilon_0 S}\right) d = \frac{Q^2 d}{2\varepsilon_0 S}$$

$$\frac{\sigma}{2\varepsilon_0}$$

外力作功使电容器能量增加

复习

1.静电平衡的条件

宏观上，电子不动，合力为零　$E_{\text{内}}=0$

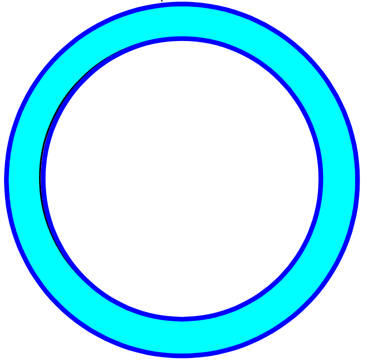
2．静电平衡的特点

$$U=c$$

$$E_{\text{内}}=0$$

场强分布：内部为零

电荷分布：只分布在外表面



3.基本性质方程

$$\oint_S \vec{E}\cdot d\vec{s}=\frac{\sum_i q_i}{\varepsilon_0} \qquad \oint_L \vec{E}\cdot d\vec{l}=0$$

4.电荷守恒定律

$$\sum_i Q_i=const.$$

5.电容器 $C=\dfrac{Q}{U}$

6.能量的计算 $W=\iiint\limits_V w_e dV=\iiint\limits_V \dfrac{1}{2}\varepsilon_0 E^2 dV$

$$W=\frac{1}{2}CU^2=\frac{1}{2}QU=\frac{Q^2}{2C}$$