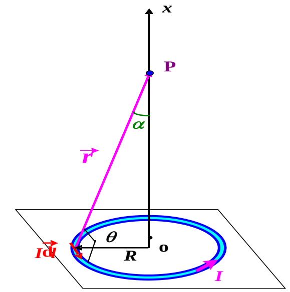
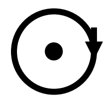
$$\mathrm{d}\vec{B}=\frac{\mu_0 I\mathrm{d}\vec{l}\times\hat{r}}{4\pi r^2}$$

无限长载流直导线 $B=\dfrac{\mu_0 I}{2\pi a}$ 若直线电流为半无限长时 $B=\dfrac{\mu_0 I}{4\pi a}$



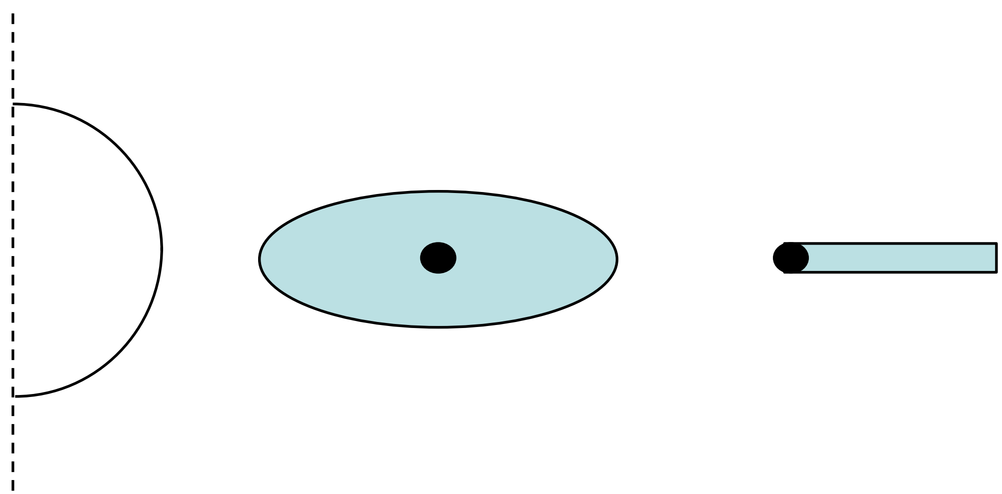
圆电流中心的场 $B=\dfrac{\mu_0 I}{2r}$



$$B=\frac{\mu_0 IR^2}{2r^3}$$

无限大均匀载流平面 $B=\dfrac{\mu_0 i}{2}$

一类：电荷转动形成电流求磁场



§7.5 磁通量 磁场的高斯定理

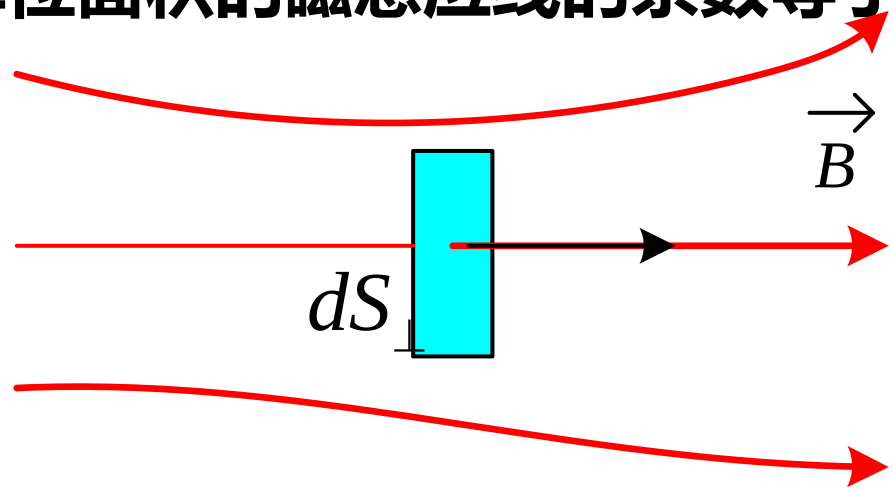
一、磁感应线(B线)

1、磁感应线:人为引入的一族有向曲线

2. 要求

(1)磁感应线上某点的**切向**和该点磁感强度**$\vec{B}$的方向**一致;

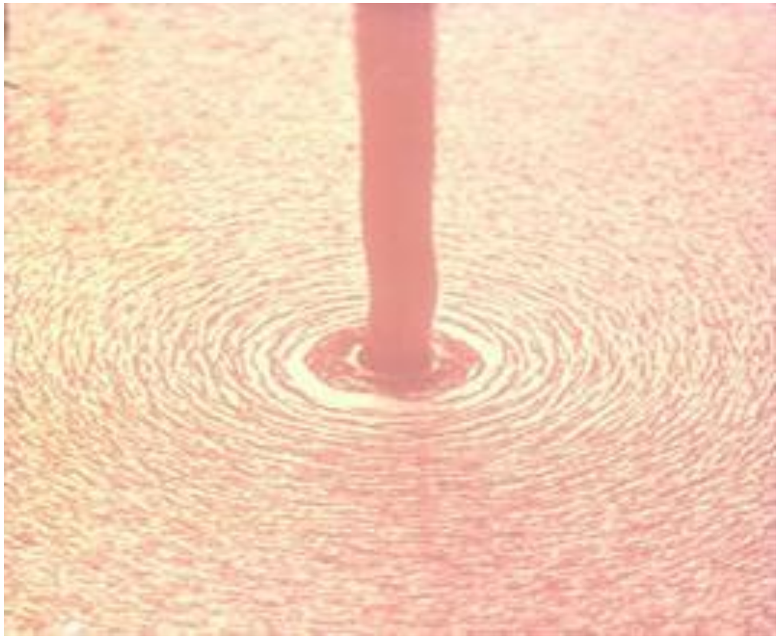
(2)通过垂直于$\vec{B}$的单位面积的磁感应线的条数等于该点$\vec{B}$的大小。

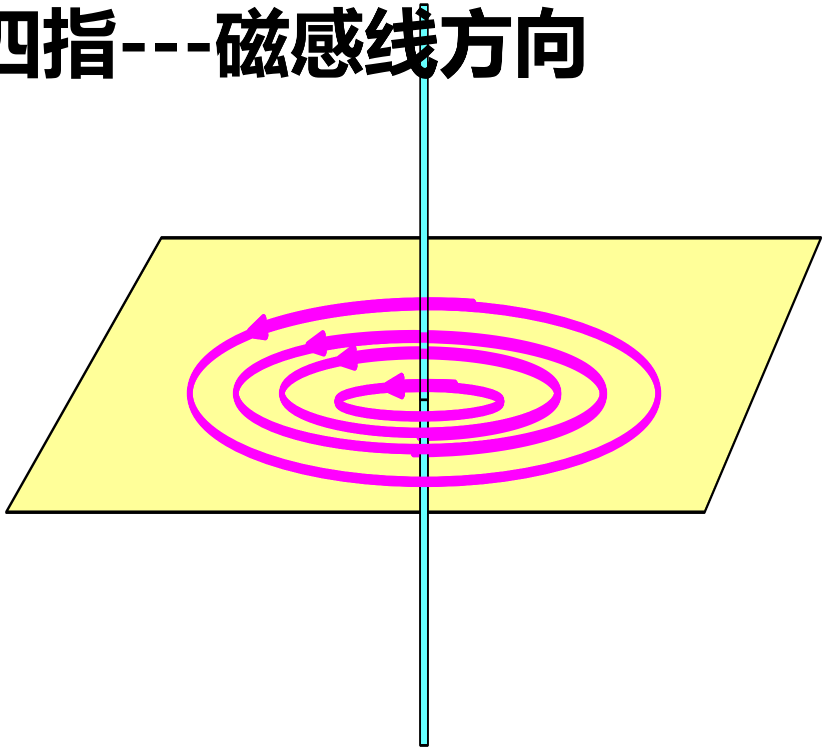


3、典型电流的磁感应线

1)载流导线的磁感应线

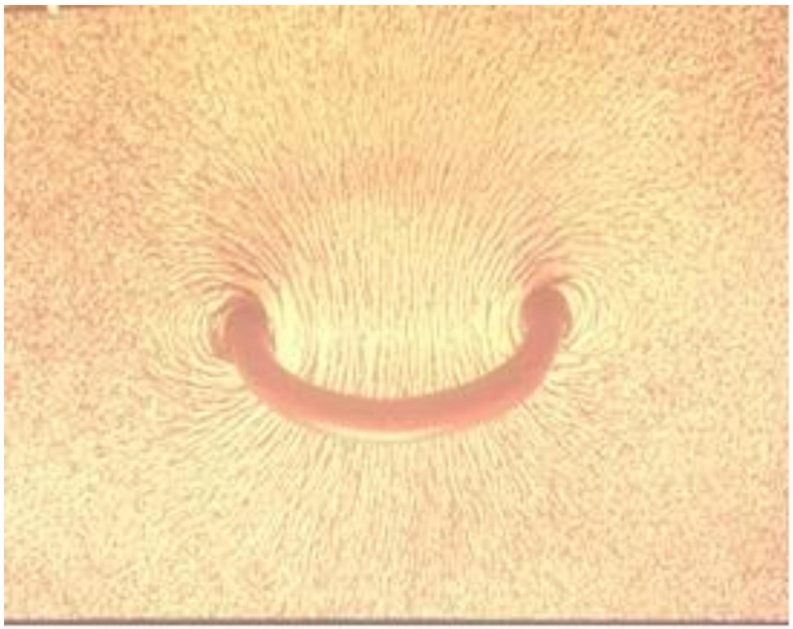
右手：拇指— I方向；四指---磁感线方向

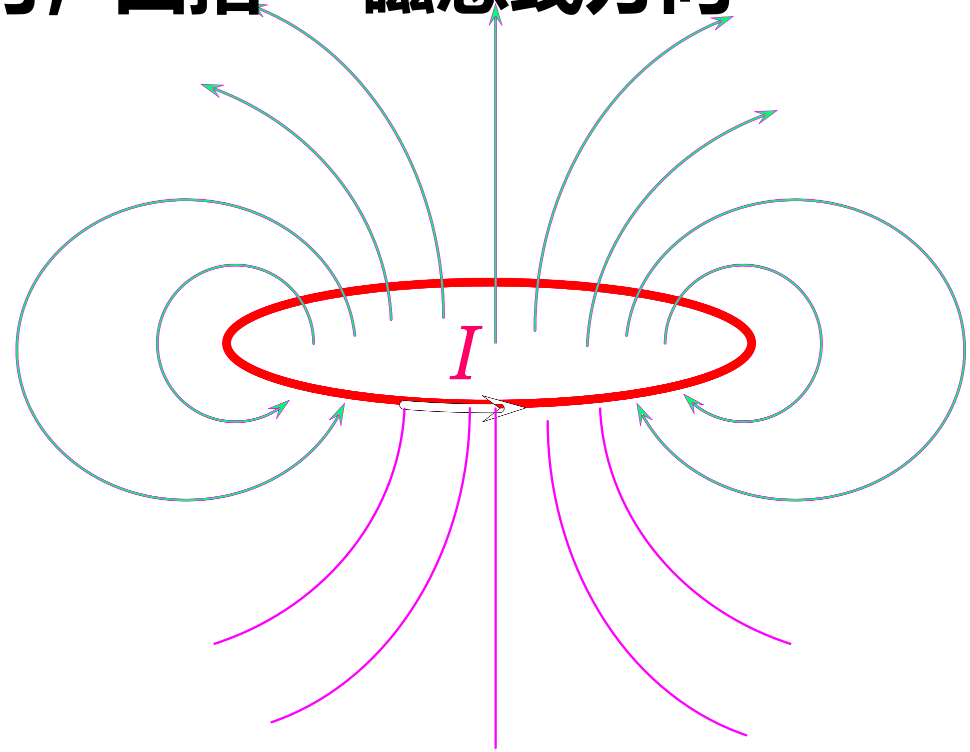




2)圆电流的磁感线

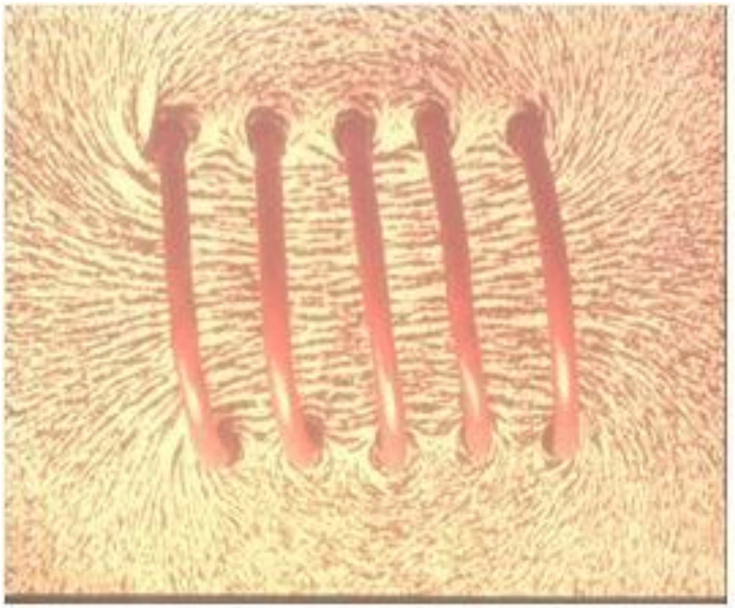
右手：拇指— /方向；四指---磁感线方向

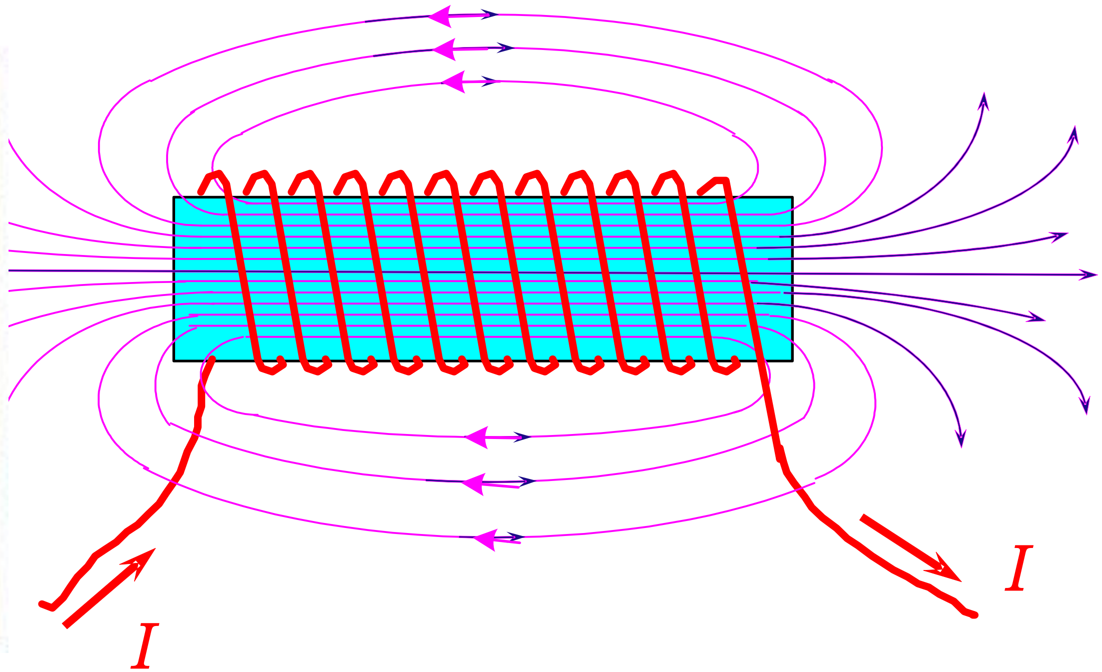




3)通电螺线管的磁感线

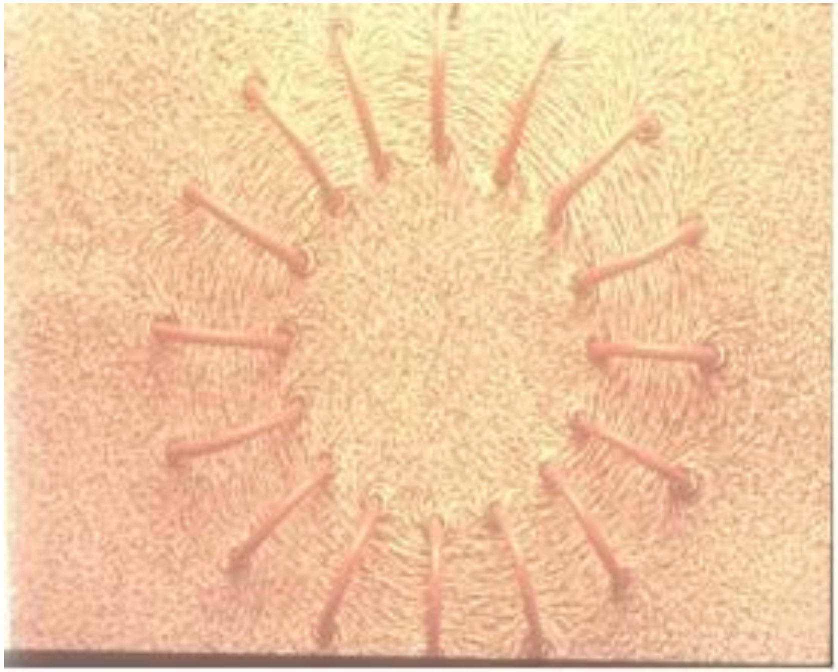
四指— /方向；拇指---线圈内部的磁感线方向

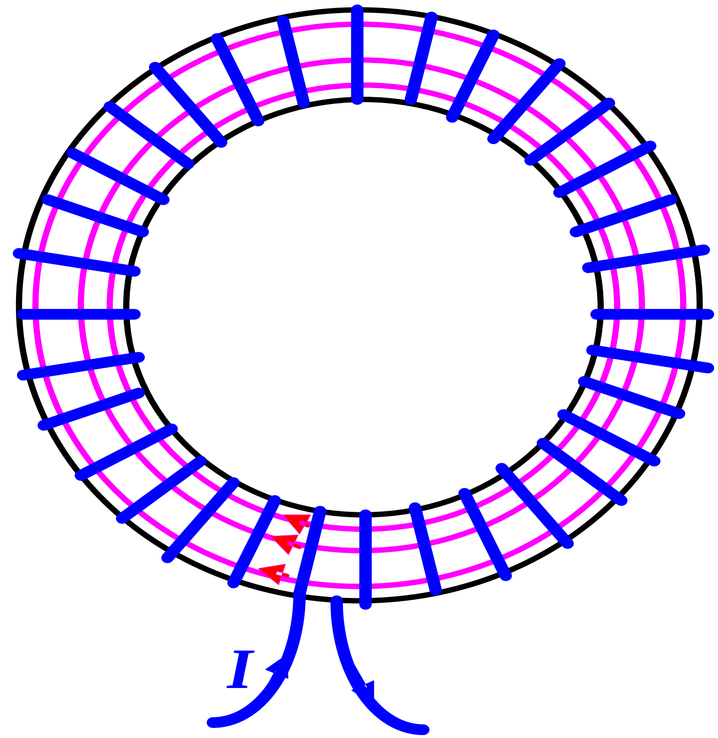




4)通电螺绕环的磁感线

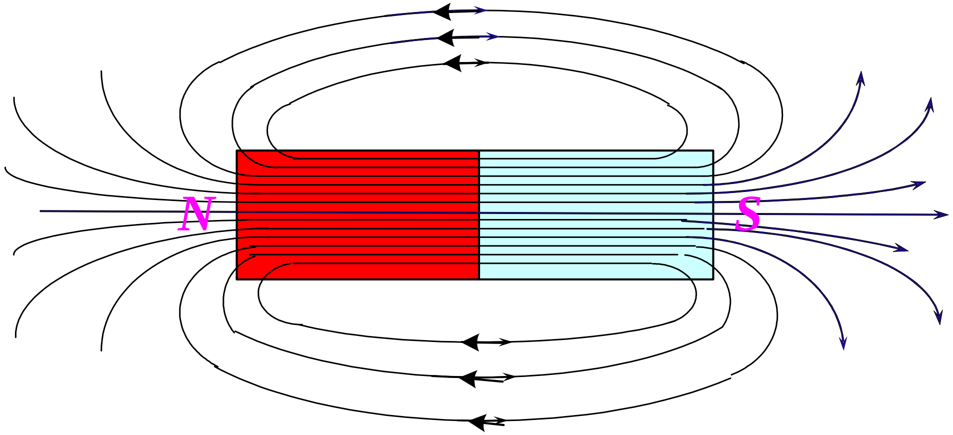
四指— /方向； 拇指---环内的磁感线方向

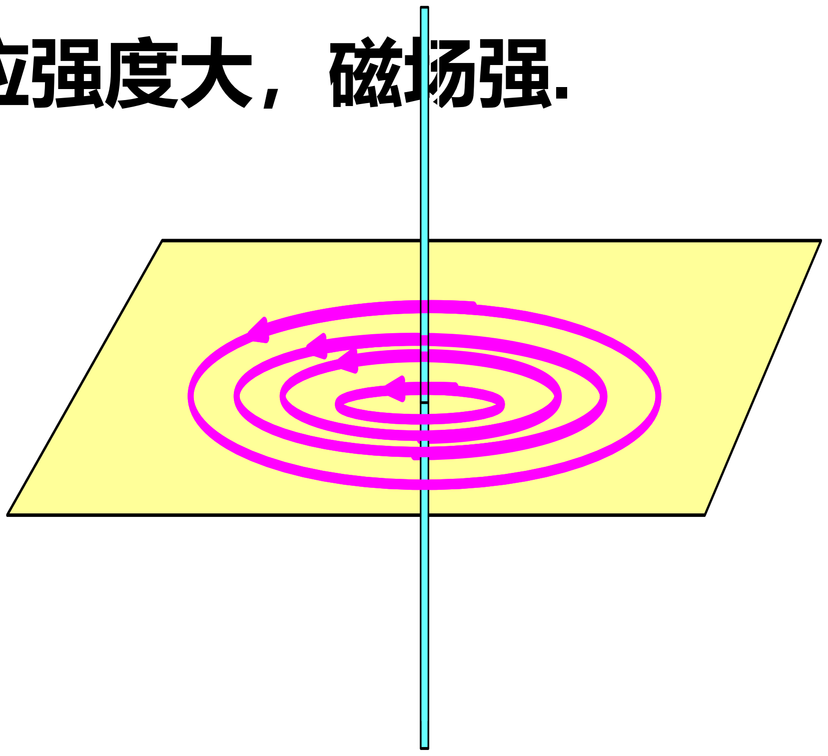




4、磁感线的性质

- 磁感线是环绕电流无头无尾的闭合曲线;
- 两条磁感线不能相交;
- 与电流成右手螺旋关系;
- 磁感线密集的地方，磁感应强度大，磁场强.

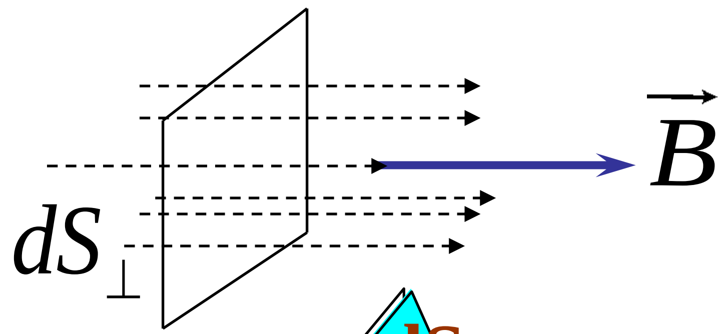




二、磁通量(类似于电场通量的定义) 韦伯(Wb)

穿过与B垂直的面的磁感线条数,也称为磁感应线的数密度

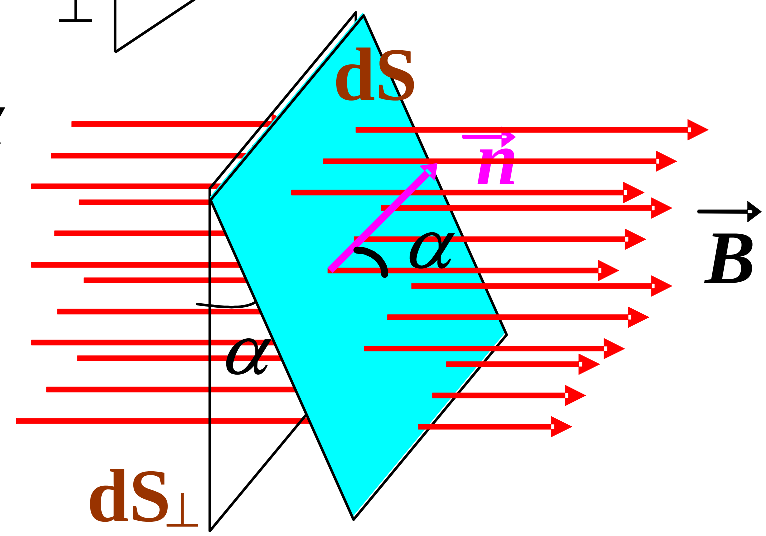
$$B = \frac{d\phi_m}{dS_\perp}$$



$$d\phi_m = BdS_\perp = BdS\cos\alpha$$

$$= \vec{B} \bullet d\vec{S}$$

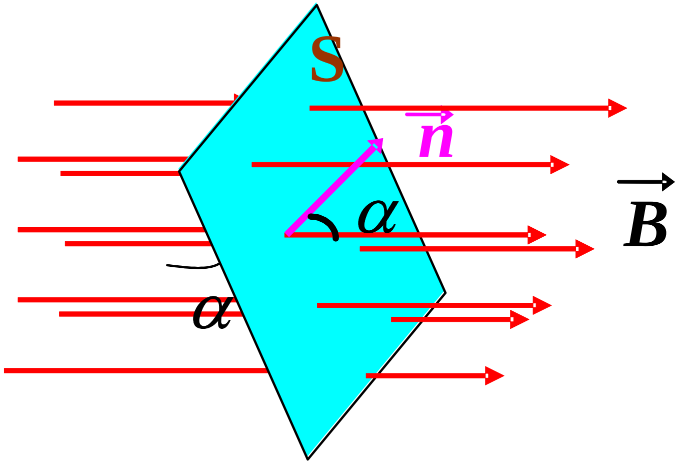
$$\boxed{d\phi_m = \vec{B} \cdot d\vec{S}}$$



通过有限曲面S的磁通量

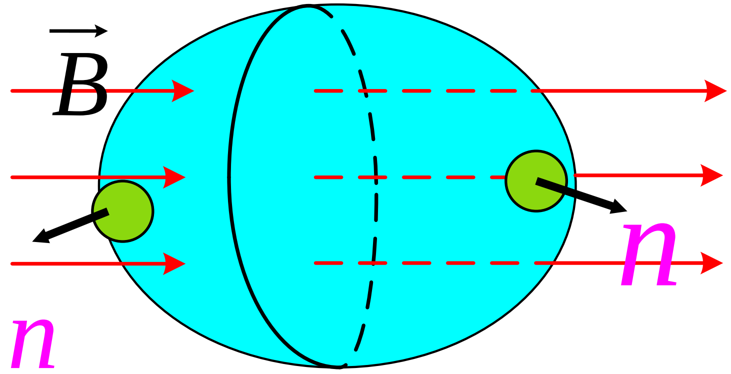
$$\phi_m = \int_s \vec{B} \cdot d\vec{S}$$

$$= \int_s B dS \cos\alpha$$



通过闭合曲面S的磁通量

$$\phi_m = \oint \vec{B} \cdot d\vec{S}$$

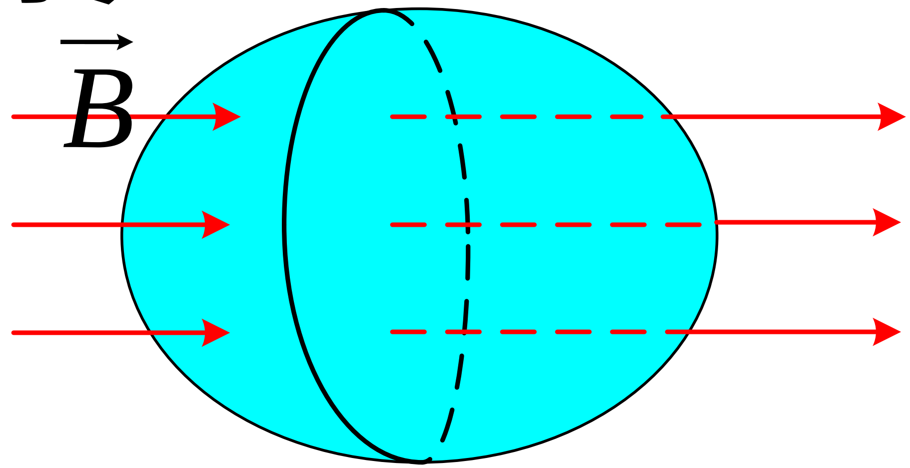


三、磁场的高斯定理（磁通连续原理）

通过任意闭合曲面的磁通量为零

$$\phi_m = \oint \vec{B} \cdot d\vec{S}$$

$$=0$$



$$\oint \vec{B} \cdot d\vec{S} = 0$$

磁场是无源场

$$\oint \vec{E} \cdot d\vec{S} = \frac{\sum_s q_{0i}}{\varepsilon_0}$$

电场是有源场

§7.6 安培环路定理

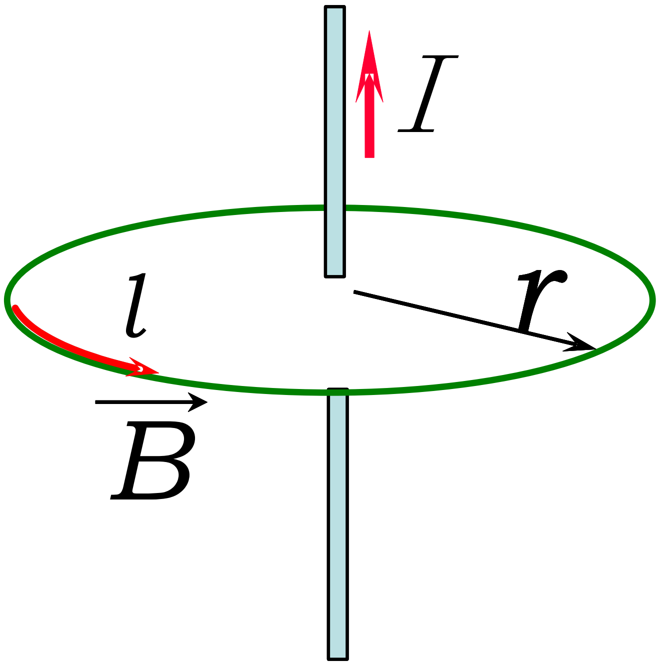
一、安培环路定理

静电场的环路定理

$$\oint_L \vec{E}\cdot d\vec{l} = 0$$

对于磁场

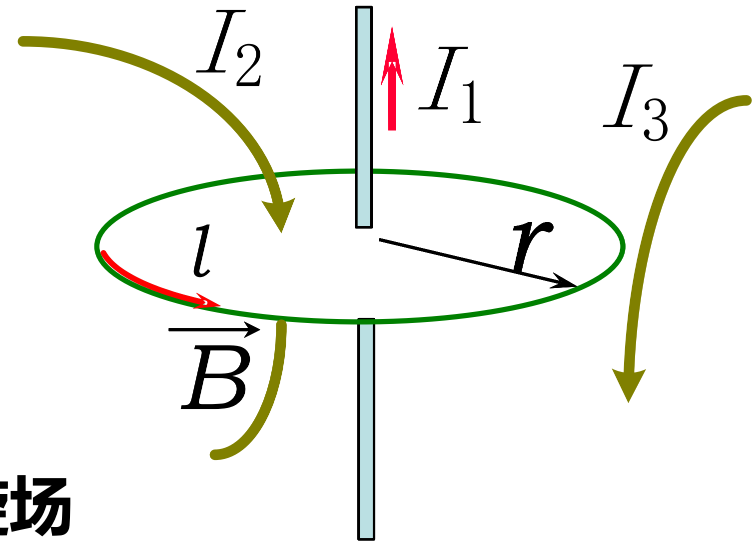
$$\oint_L \vec{B}\cdot d\vec{l} = ?$$



内容(磁场的安培环路定律)：

磁感应强度沿任意闭合路径一周的线积分等于穿过闭合路径所包围面积的电流的代数和与 μ_0 的乘积。

$$\boxed{\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum_i I_{i内}}$$



说明：

1）它只适用于稳恒电流；

2）B是全空间电流的贡献，

3）说明磁场为**非保守场**称为**涡旋场**

$\oint_L \vec{E}\cdot d\vec{l} = 0$ 静电场是保守场、无旋场

二、简证(用特例说明安培环路定理的正确性)

(1)闭合路径L环绕电流

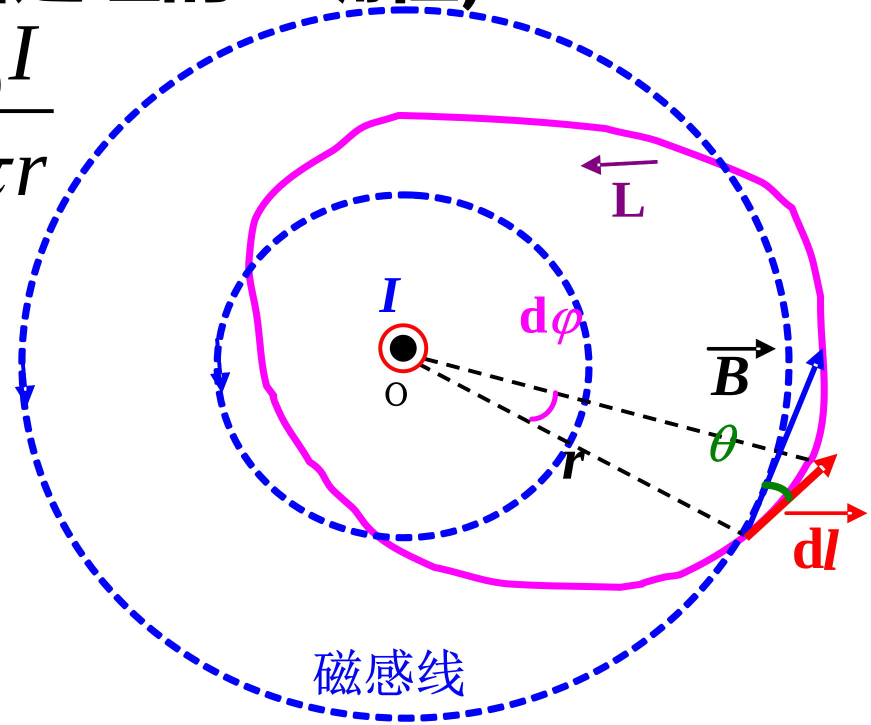
L在垂直于导线的平面内

$$B=\frac{\mu_0 I}{2\pi r}$$

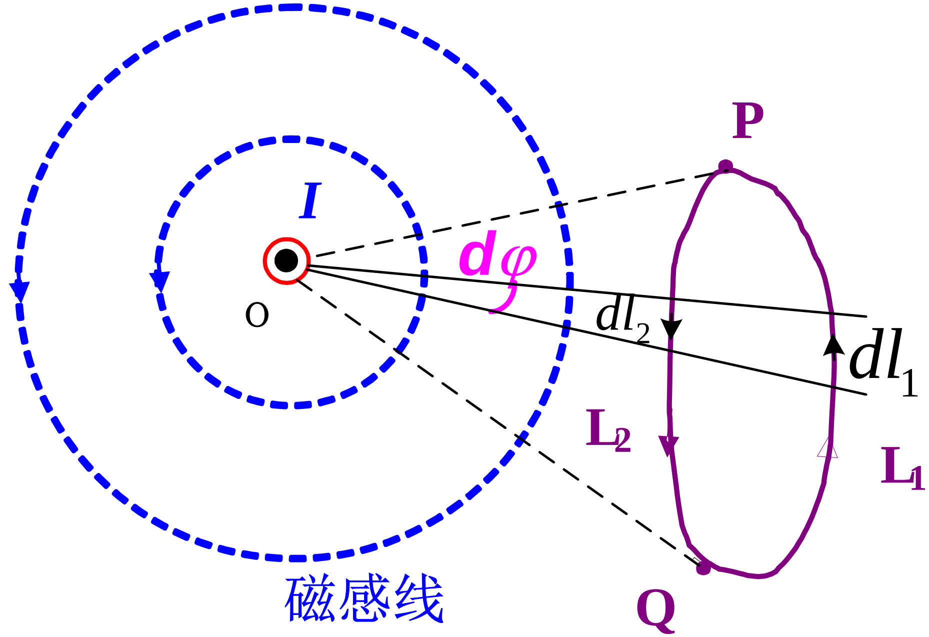
$$\oint \vec{B}\cdot d\vec{l}=\oint B\,dl\cos\theta$$

$$=\oint Br\,d\varphi=\int_0^{2\pi}\frac{\mu_0 I}{2\pi r}r\,d\varphi$$

$$=\frac{\mu_0 I}{2\pi}\int_0^{2\pi}d\varphi=\mu_0 I$$



(2)闭合路径L不包围电流



$$\oint \vec{B} \cdot d\vec{l} = \oint \vec{B} \cdot \left(d\vec{l}_1 + d\vec{l}_2 \right)$$

$$= \oint \left(\vec{B}_1 \cdot d\vec{l}_1 + \vec{B}_2 \cdot d\vec{l}_2 \right)$$

$$\vec{B}_1 \cdot d\vec{l}_1 = \frac{\mu_0 I}{2\pi r_1} r_1 d\varphi = \frac{\mu_0 I}{2\pi} d\varphi$$

$$\vec{B}_2 \cdot d\vec{l}_2 = -\frac{\mu_0 I}{2\pi r_2} r_2 d\varphi = -\frac{\mu_0 I}{2\pi} d\varphi$$

$$\therefore \vec{B}_2 \cdot d\vec{l}_1 + \vec{B}_1 \cdot d\vec{l}_2 = 0 \qquad \therefore \oint \vec{B} \cdot d\vec{l} = 0$$

三、运用安培环路定理求磁场

安培环路定理适用于任何形状恒定电流的载流体

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum_i I_{i内}$$

一般步骤:

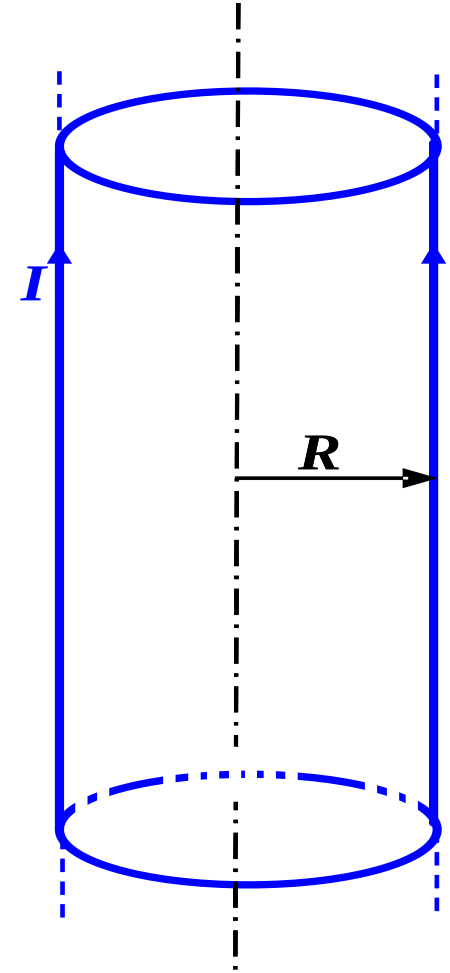
1.磁场具有对称性，分析

2.做对称性环路

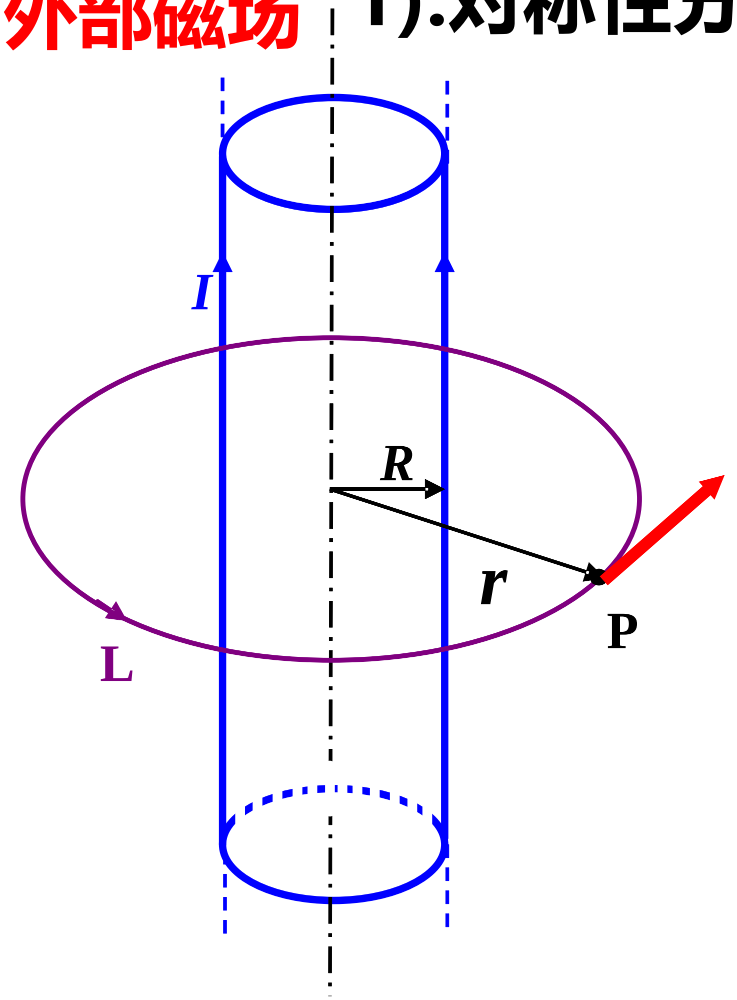
3.用环路定理解题

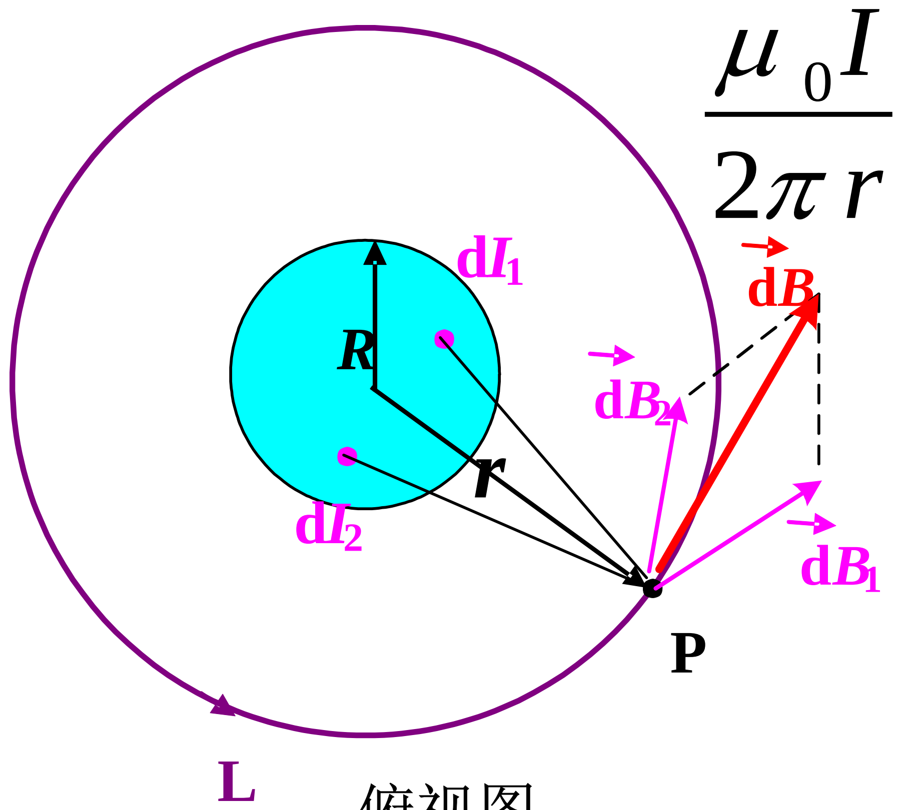
例1、均匀通电无限长直圆柱体的磁场

无限长载流圆柱体,半径 R,
电流强度为 I,电流均匀分布,
求圆柱体内外的磁场



外部磁场 1).对称性分析





俯视图

距轴线相同各点磁感应强度相同，方向与电流成右手螺旋关系

2).做对称性环路

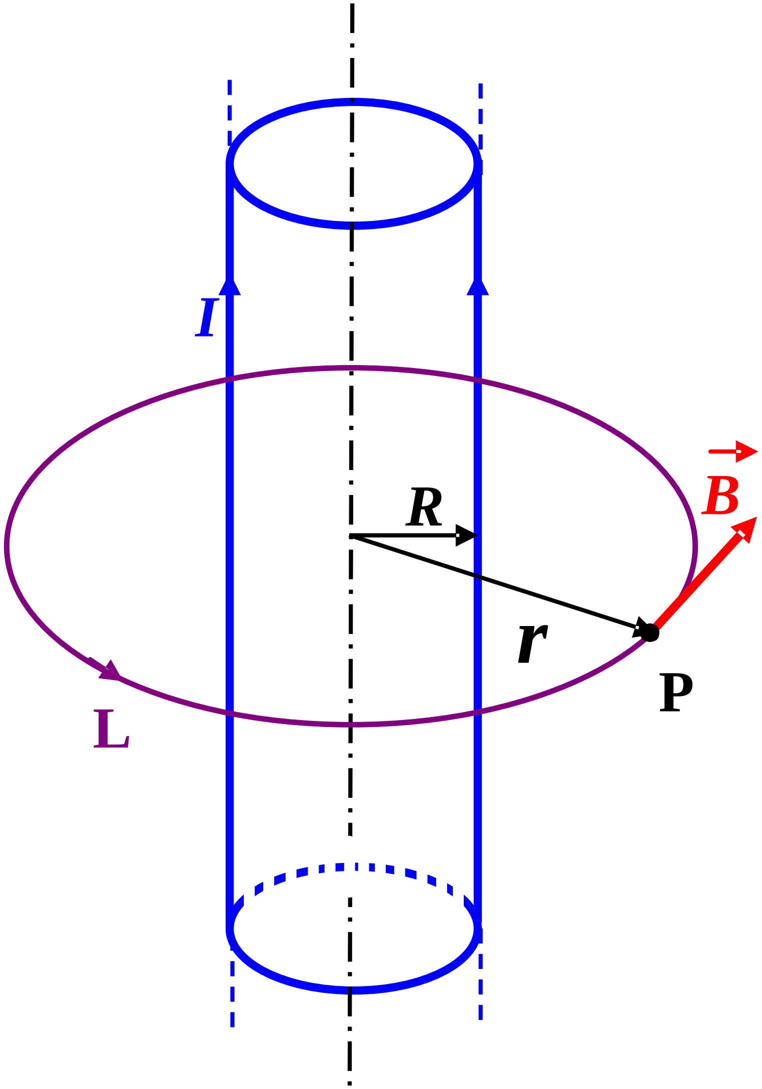
$$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos 0$$

$$= B \oint_L dl = B \cdot (2\pi r)$$

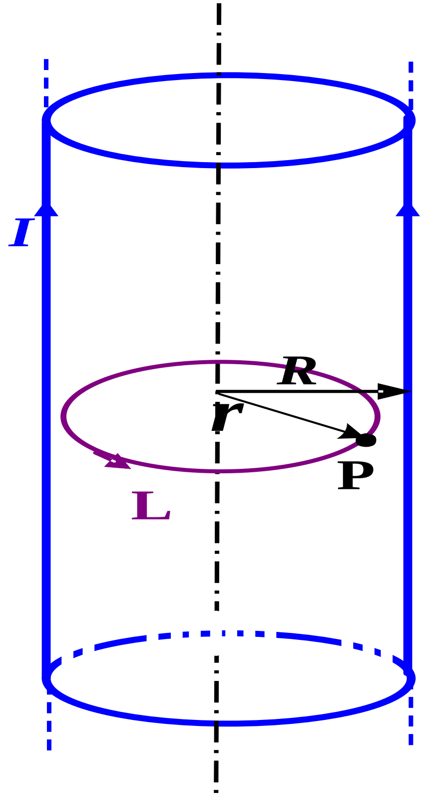
3)由环路定理 $\oint \vec{B} \cdot d\vec{l} = \mu_0 I'$

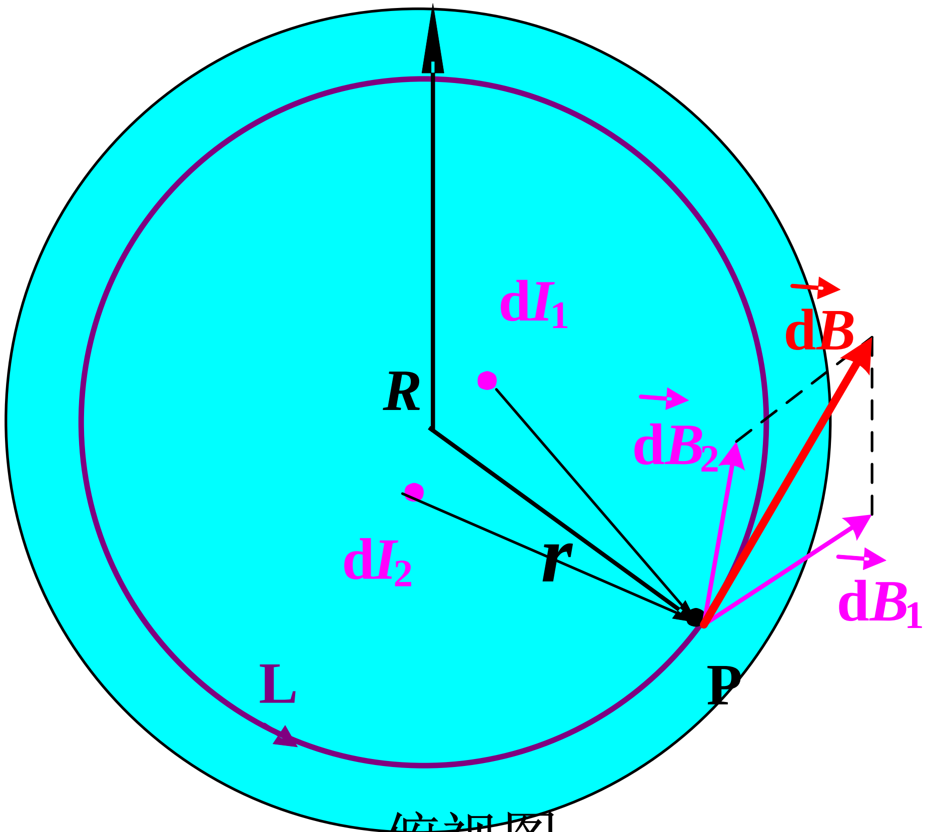
则 $B \cdot (2\pi r) = \mu_0 I$

$$\therefore B = \frac{\mu_0 I}{2\pi r}$$



内部磁场 **1).对称性分析**





俯视图

距轴线相同各点磁感应强度相同,方向与电流成右手螺旋关系

2).做对称性环路

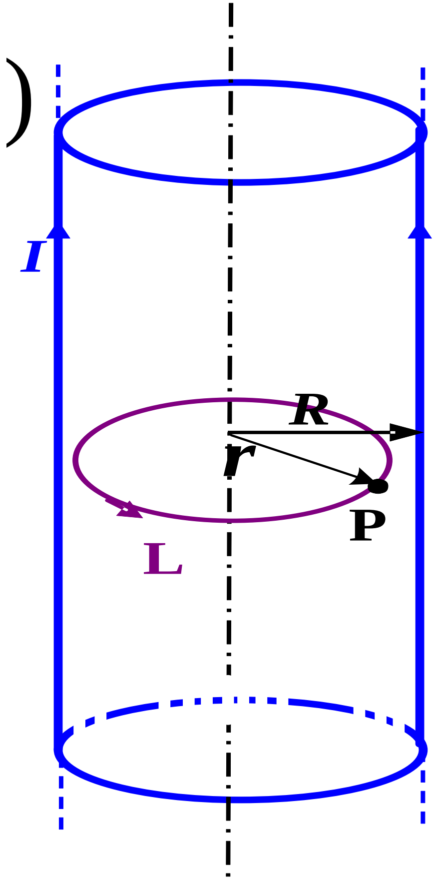
$$\oint_L \vec{B}\cdot d\vec{l} = \oint_L B\cdot dl\cdot\cos 0 = B\oint_L dl = B\cdot(2\pi r)$$

3)由环路定理 $\oint \vec{B}\cdot d\vec{l} = \mu_0 I'$

所包含的电流为 $I' = \dfrac{I}{\pi R^2}\pi r^2$

则 $B\cdot(2\pi r) = \mu_0 \dfrac{\pi r^2}{\pi R^2} I$

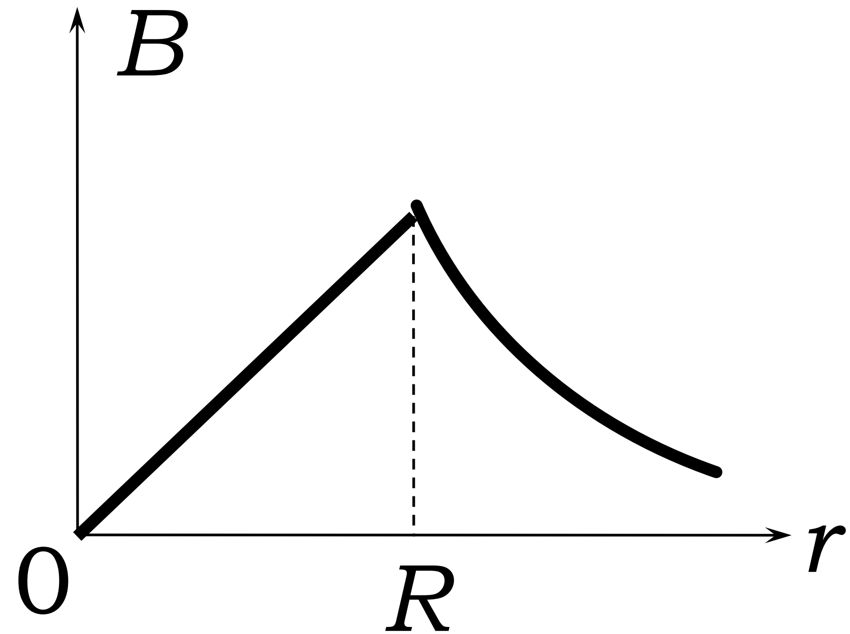
$$\therefore B = \frac{\mu_0 r}{2\pi R^2} I$$



均匀通电直长圆柱体的磁场

$$B = \frac{\mu_0 I r}{2\pi R^2} \qquad r < R$$

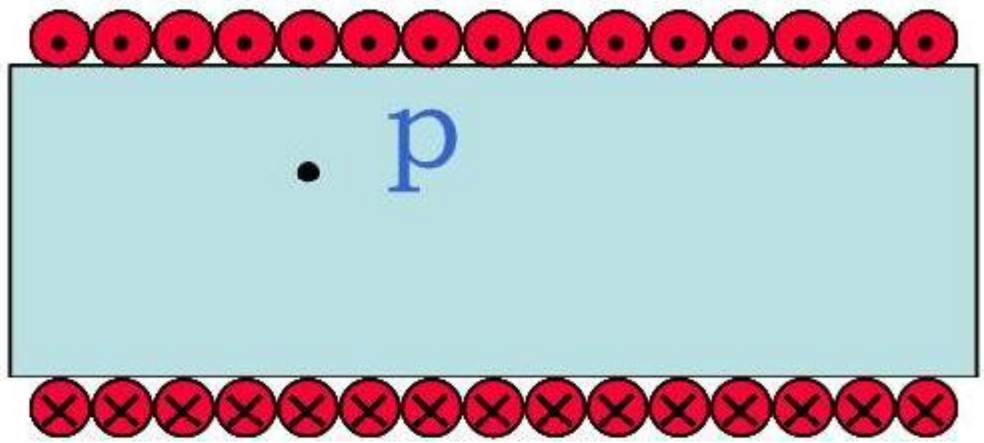
$$B = \frac{\mu_0 I}{2\pi r} \qquad r > R$$



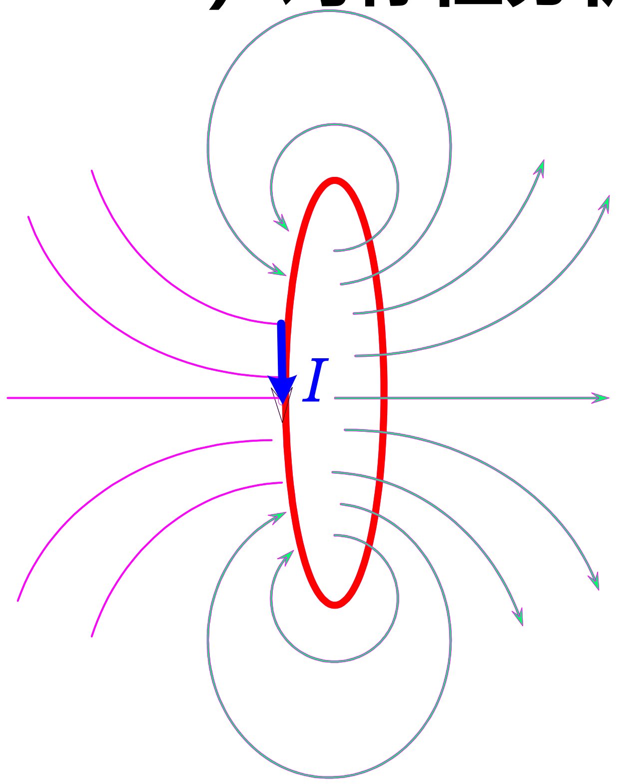
例2. 无限长螺线管的磁场

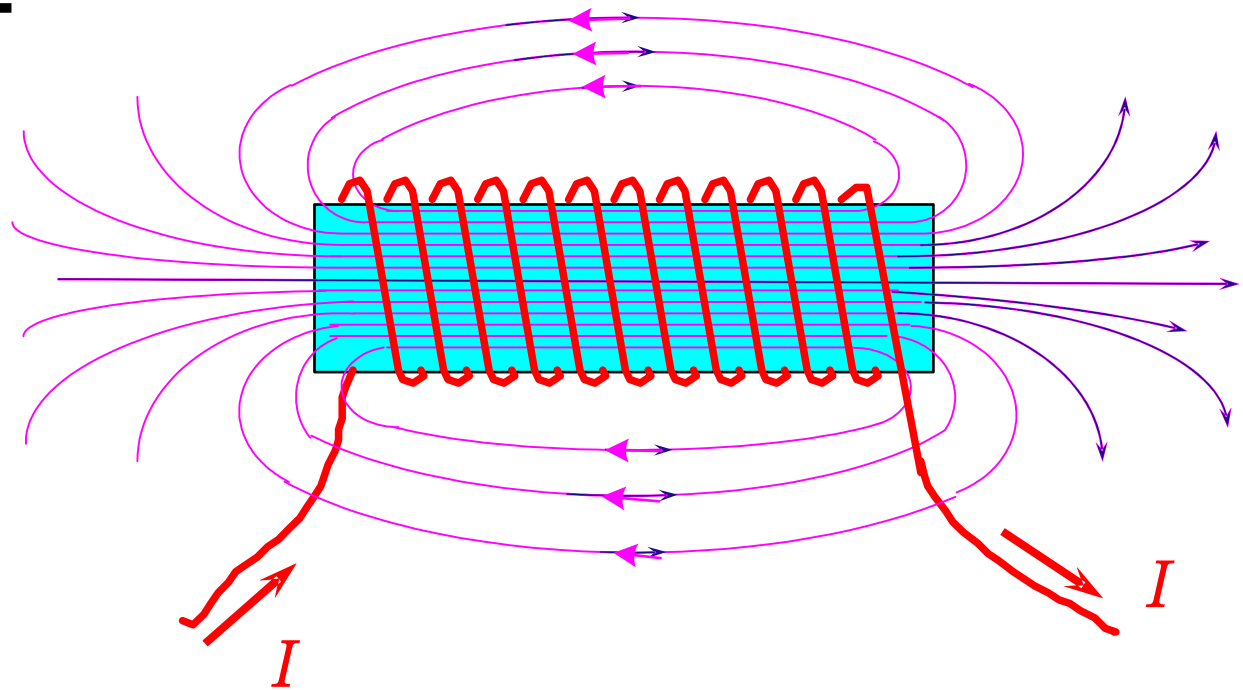
已知长直通电螺线管,半径 R,电流强度为I,单位长度的匝数为n

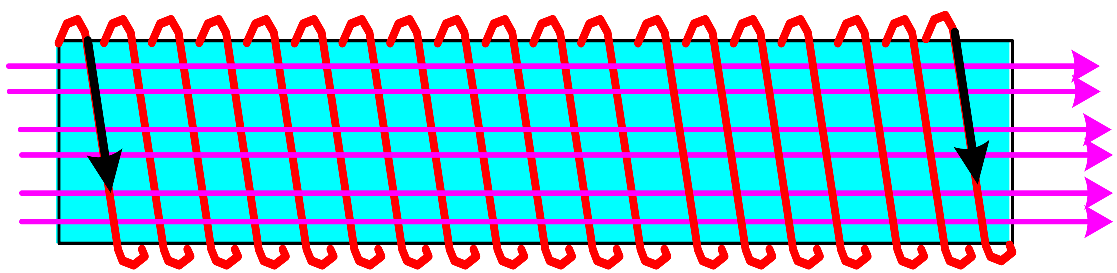
求螺线管内外任一点的磁感应强度



解：1）对称性分析

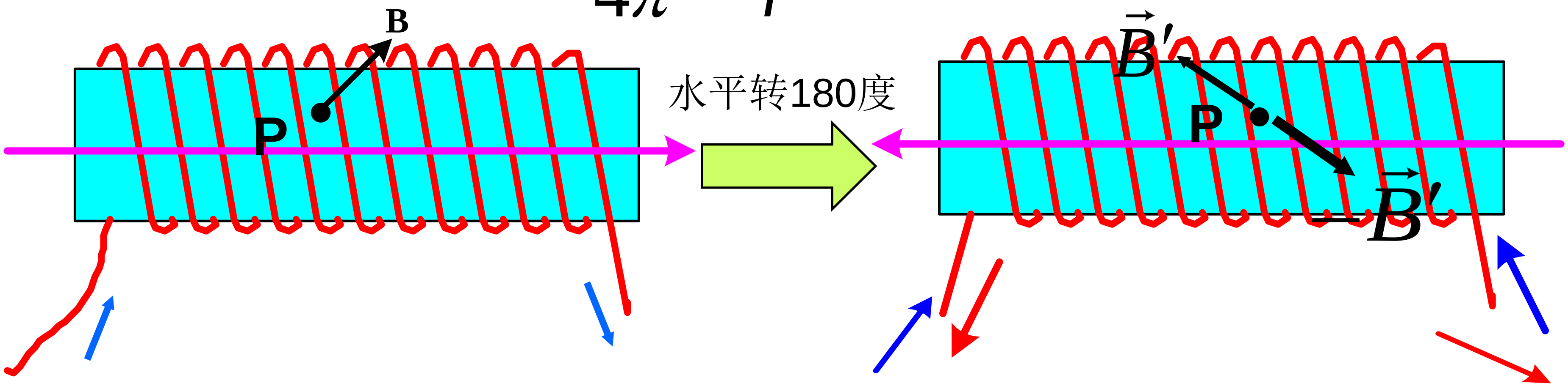






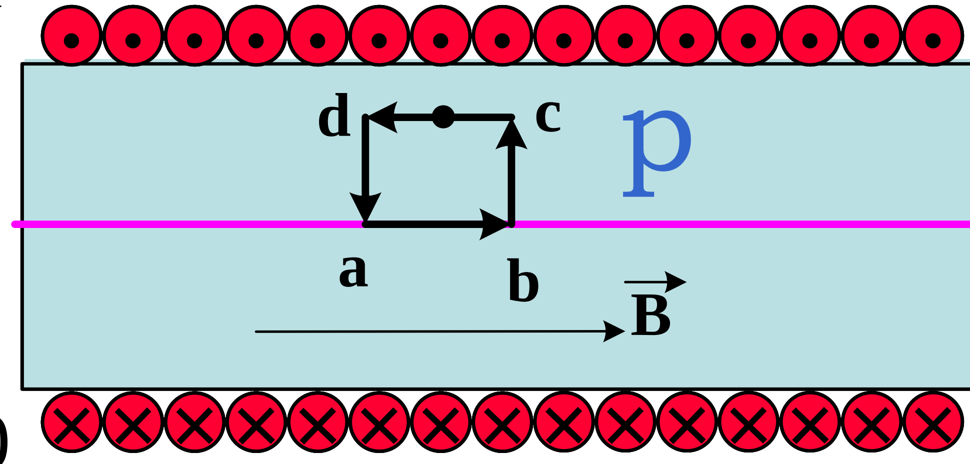
反证法

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$$



2）.做对称性环路(内部)

$$\oint_L \vec{B}\cdot d\vec{l} = \int_{ab} \vec{B}\cdot d\vec{l} + \int_{bc} \vec{B}\cdot d\vec{l} + \int_{cd} \vec{B}\cdot d\vec{l} + \int_{da} \vec{B}\cdot d\vec{l}$$



注意到:

$$\int_{bc} \vec{B}\cdot d\vec{l} = \int_{da} \vec{B}\cdot d\vec{l} = 0$$

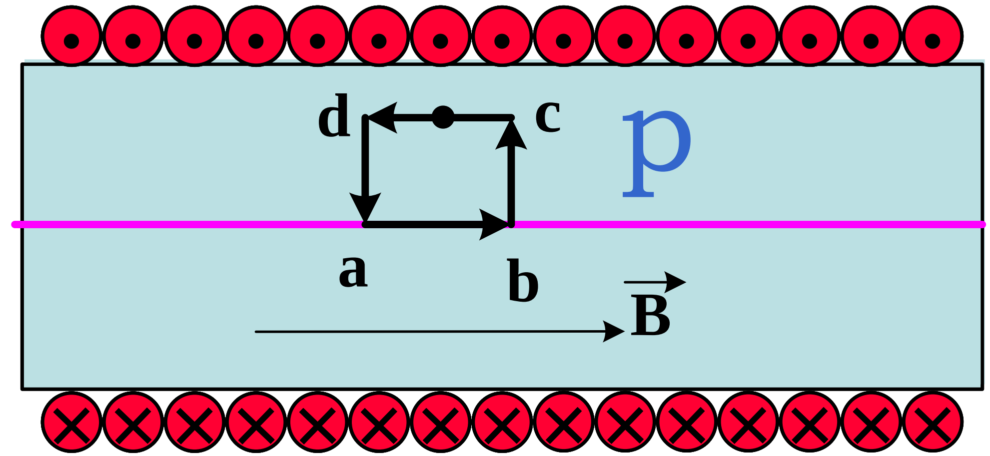
故 $$\oint_L \vec{B}\cdot d\vec{l} = \int_{ab} \vec{B}\cdot d\vec{l} + \int_{cd} \vec{B}\cdot d\vec{l} = B_{ab}l - B_{cd}l$$

$$\oint_L \vec{B}\cdot d\vec{l} = B_{ab}l - B_{cd}l$$

由环路定理

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I = 0$$

故 $B_{ab} = B_{cd} = \mu_0 nI$



无限长载流螺线内部磁场为均匀场

$$\boxed{B_{内} = \mu_0 nI}$$

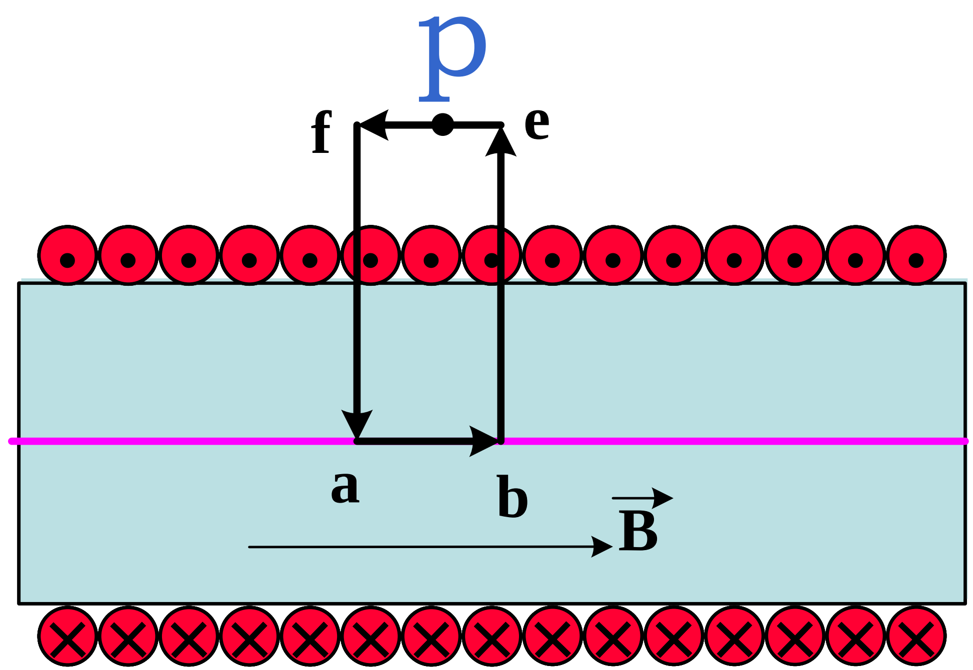
3）.做对称性环路(外部)

环路所包含的电流为nIl

由环路定理

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 nlI$$

$$\int_{ab} \vec{B}\cdot d\vec{l} + \int_{be} \vec{B}\cdot d\vec{l} + \int_{ef} \vec{B}\cdot d\vec{l} + \int_{fa} \vec{B}\cdot d\vec{l}$$



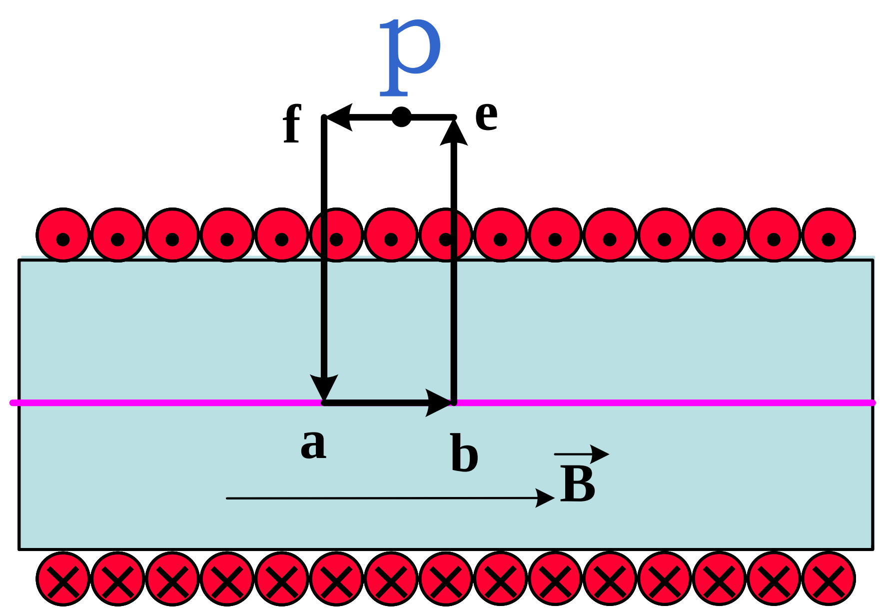
故 $\int_{ab} \vec{B} \cdot d\vec{l} + \int_{ef} \vec{B} \cdot d\vec{l} = \mu_0 n l I$

$$= B_{ab} l + B_{ef} l$$

$$= \mu_0 n I l + B_{ef} l$$

$$\Downarrow$$

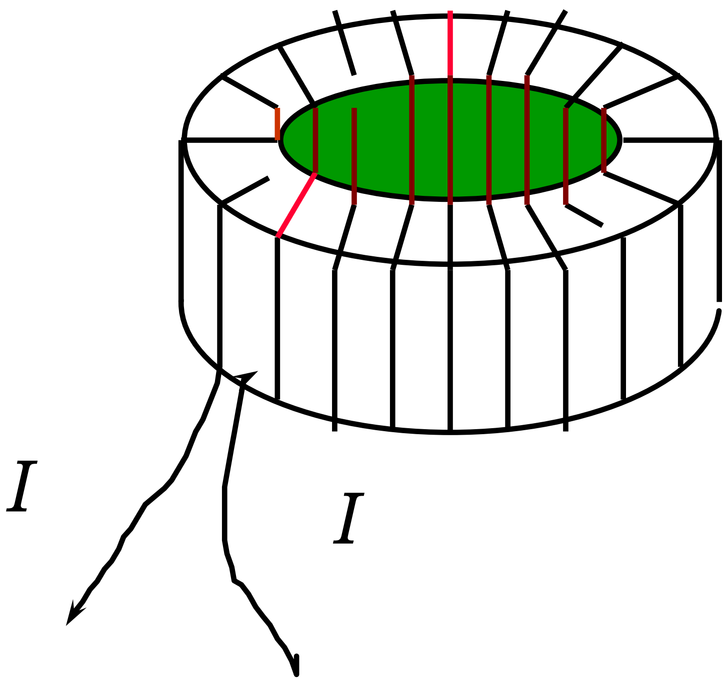
$$B_{ef} = 0$$

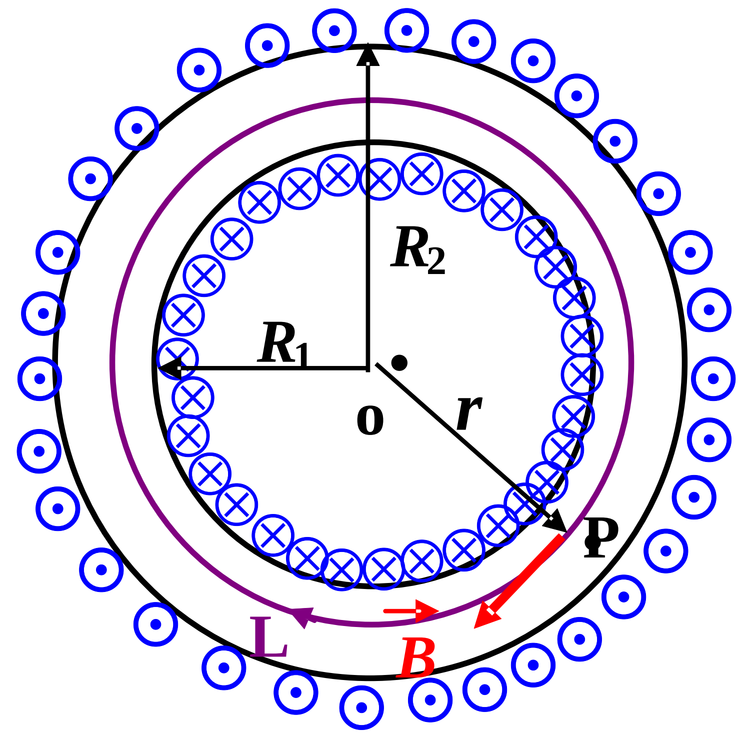


无限长载流螺线外部磁场为 $\boxed{B_{外} = 0}$

例3.通电螺绕环的磁场

设螺绕环有*N*匝线圈，线圈中通电流*I*，各尺寸如图。





由对称性，螺绕环内任一点P的磁场方向如图，
且距中心同远处，磁场的大小相同。

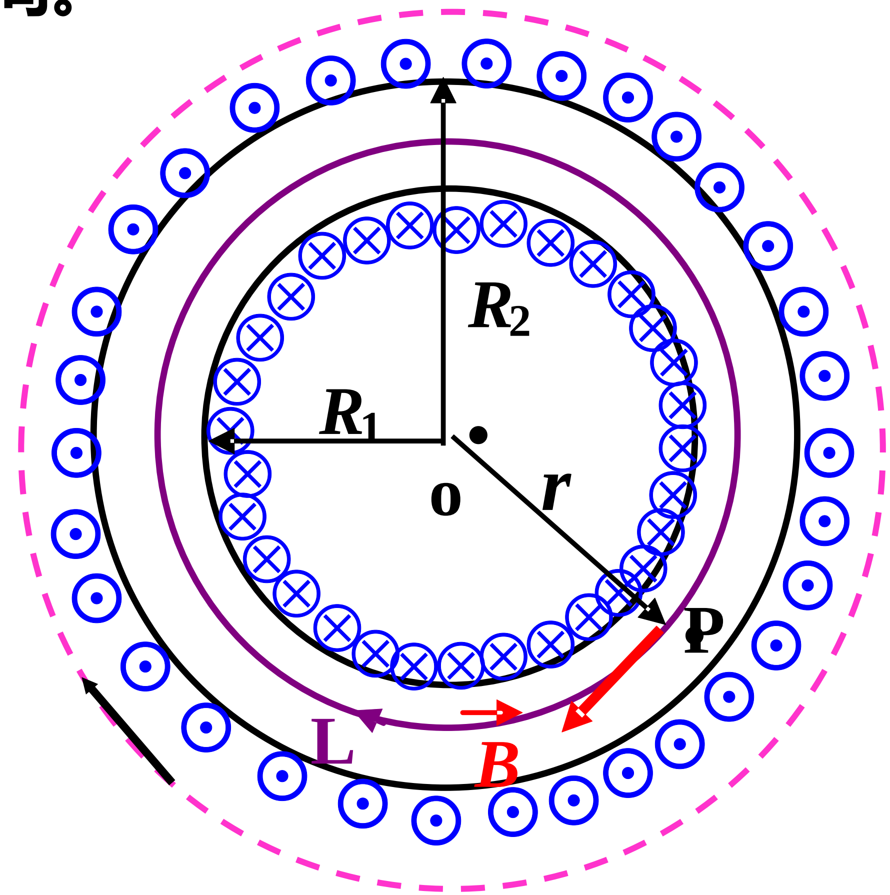
选如图的环路L 由环路定理

$$\oint \vec{B}_{内} \cdot d\vec{l} = \mu_0 NI$$

$$\therefore B \cdot (2\pi r) = \mu_0 NI$$

$$B_{内} = \frac{\mu_0 NI}{2\pi r}$$

$$B_{外} = 0$$



讨论

$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 \sum_i I_{i内}$$

$$B = \frac{\mu_0 I}{4\pi a}\left(\cos\theta_1 - \cos\theta_2\right)$$

对于长直线型载流体，安培环路定理适用于无限长的