CS5487 - Multivariate Gaussian Example

The following shows some examples of 2-d Gaussian distributions using different covariance matrices. Let's define the following covariance matrices:

$$\Sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad \Sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.25 \end{bmatrix}, \qquad \Sigma_3 = \begin{bmatrix} 0.625 & 0.375 \\ 0.375 & 0.625 \end{bmatrix} \tag{1}$$

Note that Σ_1 is an i.i.d. (or circular) Gaussian, i.e., all elements on the diagonal are the same. Σ_2 is a diagonal covariance matrix. Finally, the eigen-decomposition of Σ_3 is:

c   s   45° counterclockwise

$$\Sigma_3 = V \Lambda V^T, \quad V = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 1 & 0 \\ 0 & 0.25 \end{bmatrix}, \tag{2}$$

-s   c

which has the same eigenvalues as Σ_2. In the plots below, the first row shows the Mahalanobis distance ($\Delta = \sqrt{x^T \Sigma^{-1} x}$), where the black lines are the iso-contours at $\Delta = \{1, 2, 3, 4\}$. The second row shows the pdf of the Gaussian (red has highest density and blue lowest). The third row shows a 3D surface plot of the Gaussian. From the plots we see that Σ_1 has a circular shape, Σ_2 is elliptical, and Σ_3 is a rotated ellipse.