Implementation of Heston-Nandi model in ”Insurance +Futures” mode

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Abstract

This paper applies the Heston-Nandi model in combination with Monte Carlo simulation to pricing Asian put options in the Insurance + Futures mode. Empirical studies show that the Heston-Nandi model is closer to the out-of-money options market price than the Black-Scholes model and has better pricing precision. In addition, through the pricing results of the Insurance + Futures mode, we found relevant rules of premium rates. Using these premium rate rules will simplify the procedure of pricing agricultural insurance and options. Combined with the full text, we give relevant policy recommendations.

1. Introduction

Three Rural Problems is related to the stability of China's 900 million farmers, and is the foundation of the country's development. In the past few decades, China's agricultural development has made remarkable achievements. But when compared with developed countries, our agricultural is still relatively backward. With the deepening of economic globalization, the international market competition of farm products is becoming more and more fiercer. The price of agricultural products in China is greatly affected by the factors of the global market, which leads to drastic fluctuations in the price of farm products. Due to the immature development of China's agricultural system, the price fluctuation of farm products often lags. And due to information asymmetry and other factors, the risk of agricultural price is intensified. The extreme variation of the price of agricultural products will increase the risk of farmers' operation and cause the instability of farmers' income. At the same time, agricultural products as a necessity of people's lives, and is also the raw materials of many productions. When the price of agricultural products rises, the cost of people and enterprises will increase, and eventually, inflation will have an impact on people's living stability and national economic development.

The fluctuation of the price of agricultural products cannot be prevented, but the risk awareness of farmers can be improved. And the risk management of agricultural product prices can be carried out. In China, there are a variety of risk management methods of farm products, such as target price policy, which have promoted the development of China's agriculture. But these methods brought a particular burden for national finance. Therefore, it is particularly important to study and develop the risk management model of agricultural products such as Insurance + Futures mode.

Insurance + Futures mode is an innovative model of agricultural price risk management, which can not only effectively hedge agricultural price risk, but also contribute to the development of agricultural insurance and agricultural derivatives in China. The operation mechanism of Insurance + Futures mode is as follows: agricultural cooperatives or farmers buy agricultural insurance products provided by insurance companies; insurance companies buy put options from future companies to hedge risks; future companies copy put options to transfer risks to futures markets. The risk transfer path is: agricultural product producers to insurance companies, and insurance companies and future companies cooperate to transfer the risk from the insurance market to the futures market. It can be seen that in the whole risk transfer process, agricultural producers, insurance companies and future companies cooperate, and finally transfer the risk of agricultural price to the futures market through agricultural derivatives. Also, this mode has many advantages over other modes such as purchasing agricultural insurance or agricultural product futures. First of all, for Chinese agricultural operators, the use of options and futures is more complex, and the acceptance of insurance is higher. Secondly, compared with agricultural product futures, agricultural product insurance in Insurance + Futures mode can lock the risk of agricultural product price with less capital cost, protect farmers' basic income when the price falls, and enable farmers to obtain more profits when the price rises.

In the Insurance + Futures mode, the pricing of premiums mainly refers to the pricing of OTC options of agricultural products, so it is of great significance to find how to price OTC options of agricultural products. At present, domestic scholars do not have much exploration on OTC option pricing of agricultural products. Most scholars aim to study the characteristics, preliminary design and other aspects of ”Insurance + Futures” mode. Ning Rencong (2018) analyzed the compensation principle of agricultural products insurance and concluded that the essence of agricultural products option is an Asian option. Based on B-S formula, the pricing formula of agricultural products option was derived through the Asian option pricing model, and combined with practical cases. It showed that the options achieved excellent results in the risk management of agricultural products price. Li Yaru, Sun Rong and Liu Zhen (2018) used

Heston model to price agricultural products price insurance. Taking egg futures price insurance as an example, the pricing results of six major egg-producing areas were obtained. Nie Rong, Qian Keming and Pan Dehui (2004) give a mathematical equation for pricing European call options with mixed stochastic processes on the premise that agricultural prices follow the geometric brown-step diffusion process. Kong Liang (2007) used the actuarial method to study option pricing in the diffusion process of price subject to jump and gave the pricing model of European options for agricultural products. Wang Hui (2015) built the pricing model of agricultural products barrier options, and then hedged the risk of spot price fluctuation in agricultural products futures, options and spot market.

In this paper, Black Scholes option pricing model and Heston-Nandi volatility model are selected to price Asian agricultural options. Black Scholes option pricing model is one of the most widely used options pricing models. However, the B-S model also has an apparent deficiency. The volatility in the formula is assumed to be a constant, which is inconsistent with the market law. For this reason, many scholars have improved the pricing of B-S model and developed the stochastic volatility model and GARCH family volatility model. Among them, the GARCH family model has been widely used in asset price modeling, and its effect has been well proved in various financial literature. The GARCH option pricing model has inherent advantages. Its volatility can be observed from the discrete asset price data, and it only takes a few parameters to estimate the price and volatility sequence in a long time.

Among the options pricing models of the GARCH family, the models of Engle and Mustafa(1992), Amin and Ng(1993), Duan(1995) and Heston and Nandi(2000) are the most concerned. The Heston-Nandi model captures the randomness of volatility and the correlation between volatility and spot returns and takes into account the volatility leverage feature. Also, there is no closed-form solution for option price in most GARCH option pricing models. Heston-Nandi model develops a closed-form solution for European option.

Some scholars have tested the Heston-Nandi model. Jakob Bengtsson Ekstrom, Oscar Sjogren (2014) evaluated the performance of the Heston-Nandi option pricing model (2000) on the OMXS30(Swedish stock index) before and after the financial crisis, with the primary purpose of studying whether the Heston-Nandi model can obtain more accurate price estimation than the simpler B-S model in the calculation. The results show that the Heston-Nandi model is more accurate on the pricing of the put option. Suk Joon Byun (2011) estimated the parameters of the Heston-Nandi model by using the time series of the historical daily return of the sp 500 index and then compared the analytical formula of the Heston-Nandi model with the results of Monte Carlo simulation.

In this paper, the path obtained by the Heston-Nandi model is simulated by Monte Carlo simulation, to get the option price. The reasons for using Monte Carlo simulation instead of Heston-Nandi model closed-form solution are as follows: First of all, the analytic solution can only be applied to European option pricing, but not to American option pricing. Secondly, through our test, the result of Monte Carlo simulation is consistent with the closed-form solution on European options. Therefore, Monte Carlo simulation is finally chosen to solve the simulation path in this paper.

2. Model

2.1. The Black-Scholes-Merton Model

According to the seminal work of Black, Scholes, and Merton (1973), a revolutionary model of option pricing was established in arbitrage pricing framework. The B-S model has been the most frequently used and important model in pricing options. This model relies on two assumptions, namely that the price of the underlying asset has constant variance and subject to geometric Brownian motion

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

where W_t is standard normal distribution at time t under measure P, S is the price of the underlying asset and σ is the volatility. Pricing option under the non-arbitrage analytic form, Black and Scholes developed the Black-Scholes equation

$$F_t + \frac{1}{2}\sigma^2 S_t^2 F_{ss} + rS_t F_s - rF = 0 \tag{2}$$

where r is the risk-free interest rate and F is the value of the derivative. This stochastic differential equation has a closed-from solution, referring to Black-Scholes formula, which has been widely accepted and used in option pricing. The equation can be easily solved under the risk-neutral measure. This will be introduced in the next section.

It should be noted that the Black-Scholes model assumes that implied volatility σ is a constant, this contradicts empirical results. Usually, implied volatility rises when the price of the underlying asset is further out of the money or in the money. Therefore it shows a shape like a smile, namely Volatility Smile. Naturally, many volatility models are constructed to depict this variation.

2.2. Risk-neutral Pricing Theory and Martingale Pricing Theory

In our real world, investors always want to get an excessive expected rate of returns than the risk-free interest rate since the price of risky assets is uncertain. However, when in a risk-neutral world, investors just require the risk-free interest rate of returns. So all assets have an expected rate of return equal to the risk-free interest rate.

When pricing the option, we can assume that all investors are risk-neutral, and the pricing of the derivative will still be right in the real world. Thus the price of the derivative is the expected payoff at maturity, discounted at the risk-free interest rate. In the B-S model, under risk-neutral measure, the process of underlying asset under measure Q subject to

$$dS_t = rS_t dt + \sigma S_t dW_t \tag{3}$$

where r is the risk-free rate and W_t^* is standard normal distribution under measure Q. This method leads to a convenient way named martingale pricing theory to solve the Black-Scholes equation, raised by Harrison and Kreos (1979). Under Q measure, the present value of a European call option

$$e^{-rt}F(S_t, t) = E_t^Q[e^{-rT}(S_T - K)^+] \tag{4}$$

is a Q-martingale. The existence and uniqueness of Q-measure can be guaranteed by the fundamental theorem of asset pricing. Moreover, the dynamics of geometric Brownian motion can be obtained by Girsanov theorem when the measure is changed to an equivalent probability measure.
For example, for European call option

$$F = max(S - K), when\ t = T \tag{5}$$

and the solution is

$$c = S_t N(d_1) - Ke^{-r(T-t)} N(d_2) \tag{6}$$

where

$$d1 = \frac{log(\frac{S_t}{K}) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{7}$$

and

$$d1 = \frac{log(\frac{S_t}{K}) + (r - \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}} \tag{8}$$

c is the European call option price at time t. S_t is the price of underlying asset at time t, K is the strike price, σ is the volatility of asset, r is risk-free rate and T is the maturity of the option.

2.3. The GARCH Process

Since the B-S model was developed, many scholars have developed models that incorporate stochastic volatility. Bollerslev (1986) and Taylor (1986) developed the generalized autoregressive conditional heteroscedasticity (GARCH) process, which provides a useful extension to the ARCH model made by Engle(1982). The GARCH(p,q) model is given by

$$\begin{cases} r_t = \mu_t + a_t \\ a_t = \epsilon_t \sigma_t \\ \sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \end{cases} \tag{9}$$

where μ_t is the mean equation and σ_t is a white noise process. Unlike ARCH model where the conditional variance is only relay on the past squared error, the conditional variance in GARCH model depends on q lags of the squared error and p lags of the conditional variance. There are restrictions on the parameters if we have to ensure the GARCH (p,q) process to be weakly stationary

$$\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \tag{10}$$

If the sum of the parameters above is equal to 1, the process is referred to as the IGARCH model.

The GARCH process can depict the cluster of volatility and the fat tail of the asset price. However, the major restrictions of the GARCH model are that it enforces asymmetric response of volatility to both positive and negative innovations. Several models are developed form the GARCH framework to fit the market better.

2.4. The Heston Model

Hull, White, and Scott(1987) develop a model with stochastic volatility. Heston(1993) refines this model and assumed that the underlying asset at time T follow this form

$$dS(t) = \mu S dt + \sqrt{v(t)} S dZ_1(t) \tag{11}$$

and the variance v(t) follow the process

$$d\nu(t) = \kappa[\theta - \nu(t)]dt + \sigma\sqrt{\nu(t)} dz_2(t) \tag{12}$$

which can be regarded as a square-root process developed by Cox, Ingersoll, and Ross(1985).In this model, θ denotes the long-term rate and κ is the speed of mean reversion. However, its not possible to develop an accurate close-form formula from above stochastic differential equation. Motivated by this, Heston and Nandi(2000) extend Heston model combined with GARCH process.

2.5. The Heston-Nandi Model

Following Engle and Mustafa(1992), Duan(1995) develop an intact framework of GARCH option pricing model. Rubinstein and Brennan(1979) extend risk neturalization to a generalized form named locally risk-neutral valuation relationship(LRNVR). Duan develop the solution of GARCH option pricing model under this circumstance, this model clearly show volatility smile when its applied to empirical analysis.Motivited by Duan's work, Heston and Nandi develop a model following

$$ln(S_t) = ln(S_{t-\Delta}) + r + \lambda h_t + \sqrt{h(t)}z(t) \tag{13}$$

and the variance h_t subject to

$$h_t = \omega + \sum_{i=1}^{p} \beta_i h_{t-i\Delta} + \sum_{i=1}^{q} \alpha_i (z_{t-i\Delta} - \gamma_i \sqrt{h_{t-i\Delta}})^2 \tag{14}$$

where γ_i denotes the leverage effect of underlying assets. This model also assumes that Assumption 2: The value of the option one period prior to expiration follow the Black-Scholes-Merton formula, and from this assumption a risk-neutral GARCH process can be derived by

$$\lambda = -\frac{1}{2}, \gamma^* = \gamma + \lambda + \frac{1}{2} \tag{15}$$

There is a closed-form option pricing formula for the Heston-Nandi model. Heston and Nandi show that the conditional generating function of the asset price takes the following form

$$f(\Phi) = E_t(S_T^{\Phi}) = S_t^{\Phi} e^{A_t + B_t} \tag{16}$$

They also derive the recursion for the coefficients and the two coefficients can be calculated recursively

$$\begin{cases} A_t = A_{t+1} + \Phi r + B_{t+1}\omega - \frac{1}{2}log(1 - 2aB_{t+1}) \\ B_t = \Phi(\lambda + c) - \frac{1}{2}c^2 + bB_{t+1} + \frac{\frac{1}{2}(\Phi - c)^2}{1 - 2aB_{t+1}} \\ A_T = B_T = 0 \end{cases} \tag{17}$$

So we can derive the following option pricing formula for the European call option with strike price K that expires at time T:

$$\begin{aligned} c =& \frac{1}{2}S_t + \frac{e^{-r(T-t)}}{\pi}\int_0^\infty Re[\frac{K^{-i\Phi}f^*(i\Phi+1)}{i\Phi}]d\Phi \\ &- Ke^{-r(T-t)}(\frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re[\frac{K^{-i\Phi}f^*(i\Phi)}{i\Phi}]d\Phi) \end{aligned} \tag{18}$$

In the formula above, $f^*(i\Phi)$ is the characteristic function of the logarithm of the spot price and Re[ ] denotes the real part of a complex number.

3. Data

In this paper, we select options of egg, corn, soybean and palm oil for pricing. The reasons for the selection are as follows: First of all, in recent years, the price fluctuation of the underlying assets mentioned above is relatively large, which affects the income of farmers. Therefore, it is necessary to carry out the ”Insurance + Futures” mode for these underlying assets. Secondly, these underlying assets are essential consumer goods for residents’ daily life. Thirdly, these underlying asset futures already exist in China, and the market has the function of price discovery.

We select daily futures prices for eggs, corn, soybeans and palm oil. The data are from 26 April 2016 to 26 April 2019. The futures price unit of the egg is yuan / 500kg, and those of corn, soybean and palm oil are yuan/ton. The futures prices are all from the WIND database.

We convert the daily settlement price of all data into the daily logarithmic yield: $y_t = lnS_t - lnS_{t-1}$, and S_t is the spot price or futures price of the underlying asset at time t. Descriptive statistics of the logarithmic return rate are shown in the table below:

Table 1: Descriptive statistics of egg futures

statistics	egg	corn	palm oil	soybean
nobs	732	732	732	732
NAs	0	0	33	0
Minimum	-0.253408	-0.154702	-0.129721	-0.064234
Maximum	0.252195	0.062484	0.083859	0.109475
1. Quartile	-0.007521	-0.002878	0.000000	-0.003961
3. Quartile	0.007884	0.003245	0.000000	0.003072
Mean	0.000237	0.000052	-0.000404	-0.000127
Median	0.000000	0.000000	0.000000	0.000000
Sum	0.173422	0.037720	-0.289797	-0.093260
SE Mean	0.001201	0.000460	0.000507	0.000431
LCL Mean	-0.002120	-0.000851	-0.001400	-0.000973
UCL Mean	0.002594	0.000954	0.000592	0.000718
Variance	0.001055	0.000155	0.000184	0.000136
Stdev	0.032485	0.012436	0.013581	0.011652
Skewness	-0.197251	-3.010323	0.013581	0.943627
Kurtosis	20.452291	40.287200	23.302508	15.546650

Table 2: Jarque-Bera Test

Futures	χ-squared	p-value
egg	12843	$\leq$2.2e-16
corn	50905	$\leq$2.2e-16
soybean	7529.1	$\leq$2.2e-16
palm oil	16745	$\leq$2.2e-16

As we can see from the table, the daily logarithmic rate of return series has a sharp peak and a thick tail with a left-skewed feature. The results of Jarque-Bera statistics refuse to obey the assumption of normal distribution, so the Heston-Nandi model can better fit the characteristics of price volatility than the classical Black-Scholes model.

4. Monte Carlo Simulation

4.1. Parameter Estimation

In Heston-Nandi model, the process of underlying assets follows the GARCH model with five unknown parameters: λ, α, β, γ, ω. It should be noted that parameter measure the leverage effect of the futures price series. It is often the case that is positive, which means the futures price is negatively correlated with volatility. It is quite intuitive because when the price of futures goes down, the expected risks will rapidly increase. In our empirical analysis, this paper only focuses on one lag version of GARCH model. Consequently, our model of volatility is simplified to

$$h_t = \omega + \beta h_{t-\Delta} + \alpha(z_{t-\Delta} - \gamma\sqrt{h_{t-\Delta}})^2 \tag{19}$$

and we define Δ one day. In this way, the parameters can be easily estimated by historical futures price series. Maximum Likelihood Estimation (MLE), ordinary least square (OLS) and Generalized method of moments (GMM) approaches are commonly used to estimate the parameter of GARCH model. A plausible way to estimate the parameters presented by Heston and Nandi(2000) is OLS method. However, it requires us to obtain the historical option price series. The put options of Insurance + Futures are traded in the OTC market, so It is not quite plausible to use the price series published in the OTC market. Besides, it is quite difficult to obtain the entire datasets of these option prices. So in this paper, the MLE approach will be employed because the MLE method only uses the futures price series from historical datasets. The log likelihood function is

$$lnL = \sum_{t=1}^{T} -\frac{1}{2}[ln(2\pi h_t) + \frac{(R_t - r - \lambda h_t)^2}{h_t}] \tag{20}$$

This analysis set the risk-free interest rate as 3% (estimation of the average rate of 1-year treasury bond), through MLE method, the parameters of egg futures-soybean futurescorn futurespalm oil futures from April 26th, 2016 to April 26th, 2019 are estimated in the following table:

Table 3: Parameter Estimation

Parameter	egg	corn	soybean	palm oil
λ	0.8714	6.2274e-05	0.0018	0.0082
ω	1.3682e-05	1.2965e-05	6.1782e-05	1.1718e-04
α	5.6169e-10	2.5281e-05	8.1427e-05	4.4997e-09
β	0.9870	0.7228	0.1024	0.3674
γ	140.7054	1.0175e-04	12.7375	140.8728

The parameters of four main futures are quite different. It indicates that four agriculture futures have unique features of volatility. Significantly, parameter γ of all four futures are positive. It corroborates that the volatility of futures is negatively correlated with futures price. This assertion indicates that volatility is not constant, and Black-Scholes model will ignore this subtle nicety and cause a relatively large error when the options are deeply in the money or out of the money. The leverage effect of egg futures and palm oil futures is significantly higher than the leverage effect of corn futures and soybean futures, especially the asymmetric fluctuation of corn futures is negligible. Its reasonable because when negative shock occurs, different agriculture products have different resistance to news.

4.2. Monte Carlo Simulation

This section mainly uses egg futures price simulation as an example to show the result. After estimating the parameters, it is quite simple to simulate the path of volatility and egg price in the futures by Monte Carlo method. The historical egg futures price series from April 26th, 2016 to April 26th, 2019 are used to estimate the parameters. Our analysis set futures price at present S0 = 3781, and strike price equals to spot futures price, the risk-free interest rate is 3%, and maturity T is 1000 days. The simulated volatility curve and the futures price curve is in the following two graphs:

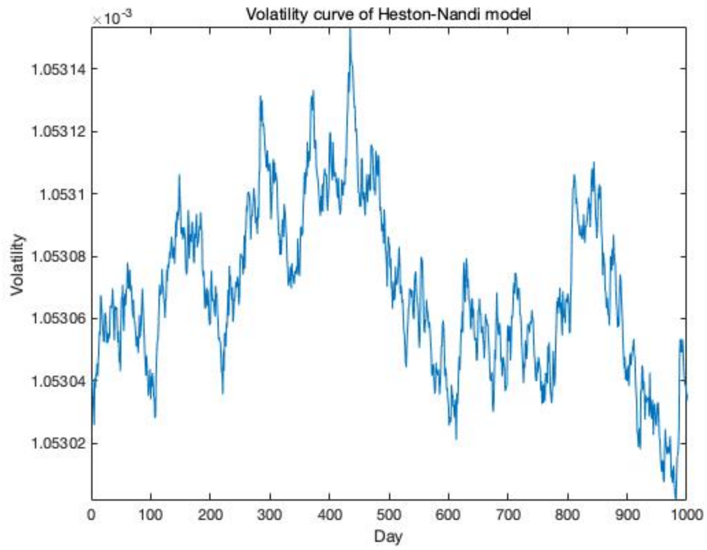

Figure 1: Volatility curve of Heston-Nandi model

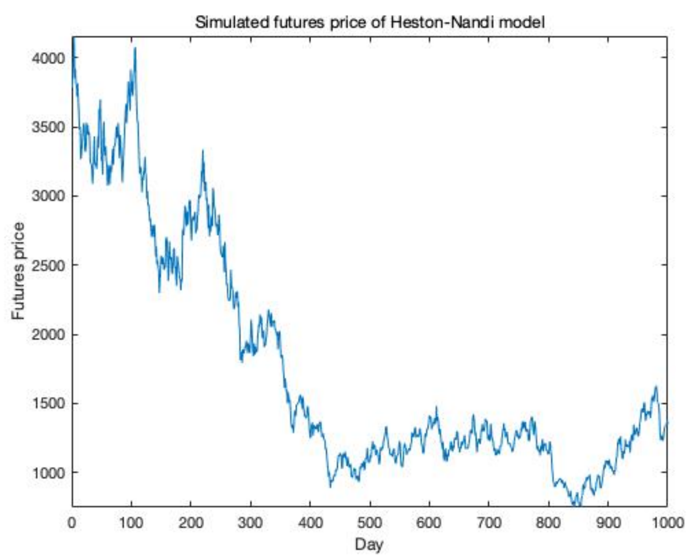

Figure 2: Simulated futures price of Heston-Nandi model

From the above picture, the volatility per day varies from 1.053e-03 to 1.05316e-03, Therefore the volatility is mutable in Heston-Nandi model. It can capture more precise feature of the implied volatility than Black-Scholes-Merton models. In the next session, this paper will provide the result of option pricing with path simulated above.

4.3. Insurance + Futures pricing

This section will highlight on the pricing a put option of future in Insurance + Futures mode. And we only concentrate on the pricing procedure of the put option and ignore some extra fee or trade cost during the trading among farmers, insurance companies and futures companies. We define that protective price is the strike price of the put option, it indicates that when the futures price goes down, the insurance companies are capable of using the option to meet the loss from reduction of the futures price. However, setting an appropriate protective price is quite tricky. If the protective price is too low, it is not even possible to trigger the option exercise, the effect of ”Insurance + Futures” will be suppressed. And if the protective price is too high, according to the relationship between the strike price and option value, the premium will be quite high, and farmers are willing to take risks rather than purchasing this product. Hence, the following empirical analysis will set a range of reasonable protective price (strike price) and calculate the value of the put option, namely the premium of Insurance + Futures product.

BecauseInsurance + Futures product is aimed to prevent farmers from unpredictable loss of agriculture products, this product need to cover a wide range of agriculture products, such as egg, wheat, maize, etc. Nonetheless, different kinds of agriculture products have different features. Therefore, it is doubtful whether Insurance + Futures will be applicative to all kinds of products. Consequently, it is imperative to verify the universality of ”Insurance + Futures” product. In the following part, this paper will apply a variety of agriculture product into this Heston-Nandi pricing model.

In most cases, Insurance + Futures often use Asian option instead of vanilla options. Because it is not reasonable only to use futures price of expiration day as the benchmark, it is more appropriate to judge whether farmers need the subsidy based on the average level of the futures price. And Asian option has corresponding features, so it is an ideal choice for Insurance + Futures. However, the closed-form formula of Heston-Nandi model presented in Heston and Nandis paper contrapose vanilla call option, and it is relatively challenging to deduce the closed-form formula of Asian option in this situation. Hence, a practical alternative to calculating the value of ”Insurance + Futures” is using the Monte Carlo method. For convenience, this analysis will use the arithmetic average of the whole period in Asian option pricing.

In short, this part will concentrate on the pricing procedure of Insurance + Futures product with Heston-Nandi model. The protective price and underlying agriculture product will vary in the analysis, in order to testify the broad applicability of Insurance + Futures.

In our pricing procedure, the risk-free interest rate is always set at 3% per year. Maturity is 30 trading daysthe spot price of futures S0 is the price on April 26th, 2019. Strike price or protective price K will vary from 0.5*S0 to 1.5*S0, and the step is 0.1*S0. In this way, the relationship between protective price and premium can be discerned. The result is as follows:

Table 4: Premium of egg insurance

Protective price	Premium	Premium rate
0.95*S0	71.6141	1.98%
0.96*S0	84.6832	2.31%
0.97*S0	97.7775	2.68%
0.98*S0	115.3554	3.09%
0.99*S0	132.5680	3.53%
1.00*S0	151.7504	4.00%
1.01*S0	170.7035	4.49%
1.02*S0	193.8577	4.99%

1.03*S0	217.5914	5.52%
1.04*S0	240.2257	6.12%
1.05*S0	266.6599	6.74%

Table 5: Premium of corn insurance

Protective price	Premium	Premium rate
0.95*S0	2.4252	0.14%
0.96*S0	4.2027	0.23%
0.97*S0	7.1916	0.40%
0.98*S0	11.5412	0.63%
0.99*S0	17.4925	0.95%
1.00*S0	25.4425	1.36%
1.01*S0	36.1054	1.92%
1.02*S0	48.4420	2.55%
1.03*S0	62.7733	3.27%
1.04*S0	77.5707	4.00%
1.05*S0	93.8619	4.80%

Table 6: Premium of soybean insurance

Protective price	Premium	Premium rate
0.95*S0	6.2257	0.20%
0.96*S0	10.2095	0.32%
0.97*S0	15.8573	0.50%
0.98*S0	24.1176	0.75%
0.99*S0	34.7202	1.07%
1.00*S0	49.1354	1.50%
1.01*S0	66.7581	2.01%
1.02*S0	88.5725	2.64%
1.03*S0	111.9962	3.31%
1.04*S0	138.8481	4.07%
1.05*S0	167.1427	4.85%

Table 7: Premium of palm oil insurance

Protective price	Premium	Premium rate
0.95*S0	9.0611	0.22%
0.96*S0	15.1627	0.37%
0.97*S0	23.5128	0.57%
0.98*S0	35.1865	0.84%
0.99*S0	50.7821	1.20%
1.00*S0	69.7707	1.64%
1.01*S0	92.7802	2.16%
1.02*S0	120.2354	2.77%
1.03*S0	151.1951	3.45%
1.04*S0	183.7694	4.15%
1.05*S0	219.8117	4.91%

From the four tables listed above, it is incontestable that the premium is positively correlated with the protective price. This agrees with the conclusion derived from the Black-Scholes model. It is cogent that premium rate is more precise to reflect farmers cost because it shows the relative cost to the protective price, consequently, Table 4 display premium rate in the last column, analogous to premium, the rate is positively correlated with the protective price. Moreover, it is clear that the premium rate of corn futures, soybean futures, and palm oil futures are similar and all of them range from approximately 0.2% to 5%. However, the premium rate of egg futures is relatively higher than the other three products. The following picture depicts the premium rate of four futures with different protective prices:

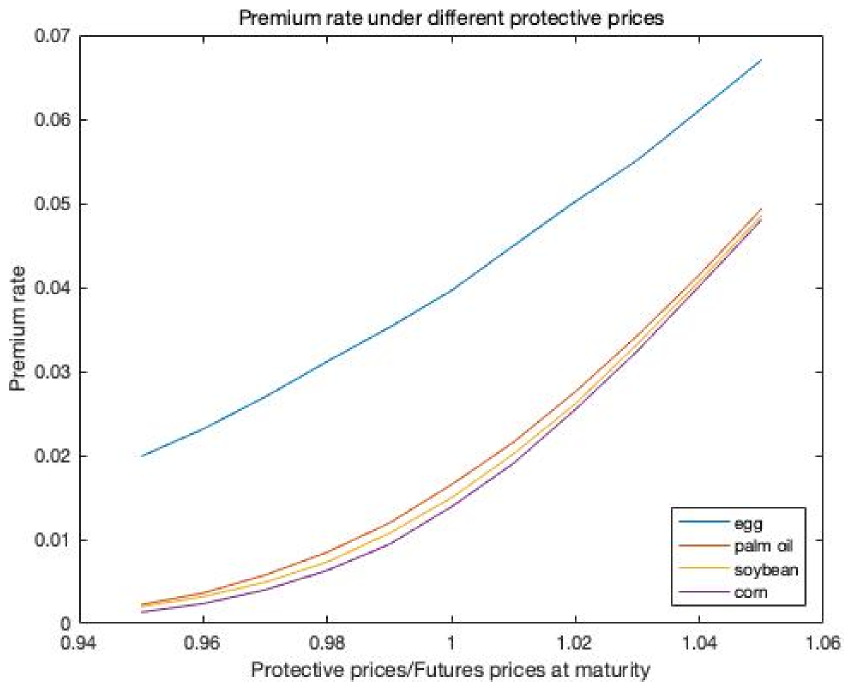

Figure 3: Premium rate under different protective prices

This graph shows that the premium rate of egg futures is higher than the others, and the premium rate of palm oil futures is slightly higher than that of soybean futures and the premium rate of soybean futures is also a little higher than that of corn futures. It is significant that all premium rates have a similar trend when the protective set from 95% of futures prices to 105% of the futures prices. If farmer choose to fix the price at present future price S0the costs of soybean, palm oil and corn insurances are roughly 1.5% and that of egg insurances is approximately 4.0%. it is an acceptable premium for farmers. Hence, it is a feasible solution for the insurance company to construct a valid product to prevent farmers from huge losses.

The result delineates the costs when farmers choose different protective prices. There is no wonder that the premium is different for different products, and the price of the egg usually fluctuates sharply because there are more risks in the egg industry. Therefore, it is tenable to charge more for egg insurance. Its relatively flexible for farmers to choose insurance at different level, correspondingly, the farmer needs to pay different premium due to the different costs of put option included in the Insurance + Futures, and different policy of supporting agriculture can be lain down according to the application of Insurance + Futures, This will expatiate in the following section.

What should also be considered is that strike prices are set proportional to S0-the spot future prices. However, its still unknown whether the premium rates are sensitive to spot future prices, its plausible to conduct a sensitivity analysis of S0. Exemplifying sensitivity with egg futures, the chart manifests premium rates under different spot futures prices and protective prices, other parameters remain invariant:

Table 8: Sensitivity analysis of spot futures price

	S0=3381	S0=3581	S0=3781	S0=3981	S0=4181
K=0.95*S0	2.00%	2.01%	1.98%	2.00%	1.97%
K=0.96*S0	2.31%	2.31%	2.31%	2.35%	2.30%
K=0.97*S0	2.69%	2.69%	2.68%	2.69%	2.71%
K=0.98*S0	3.09%	3.09%	3.09%	3.12%	3.08%
K=0.99*S0	3.52%	3.51%	3.53%	3.51%	3.53%
K=1.00*S0	3.98%	3.98%	4.00%	4.03%	4.01%
K=1.01*S0	4.48%	4.48%	4.49%	4.47%	4.49%
K=1.02*S0	5.03%	4.98%	4.99%	5.00%	4.96%
K=1.03*S0	5.58%	5.54%	5.52%	5.56%	5.53%
K=1.04*S0	6.12%	6.13%	6.12%	6.10%	6.13%
K=1.05*S0	6.71%	6.69%	6.74%	6.70%	6.72%

The premium rates roughly remain the same when spot futures prices change. It reflects an important proposition: the stability of spot futures rates are retained regardless of the variation of spot price. Egg futures, soybean futures and palm oil futures comply with this property as well, in general term, its a quite robust conclusion that the premium rates of corn futures, soybean futures and palm oil futures vary approximately from 0.2% to 5% and those of egg futures are slightly higher, the universality of above conclusion are largely assured.

5. Comparison of Heston-Nandi Model and B-S Model

In this section, we compare different pricing models with exchange-traded options, using soybean meal options, corn options, white sugar options, and cotton No.1 options, all of which are from Dalian Commodity Exchange (hereafter referred to as DCE) and Zhengzhou Commodity Exchange (hereafter referred to as ZCE). While pricing options with various methods, the volatility time interval is consistent with section 3, from 26th April 2016 to 26th April 2019. Moreover, we calculate the predicted price on 26th April 2019. As for the risk-free rate, we choose one month Shanghai Interbank Offered Rate(Shibor) in the model. We use Monte Carlo simulation and Heston-Nandi model, closed-form formula of Heston-Nandi model and Black-Scholes model. Based on the price of exchange-traded put options, we can get which method is more appropriate to price options in Insurance + Futures products in the OTC market. Price with different pricing models are shown in the figure below:

Table 9: Price of soybean meal options

Strike price	B-S model	Solution of HN	MC simulation of HN	Exchange-trade options M1907-P-****
2300	13.79	13.87	13.07	1.00
2350	23.01	23.12	22.15	4.00
2400	36.17	36.32	34.86	10.00
2450	53.88	54.06	51.89	22.50
2500	76.50	76.71	74.21	42.50
2550	104.08	104.32	101.60	71.50
2600	136.36	136.62	133.29	107.50
2650	172.85	173.14	169.59	150.00
2700	212.91	213.22	208.77	196.00

2750	255.85	256.18	252.10	244.50
2800	300.98	301.35	296.66	294.50
2850	347.73	348.13	343.46	344.00
2900	395.60	396.04	391.39	394.00
2950	444.23	444.72	439.74	444.00
3000	493.35	493.88	488.45	494.00
3050	542.78	543.36	538.16	544.00
3100	592.39	593.01	588.43	594.00
3150	642.10	642.78	637.34	644.00
3200	691.88	692.60	686.93	694.00

Table 10: Price of corn options

Strike price	B-S model	Solution of HN	MC simulation of HN	Exchange-trade options C1907-P-****
1700	2.09	1.79	1.71	0.50
1720	3.23	2.72	2.52	0.50
1740	4.84	4.06	3.77	0.50
1760	7.05	5.93	5.55	0.50
1780	9.99	8.48	8.04	0.50
1800	13.80	11.88	11.26	1.50
1820	18.61	16.27	15.30	3.00
1840	24.54	21.82	20.79	6.00
1860	31.68	28.66	27.20	10.50
1880	40.09	36.88	34.93	17.50
1900	49.80	46.52	44.46	26.50
1920	60.79	57.59	55.80	38.00
1940	73.04	70.04	67.60	52.00
1960	86.47	83.76	81.33	68.00
1980	100.99	98.65	95.60	85.00
2000	116.49	114.55	111.03	103.50
2020	132.86	131.31	128.02	122.50
2040	149.97	148.80	145.02	142.00

2060	167.70	166.87	163.65	162.00
2080	185.94	185.41	181.05	182.00

Table 11: Price of white sugar options

Strike price	B-S model	Solution of HN	MC simulation of HN	Exchange-trade options SR907P****
4200	0.00	0.00	0.00	1.00
4300	0.00	0.00	0.00	1.00
4400	0.00	0.00	0.00	1.50
4500	0.01	0.02	0.02	2.00
4600	0.09	0.10	0.07	3.00
4700	0.47	0.49	0.42	3.50
4800	1.88	1.93	1.70	5.00
4900	6.11	6.24	5.90	6.50
5000	16.43	16.67	15.36	7.50
5100	37.27	37.63	35.11	21.00
5200	72.87	73.33	68.71	54.00
5300	125.46	125.97	119.87	109.00
5400	194.27	194.77	185.98	182.50
5500	276.04	276.53	267.25	269.50
5600	366.56	367.05	358.34	363.50
5700	462.13	462.64	453.16	461.50
5800	560.19	560.76	552.60	561.00
5900	659.31	696.97	649.60	661.00
6000	758.84	759.58	749.58	761.00
6100	858.50	859.33	849.19	861.00
6200	958.19	959.12	949.17	961.00

Table 12: Price of cotton No.1 options

Strike price	B-S model	Solution of HN	MC simulation of HN	Exchange-trade options CF907P****
14000	14.62	18.55	17.09	8.00
14200	27.20	31.94	29.48	14.00
14400	47.60	52.73	48.54	23.00
14600	78.70	83.55	78.30	38.00
14800	123.46	127.21	120.87	61.00
15000	184.51	186.42	175.75	97.00
15200	263.79	263.35	248.06	152.00
15400	362.26	359.35	343.93	241.00
15600	479.72	474.71	454.71	377.00
15800	614.99	608.55	588.94	534.00
16000	766.01	758.96	732.71	707.00
16200	930.26	923.57	898.15	890.00
16400	1105.09	1099.33	1067.60	1,079.00
16600	1287.94	1283.50	1253.05	1,272.00
16800	1476.61	1473.61	1447.05	1,468.00
17000	1669.31	1667.65	1636.27	1,666.00

Since the B-S formula was proposed, the B-S model has been used for option pricing for the longest time. Moreover, the pricing formulas for different options are also presented in various articles. In our study, the B-S formula shows good results compared with the market price of options, however, the dispersions between them are not ignorable. There is always significant differences between BS formula and real prices, maybe due to the omission of transaction costs, the choice of risk-free interest rate different from the benchmark chosen by the exchange. price from BS formula is significantly higher than the real options price, and this problem has been relieved as strike prices rise.

Besides the Black-Scholes model, the results from the Monte Carlo simulation of Heston-Nandi model and closed-form formula of Heston-Nandi model show consistency. Firstly, the results with only minor differences between Monte Carlo simulation and close-form formula of Heson-Nandi model, show that Monte Carlo simulation has a good pricing effect on the Heston-Nandi model. Nonetheless, like Black-Scholes formula, there are also slight differences between Heston-Nandi results and real market prices.

From the above results, the Black-Scholes model has shown quite plausible pricing results and it can be applied effectively to option pricing. Monte Carlo simulation of Heston-Nandi model and closed-form formula of Heston-Nandi model, however, show some improvements over the BS model, while the B-S model shows deviations and Heston-Nandi model narrows the gaps. Price from Heston-Nandi model is closer to the price of exchange-traded options. From this point of view, Heston-Nandi model is quite ideal for pricing Insurance + Futures products in the OTC markets, guiding the pricing in the OTC market and generating less option pricing errors.

In conclusion, there is a small difference in the result between Monte Carlo simulation of Heston-Nandi model and close-form formula of Heston-Nandi model, which is also close to the result of BS formula. The result of Heston-Nandi model, however, has a better characterization effect on deeply in-the-money and out-of-the-money options. And for various agricultural products, the best pricing models varies from each other, indicating that different pricing methods will be preferred in different situations, however, generally, the Heston-Nandi model shows more precise pricing results on average level.

6. Case study: Corn futures in Liaoning Province

Liaoning Province is a vital corn planting area in China, with an average annual planting area of 22.667 million hectares and an average annual output of 13.842 million tons, accounting for 6.55% of the national average annual corn production. The cancellation of the 2016 corn purchase policy has reduced corn planting income, farmers' intention for planting corn has declined, and how to more effectively protect the corn planter's planting income is a major agricultural issue.

In order to support the planting of corn, Xinhuruifeng Co. Ltd. starts anInsurance + Futures project. The size of the contract is 18300 tons, and the start date is August 8th, 2016. Moreover, the maturity is December 16th, 2016, the number of trading days is 87 days, and the protective price of this contract is 1650 yuan per ton. The put option in the contract is a typical Asian option and use the average price of the last 45 days before maturity, namely the average price from October 17th, 2016 to December 16th, 2016. The actual premium of this insurance is 245.9 yuan per ton. However, the actual option price is 208 yuan per ton, and the difference between the premium and the option price is the profit of the insurance company. In the following context, the option price will be re-estimated by our Monte Carlo simulation with Heston-Nandi model, A comparison between two results will be shown, and some interpretations of the difference between our model and market price will be stated as well. The risk-free interest rate is 3%, the spot price of corn futures at August,8th, 2016 is 1460. The paths of the simulation are 100000, and the pricing procedure is repeated 100 times in order to depict the distribution of the estimated put option price, the histogram below will show the result visually:

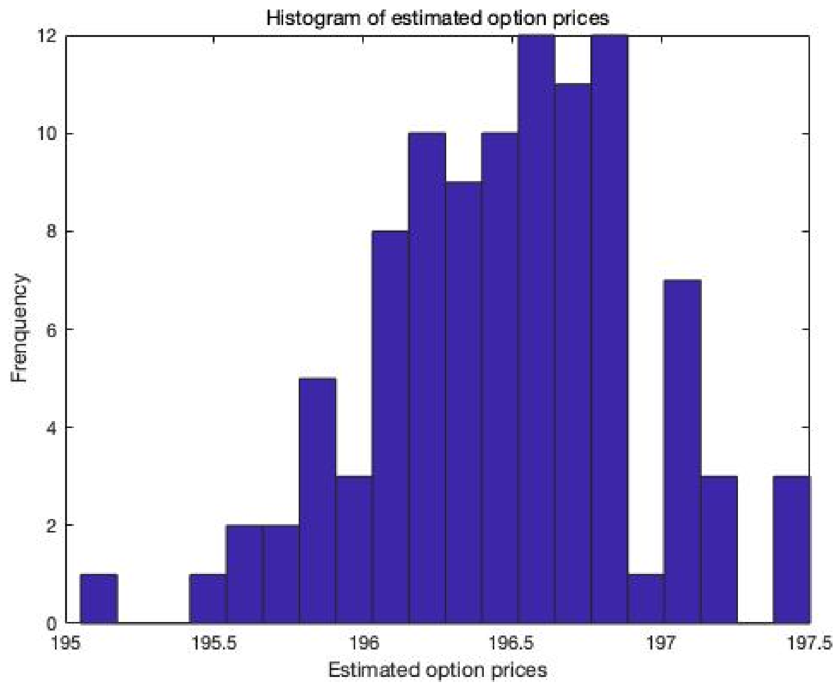

Figure 4: Histogram of estimated option prices

The estimated put option price is approximately 196.5 yuan per ton. It is quite close to the real option price in the OTC market which equals to 208. Nonetheless, the estimated option price is slightly lower than the option price in the market, and the difference required to be interpreted remains significant. There are some possible explanations for the diversion between the theoretical option price and real option price:

Firstly, OTC market develops insufficiently, and option prices have not achieved equilibrium. Exchange-traded options are normalized and traded frequently, and there are masses of arbitragers seeking for opportunities to arbitrage. Therefore, if the price is misjudged, it will return to equilibrium rapidly. By contrast, the OTC option has quite low liquidity and inactive trading. Therefore the option prices are determined by subjective expectation of buyers and sellers, and there is no corresponding mechanism to correct the straying price. Moreover, considering low liquidity, people who sell options expect higher option price to offset the liquidity risks.

Secondly, the implied volatility may be misjudged. In the OTC market, various options are priced by investment banks, and the implied volatility is estimated by historical data or various GARCH models. A different approach will lead to a slightly different result. Therefore, the diversion between our model and the real price may originate from different estimation methods. However, the divergence is not very large indicated the common validity of different volatility estimating approaches.

Above analysis applies our model to a practical case. The proximity of our result and market option price testify the efficacy of Heston-Nandi pricing model. However, the dispersion between these two prices indicates that various factors impact option prices in the OTC market, and it is fairly not possible to eliminate all the dispersion by complicating the model. But the results are still very meaningful because they act as a guide to priceInsurance + Futures product and formulate a policy.

In conclusion, the application of Heston-Nandi model above sufficiently corroborates the validity of the model in practical situations. This model is capable to capture more precise patterns of volatility. Therefore, the pricing results are quite accurate. In this case, government is able to collaborate with local insurance companies to popularize Insurance+ Futures products. In this way, farmers only need to defray a stable amount of premium but they will be effectively prevented from huge losses. When the prices of agriculture products fluctuate, Insurance+ Futures mode is a quite practical solution to eliminate farmers risks.

7. Conclusion

In this paper, the Heston-Nandi model and Monte Carlo simulation are used to price Asian agricultural options. Through the comparison in section six, we can find that the Heston-Nandi model combined with Monte Carlo simulation has a great pricing effect.

Compared with the closed-form solution of Heston-Nandi, the result of combining the Heston-Nandi model with Monte Carlo simulation for option pricing is closer to the market price. We compare the pricing results of the B-S formula with the pricing results of the Heston-Nandi model and Monte Carlo simulation. When pricing the out-of-the-money options, the Heston-Nandi pricing results are closer to the market value. Taken together, the Heston-Nandi model works well for pricing.

Based on the research in this paper, we can draw the following conclusions about the ”Insurance + Futures” mode. First of all, for different agricultural products, the premium rate is different. In this article, we price the egg, corn, soybean and palm oil options. The result shows that the premium rate of the egg is higher than the other three kinds produce. This is because the volatility of the egg's price is high, and the other three agricultural products volatility is relatively low. When deciding insurance cost, we must consider the kind of produce. Secondly, we can fix a reasonable executive price according to the rate. If the premium rate is high, farmers may prefer to take the risk rather than buy insurance. So we can choose a reasonable premium rate and then determine the strike price. Thirdly, through the case study, we find that the premium cost decides by our model is very close to the realistic premium cost. This can prove that our model is useful. In the future, we may consider using our model for pricing.

”Insurance + Futures” mode requires both accurate pricing and relevant policies and measures matching product pricing. To this end, we have put forward the following policy recommendations. First of all, we will increase the kinds of agricultural product options and futures. At present, there are too few kinds of options for domestic agricultural products. Moreover, OTC options are not regulated, and futures companies are at high risk of default. Also, higher fees for OTC options raise insurers' costs. Therefore, we need to continue to increase the kinds of agricultural options and futures. Secondly, insurance companies and futures companies should strengthen interaction and cooperation. Good communication is the guarantee of successful cooperation. They should work together to promote the ”Insurance + Futures” model. Thirdly, the government and the media should strengthen the publicity and popularization of financial knowledge to make it easier for farmers to accept the ”Insurance + Futures” mode of risk management. Fourth, the government should give full policy support. At present, most domestic futures companies do not have the corresponding capital and technology to support the ”Insurance + Futures” model, which requires the government to give technical and policy support to the futures industry. The government also needs to promote the pilot work and improve the ”Insurance + Futures” mode.

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